Photon Mapping

UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li
SDS light paths

recall: it’s impossible for a bidirectional path tracer to render this!
Idea 1: allow "near miss"

- pinhole camera
- diffuse
- mirror (specular)
- point light
- mirror (specular)
Idea 2: share light subpaths among different pixels
Photon mapping

1. trace random light subpaths
1. trace random light subpaths

2. store **photons** on diffuse surfaces
1. trace random light subpaths

2. store photons on diffuse surfaces
Photon mapping

1. trace random light subpaths
2. store photons on diffuse surfaces
3. trace random camera subpaths
1. trace random light subpaths

2. store photons on diffuse surfaces

3. trace random camera subpaths

4. reconstruct path contribution from photons

Photon mapping
Photon mapping

1. trace random light subpaths
2. store **photons** on diffuse surfaces
3. trace random camera subpaths
4. reconstruct path contribution from photons

**Bidirectional Photon Mapping**

Jiří Vorba
*Supervised by: Jaroslav Křivánek*

Charles University, Prague
Photon mapping

1. trace random light subpaths
2. store photons on diffuse surfaces
3. trace random camera subpaths
4. reconstruct path contribution from photons
Math formulation: blurring path contribution

\[ \int \text{light paths} \ f(\vec{x})d\vec{x} \quad \rightarrow \quad \int \int \text{surface} \int \text{light paths} \ k(x_2, x'_2)f(\vec{x}')d\vec{x}dx_2' \]

\(k\): convolution kernel
e.g. a disk kernel \(\frac{1}{\pi r^2}\)
Math formulation: blurring path contribution

\[ \int_{\text{light paths}} f(\vec{x}) d\vec{x} \quad \longrightarrow \quad \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x'_2) f(\vec{x'}) d\vec{x} dx'_2 \]

\( k \): convolution kernel
e.g. a disk kernel \( \frac{1}{\pi r^2} \)
Sidetrack: blurring an integrand does *not* necessarily change its integral!

recall: integration = taking DC in frequency domain

\[ \int f(x) dx = \hat{f}(0) \]

blurring = multiply the DCs in frequency domain

\[ \int \int k(x, y)f(x) dx dy = \hat{f}(0)\hat{k}(0) \]

as long as \( \hat{k}(0) = 1 \), the integral is preserved!
Photon mapping: estimating the blurring integral using camera subpaths & light subpaths

\[ \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x_2') f(\bar{x}') d\bar{x} dx_2' \approx \frac{k(x_2, x_2') f(\bar{x}')}{p(x_0 \rightarrow x_1 \rightarrow x_2) p(x_4 \rightarrow x_3 \rightarrow x_2')} \]
Density estimation interpretation of photon mapping

- reconstructing radiance at position $x$ using randomly sampled photons at position $x_i$

$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} k(x_i, x) \gamma_i$$

important:

$N = \text{all photons, not just photon nearby to } x!$

photon contribution $\times$ BSDF($x$)
Bias-variance trade-off in photon mapping
Bias-variance trade-off in photon mapping

large radius
high bias,
low variance

small radius
low bias,
higher variance
Bias-variance trade-off in photon mapping

how do we analyze the effect of the interpolation radius?

large radius
high bias,
low variance

small radius
low bias,
higher variance
Bias-variance analysis of photon mapping

\[
L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i
\]

normalize kernel s.t. \( x' - x \)
is constrained to a unit circle

\[
\frac{1}{r^2} \int k(t) dt = 1 \quad \int tk(t) dt = 0
\]
Bias-variance analysis of photon mapping

\[
\text{bias} = E \left[ \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \right] - L(x, \omega)
\]

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Bias-variance analysis of photon mapping

\[ \text{bias} = E \left[ \frac{1}{r^2} k \left( \frac{X-x}{r} \right) \right] E[\gamma] - L(x, \omega) \]

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

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\[ \text{bias} = E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega) \]

\[ E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] = \frac{1}{r^2} \int k \left( \frac{X - x}{r} \right) p(X) dX \]

normalize kernel s.t. \( x' - x \)

is constrained to a unit circle

\[ \frac{1}{r^2} \int k(t) dt = 1 \quad \int tk(t) dt = 0 \]

p(X): PDF of a photon landing at location X
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

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\[
\text{bias} = E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega)
\]

\[
E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] = \frac{1}{r^2} \int k(t)p(x + rt)dt
\]

\[
t = \frac{X - x}{r}
\]

normalize kernel s.t. \(x' - x\)

is constrained to a unit circle

\[
p(x + rt) \approx p(x) + rt \nabla p(x) + r^2 t^T H_p(x)t
\]

\[
\frac{1}{r^2} \int k(t)dt = 1 \quad \int tk(t)dt = 0
\]

\(p(X):\) PDF of a photon landing at location \(X\)
Bias-variance analysis of photon mapping

\[
L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i
\]

normalize kernel s.t. \( x' - x \) is constrained to a unit circle

\[
\frac{1}{r^2} \int k(t) dt = 1 \quad \text{and} \quad \int tk(t) dt = 0
\]

\[
\text{bias} = E \left[ \frac{1}{r^2} k \left( \frac{X-x}{r} \right) \right] E[\gamma] - L(x, \omega)
\]

\[
E \left[ \frac{1}{r^2} k \left( \frac{X-x}{r} \right) \right] = \frac{1}{r^2} \int k(t) p(x + rt) dt
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t = \frac{X-x}{r}
\]

\[
p(x + rt) \approx p(x) + rt \nabla p(x) + r^2 t^T H_p(x) t
\]

\[
\int k(t) p(x + rt) dt \approx p(x) + r^2 \cdot \int t^T H_p(x) t dt
\]

\( p(X) \): PDF of a photon landing at location \( X \)
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

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normalize kernel s.t. \( x' - x \) is constrained to a unit circle

\[ \frac{1}{r^2} \int k(t) \, dt = 1 \quad \int tk(t) \, dt = 0 \]

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\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

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\[ E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] \approx p(x) + r^2 \cdot \int t^T H_p(x) t dt \]

\[ L(x, \omega) = E[\gamma] E[\delta (X - x)] = E[\gamma] p(x) \]

\[ \frac{1}{r^2} \int k(t) dt = 1 \quad \int t k(t) dt = 0 \]

\( p(X) \): PDF of a photon landing at location \( X \)
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

\[ \text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t \, dt \]
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k\left(\frac{x_i - x}{r}\right) \gamma_i \]

\[ \text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t \, dt \]

\[ \text{variance} \approx \left( \text{Var}[\gamma] + E[\gamma]^2 \right) \frac{p(x)}{N r^2} \int k(t)^2 \, dt \]
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

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\text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 dt

Observation:
• variance reduces with \( N \), bias does not
• bias increases with \( r \), but variance reduces with \( r \)
Bias-variance analysis of photon mapping

\[ \text{bias} \propto r^2 \quad \text{variance} \propto \frac{1}{Nr^2} \]

Observation:
- variance reduces with N, bias does not
- bias increases with r, but variance reduces with r
Bias-variance analysis of photon mapping

**quiz:** is photon mapping a consistent estimator?

\[
\text{bias } \propto r^2 \quad \quad \quad \quad \quad \quad \text{variance } \propto \frac{1}{N r^2}
\]

Observation:

- variance reduces with N, bias does not
- bias increases with r, but variance reduces with r
Bias-variance trade-off in photon mapping
Bias-variance trade-off in photon mapping

- Large radius:
  - High bias,
  - Low variance

- Small radius:
  - Low bias,
  - Higher variance
Epanechnikov kernel minimizes the MSE

\[ k(t) = \begin{cases} 
\frac{3}{4\sqrt{5}} \left( 1 - \frac{1}{5}t^2 \right) & -\sqrt{5} \leq t \leq \sqrt{5} \\
0 & \text{otherwise}
\end{cases} \]

\[ \text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{N^r} \int k(t)^2 dt \]

\[ \begin{align*}
\text{minimize} & \quad \int k(t)^2 dt \\
\text{s.t.} & \quad \frac{1}{r^2} \int k(t) dt = 1 \\
& \quad \int tk(t) dt = 0 \\
& \quad \int t^2 k(t) dt = 1
\end{align*} \]

Silverman 1986
Progressive photon mapping:  
a consistent photon mapping estimator

\[ L \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

\[ \text{bias } \propto r^2 \]

\[ \text{variance } \propto \frac{1}{Nr^2} \]

can we eliminate bias when N goes to infinity?
Progressive photon mapping: a consistent photon mapping estimator

- key idea: select a sequence $r_i$ with gradually reduced radius to remove bias

- can’t reduce too fast, can’t reduce too slow

$$L \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r_i^2} k \left( \frac{x_i - x}{r_i} \right) \gamma_i$$

bias $\propto r^2$

variance $\propto \frac{1}{Nr^2}$
Progressive photon mapping:
a consistent photon mapping estimator

- key idea: select a sequence $r_i$ with gradually reduced radius to remove bias
- can’t reduce too fast, can’t reduce too slow

$$L \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r_i^2} k \left( \frac{x_i - x}{r_i} \right) \gamma_i$$

for each iteration $i$

$$\text{bias} \propto r_i^2$$

$$\text{variance} \propto \frac{1}{r_i^2}$$
Progressive photon mapping: a consistent photon mapping estimator

goal: decrease $r$ so that bias goes to 0, but variance does not go to infinity

for each iteration $i$

\[
\text{bias } \propto r_i^2 \\
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\]
Progressive photon mapping: a consistent photon mapping estimator

goal: decrease $r$ so that bias goes to 0, but variance does not go to infinity

idea: set $r_i$ such that

$$\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1} \quad (\alpha \in (0,1))$$

for each iteration $i$

bias $\propto r_i^2$

variance $\propto \frac{1}{r_i^2}$
Progressive photon mapping: a consistent photon mapping estimator

goal: decrease $r$ so that bias goes to 0, but variance does not go to infinity

idea: set $r_i$ such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}$ \hspace{1cm} (\alpha \in (0,1))

for each iteration $i$

$\text{bias} \propto r_i^2$

$\text{variance} \propto \frac{1}{r_i^2}$

$\frac{\text{Var}_{i+1}}{\text{Var}_i} = \frac{i + 1}{i + \alpha}$  \hspace{1cm} $\frac{\text{Bias}_{i+1}}{\text{Bias}_i} = \frac{i + \alpha}{i + 1}$
Progressive photon mapping: a consistent photon mapping estimator

goal: decrease $r$ so that bias goes to 0, but variance does not go to infinity

idea: set $r_i$ such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}$ \[(\alpha \in (0,1))\]

$\frac{\text{Var}_{i+1}}{\text{Var}_i} = \frac{i + 1}{i + \alpha}$ \hspace{1cm} $\frac{\text{Bias}_{i+1}}{\text{Bias}_i} = \frac{i + \alpha}{i + 1}$

$\text{Var} = \frac{1}{N^2} \sum_i \text{Var}_i = O(N^{-\alpha})$ \hspace{1cm} $\text{Bias} = \frac{1}{N} \sum_i \text{Bias}_i = O(N^{1-\alpha})$

for each iteration $i$:

$\text{bias} \propto r_i^2$

$\text{variance} \propto \frac{1}{r_i^2}$
Progressive photon mapping:  
a consistent photon mapping estimator

Goal: decrease $r$ so that bias goes to 0,  
but variance does not go to infinity

Idea: set $r_i$ such that  
$$ \frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1} \quad (\alpha \in (0,1)) $$

$$ \text{Var} = \frac{1}{N^2} \sum_i \text{Var}_i = O \left( N^{-\alpha} \right) $$

$$ \text{Bias} = \frac{1}{N} \sum_i \text{Bias}_i = O \left( N^{1-\alpha} \right) $$
Progressive photon mapping: a consistent photon mapping estimator

Goal: decrease $r$ so that bias goes to 0, but variance does not go to infinity.

Idea: set $r_i$ such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}$ ($\alpha \in (0, 1)$)

$\text{Var} = \frac{1}{N^2} \sum_i \text{Var}_i = O\left(N^{-\alpha}\right)$

$\text{Bias} = \frac{1}{N} \sum_i \text{Bias}_i = O\left(N^{1-\alpha}\right)$

Quiz: what is the asymptotically optimal $\alpha$?
Progressive photon mapping: a consistent photon mapping estimator

goal: decrease \( r \) so that bias goes to 0, but variance does not go to infinity

idea: set \( r_i \) such that \( \frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1} \) \((\alpha \in (0,1))\)

\[
\text{Var} = \frac{1}{N^2} \sum_i \text{Var}_i = O\left(N^{-\alpha}\right)
\]

\[
\text{Bias} = \frac{1}{N} \sum_i \text{Bias}_i = O\left(N^{1-\alpha}\right)
\]

\[
\alpha = \frac{2}{3} \text{ gives optimal mean square error } = \text{bias}^2 + \text{variance}
\]
Photon mapping is good at SDS light paths.
Photon mapping is good at SDS light paths
Alternative: directly set $r$ to minimize mean square error

\[
\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t dt
\]

\[
\text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 dt
\]

mean square error = bias^2 + variance

Adaptive Progressive Photon Mapping

ANTON S. KAPLANYN and CARSTEN DACHSBACHER
Karlsruhe Institute of Technology

(a) progressive photon mapping  (b) adaptive PPM (our method)  (c) PPM  (d) our method

Anton’s method    Anton’s method
Combining with bidirectional path tracing (VCM/UPS)

- apply multiple importance sampling
Combining with bidirectional path tracing (VCM/UPS)

- apply multiple importance sampling

- challenge: photon mapping has one more vertex ($x'_2$ in this case), can’t compare PDFs

path tracing: $x_0x_1x_2x_3x_4$

photon mapping: $x_0x_1x_2x'_2x_3x_4$
Combining with bidirectional path tracing (VCM/UPS)

- apply multiple importance sampling
- challenge: photon mapping has one more vertex ($x'_2$ in this case), can’t compare PDFs
- idea: perturb the bidirectional path tracing vertex to match, approximate perturbation probability as $\frac{1}{\pi r^2}$

path tracing: $x_0 x_1 x_2 x_3 x_4$
photon mapping: $x_0 x_1 x_2 x'_2 x_3 x_4$
Photon mapping is good at SDS paths
BPT is better at non SDS paths
Can we make photon mapping unbiased?

• surprisingly — yes!

• recall: blurring the integrand doesn’t change the integral if the kernel is properly normalized

• why is photon mapping biased?

  • it usually uses fake BSDF & visibility

  • kernel is not normalized w.r.t. visibility
Unbiased photon mapping:
trace rays to the photon to debias

\[
\int_{\text{surface}} \int_{\text{light paths}} k(x_2, x'_2)f(x') dx'dx'_2 
\approx \frac{k(x_2, x'_2)f(x')}{p(x_0 \to x_1 \to x_2)p(x_4 \to x_3 \to x'_2) \int k(x_2, x'_2) dx'_2}
\]
Unbiased photon mapping: trace rays to the photon to debias

\[
\int_{\text{surface}} \int_{\text{light paths}} k(x_2, x'_2) f(\bar{x}') d\bar{x} dx'_2 \\
\approx \frac{k(x_2, x'_2) f(\bar{x}')}{p(x_0 \rightarrow x_1 \rightarrow x_2)p(x_4 \rightarrow x_3 \rightarrow x'_2)} \int k(x_2, x'_2) dx'_2
\]

challenge: taking reciprocal of a Monte Carlo estimator leads to bias!
Unbiased estimation of a reciprocal integral

\[ \frac{1}{\int g(x)dx} \neq E \left[ \frac{1}{\frac{1}{N} \sum_{i=1}^{N} g(x_i)} \right] \]

similar to the problem we faced when estimating transmittance
Unbiased estimation of a reciprocal integral

Idea: rewrite the reciprocal using an infinite series

\[
\frac{1}{\int g(x)dx} = \frac{1}{1 - G} = 1 + G + G^2 + \cdots
\]

can be estimated using Russian roulette
Unbiased photon mapping converges faster, but can't do pure specular paths
Photon beams for volumetric rendering

- treat a light subpath as infinitely many photons
- treat a camera subpath as infinitely many query points

Progressive Photon Beams

<table>
<thead>
<tr>
<th>Wojciech Jarosz(^1)</th>
<th>Derek Nowrouzezahrai(^1)</th>
<th>Robert Thomas(^1)</th>
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<td>(^1)Disney Interactive Studios</td>
<td>(^1)Disney Research Zürich</td>
<td>(^2)University of Bern</td>
<td>(^3)University of Bern</td>
</tr>
</tbody>
</table>
Combining photon beams, points, and bidirectional path tracing

**Unifying Points, Beams, and Paths in Volumetric Light Transport Simulation**

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¹Charles University in Prague  ²Light Transportation Ltd.  ³Aarhus University  ⁴Université de Montréal  ⁵Disney Research Zürich
Photon planes and photon volumes

• infinitely many photons in planes & volumes
Photon cones/cylinders/spheres and photon bunnies

Photon surfaces for robust, unbiased volumetric density estimation

Xi Deng\textsuperscript{1}\textsuperscript{,} Shaojie Jiao\textsuperscript{1}\textsuperscript{,} Benedikt Bitterli\textsuperscript{1}\textsuperscript{,} Wojciech Jarosz\textsuperscript{1}

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UC San Diego

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Toshiya Hachisuka, Wojciech Jarosz, Henrik Wann Jensen
UC San Diego, Disney Research Zurich

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TOSHIYA HACHISUKA and HENRIK WANN JENSEN
University of California, San Diego
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Xin Sun1, Kun Zhou1, Stephen Lin1, and Baining Guo2
1Microsoft Research Asia, 2State Key Lab of CAD&CG, Zhejiang University

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Iliyan Georgiev1, Jaroslav Křivinka1, Tomáš Davidovič1, and Philippe Slusallek1
1Charles University, Prague, 2Saoft University, Zürich, 3Saoft University, Saarbrücken

2012

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Toshiya Hachisuka1,2,1,2
1Aarhus University, 2NVIDIA Research

Path Space Regularization for Holistic and Robust Light Transport
Anton S. Kaplanyan and Carsten Dachsbacher
Karlsruhe Institute of Technology, Germany

Progressive photon beams
Wojciech Jarosz1, Derek Nowrouzezahrai1,2,3, Robert Thomas1,2, Peter-Pike Sloan1,2, and Matthias Zwicker3
1Disney Research Zürich, 2Disney Interactive Studios, 3University of Bern

In ACM Transactions on Graphics (Proceedings of SIGGRAPH Asia), 2013

Unifying points, beams, and paths in volumetric light transport simulation
Jaroslav Křivinka1,2, Iliyan Georgiev1,2, and Wojciech Jarosz1
1Charles University, Prague, 2Light Transport Ltd, 3University of Bern, Zürich

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2010

2013
History/bibliography

Unbiased Photon Gathering for Light Transport Simulation

Hao Qin*, Xin Sun†, Qiming Hou‡, Baining Guo†, Kun Zhou*

*State Key Lab of CAD&CG, Zhejiang University  †Microsoft Research Asia

2015

Gradient-Domain Photon Density Estimation

Binh-Son Hua1, Adrien Gruson2, Derek Nowrouzezahrai3, Toshiya Hachisuka2

1Singapore University of Technology and Design  2The University of Tokyo  3McGill University

2017

Hierarchical Neural Reconstruction for Path Guiding Using Hybrid Path and Photon Samples

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ZEIXANG XU, Adobe Research, USA
TIANCHENG SUN, University of California San Diego, USA
ALEXANDR KUZNETSOV, University of California San Diego, USA
MARK MEYER, Pixar Animation Studios, USA
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HAO SU, University of California San Diego, USA
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2021

CPPM: Chi-squared Progressive Photon Mapping

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2020
Next time: Metropolis light transport