$$\min_x f(x) \quad \text{s.t. } x \in C$$

Thm. (first order optimality condition)

x is optimal $\iff \nabla f(x)^T (y-x) \geq 0$ for all $y \in C$

$N_C(x) = \{ g : g^T x \leq g^T y \quad \forall y \in C \}$

x is optimal iff. $\nabla f(x) \in N_C(x)$

Special case1 : when $C = R^n$ the $\nabla f(x) = 0$

Special case2 : when $C = \{ x \mid Ax = b \}$

$\exists u$ s.t. $\nabla f(x) + A^T u = 0$

Goemans-Williamson Alg for approx Maxcut

$\text{maxcut}(G) \leq \widetilde{\text{maxcut}}(G)$

every node i is described by $u_i \in \mathbb{R}^n$

$$E[\text{cut}(G,W)] = \sum_{(i,j) \in E} w_{ij} E[\mathbb{1}(u_i \text{ is on one side of } \{v^T x \geq 0\},\ u_j \text{ is on the other side})]$$

$$= \sum_{(i,j) \in E} w_{ij} E\left[\frac{\angle(u_i, u_j)}{\pi}\right] = \sum_{(i,j) \in E} w_{ij} E\left[\frac{\theta_{ij}}{\pi}\right]$$

$$= \sum w_{ij} E\left[\frac{\arccos(\theta_{ij})}{\pi}\right]$$

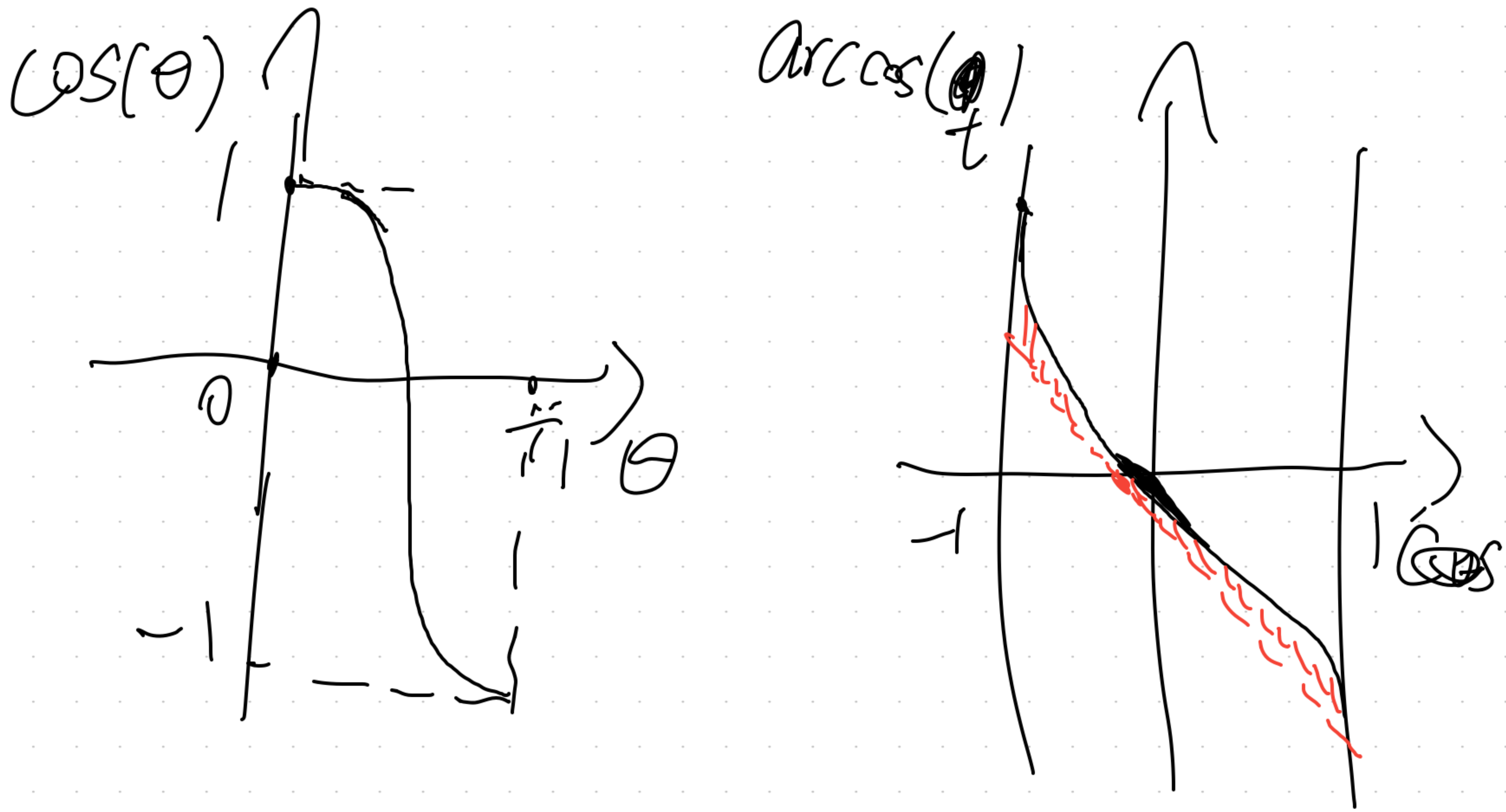
$$\geq \sum w_{ij} \beta \frac{(1 - \angle u_i, u_j)}{\pi}$$

$\beta \simeq 0.878$

$\theta_{ij} \sim \angle u_i, u_j \rangle = \cos(\theta_{ij})$

$\cos(\theta_{ij}) = Y_{ij}$

$$\geq \frac{\beta}{\pi} \underbrace{\sum w_{ij}(1 - Y_{ij})}_{\text{maxcut}(G)} = \frac{\beta}{\pi} \text{maxcut}(G)$$


cos(θ)
1
0
-1
π
θ
arccos(t)
-1
1
cos