Handling control flow in AD

UCSD CSE 291
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Today: differentiating control flow

we will focus on reverse mode: handling control flow in forward mode is trivial

```
if …:
…
else:
…
```

if/else conditions

```
for i in range(…):
   …

while (…):
   …
```

loops

```
f(…):
```

function calls

Today: differentiating control flow

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```
if …:
…
else:
…
```

```
for i in range(…):
  …
```

```
while (…):
  …
```

```
f(…):
```

if/else conditions

loops

function calls

Differentiating if/else statements

```
z : float = 0
if x > y:
   z = z0
else:
   z = z1
```

?

Differentiating if/else statements

```
z : float = 0
if x > y:
  z = z0
else:
  z = z1
```

```
z : float = 0
dz : float = 0
if x > y:
  z = z0
else:
  z = z1

if x > y:
  dz0 += dz
else:
  dz1 += dz
```

Differentiating if/else statements

```
z : float = 0
if x > y:
   t : float = z0 * z0
   z = t * t
else:
   z = z1
```

?

Differentiating if/else statements

```
z : float = 0
if x > y:
  t : float = z0 * z0
  z = t * t
else:
  z = z1
```

```
z : float = 0
dz : float = 0
if x > y:
  t : float = z0 * z0
  z = t
else:
  z = z1

if x > y:
  t : float = z0 * z0
  dt : float = 0
  dt += dz * 2 * t
  dz0 += dt * 2 * z0
else:
  dz1 += dz
```

Differentiating if/else statements

```
z : float = 0
if x > y:
  t : float = z0 * z0
  z = t * t
else:
  z = z1
```

```
z : float = 0
dz : float = 0
if x > y:
  t : float = z0 * z0
  z = t
else:
  z = z1

if x > y:
  t : float = z0 * z0
  dt : float = 0
  dt += dz * 2 * t
  dz0 += dt * 2 * z0
else:
  dz1 += dz
```

duplicated computation!

Differentiating if/else statements

```
z : float = 0
if x > y:
  t : float = z0 * z0
  z = t * t
else:
  z = z1
```

?

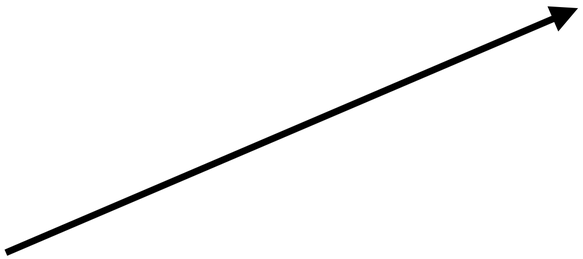
```
z : float = 0
t : float
if x > y:
  t = z0 * z0
  z = t * t
else:
  z = z1
```

Differentiating if/else statements

```
z : float = 0
if x > y:
  t : float = z0 * z0
  z = t * t
else:
  z = z1
```



```
z : float = 0
t : float
if x > y:
  t = z0 * z0
  z = t * t
else:
  z = z1
```



```
z : float = 0
dz : float = 0
t : float
dt : float = 0
if x > y:
  t = z0 * z0
  z = t
else:
  z = z1

if x > y:
  dt += dz * 2 * t
  dz0 += dt * 2 * z0
else:
  dz1 += dz
```

Differentiating if/else statements

in loma, we disallow variable declaration inside if/else statements

```
z : float = 0
if x > y:
  t : float = z0 * z0
  z = t * t
else:
  z = z1
```

compile error

```
z : float = 0
t : float
if x > y:
  t = z0 * z0
  z = t * t
else:
  z = z1
```

Ok

Differentiating if/else statements

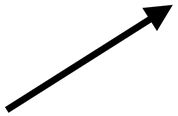
```
def f(x : In[float]) -> float:
  if x > 0:
    return 2 * x
  y : float = x * x
  return y * y
```

?

Differentiating if/else statements

```
def f(x : In[float]) -> float:
  if x > 0:
    return 2 * x
  y : float = x * x
  return y * y
```

```
def f(x : In[float]) -> float:
  ret : float
  if x > 0:
    ret = 2 * x
  y : float = x * x
  if x <= 0:
    ret = y * y
  return ret
```



```
def DTf(x : In[float], dx : Out[float],
        dreturn : In[float]):
  ret : float
  dret : float = 0
  if x > 0:
    ret = 2 * x
  y : float = x * x
  dy : float = 0
  if x <= 0:
    ret = y * y

  dret += dreturn
  if x <= 0:
    dy += dret * 2 * y
  dx += dy * 2 * x
  if x > 0:
    dx += dret * 2
```

Differentiating if/else statements

in loma, we disallow early return of functions

```
def f(x : In[float]) -> float:
  if x > 0:
    return 2 * x
  y : float = x * x
  return y * y
```

compile error

```
def f(x : In[float]) -> float:
  ret : float
  if x > 0:
    ret = 2 * x
  y : float = x * x
  if x <= 0:
    ret = y * y
  return ret
```

Ok

Today: differentiating control flow

we will focus on reverse mode: handling control flow in forward mode is trivial

```
if …:
  …
else:
  …
```

```
for i in range(…):
  …

while (…):
  …
```

```
f(…):
```

if/else conditions

loops

function calls

Differentiating loops

```
for i in range(10):
  x = sin(i * x)
```


?

Differentiating loops

```
for i in range(10):
  x = sin(i * x)
```



```
for i in range(10):
  stack.push(x)
  x = sin(i * x)

for i in reversed(range(10)):
  x = stack.pop()
  dx_ : float = dx * cos(i * x) * i
  dx = 0
  dx += dx_
```

Differentiating loops

```
for i in range(10):
  x = sin(i * x)
```



```
for i in range(10):
  stack.push(x)
  x = sin(i * x)

for i in reversed(range(10)):
  x = stack.pop()
  dx_ : float = dx * cos(i * x)
  dx = 0
  dx += dx_
```

dynamic heap allocation — slow!

Differentiating loops

in loma, we always ask user to provide an upper bound of the loop, so we can preallocate the stack

```
for i in range(10):
  x = sin(i * x)
```

```
stack.reserve(10)
for i in range(10):
  stack.push(x)
  x = sin(i * x)

for i in reversed(range(10)):
  x = stack.pop()
  dx_ : float = dx * cos(i * x)
  dx = 0
  dx += dx_
```

Differentiating loops

```
while (x > 0, max_iter := 10):
  x = sin(x)
```


?

Differentiating loops

```
while (x > 0, max_iter := 10):
  x = sin(x)
```

```
stack.reserve(10)
ctr : int = 0
while (x > 0, max_iter := 10):
  stack.push(x)
  x = sin(x)
  ctr += 1

while (ctr > 0, max_iter := 10):
  x = stack.pop()
  dx_ : float = dx * cos(x)
  dx = 0
  dx += dx_
  ctr -= 1
```

Differentiating loops

```
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    …
```


?

Differentiating loops

```
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    …
```



```
stack.reserve(100)
ctr0 : int = 0
ctr1 : int = 0
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    …
    ctr1 += 1
  ctr0 += 1

while (ctr0 > 0, max_iter := 10):
  while (ctr1 > 0, max_iter := 10):
    …
    ctr1 -= 1
  ctr0 -= 1
```

is this correct?

Differentiating loops

```
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    …
```



```
stack.reserve(100)
ctr0 : int = 0
ctr1 : Array[int, 10]
ctr1_ptr : int = 0
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    …
    ctr1[ctr1_ptr] += 1
  ctr1_ptr += 1
  ctr0 += 1

while (ctr0 > 0, max_iter := 10):
  ctr1_ptr -= 1
  while (ctr1[ctr1_ptr] > 0, max_iter := 10):
    …
    ctr1[ctr1_ptr] -= 1
  counter0 -= 1
```

need to construct counter arrays for nested while loops

Differentiating loops

```
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    while (cond2, max_iter := 10):
    …
```


?

Differentiating loops

```
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    while (cond2, max_iter := 10):
    …
```



```
stack.reserve(100)
ctr0 : int = 0
ctr1 : Array[int, 10]
ctr1_ptr : int = 0
ctr2 : Array[int, 100]
ctr2_ptr : int = 0
while (cond0, max_iter := 10):
  while (cond1, max_iter := 10):
    while (cond2, max_iter := 10):
      …
      ctr2[ctr2_ptr] += 1
    ctr2_ptr += 1
    ctr1[ctr1_ptr] += 1
  ctr1_ptr += 1
  ctr0 += 1

while (ctr0 > 0, max_iter := 10):
  ctr1_ptr -= 1
  while (ctr1[ctr1_ptr] > 0, max_iter := 10):
    ctr2_ptr -= 1
    while (ctr2[ctr2_ptr] > 0, max_iter := 10):
      …
      ctr2[ctr2_ptr] -= 1
    ctr1[ctr1_ptr] -= 1
  ctr0 -= 1
```

Differentiating loops

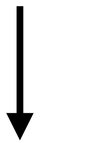
```
while (True, max_iter := 10):
  if x > 0:
    break
  …
```


?

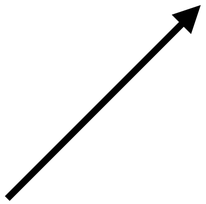
Differentiating loops

in loma, we disallow break statements

```
while (True, max_iter := 10):
  if x > 0:
    break
  …
```



```
flag : int = True
while (flag, max_iter := 10):
  if x > 0:
    flag = False
  if flag:
    …
```



…

Differentiating loops

```
while (cond, max_iter := 10):
  if x > 0:
    continue
  …
```

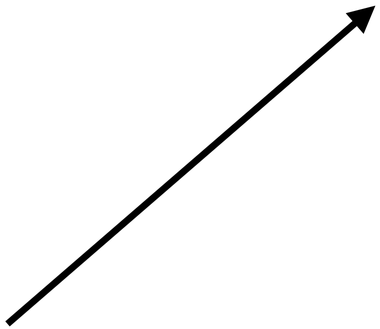

?

Differentiating loops

in loma, we disallow continue statements

```
while (cond, max_iter := 10):
  if x > 0:
    continue
  …
```

```
while (cond, max_iter := 10):
  while (True, max_iter := 1):
    if x > 0:
      break
    …
    break
```



?

For loops vs while loops

```
for i in range(n):
  for j in range(i, n):
```

v.s.

```
i : int = 0
while i < n:
  j : int = i
  while j < n:
    …
    j += 1
  i += 1
```

For loops vs while loops

```
for i in range(n):
  for j in range(i, n):
```

v.s.

```
i : int = 0
while i < n:
  j : int = i
  while j < n:
    …
    j += 1
  i += 1
```

for loops provide more structures for us to revert them

while loops can require more memory to store the loop counters

For loops vs while loops

take home message: program abstraction is important for the AD system to figure out the right optimization!!

```
for i in range(n):
  for j in range(i, n):
```

v.s.

```
i : int = 0
while i < n:
  j : int = i
  while j < n:
    …
    j += 1
  i += 1
```

for loops provide more structures for us to revert them

while loops can require more memory to store the loop counters

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  …
else:
  …
```

if/else conditions

```
for i in range(…):
  …
```

```
while (…):
  …
```

loops

```
f(…):
```

function calls

Differentiating function calls

```
# def f(x : In[float]) -> float
y = f(x)
```


?

Differentiating function calls

```
# def f(x : In[float]) -> float
y = f(x)
```

```
y = f(x)
DTf(x, dx, dy)
```

Differentiating function calls

```
# def f(x : In[float], y : Out[float])
f(x, y)
```


?

Differentiating function calls

```
# def f(x : In[float], y : Out[float])

f(x, y)
```

```
stack.push(y)
f(x, y)

y = stack.pop()
DTf(x, dx, dy)
```

Differentiating function calls

```
# def f(x : In[float], y : Out[float])
f(2 + x, y)
```


?

Differentiating function calls

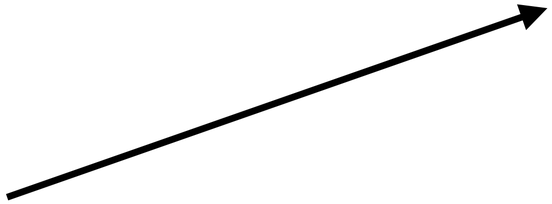
we provide the utility code for doing this transformation in your homework

```
# def f(x : In[float], y : Out[float])

f(2 + x, y)
```

…

```
t = 2 + x
f(t, y)
```



Differentiating recursion

```
def f(x : In[float], n : In[int]) -> float:
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x
  return ret
```

?

Differentiating recursion

```
def f(x : In[float], n : In[int]) -> float:
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x
  return ret
```

```
def DTf(x : In[float],
        dx : Out[float],
        n : In[int]
        dret : Out[float]):
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x

  if n <= 0:
    dx += dret
  else:
    DTf(x, dx, n-1, dret * x)
    dx += dret * f(x, n - 1)
```

is this correct?

Differentiating recursion

```
def f(x : In[float], n : In[int]) -> float:
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x
  return ret
```

naive differentiation of recursion violates cheap gradient principle!!

```
def DTf(x : In[float],
        dx : Out[float],
        n : In[int]
        dret : Out[float]):
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x

  if n <= 0:
    dx += dret
  else:
    DTf(x, dx, n-1, dret * x)
    dx += dret * f(x, n - 1)
```

O(n)

O(n^2)!!

is this correct?

Differentiating recursion

```
def f(x : In[float], n : In[int]) -> float:
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x
  return ret
```

```
def DTf(x : In[float],
        dx : Out[float],
        n : In[int]
        dret : Out[float],
        f_cache = {}):
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f_cache.lookup(x, n - 1) * x

  if n <= 0:
    dx += dret
  else:
    DTf(x, dx, n-1, dret * x)
    dx += dret * f_cache.lookup(x, n - 1)
```

Differentiating recursion

```
def f(x : In[float], n : In[int]) -> float:
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x
  return ret
```

```
def DTf(x : In[float],
        dx : Out[float],
        n : In[int]
        dret : Out[float],
        f_cache = {}):
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f_cache.lookup(x, n - 1) * x

  if n <= 0:
    dx += dret
  else:
    DTf(x, dx, n-1, dret * x)
    dx += dret * f_cache.lookup(x, n - 1)
```

for a better way to construct f_cache, see this paper
the trick is to apply "Continuation Passing Style" in functional programming to store variables in a *continuation*

Demystifying Differentiable Programming: Shift/Reset the Penultimate Backpropagator

FEI WANG, Purdue University, USA
DANIEL ZHENG, Purdue University, USA
JAMES DECKER, Purdue University, USA
XILUN WU, Purdue University, USA
GRÉGORY M. ESSERTEL, Purdue University, USA
TIARK ROMPF, Purdue University, USA

Differentiating recursion

alternatively, recursions can always be rewritten as iterative loops
(Church-Turing thesis)

```
def f(x : In[float], n : In[int]) -> float:
  ret : float
  if n <= 0:
    ret = x
  else:
    ret = f(x, n - 1) * x
  return ret
```

```
ret = 1
for i in range(n):
  ret *= x
```

Food for thoughts

if we can differentiate general function calls (with recursion), do we need to handle anything else?

```
z = x + y
```

```
add x y z
```

```
for i in range(10):
  …
```

```
scan … range(10)
```

```
if x:
  a
else:
  b
```

```
x a b
```

(Church boolean)