To Do

- Start working on final projects (initial results and proposal due in a week). Ask me if problems
- Volumetric rendering (this lecture) may be one component of the final project (but hard, be careful)
- Increasingly accurate appearance requires volumetric scattering (even for skin, hair, fur)
- Continues to be an active area of research

Many slides courtesy Pat Hanrahan/Matt Pharr (Stanford CS 348b) and Steve Rotenberg, Henrik Wann Jensen (UCSD CSE 168)

Volumetric Scattering

- Participating Media (light participates via scattering)
  - Volumetric phenomena like clouds, smoke, fire
  - Subsurface scattering, translucency (wax, human skin)
  - These are not surfaces with well-defined BRDFs
  - Rather volumes where light can scatter
  - Medium is often known as a participating medium

- Surface Rendering: Radiance Constant along Ray
  - Only true in absence of participating media
  - No longer true for volumetric scattering
  - Often replace ray tracing with ray marching in medium

- Volumetric Properties
  - BRDF replaced by phase function
  - Must consider absorption and scattering in medium
### Homogeneous vs Heterogeneous

- **Homogeneous**: Properties constant everywhere
  - Example: Fog often represented as homogeneous
- **Heterogeneous**: Varies across space
  - Example: Smoke, fire etc.
  - Sometimes called inhomogeneous
- Homogeneous volumes often easier
  - Some computational shortcuts (transmittance etc.)
  - Some analytic formulae

### Volumetric Interactions

- 4 different processes affect radiance of a beam
  - Absorption
  - Out-Scattering
  - Emission
  - In-Scattering

### Absorption

\[ \frac{dL(p, \omega)}{dL(p, \omega)} = -\sigma_a(p) \ L(p, \omega) \ ds \]

Absorption cross section: \( \sigma_a(p) \)
- Probability of being absorbed per unit length
- Units: 1/distance

### Transmittance

\[ \log L(p + s\omega, \omega) = -\int_0^s \sigma_a(p + s'\omega) \ ds' = -\tau(s) \]

**Optical distance (depth)**: \( \tau(s) = \int_0^s \sigma_a(p') \ ds' \)

\[ p' = p + s' \omega \]

**Homogeneous medium constant** \( \sigma_a \tau(s) = \sigma_a s \)
Transmittance and Opacity

\[ \frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) L(p, \omega) \, ds \]
\[ \frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_s(p) \, ds \]
\[ \log L(p + s \omega, \omega) = - \int_0^s \sigma_a(p + s' \omega, \omega) \, ds' = -\tau(s) \]
\[ L(p + s \omega, \omega) = e^{-\tau(s)} L(p, \omega) = T(s) L(p, \omega) \]

Transmittance: \( T(s) = e^{-\tau(s)} \)
Opacity: \( \alpha(s) = 1 - T(s) \)

Out-Scattering

\[ L(p, \omega) \xrightarrow{\sigma_s(p)} L + dL \]
\[ dL(p, \omega) = -\sigma_s(p) L(p, \omega) \, ds \]

Scattering cross-section: \( \sigma_s \)
- Probability of being scattered per unit length

Extinction

\[ L(p, \omega) \xrightarrow{\sigma_a(p)} L + dL \]
\[ dL(p, \omega) = -\sigma_a(p) L(p, \omega) \, ds \]

Total cross section: \( \sigma_t = \sigma_a + \sigma_s \)
Albedo: \( W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_a}{\sigma_a + \sigma_s} \)
Optical distance from absorption and scattering:
\[ \tau(s) = \int_0^s \sigma_a(p') \, ds' \]

Ray Marching for Transmittance

\[ \tau(s) = \int_0^s \sigma_a(p + s' \omega) \, ds' \]
\[ T(s) = e^{-\tau(s)} \]

Monte Carlo not necessary for 1D—can use a Riemann sum:
\[ \tau(s) \approx \frac{\Delta}{N} \sum_{i=1}^{N} \sigma_a(x_i) \]
\[ x_i = x + \frac{i + 0.5}{N} \omega \]

Emission

\[ L(x, \omega) \xrightarrow{\sigma_s(x)} L + dL \]
\[ dL(p, \omega) = \sigma_s(p) L_e(p, \omega) \, ds \]

In-Scattering

\[ L(x, \omega) \xrightarrow{\sigma_s(x)} L + dL \]
\[ S(p, \omega) = \sigma_s(p) \int_{\omega'} p(\omega' \rightarrow \omega) L(p, \omega') \, d\omega' \]

Phase function: \( p(\omega' \rightarrow \omega) \)
Reciprocity: \( p(\omega' \rightarrow \omega) = p(\omega \rightarrow \omega') \)
Energy conservation: \( \int_{\omega'} p(\omega' \rightarrow \omega) \, d\omega' = 1 \)
Scattering Phase Functions

- Light interacts with volume, scatters in some spherical distribution
- Similar to light scattering off a surface
- Phase function analogous to a surface BRDF
- Depends only on cosine of incident-outgoing
- Like BRDFs, volumetric phase functions must be reciprocal and conserve energy
- Similar to BRDFs, we will want to do importance sampling and evaluation of phase functions

Rayleigh Scattering

- Rayleigh scattering describes the scattering of light by particles much smaller than the wavelength
  \[ p(\cos \theta) = \frac{3}{16\pi} (1 + \cos^2 \theta) \]
  \[ \sigma = \frac{2\pi^3 d^4}{3} \left( \frac{n^2 - 1}{n^2 + 2} \right)^2 \]
- Where \( \lambda \) is the wavelength of light, \( d \) is the diameter of the particle, and \( n \)
  is the index of refraction of the particle
- The strong dependence on wavelength (\( \lambda^n \)) causes greater scattering towards the blue end of the spectrum
- The blue color of the sky is caused by Rayleigh scattering of sunlight by air molecules

Mie Scattering

- Scatter electromagnetic waves by spherical particles
- Size of particles same scale as wavelength of light
- Water droplets in atmosphere, fat droplets in milk
- After Gustave Mie, Ludvig Lorenz

Empirical Mie Approximation

- The following empirical function is often used to approximate the shape of Mie scattering
  \[ p(\cos \theta) = \frac{1}{4\pi} \left( \frac{1}{2} + \frac{z + 1}{2} \left( \frac{1 + \cos \theta}{2} \right)^{z/2} \right) \]
Henyey-Greenstein Function

- The Henyey-Greenstein phase function is an empirical function originally designed to model the scattering in galactic dust clouds.

\[ p(\cos \theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos \theta)^{1.5}} \]

- It uses an anisotropy parameter \( g \) that ranges between -1 (full backscatter) and 1 (full forward scatter), and is isotropic for \( g = 0 \).

Direct Illumination in a Volume

\[ S_0(y', \omega) = \sigma_0(y') \int \sigma((y', \omega) \to \omega) L_0(y', \omega') \, d\omega' \]

Estimator:

\[ \sigma_0(y') \frac{1}{N} \sum_{i=1}^{N} \sigma((y, \omega) \to \omega) L_0(y', \omega_i) \]

Computing direct lighting, \( L_0 \), can be expensive.

Not just a shadow ray—need to compute transmittance.

Transmittance for Shadow Rays

Besides Monte Carlo, precomputed transmittance can be faster for point, distant lights.

Single-Scattering

Minneart: Color and Light In The Open Air

pbpt: Spot-Lit Ball In The Fog
The Volume Rendering Equation

Integro-differential equation:
\[ \frac{\partial L(p, \omega)}{\partial s} = -\sigma_A L(p, \omega) + S(p, \omega) \]

Integro-integral equation:
\[ L(p, \omega) = \int_0^\infty T(p', \omega') S(p', \omega') \, ds' \]

Attenuation: absorption and scattering
\[ e^{-\int_0^s \sigma_A(p') \, ds'} \]

Sources: in-scattering (and emission)
\[ \sigma_s(p') \int_{S^2} p(\omega' \rightarrow \omega) L(p', \omega') \, d\omega' \]

Evaluating the Estimator: S

Include indirect illumination in the source term:
\[ S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') \, d\omega' \]
\[ L(x, \omega') = L_0(x, \omega') + L_a(x, \omega') \]

- Compute direct lighting as before
- Sample incident direction from the phase function's distribution, trace a ray recursively...
\[ L_a(x, \omega') \approx \frac{p(\omega'' \rightarrow \omega') L(x, \omega'')}{p(\omega'')} \]

Uniform spherical directions: \[ p(\omega'') = \frac{1}{4\pi} \]

Linear Sampling of T

We want samples along a finite ray \([0, t_{\text{max}}]\).

- Uniform probability along the ray:
\[ p(t) = \frac{1}{t_{\text{max}}} \]

- Sampling recipe:
\[ \xi = \int_0^t p(t) \, dt \quad t = \xi t_{\text{max}} \]

Exact Sampling of Uniform T

We want samples along a finite ray \([0, t_{\text{max}}]\), \[ p(t) \propto e^{-\sigma t} \]

- Normalize to find PDF:
\[ \int_0^{t_{\text{max}}} e^{-\sigma t} \, dt = -\frac{1}{\sigma} (e^{-\sigma t_{\text{max}}} - 1) = e \quad p(t) = \sigma e^{-\sigma t} \]

- Invert to find \( t \) for a random sample:
\[ \xi = \int_0^t p(t) \, dt \]
\[ t = -\frac{1}{\sigma} \log(1 - \xi (1 - e^{-\sigma t_{\text{max}}})) \]
Volumetric Path Tracing

Integro-integral equation:
\[ L(p, \omega) = \int_0^{\infty} T(p', s') \, S(p', \omega) \, ds' \]

Monte Carlo integration: sample \( s' \sim p(s) \)

Estimator:
\[ \frac{T(p', s') \, S(p', \omega)}{p(s')} \]

Multiple Scattering

Clouds

Translucency

- Translucency is a volumetric lighting effect with additional effects at the surface (usually rough dielectric type interaction)
- These can be modeled through standard volumetric lighting techniques, or can be optimized through some further methods designed specifically for sub-surface scattering

Fire

- "Physically Based Modeling and Animation of Fire", Nguyen, Fedkiw, Jensen, 2003
Sky Rendering

- “A Practical Analytical Model for Daylight”, Preetham, Shirley, Smits, 1999
- “Precomputed Atmospheric Scattering”, Bruneton, Neyret, 2008

Volumetric Caustics


Rainbows

- “Physically Based Simulation of Rainbows”, Sadeghi, Munoz, Laven, Jarosz, Seron, Gutierrez, Jensen, 2012

Atmospheric Phenomena

- Corona
- Ice Crystal Halo
- Glory