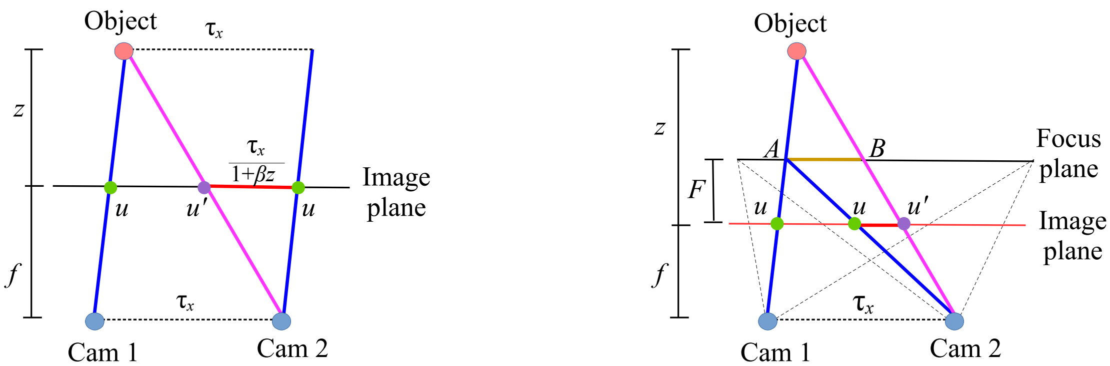
Supplementary Document

1 Optical Flow for General Focus Distance



(a) Cameras focused at infinity

(b) Cameras focused at depth F

Figure 1: *Comparison between cameras focused at infinity and focused at some finite depth.*

In the main text we assumed all cameras are looking straight ahead. Assume the camera is focused at depth F instead. The view change stays the same as the previous case. The spatial change, however, is different since the change in pixel coordinates (distance between u' and u) changes. From Fig. 1b, it can be seen that the distance between u' and u becomes

$$\overrightarrow{u'u} = -\frac{f}{f+F}\overrightarrow{AB} = -\frac{f}{f+F} \cdot \frac{z-F}{z+f}\tau_x \tag{1}$$

Comparing it to Fig. 1a, it can be seen that $1/(1+\beta z)$ is replaced by

$$\frac{-f}{F+f}\frac{z-F}{z+f} \tag{2}$$

Note that when $F \to \infty$, i.e., all cameras are looking straight ahead, the above expression reduces to $1/(1+\beta z)$. We can then replace $1/(1+\beta z)$ in the linear system with this new term. Using the same procedure, it will still lead to the same PDE as in the main paper, only with different κ values.

2 Optimization Objective

The final optimization goal for our system consists of a data term D that ensures the image patch satisfies the PDE, and a smoothness term S that ensures neighboring normals and depths (a_4 to a_6) to be similar,

$$a = \arg\min_a \sum_i D_i^2 + \eta \sum_j S_j^2 \tag{3}$$

where

$$D_i = \left[\begin{array}{cc} \mathbf{a}^\top & 1 \end{array} \right] M_i \left[\begin{array}{c} \mathbf{a} \\ 1 \end{array} \right] \qquad i = 1, 2, ..., r^2 \tag{4}$$

$$S_j = a_j - a_j^0 \qquad j = 4, 5, 6 \tag{5}$$

where r^2 is the size of the image patch (5×5 in our experiment), M_i is a 7×7 symmetric matrix at pixel i, a_j^0 is the average a_j of its 4-neighbors that have already been computed, and η is the weight. To compute M, we plug (19)-(20) into (22) in the main paper. Namely, we replace z, n_u and n_v in the PDE

$$(\kappa_1 + \kappa_2 z)n_u + (\kappa_3 + \kappa_4 z)n_v + (\kappa_5 + \kappa_6 z)\big(1 + \beta(3z - 2a_6 - a_4 u - a_5 v)\big) = 0 \tag{6}$$

with

$$z = a_1 u^2 + a_2 v^2 + a_3 uv + a_4 u + a_5 v + a_6 \tag{7}$$

$$n_u = 2a_1 u + a_3 v + a_4, \quad n_v = 2a_2 v + a_3 u + a_5 \tag{8}$$

This results in a quadratic equation in $a_1, ..., a_6$, and can be factored as (23) in the main paper,

$$\begin{bmatrix} \mathbf{a}^\top & 1 \end{bmatrix} M \begin{bmatrix} \mathbf{a} \\ 1 \end{bmatrix} = 0 \tag{9}$$

Since M is symmetric, we provide only half the matrix components here,

$M_{11} = 3\beta\kappa_6 u^4 + 2\kappa_2 u^3$, $M_{12} = 3\beta\kappa_6 u^2 v^2 + \kappa_4 u^2 v + \kappa_2 uv^2$, $M_{13} = \kappa_4 u^3/2 + 3\kappa_2 u^2 v/2 + 3\beta\kappa_6 u^3 v$,
$M_{14} = 3\beta\kappa_6 u^3 + 3\kappa_2 u^2/2$, $M_{15} = \kappa_4 u^2/2 + \kappa_2 uv + 3\beta\kappa_6 u^2 v$, $M_{16} = 2\beta\kappa_6 u^2 + \kappa_2 u$,
$M_{17} = \kappa_1 u + \kappa_6 u^2/2 + 3\beta\kappa_5 u^2/2$,
$M_{22} = 3\beta\kappa_6 v^4 + 2\kappa_4 v^3$, $M_{23} = \kappa_2 v^3/2 + 3\kappa_4 uv^2/2 + 3\beta\kappa_6 uv^3$, $M_{24} = \kappa_2 v^2/2 + \kappa_4 uv + 3\beta\kappa_6 uv^2$,
$M_{25} = 3\beta\kappa_6 v^3 + 3\kappa_4 v^2/2$, $M_{26} = 2\beta\kappa_6 v^2 + \kappa_4 v$, $M_{27} = \kappa_3 v + \kappa_6 v^2/2 + 3\beta\kappa_5 v^2/2$,
$M_{33} = 3\beta\kappa_6 u^2 v^2 + \kappa_4 u^2 v + \kappa_2 uv^2$, $M_{34} = \kappa_4 u^2/2 + \kappa_2 uv + 3\beta\kappa_6 u^2 v$, $M_{35} = \kappa_2 v^2/2 + \kappa_4 uv + 3\beta\kappa_6 uv^2$,
$M_{36} = \kappa_4 u/2 + \kappa_2 v/2 + 2\beta\kappa_6 uv$, $M_{37} = (\kappa_3 u + \kappa_1 v + \kappa_6 uv + 3\beta\kappa_5 uv)/2$,
$M_{44} = 3\beta\kappa_6 u^2 + \kappa_2 u$, $M_{45} = \kappa_4 u/2 + \kappa_2 v/2 + 3\beta\kappa_6 uv$, $M_{46} = \kappa_2/2 + 2\beta\kappa_6 u$,
$M_{47} = (\kappa_1 + \kappa_6 u + 3\beta\kappa_5 u)/2$,
$M_{55} = 3\beta\kappa_6 v^2 + \kappa_4 v$, $M_{56} = \kappa_4/2 + 2\beta\kappa_6 v$, $M_{57} = (\kappa_3 + \kappa_6 v + 3\beta\kappa_5 v)/2$,
$M_{66} = \beta\kappa_6$, $M_{67} = (\kappa_6 + \beta\kappa_5)/2$,
$M_{77} = \kappa_5$