

## Computer Graphics

CSE 167 [Win 22], Lecture 4: Transformations 2

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<http://viscomp.ucsd.edu/classes/cse167/wi22>

## To Do

- Start doing HW 1
  - Time is short, but needs only little code [Due Jan 19]
  - Ask questions or clear misunderstandings by next lecture
- Specifics of HW 1
  - Last lecture covered basic material on transformations in 2D. Likely need this lecture to understand full 3D transformations
  - Last lecture had full derivation of 3D rotations. You only need final formula
  - gluLookAt derivation this lecture helps clarifying some ideas
- Read and post on Piazza re questions
- Any remaining issues with UCSD Online graders, submission of homeworks?

## Outline

- Translation: Homogeneous Coordinates
- Combining Transforms: Scene Graphs
- Transforming Normals
- Rotations revisited: coordinate frames
- gluLookAt (quickly)

## Translation

- E.g. move x by +5 units, leave y, z unchanged
- We need appropriate matrix. What is it?

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+5 \\ y \\ z \end{pmatrix}$$

Transformations game demo

## Homogeneous Coordinates

- Add a fourth homogeneous coordinate ( $w=1$ )
- 4x4 matrices very common in graphics, hardware
- Last row always 0 0 0 1 (until next lecture)

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x+5 \\ y \\ z \\ 1 \end{pmatrix}$$

## Representation of Points (4-Vectors)

Homogeneous coordinates

$$P = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x/w \\ y/w \\ z/w \\ 1 \end{pmatrix}$$

- Divide by 4<sup>th</sup> coord ( $w$ ) to get (inhomogeneous) point
- Multiplication by  $w > 0$ , no effect
- Assume  $w \geq 0$ . For  $w > 0$ , normal finite point. For  $w = 0$ , point at infinity (used for vectors to stop translation)

### Advantages of Homogeneous Coords

- Unified framework for translation, viewing, rot...
- Can concatenate any set of transforms to 4x4 matrix
- No division (as for perspective viewing) till end
- Simpler formulas, no special cases
- Standard in graphics software, hardware

### General Translation Matrix

$$T = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & T \\ 0 & 1 \end{pmatrix}$$

$$P' = TP = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + T_x \\ y + T_y \\ z + T_z \\ 1 \end{pmatrix} = P + T$$

### Combining Translations, Rotations

- Order matters!! TR is not the same as RT (demo)
- General form for rigid body transforms
- We show rotation first, then translation (commonly used to position objects) on next slide. Slide after that works it out the other way
- Demos with applet, homework 1

### Combining Translations, Rotations

$$P' = (TR)P = MP = RP + T$$

$$M = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & T_x \\ R_{21} & R_{22} & R_{23} & T_y \\ R_{31} & R_{32} & R_{33} & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix}$$

Transformations game demo

### Combining Translations, Rotations

$$P' = (RT)P = MP = R(P + T) = RP + RT$$

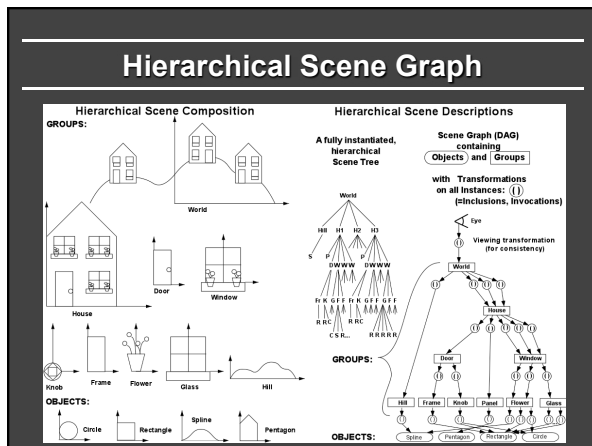
$$M = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R_{3 \times 3} & R_{3 \times 3} T_{3 \times 1} \\ 0_{1 \times 3} & 1 \end{pmatrix}$$

Transformations game demo

### Outline

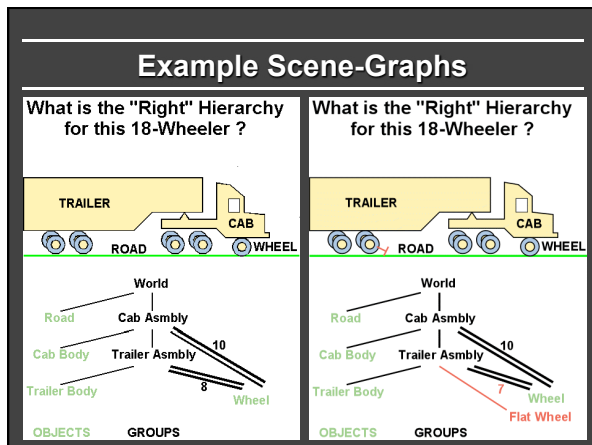
- Translation: Homogeneous Coordinates
- *Combining Transforms: Scene Graphs*
- Transforming Normals
- Rotations revisited: coordinate frames
- gluLookAt (quickly)

Slides for this part courtesy Prof. O'Brien



## Drawing a Scene Graph

- Draw scene with pre-and-post-order traversal
  - Apply node, draw children, undo node if applicable
- Nodes can carry out any function
  - Geometry, transforms, groups, color, ...
- Requires stack to "undo" post children
  - Transform stacks in OpenGL
- Caching and instancing possible
- Instances make it a DAG, not strictly a tree



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## Normals

- Important for many tasks in graphics like lighting
- Do not transform like points e.g. shear
- Algebra tricks to derive correct transform

Incorrect to transform like points

## Finding Normal Transformation

$$t \rightarrow Mt \quad n \rightarrow Qn \quad Q = ?$$

$$n^T t = 0$$

$$n^T Q^T M t = 0 \Rightarrow Q^T M = I$$

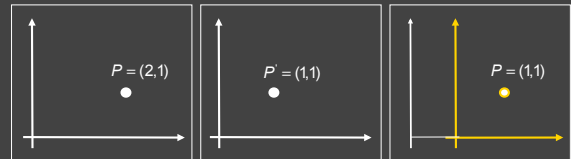
$$Q = (M^{-1})^T$$

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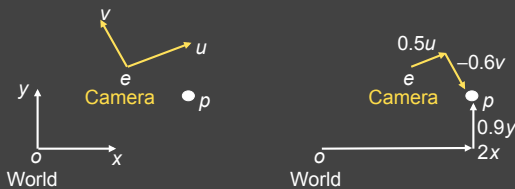
## Coordinate Frames

- All of discussion in terms of operating on points
- But can also change coordinate system
- Example, motion means either point moves backward, or coordinate system moves forward

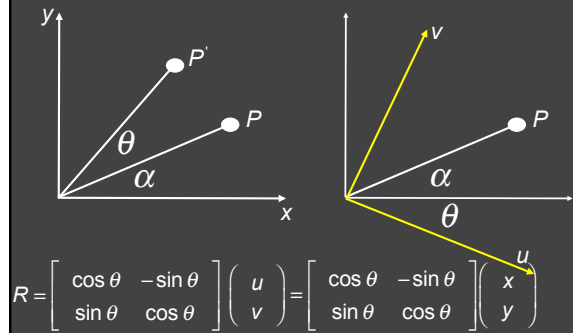


## Coordinate Frames: In general

- Can differ both origin and orientation (e.g. 2 people)
- One good example: World, camera coord frames (H1)



## Coordinate Frames: Rotations



## Outline

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## Geometric Interpretation 3D Rotations

- Rows of matrix are 3 unit vectors of new coord frame
- Can construct rotation matrix from 3 orthonormal vectors

$$R_{uvw} = \begin{pmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{pmatrix} \quad u = x_u X + y_u Y + z_u Z$$

## Axis-Angle formula (summary)

$$(b \setminus a)_{ROT} = (I_{3 \times 3} \cos \theta - aa^T \cos \theta)b + (A^* \sin \theta)b$$

$$(b \rightarrow a)_{ROT} = (aa^T)b$$

$$R(a, \theta) = I_{3 \times 3} \cos \theta + aa^T (1 - \cos \theta) + A^* \sin \theta$$

$$R(a, \theta) = \cos \theta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + (1 - \cos \theta) \begin{pmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{pmatrix} + \sin \theta \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$$

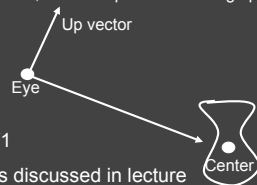
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## Case Study: Derive gluLookAt

Defines camera, fundamental to how we view images

- `gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)`
- Camera is at eye, looking at center, with the up direction being up



- May be important for HW1
- Combines many concepts discussed in lecture
- Core function in OpenGL for later assignments

## Steps

- `gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)`
- Camera is at eye, looking at center, with the up direction being up
- First, create a coordinate frame for the camera*
- Define a rotation matrix
- Apply appropriate translation for camera (eye) location

## Constructing a coordinate frame?

We want to associate  $w$  with  $a$ , and  $v$  with  $b$

- But  $a$  and  $b$  are neither orthogonal nor unit norm
- And we also need to find  $u$

$$w = \frac{a}{\|a\|}$$

$$u = \frac{b \times w}{\|b \times w\|}$$

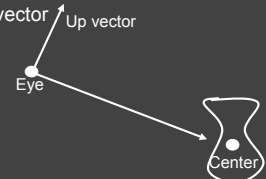
$$v = w \times u$$

from lecture 2

## Constructing a coordinate frame

$$w = \frac{a}{\|a\|} \quad u = \frac{b \times w}{\|b \times w\|} \quad v = w \times u$$

- We want to position camera at origin, looking down  $-Z$  dirn
- Hence, vector  $a$  is given by **eye - center**
- The vector  $b$  is simply the **up vector**



## Steps

- `gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)`
- Camera is at eye, looking at center, with the up direction being up
- First, create a coordinate frame for the camera
- *Define a rotation matrix*
- Apply appropriate translation for camera (eye) location

## Geometric Interpretation 3D Rotations

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## Steps

- `gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)`
- Camera is at eye, looking at center, with the up direction being up
- First, create a coordinate frame for the camera
- Define a rotation matrix
- *Apply appropriate translation for camera (eye) location*

## Translation

- `gluLookAt(eyex, eyey, eyez, centerx, centery, centerz, upx, upy, upz)`
- Camera is at eye, looking at center, with the up direction being up
- *Cannot* apply translation after rotation
- The translation must come first (to bring camera to origin) before the rotation is applied

## Combining Translations, Rotations

$$P' = (RT)P = MP = R(P + T) = RP + RT$$

$$M = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R_{3 \times 3} & R_{3 \times 3} T_{3 \times 1} \\ 0_{1 \times 3} & 1 \end{pmatrix}$$

## gluLookAt final form

$$\begin{pmatrix} x_u & y_u & z_u & 0 \\ x_v & y_v & z_v & 0 \\ x_w & y_w & z_w & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x_u & y_u & z_u & -x_u e_x - y_u e_y - z_u e_z \\ x_v & y_v & z_v & -x_v e_x - y_v e_y - z_v e_z \\ x_w & y_w & z_w & -x_w e_x - y_w e_y - z_w e_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$