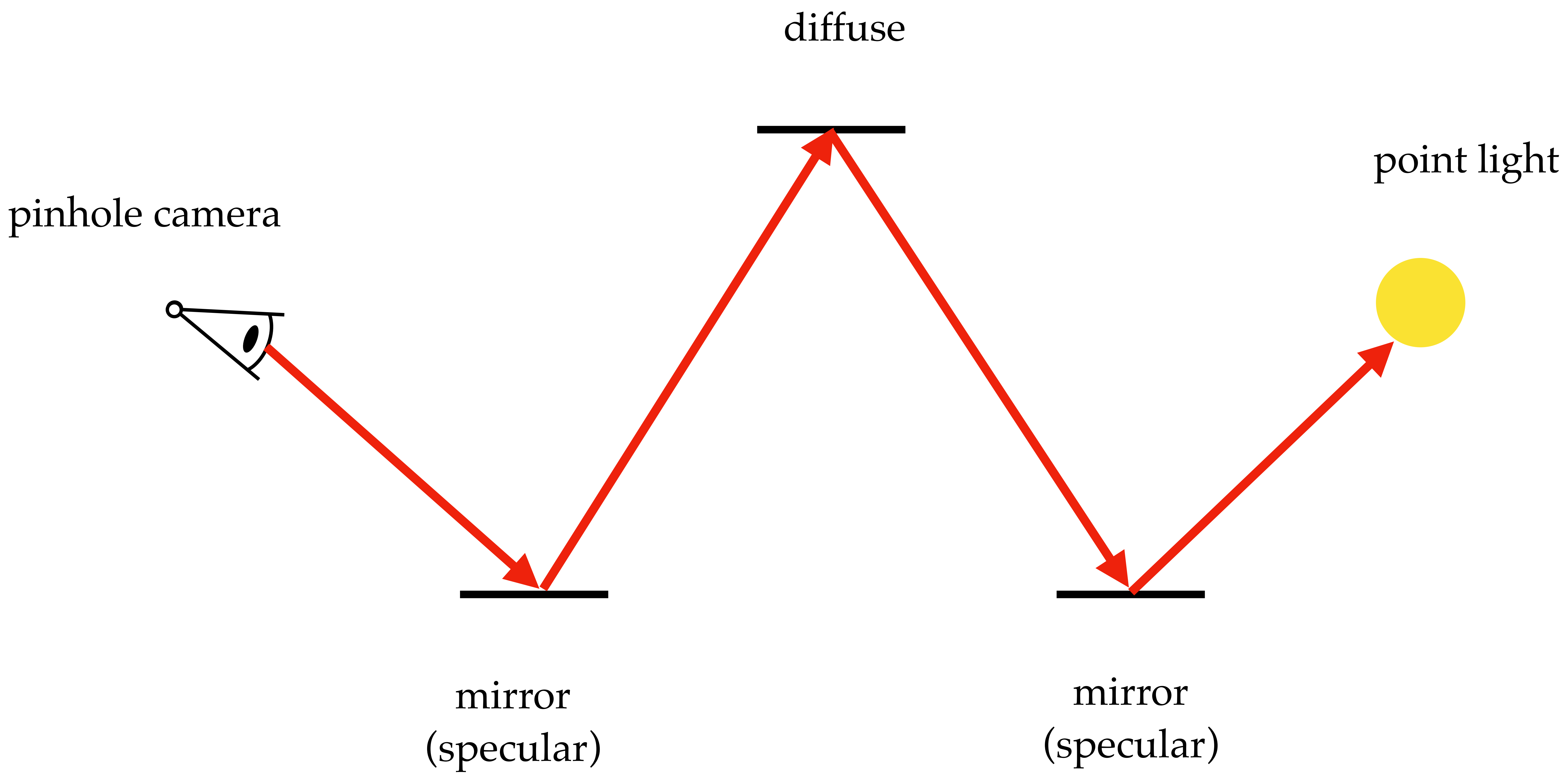
Rendering specular light paths

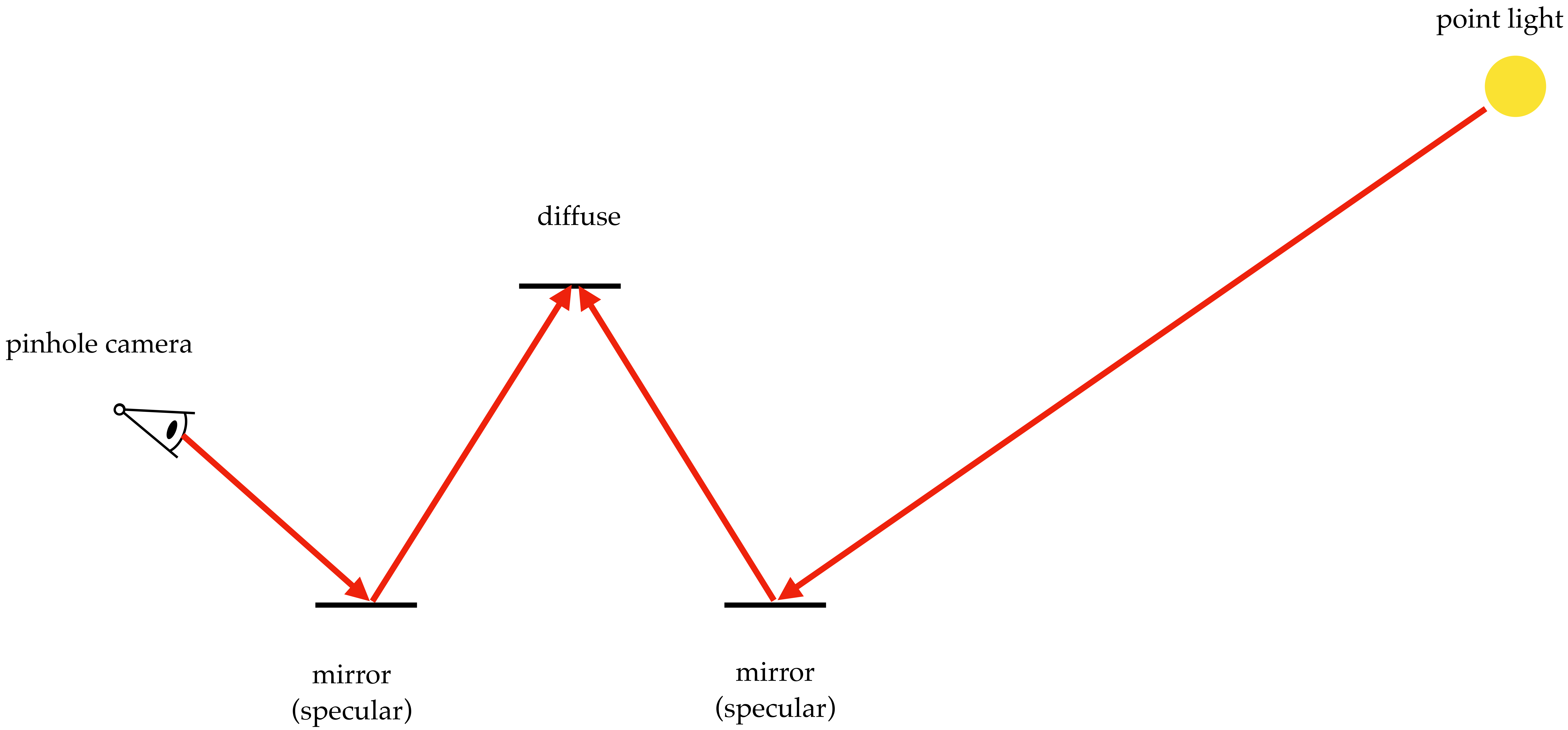
UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li

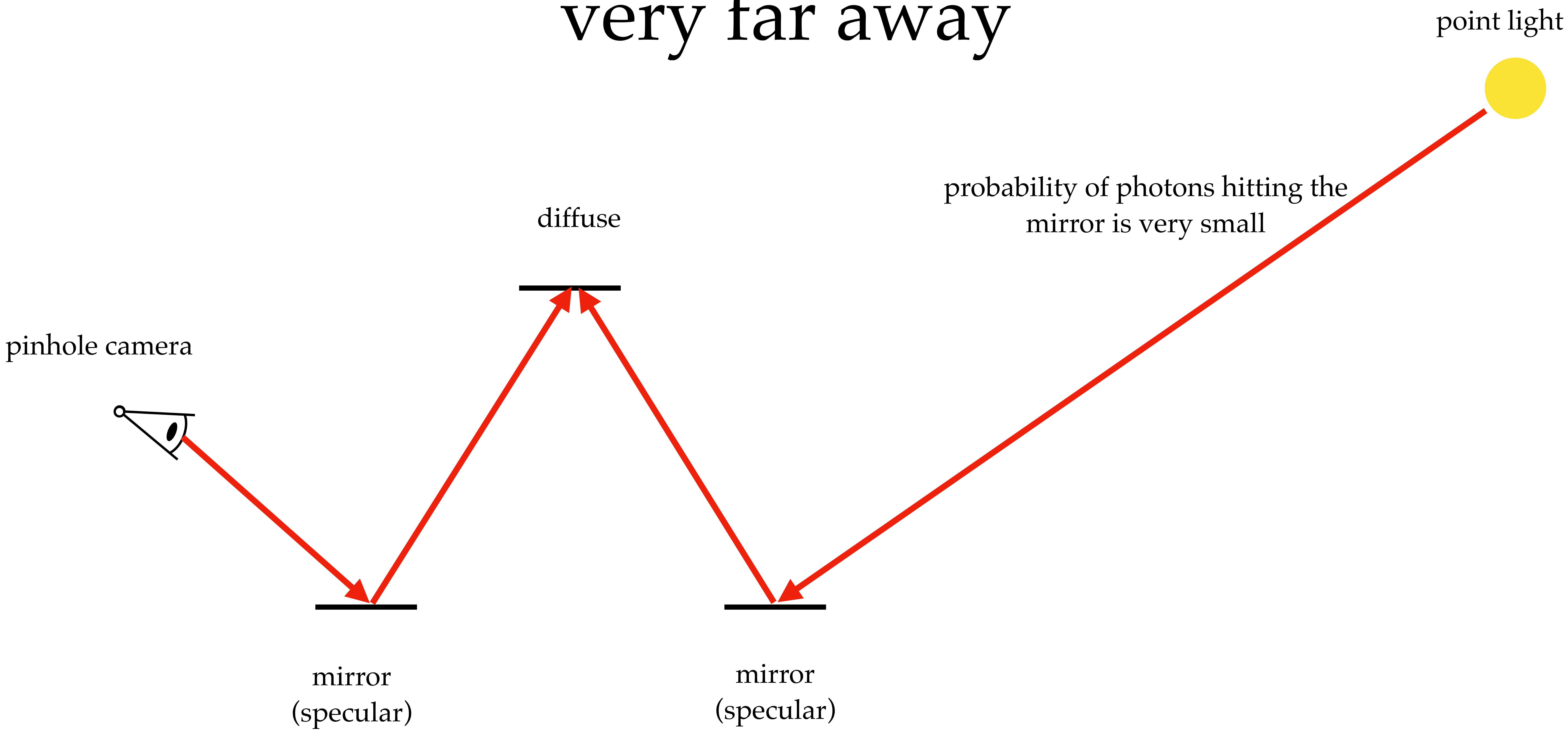
Back to SDS light paths



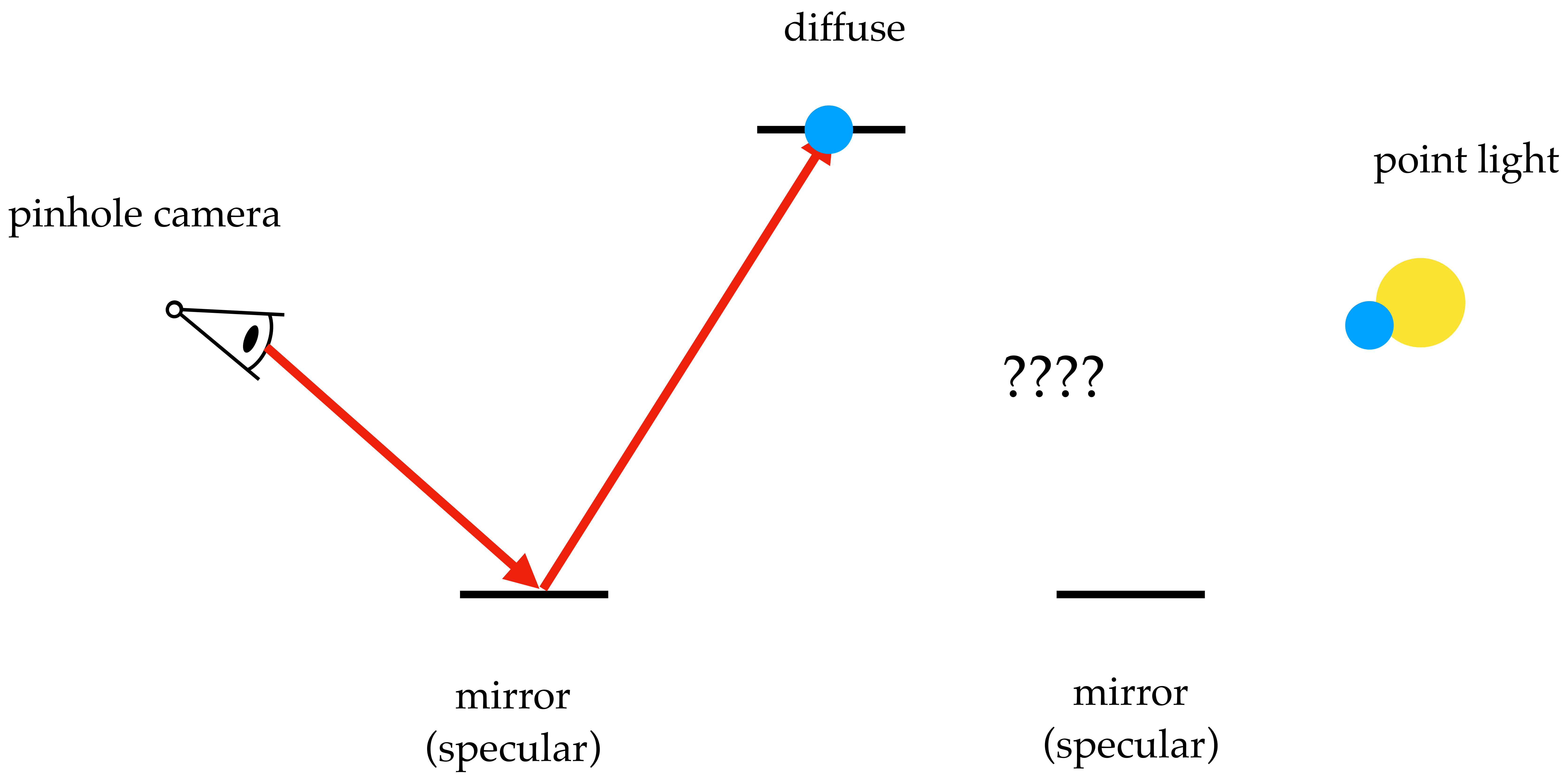
Does photon mapping work in this scenario?



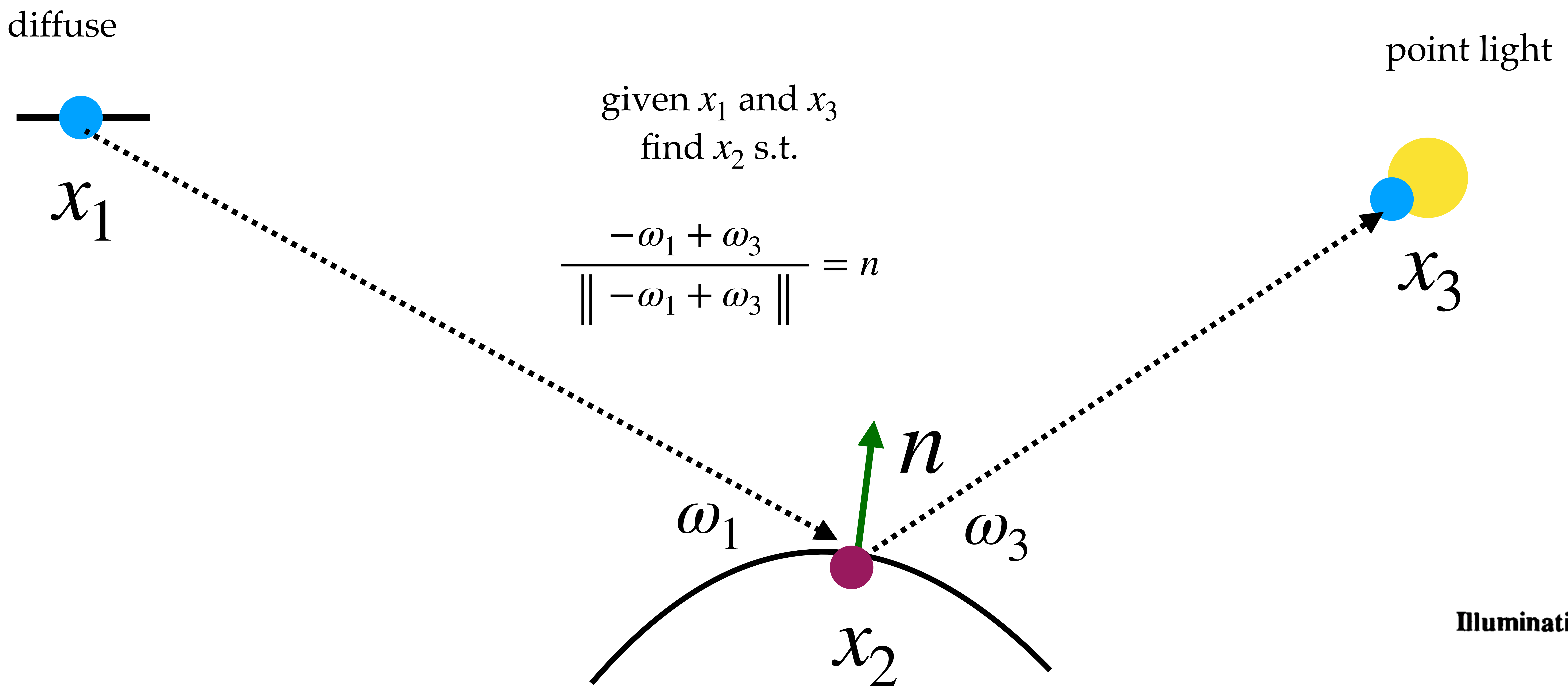
Photon mapping fails when light source is very far away



Can we directly find this light path?



Idea: find light paths by solving a non-linear equation



given x_1 and x_3
find x_2 s.t.

$$\frac{-\omega_1 + \omega_3}{\| -\omega_1 + \omega_3 \|} = n$$

Illumination from Curved Reflectors

Don Mitchell †
Pat Hanrahan ‡

† AT&T Bell Laboratories
‡ † Princeton University

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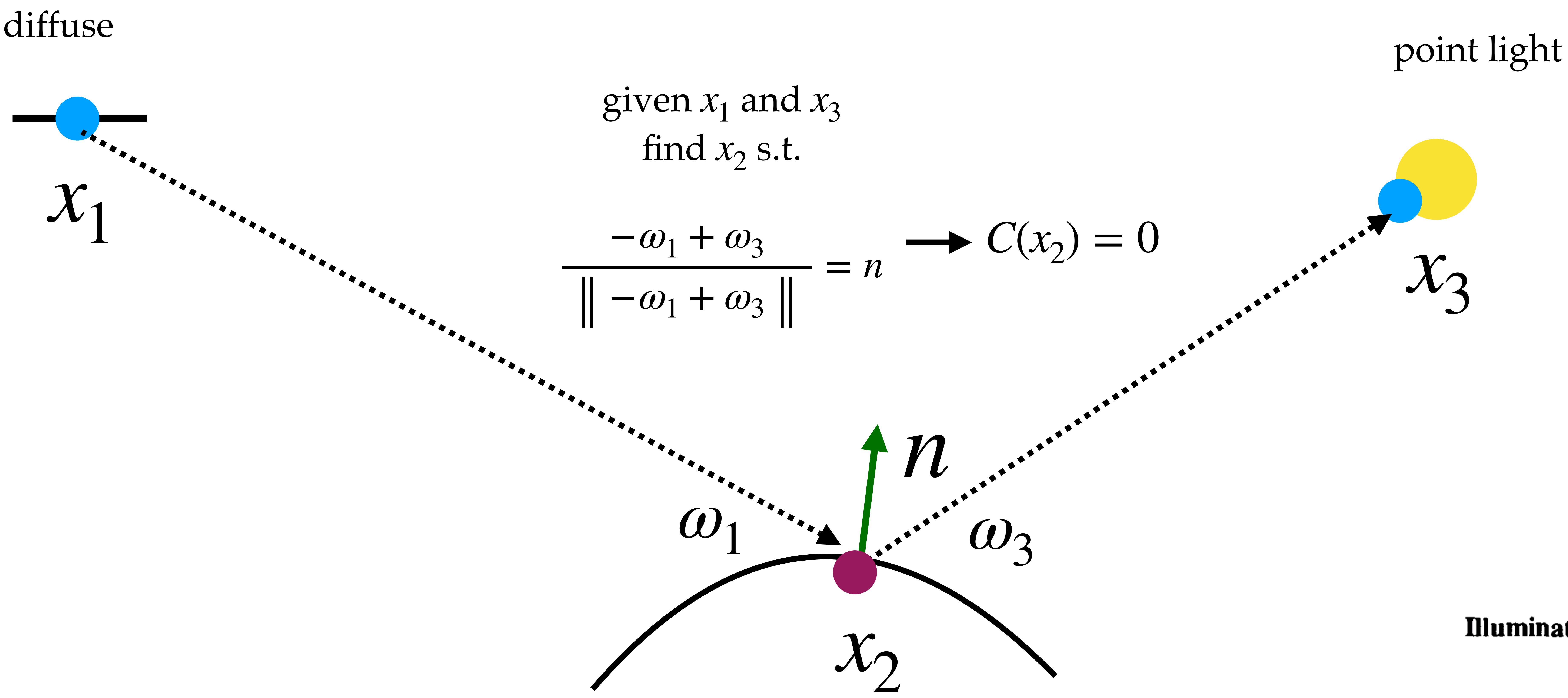

given x_1 and x_3
find x_2 s.t.

$$\frac{-\omega_1 + \omega_3}{\| -\omega_1 + \omega_3 \|} = n \quad \longrightarrow \quad C(x_2) = 0$$

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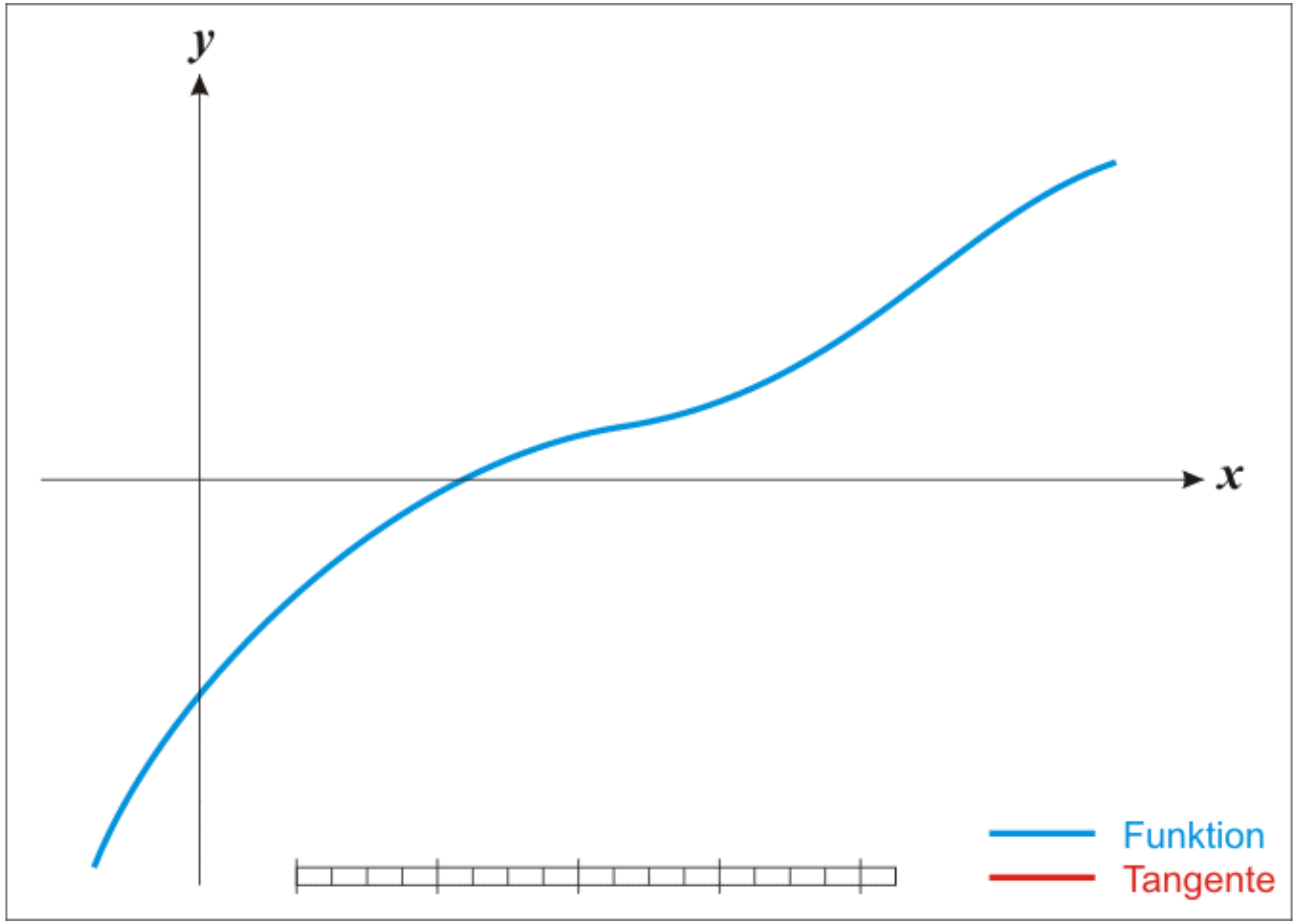
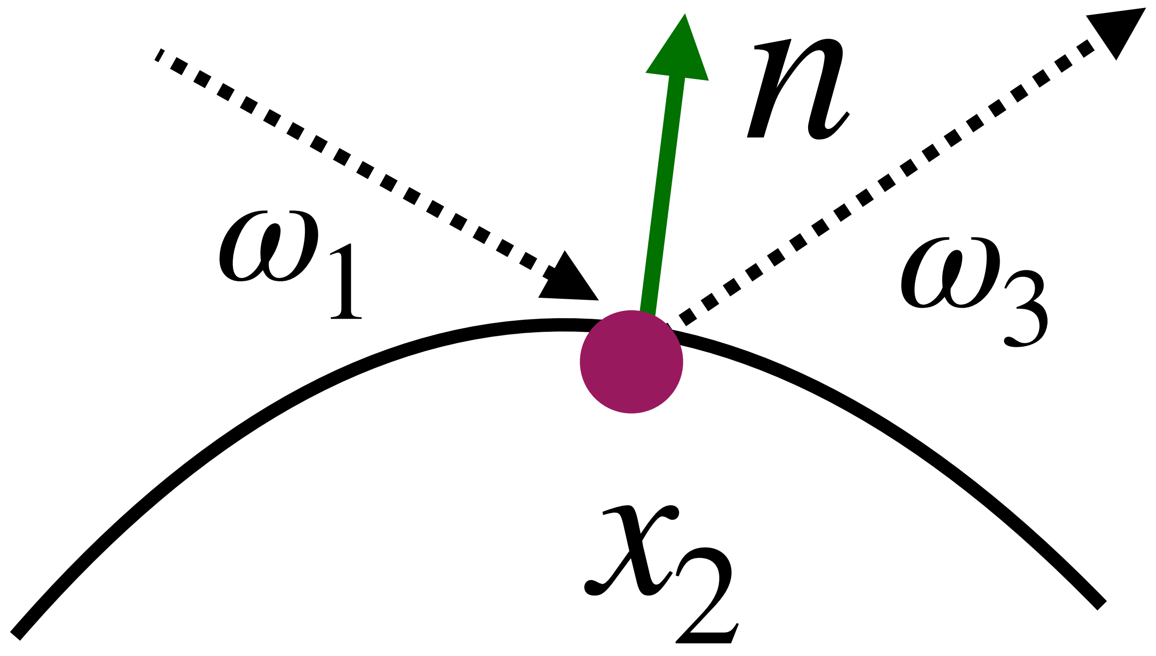
Idea: find light paths by solving a non-linear equation

- solve x_2 using Newton's method: start from an initial guess, iteratively improve



$$C(x_2) = 0$$



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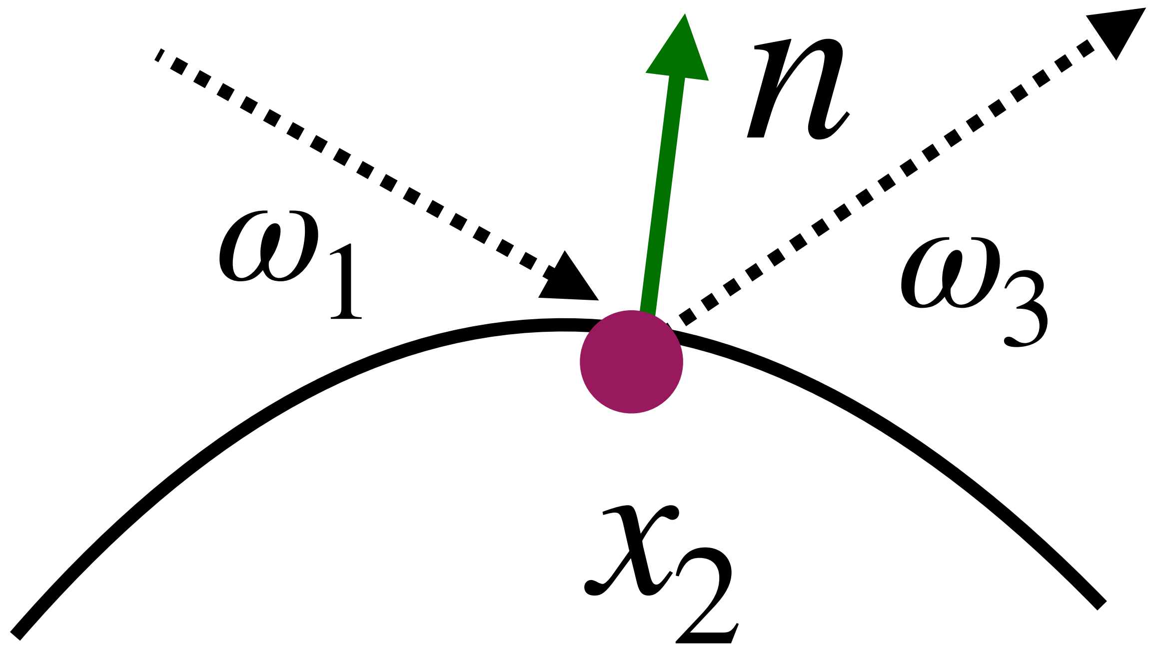
- solve x_2 using Newton's method: start from an initial guess, iteratively improve

$$C(x_2 + \Delta x_2) \approx C(x_2) + J_C(x_2)\Delta x_2 = 0$$

$$\Delta x_2 = -J_C(x_2)^{-1}C(x_2)$$

$$C(x_2) = 0$$



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Idea: find light paths by solving a non-linear equation

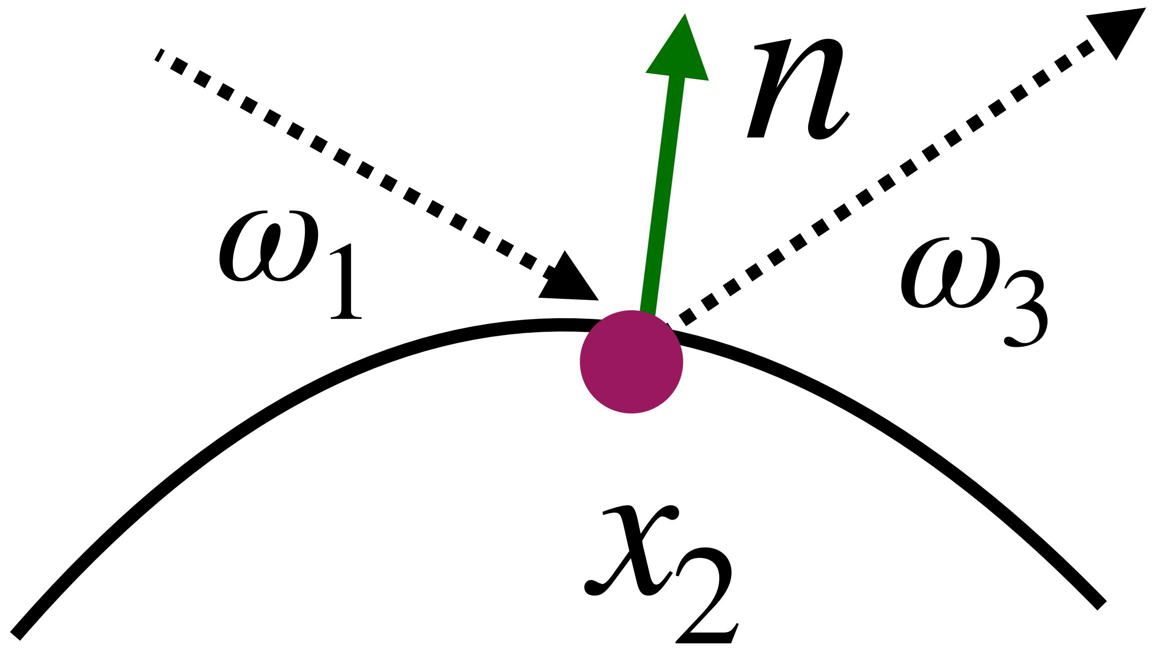
- solve x_2 using Newton's method: expand constraint C using first-order Taylor expansion

$$C(x_2' + \Delta x_2) \approx C(x_2') + J_C(x_2')\Delta x_2 = 0$$

$$\Delta x_2 = -J_C(x_2')^{-1} C(x_2')$$

```
start from an initial guess x_2'
while ‖C(x_2')‖ > ϵ:
  x_2' = x_2' - J_C(x_2')^{-1} C(x_2')
x_2 = x_2'
```

$$C(x_2) = 0$$



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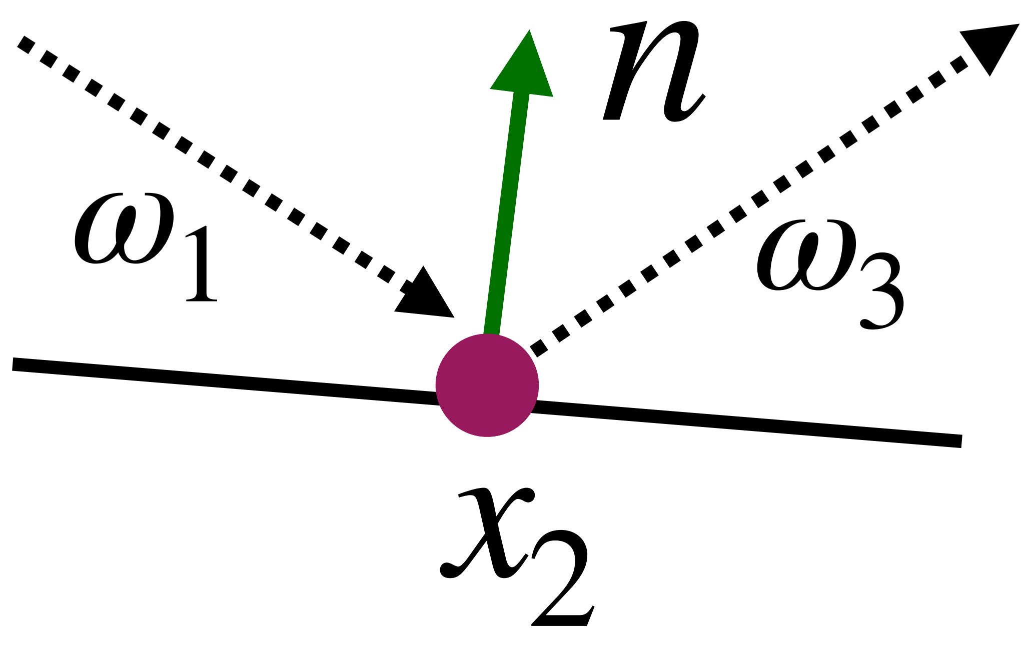
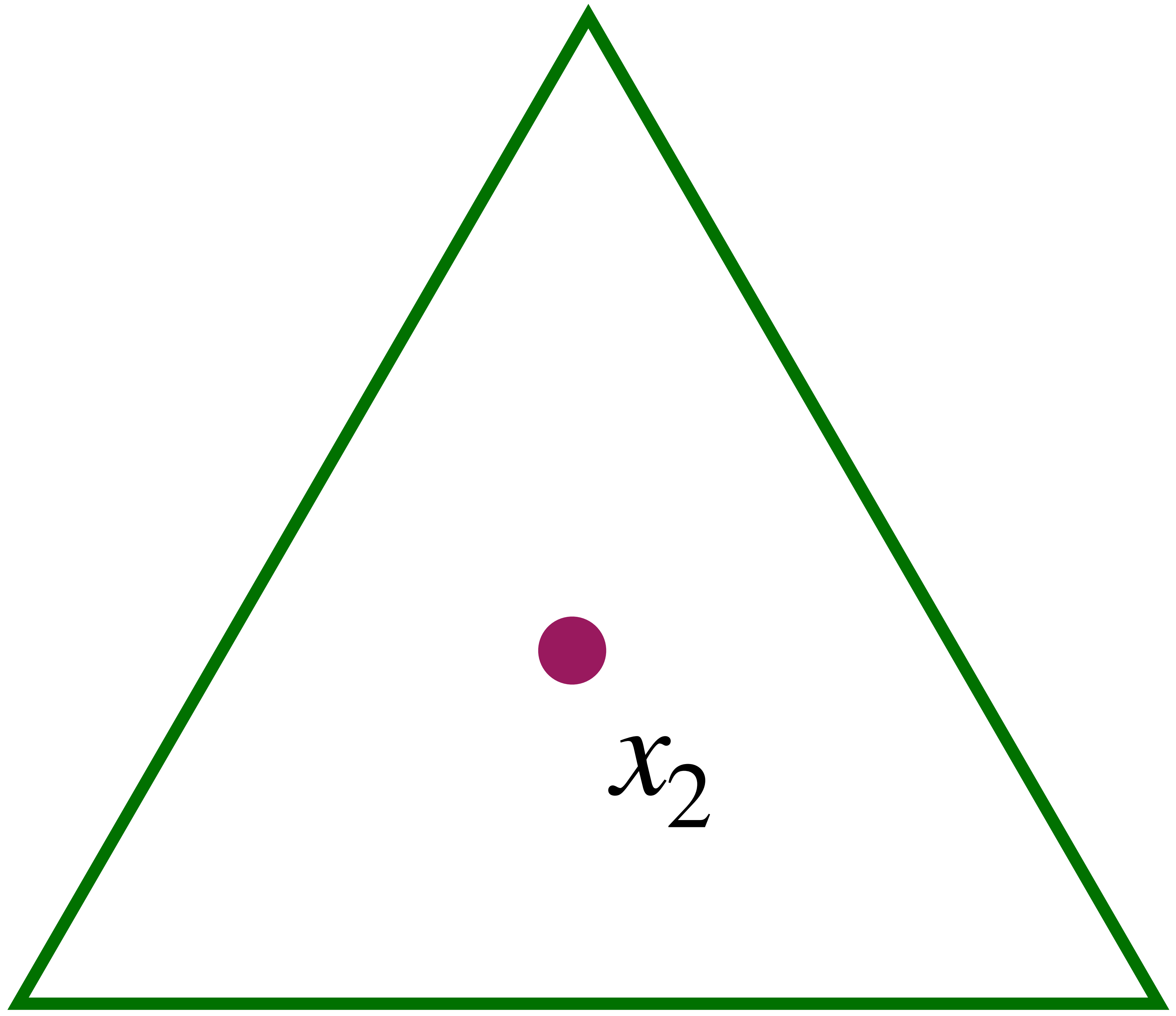
A single triangle case without shading normal

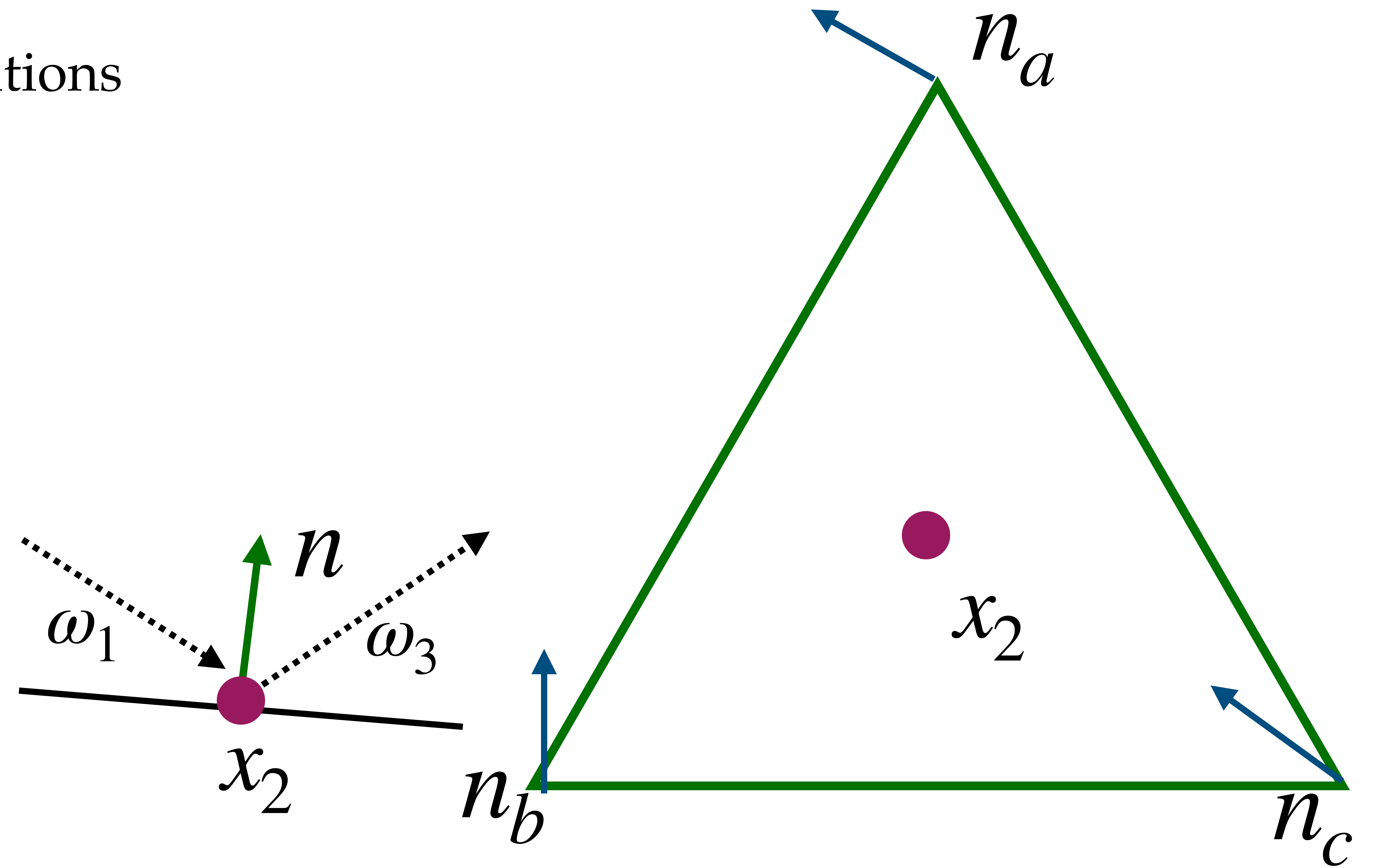
- n is fixed, find a point x_2 on the plane s.t. the constraint is satisfied
- unique solution exists



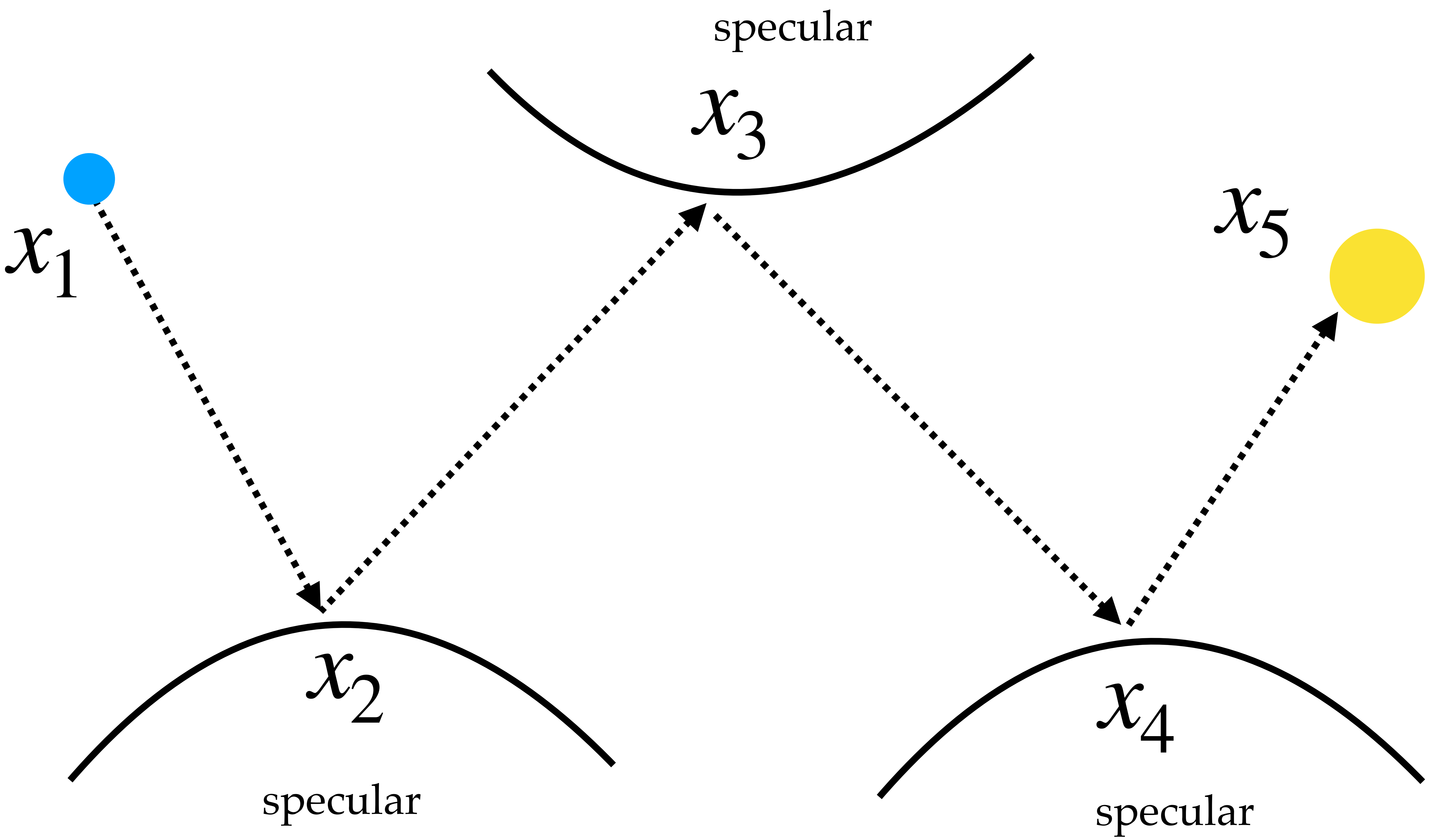


A single triangle case with shading normal

- n interpolates n_a, n_b, n_c based on the position of x_2
- may have zero, one, or multiple solutions



Easily generalizable to multiple specular surfaces



given x_1 and x_5
find x_2, x_3, x_4 s.t.

$$\frac{-\omega_1 + \omega_3}{\| -\omega_1 + \omega_3 \|} = n_2$$
$$\frac{-\omega_2 + \omega_4}{\| -\omega_2 + \omega_4 \|} = n_3$$
$$\frac{-\omega_3 + \omega_5}{\| -\omega_3 + \omega_5 \|} = n_4$$

$$C(x_2, x_3, x_4) = 0$$

Theory and Application of Specular Path Perturbation

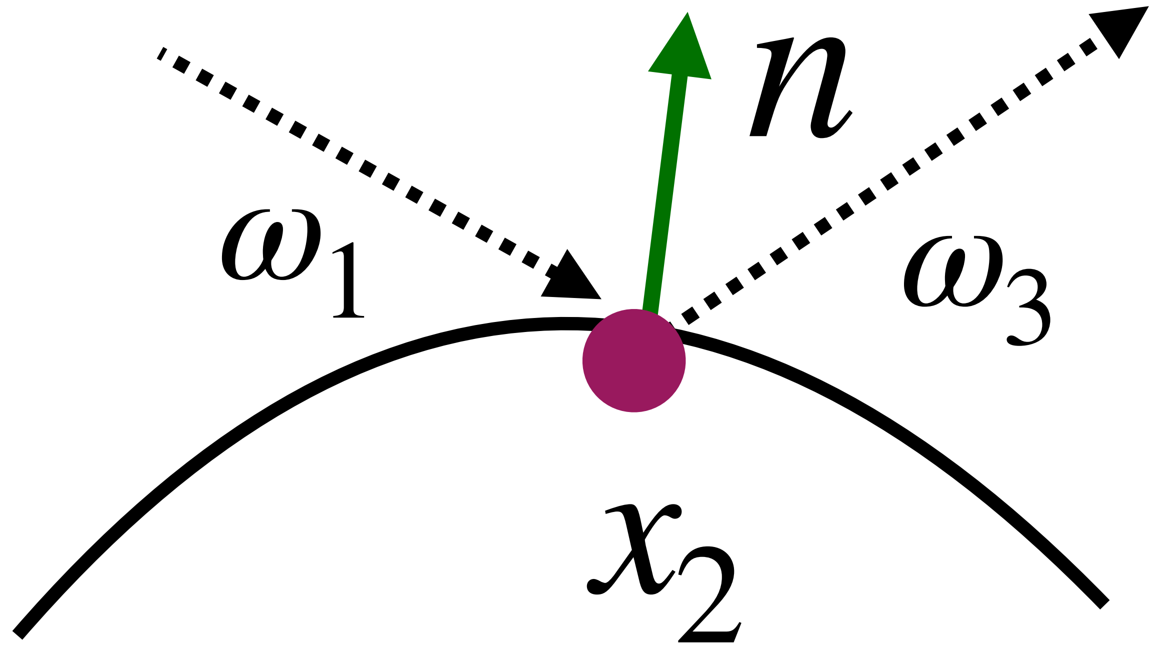
MIN CHEN and JAMES ARVO
California Institute of Technology

Challenge: incorporate Newton's method in a Monte Carlo renderer

- how do we generate initial guesses?
- how do we handle a large number of triangles?
- what is the probability density?

start from an initial guess x_2'
while $\| C(x_2') \| > \epsilon$:
  x_2' = x_2' - $J_C(x_2')^{-1} C(x_2')$
x_2 = x_2'

$$p(x_2) = ?$$

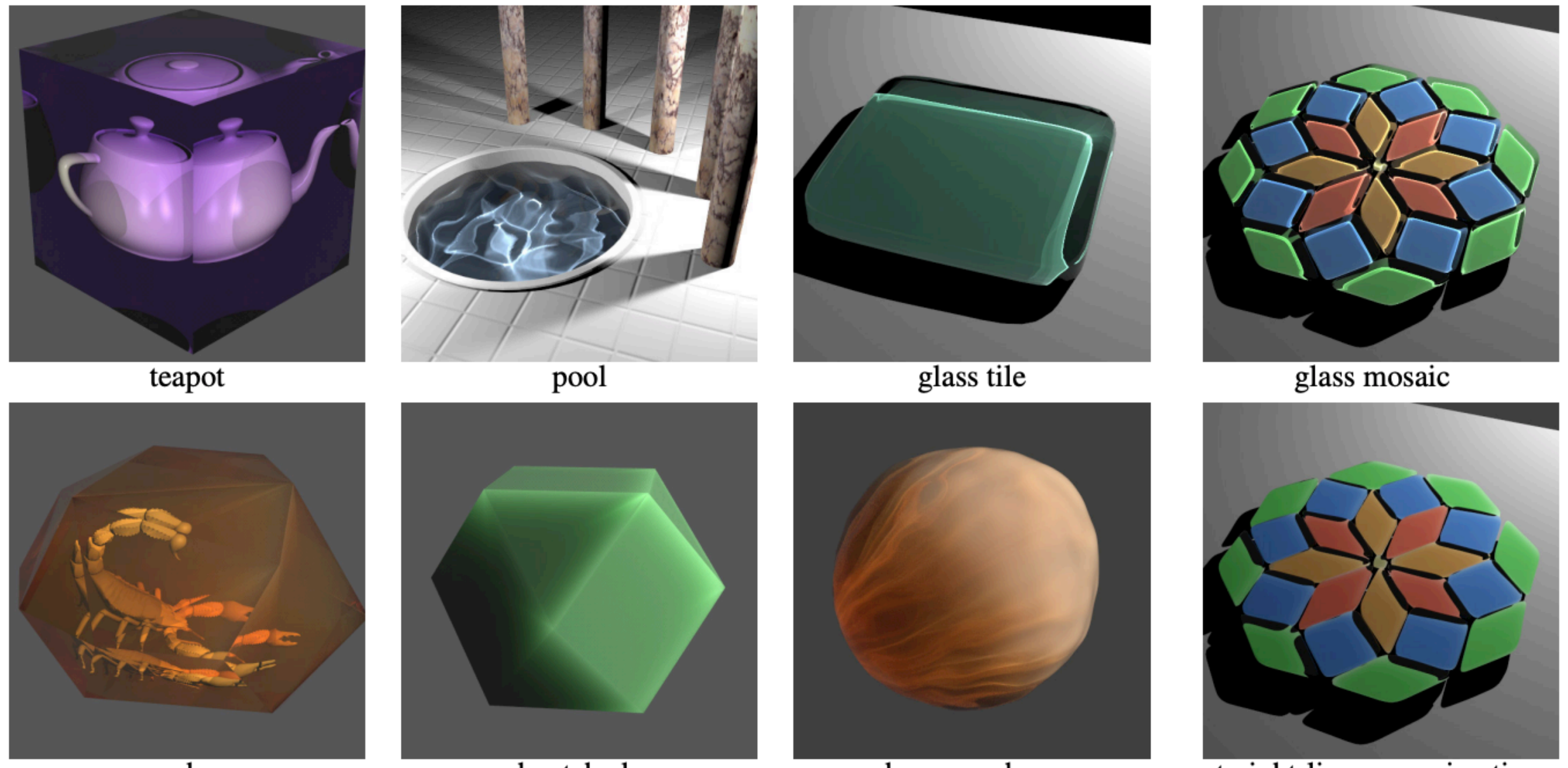


Three strategies to incorporate Newton's method into a renderer

use new data structure to enumerate roots

Single Scattering in Refractive Media with Triangle Mesh Boundaries

Bruce Walter, *Cornell University*
Shuang Zhao, *Cornell University*
Nicolas Holzschuch, *INRIA – LJK*
Kavita Bala, *Cornell University*



teapot | pool | glass tile | glass mosaic
amber | cuboctahedron | bumpy sphere | straight-line approximation

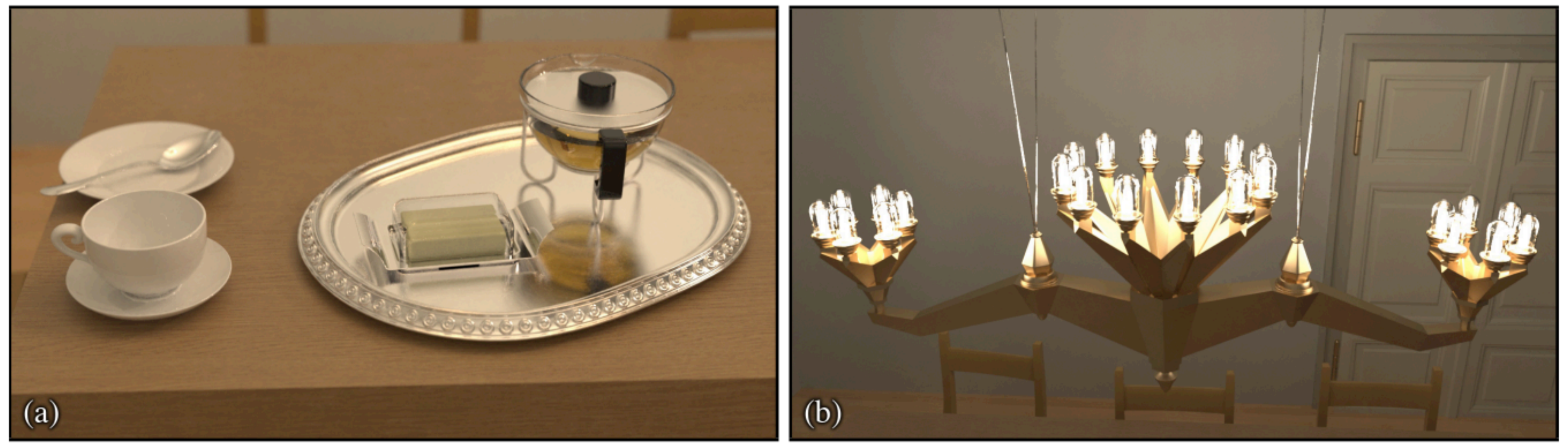
Path Cuts: Efficient Rendering of Pure Specular Light Transport

BEIBEI WANG, School of Computer Science and Engineering, Nanjing University of Science and Technology
MILOŠ HAŠAN, Adobe Research
LING-QI YAN, University of California, Santa Barbara

Our Result (TT + environment lighting) | Our Result (TT), 7.36s, Glint count: 159

Manifold Exploration: A Markov Chain Monte Carlo technique for rendering scenes with difficult specular transport

Wenzel Jakob, Steve Marschner
Cornell University



(a) (b)

Metropolis light transport

Specular Manifold Sampling for Rendering High-Frequency Caustics and Glints

TIZIAN ZELTNER, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland
ILIYAN GEORGIEV, Autodesk, United Kingdom
WENZEL JAKOB, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland

randomized initialization using Monte Carlo sampling

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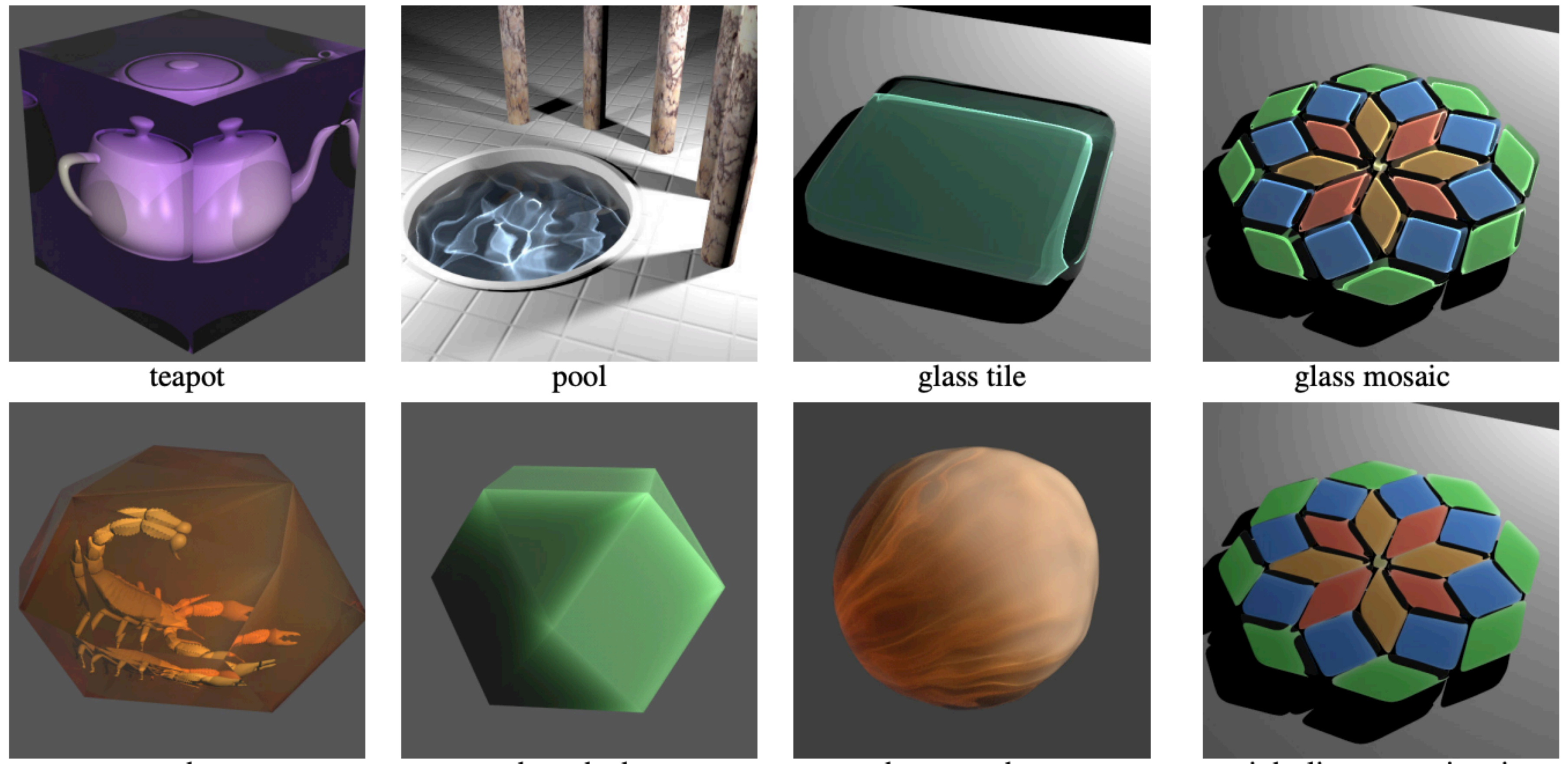

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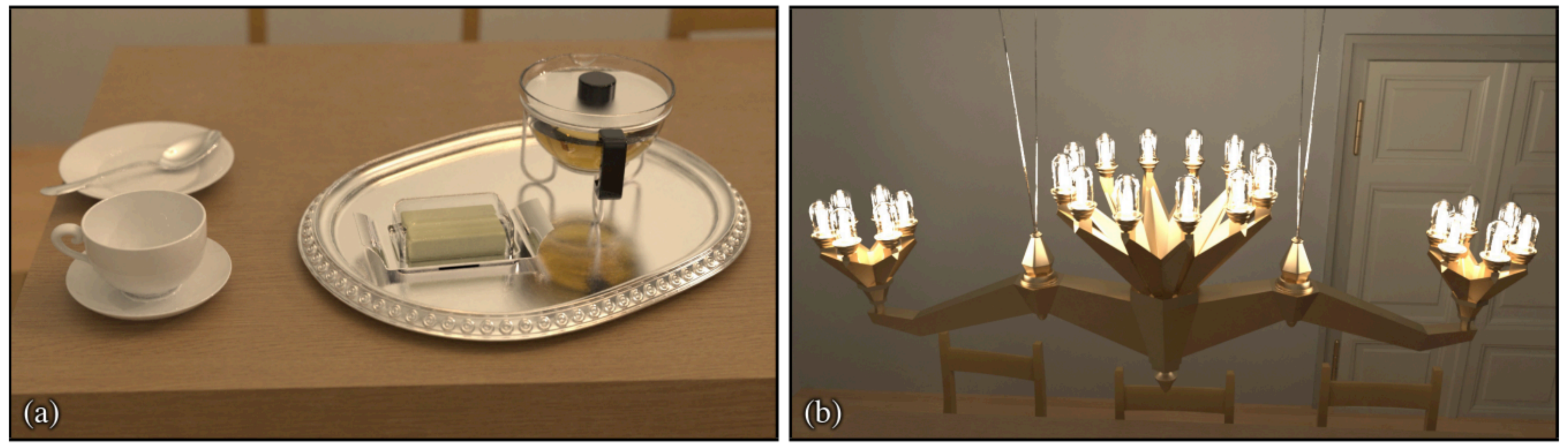

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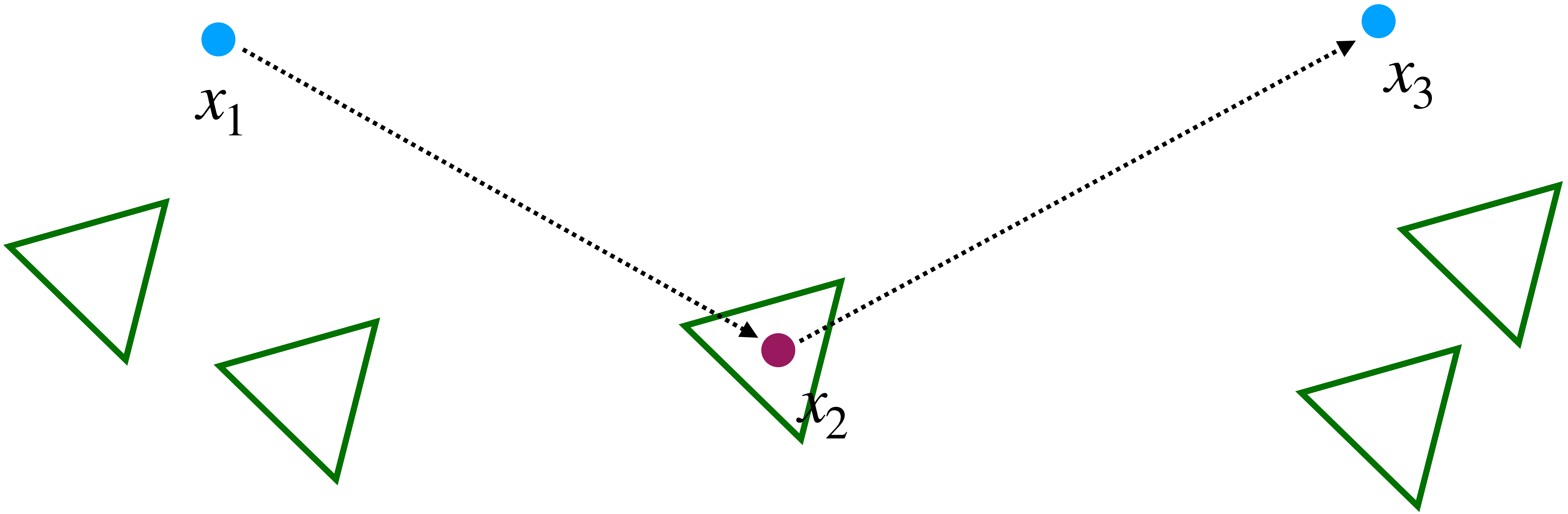
randomized initialization using Monte Carlo sampling

Idea: enumerate roots using a data structure

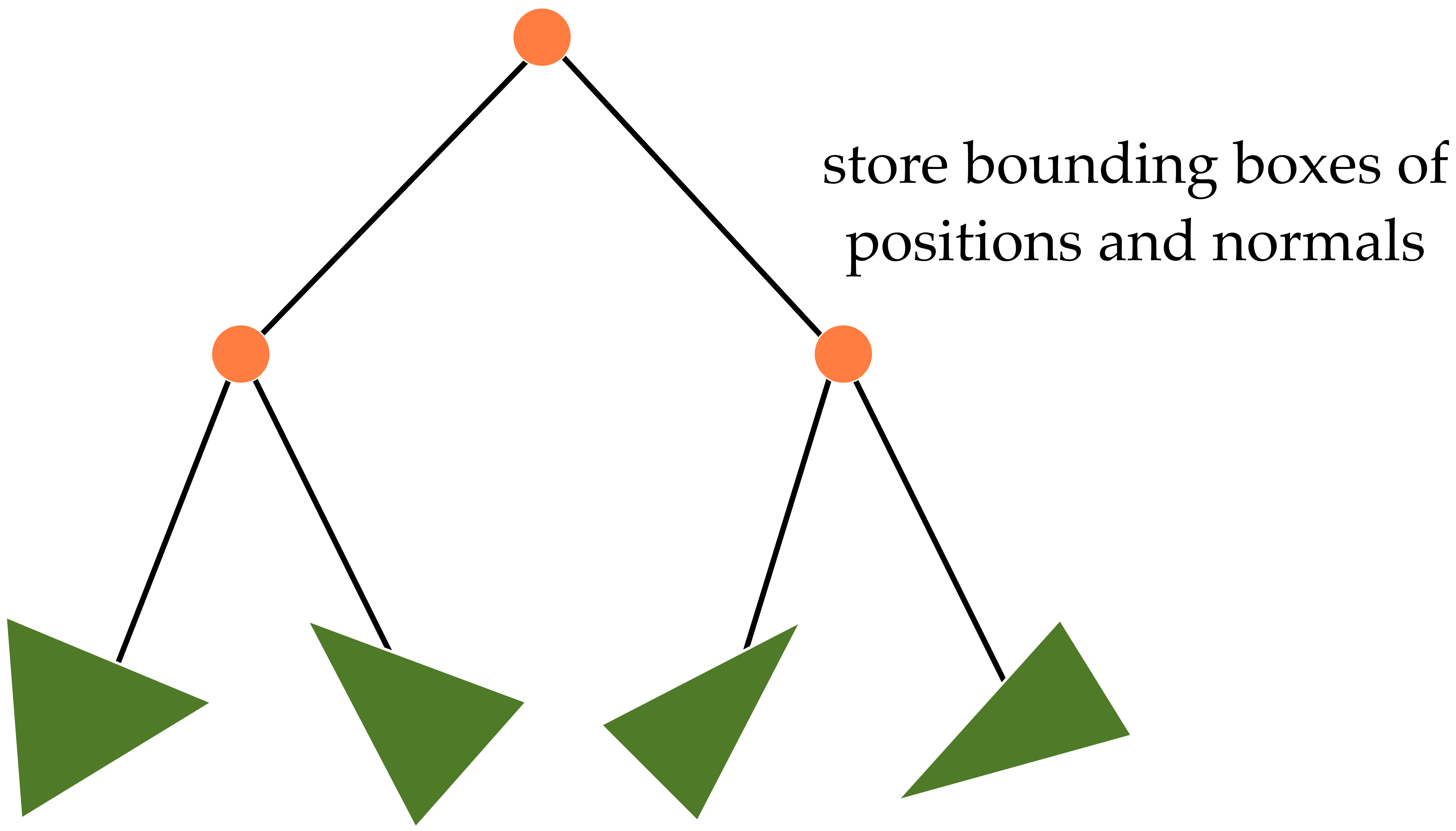
- observation: most triangles contain no solution given x_1 and x_3



note: there are only countably many roots if the scene is made of triangle meshes

Hierarchical pruning using a 6D tree

- skip the whole subtree if it is impossible that half-vector would be the same as the normal



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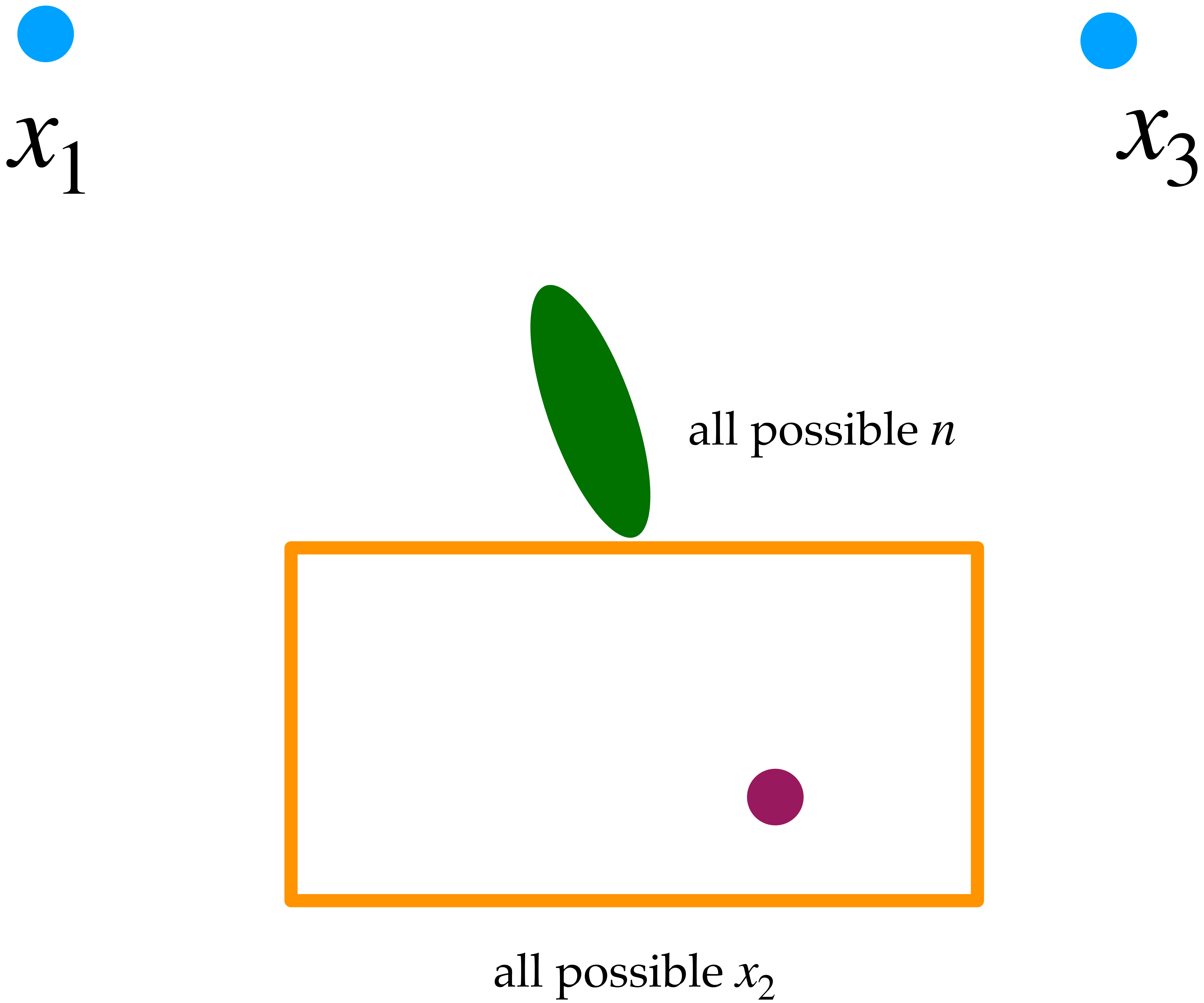
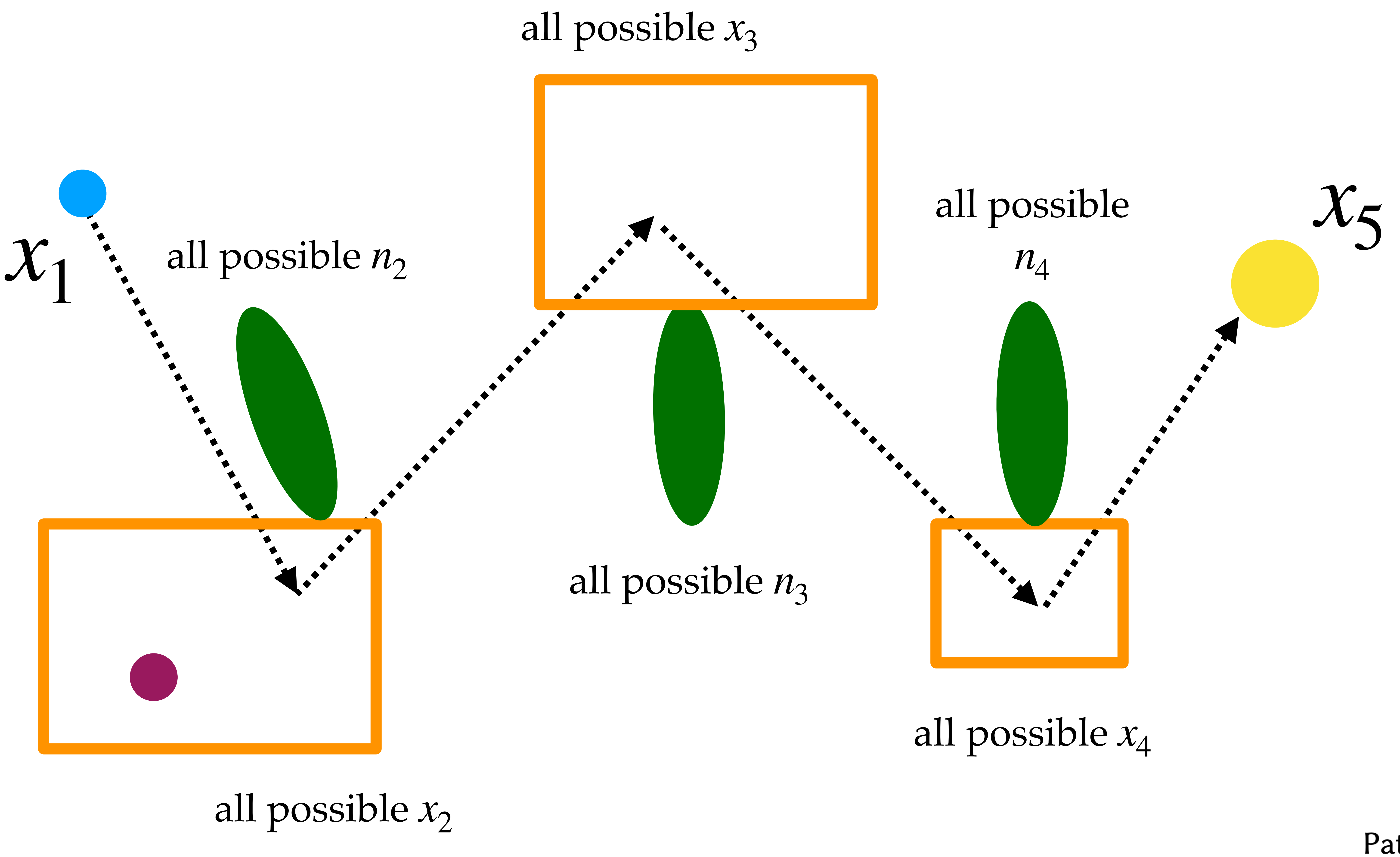


Hierarchical pruning generalizes to multiple bounces



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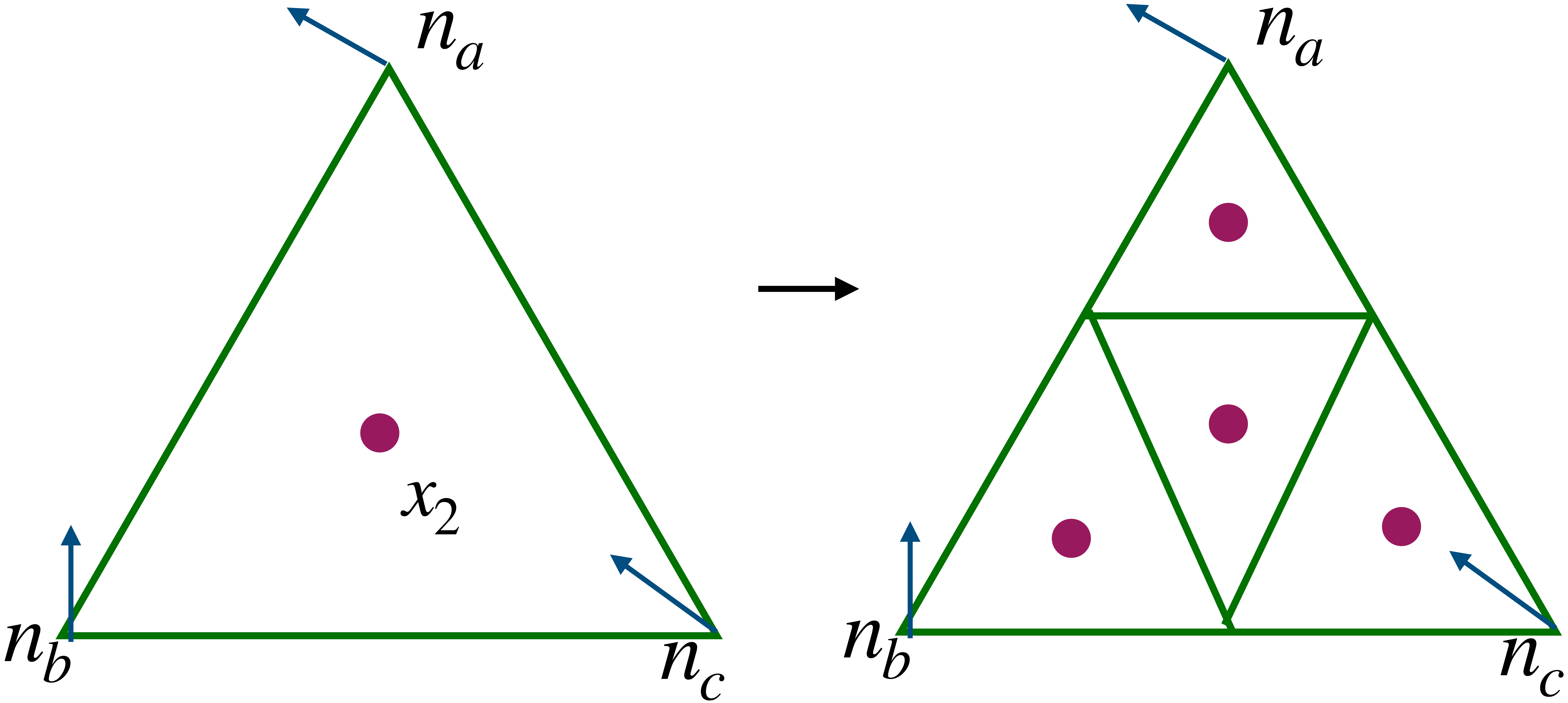
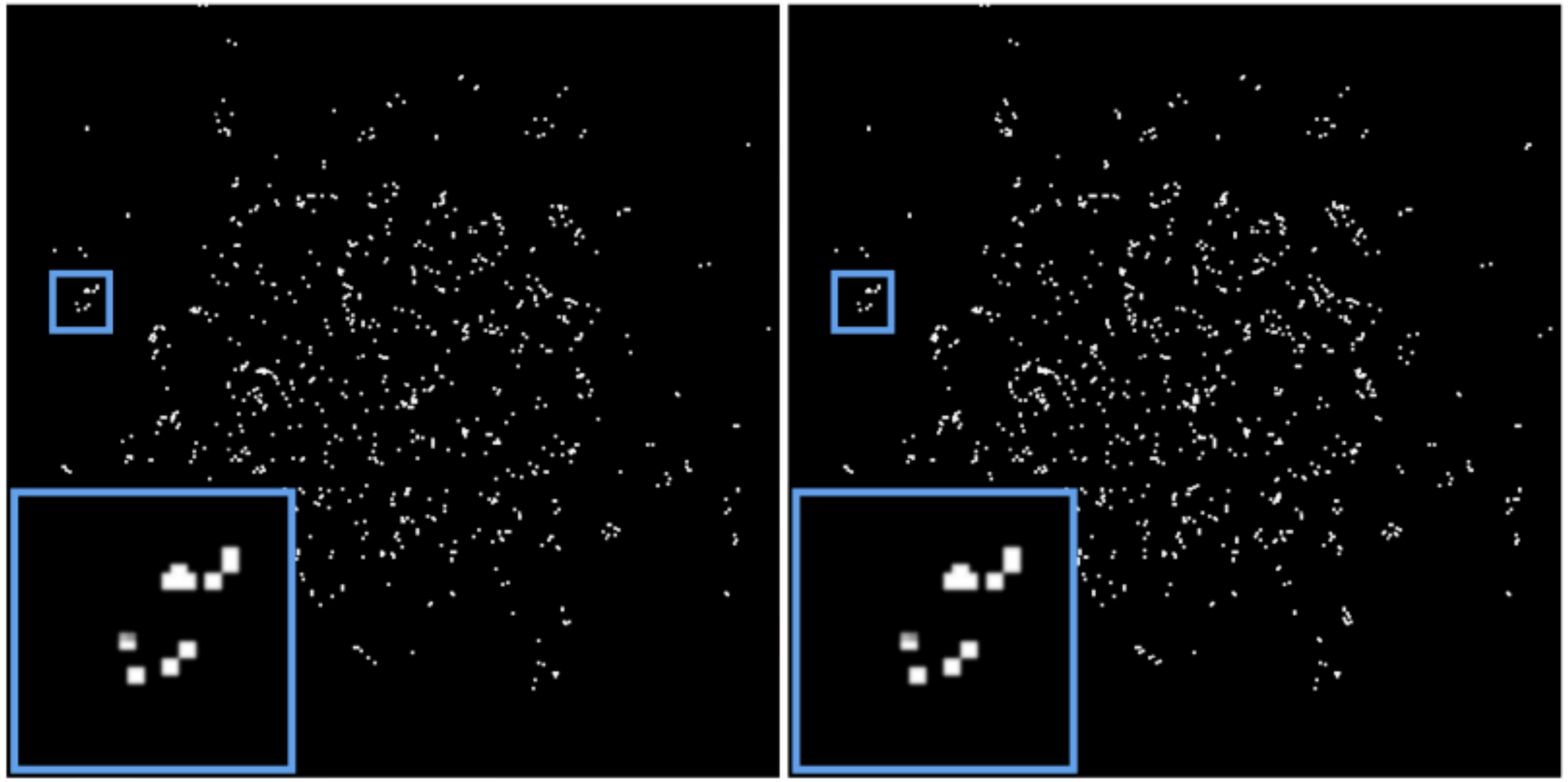
Optional: subdivide triangles with shading normals for more accurate results

- can subdivide until the constraint is provably convex



In practice, triangle subdivision is usually not worth it

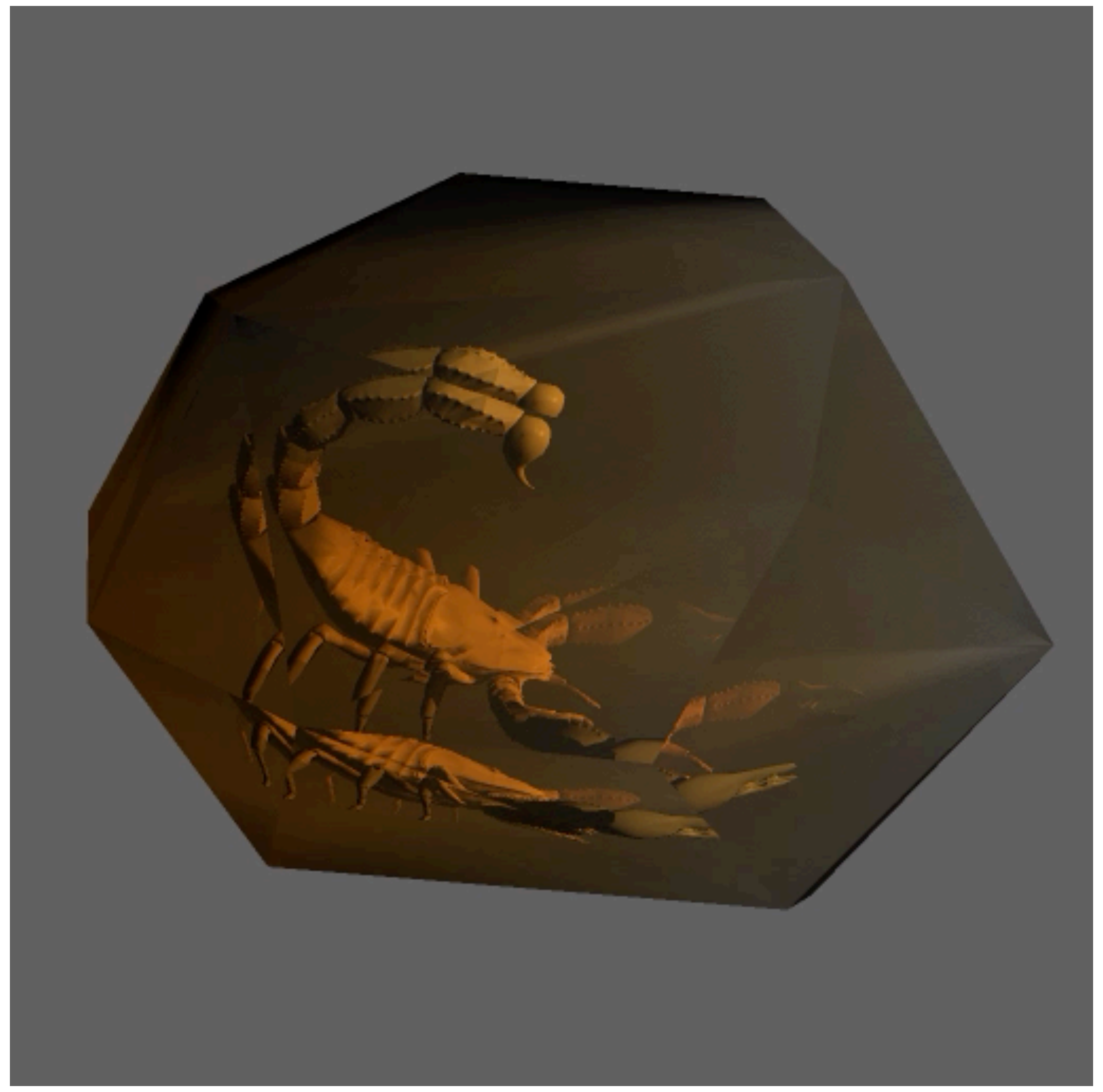
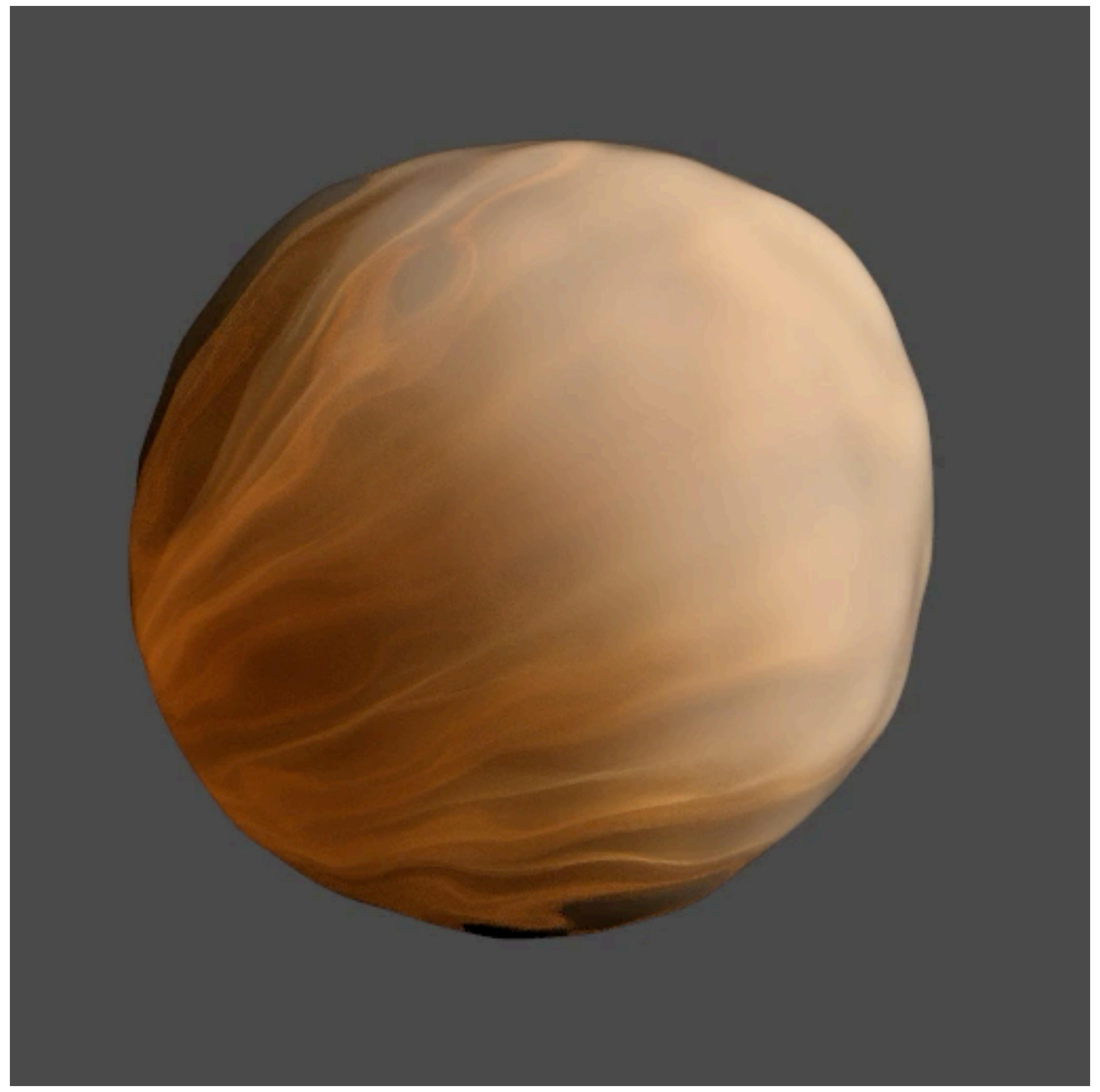
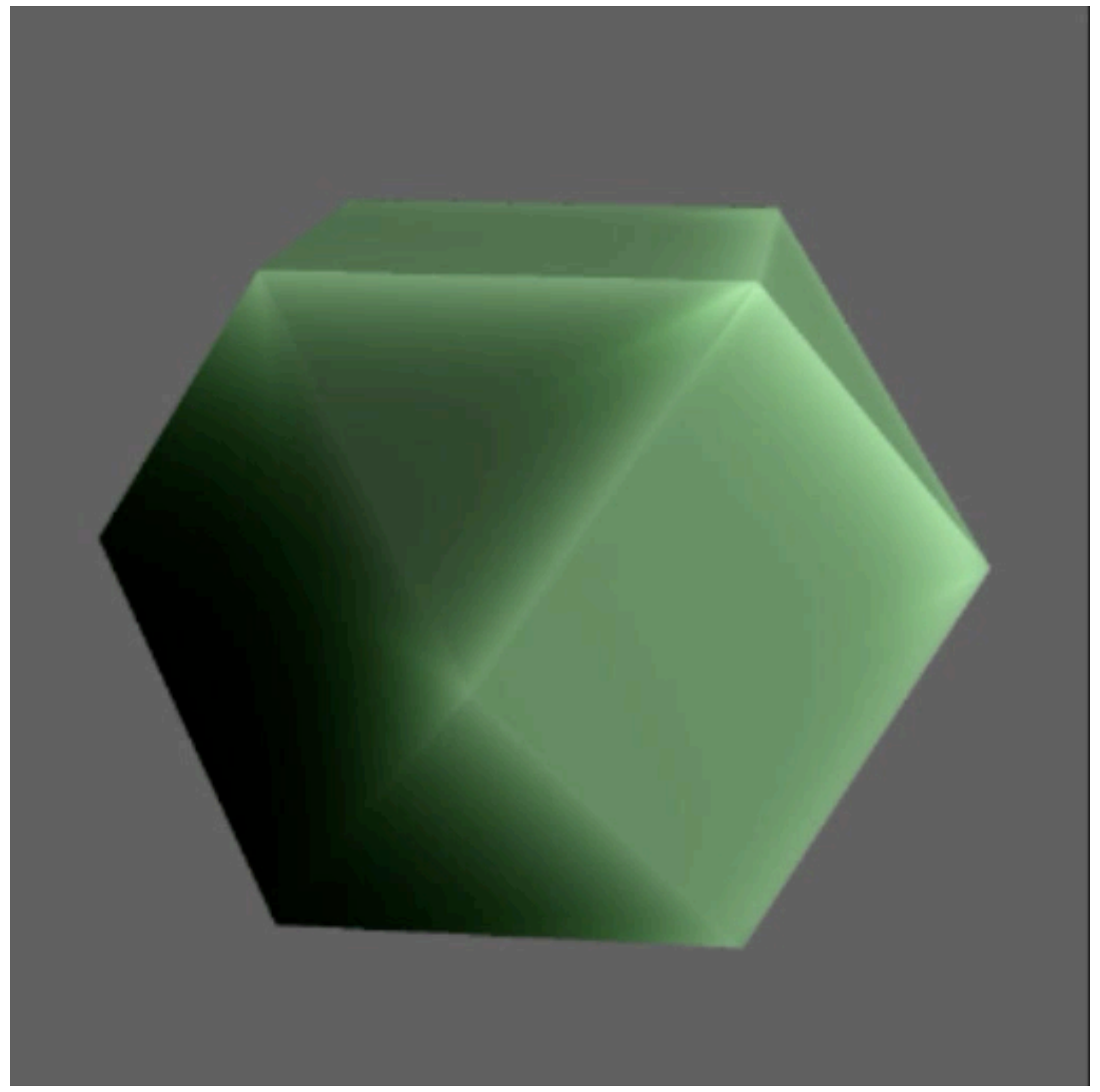
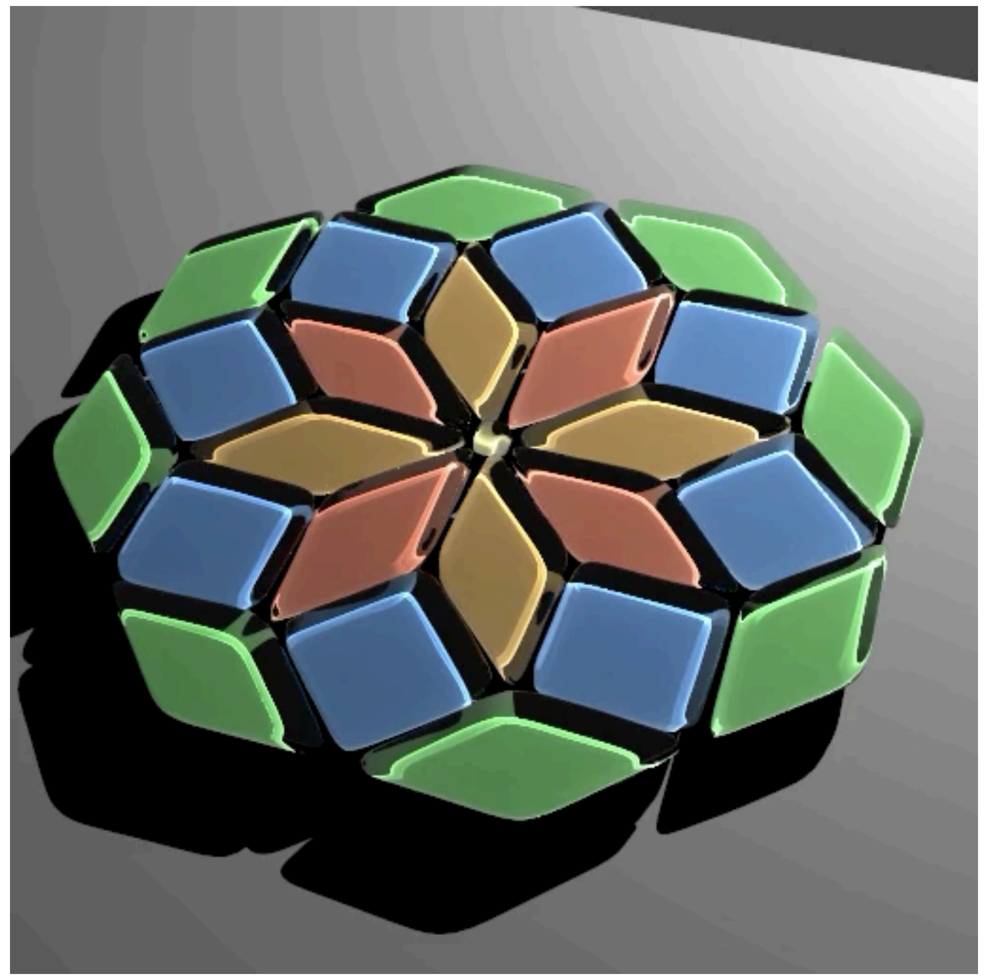
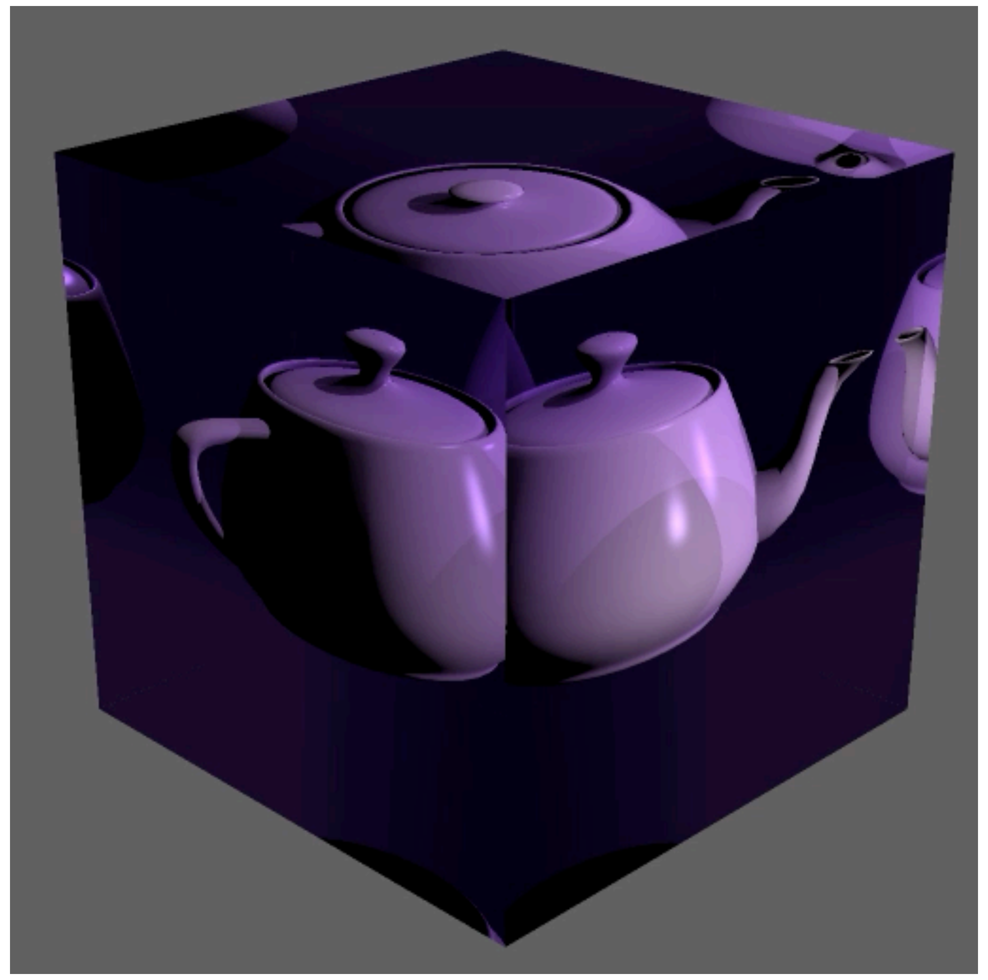
- subdivision can find a few more paths, but usually gives visually similar results



w/o Interval Netwon (TT)
Glint count: 1036,
Time: 0.62s

w/ Interval Netwon (TT)
Glint count: 1052,
Time: 23h

Fancy animations

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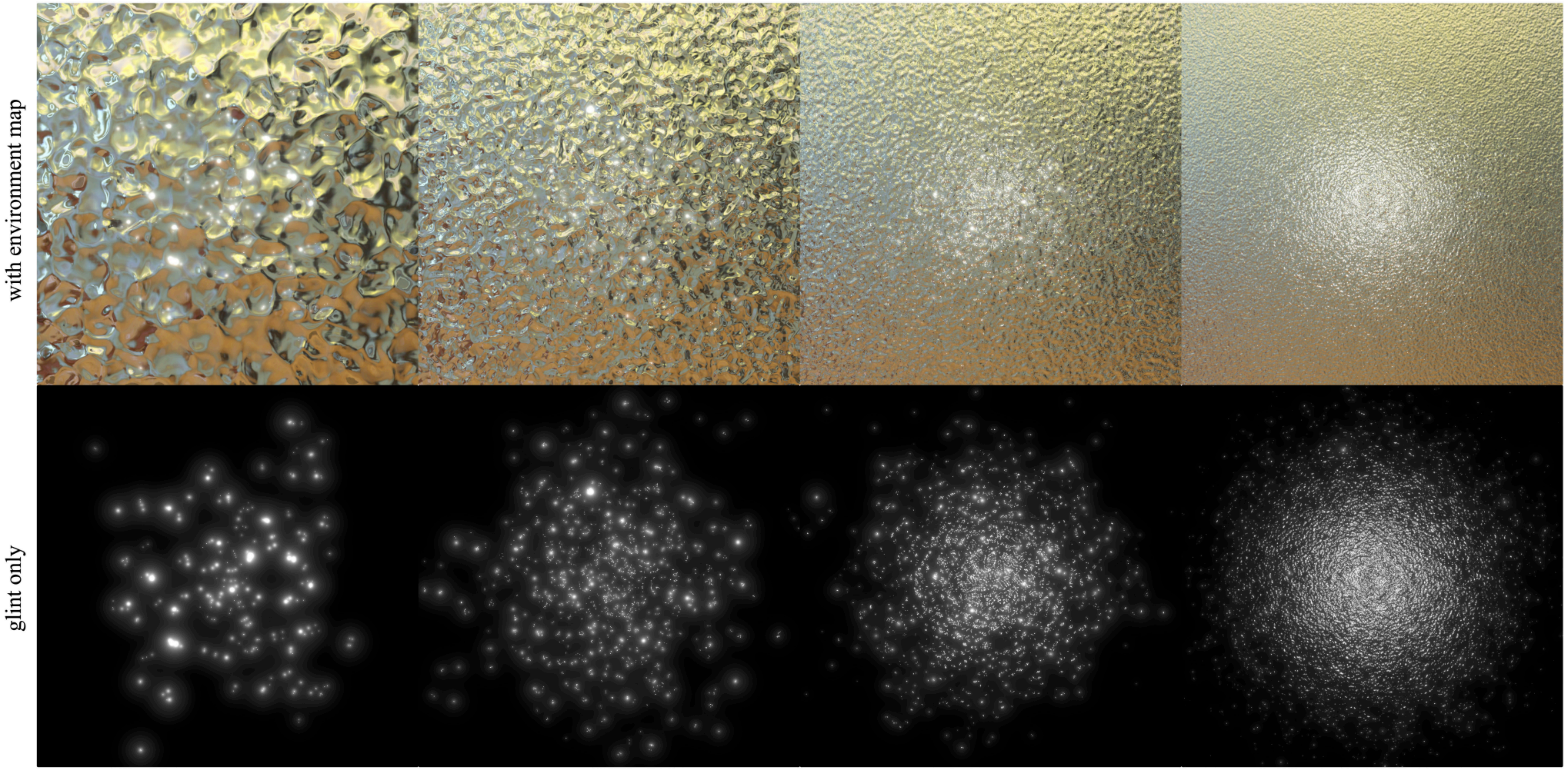
Fancy images



Triangle Count: 64516
Time: 0.49s, Glint Count: 284

Triangle Count: 260100
Time:0.72s, Glint Count: 2358

Triangle Count: 1044484
Time: 1.42s, Glint Count: 16323

Triangle Count: 4186116
Time: 10.69s, Glint Count: 305693

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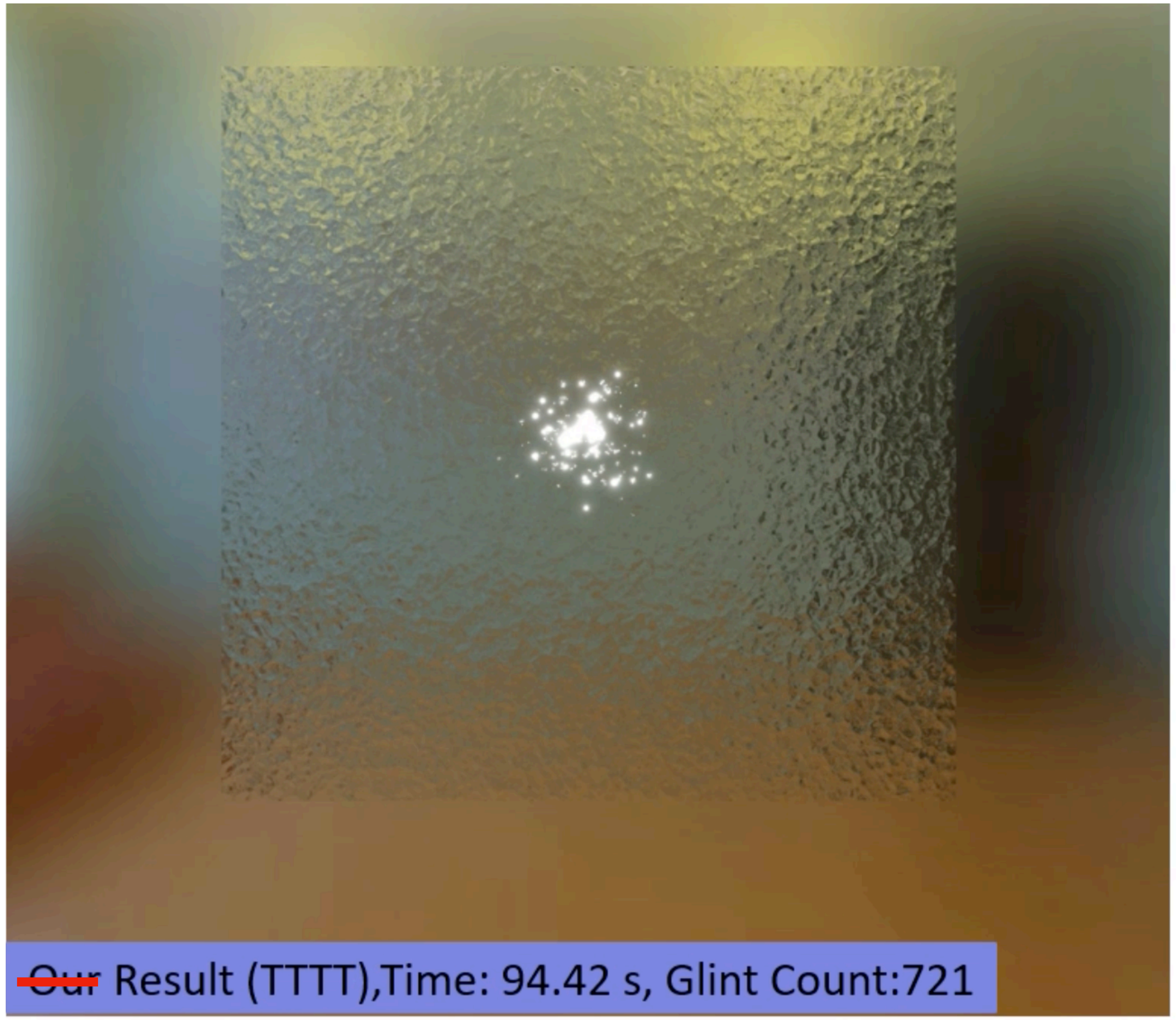

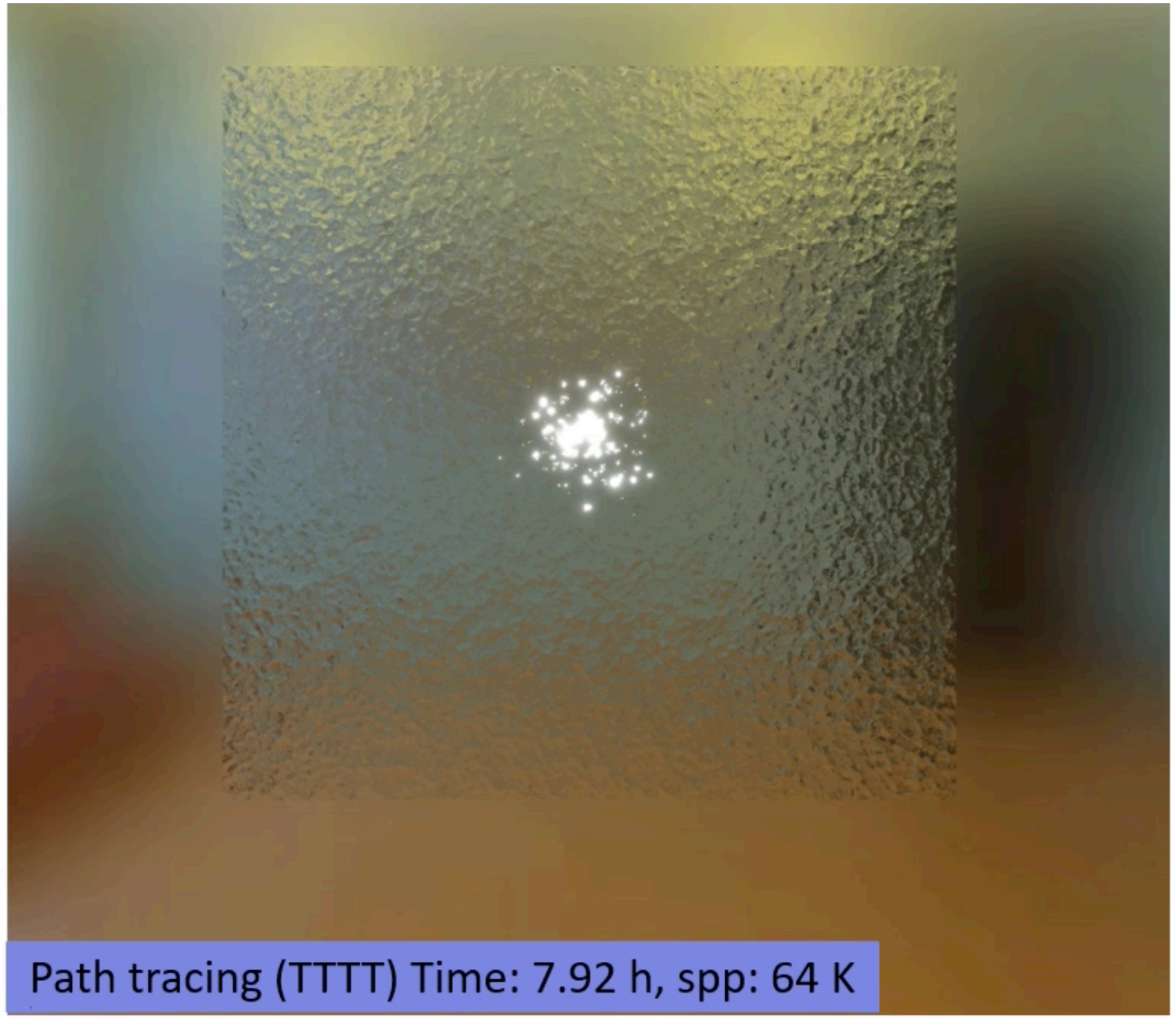

Wang et al.

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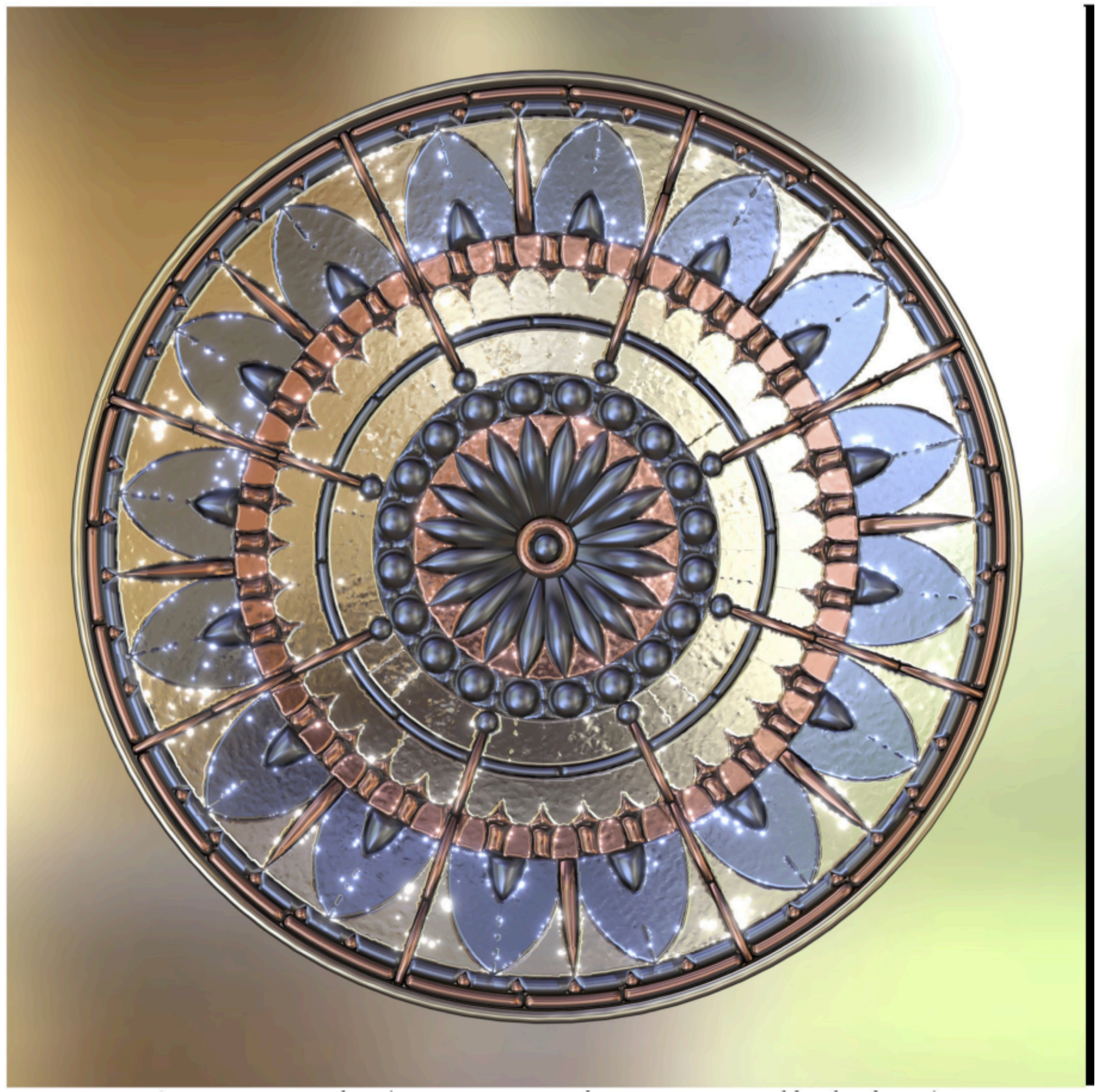

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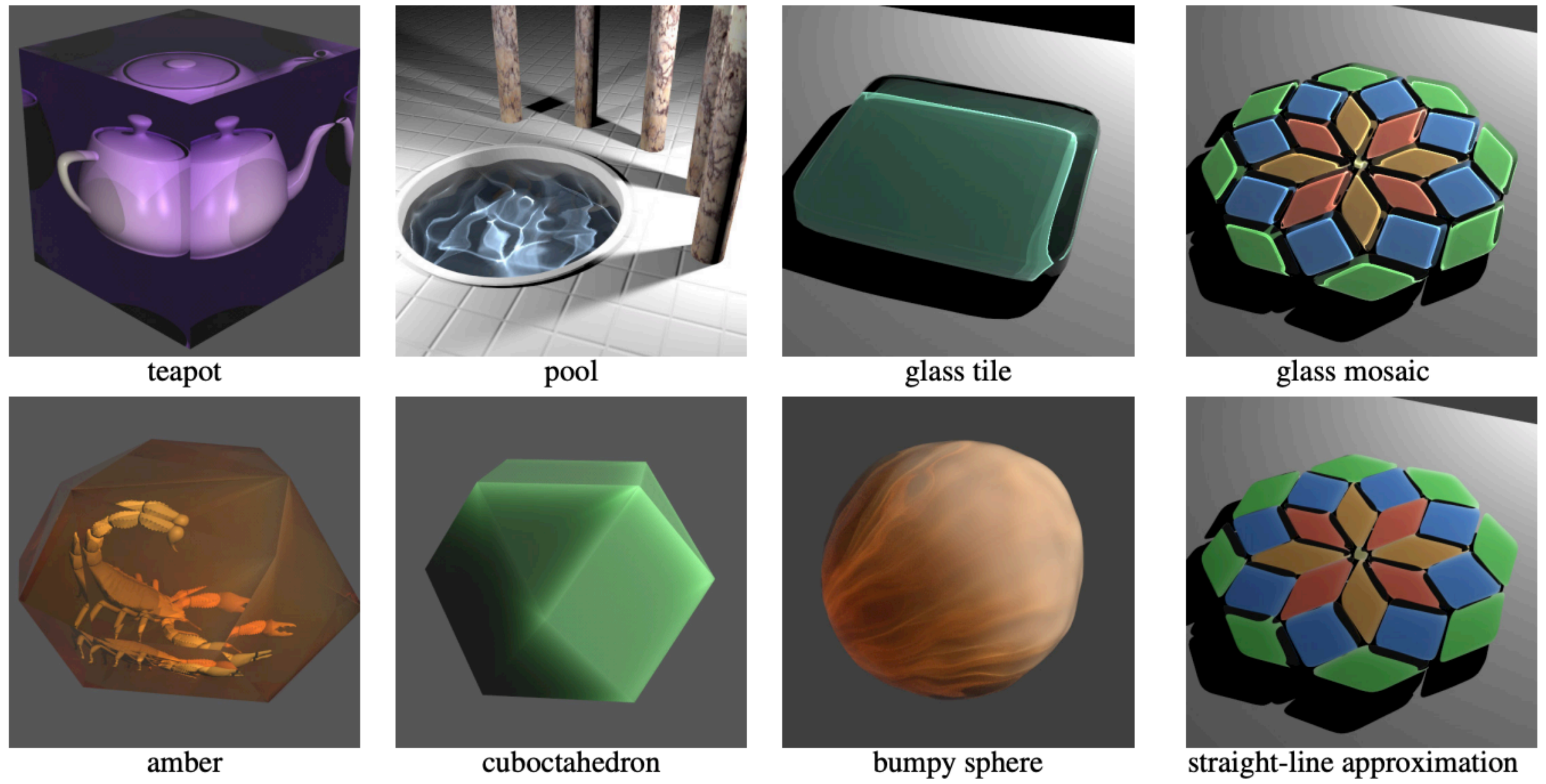

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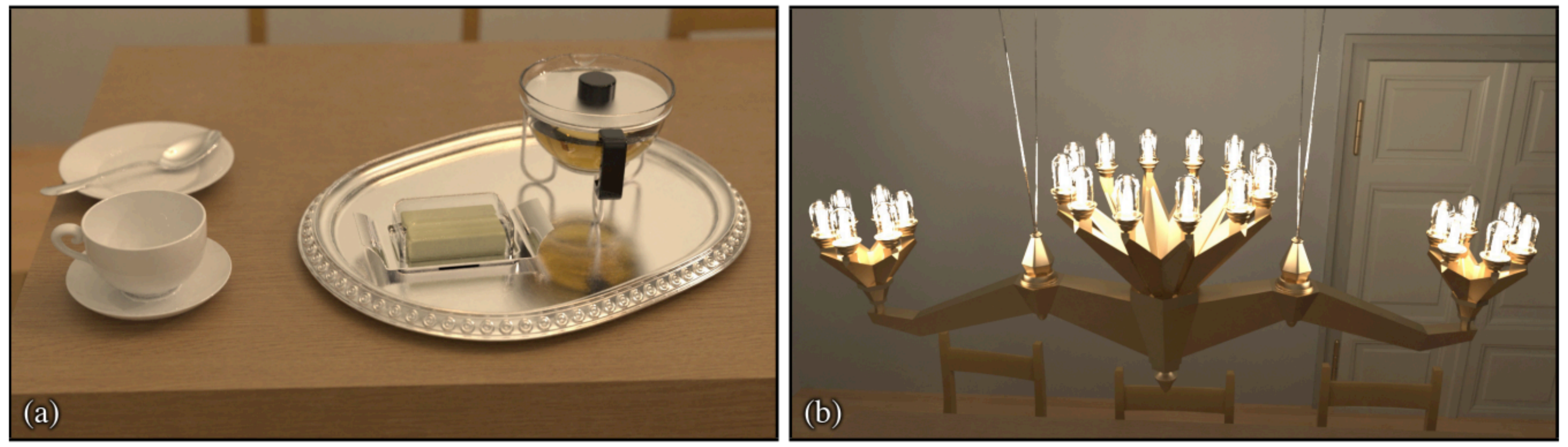

Metropolis light transport

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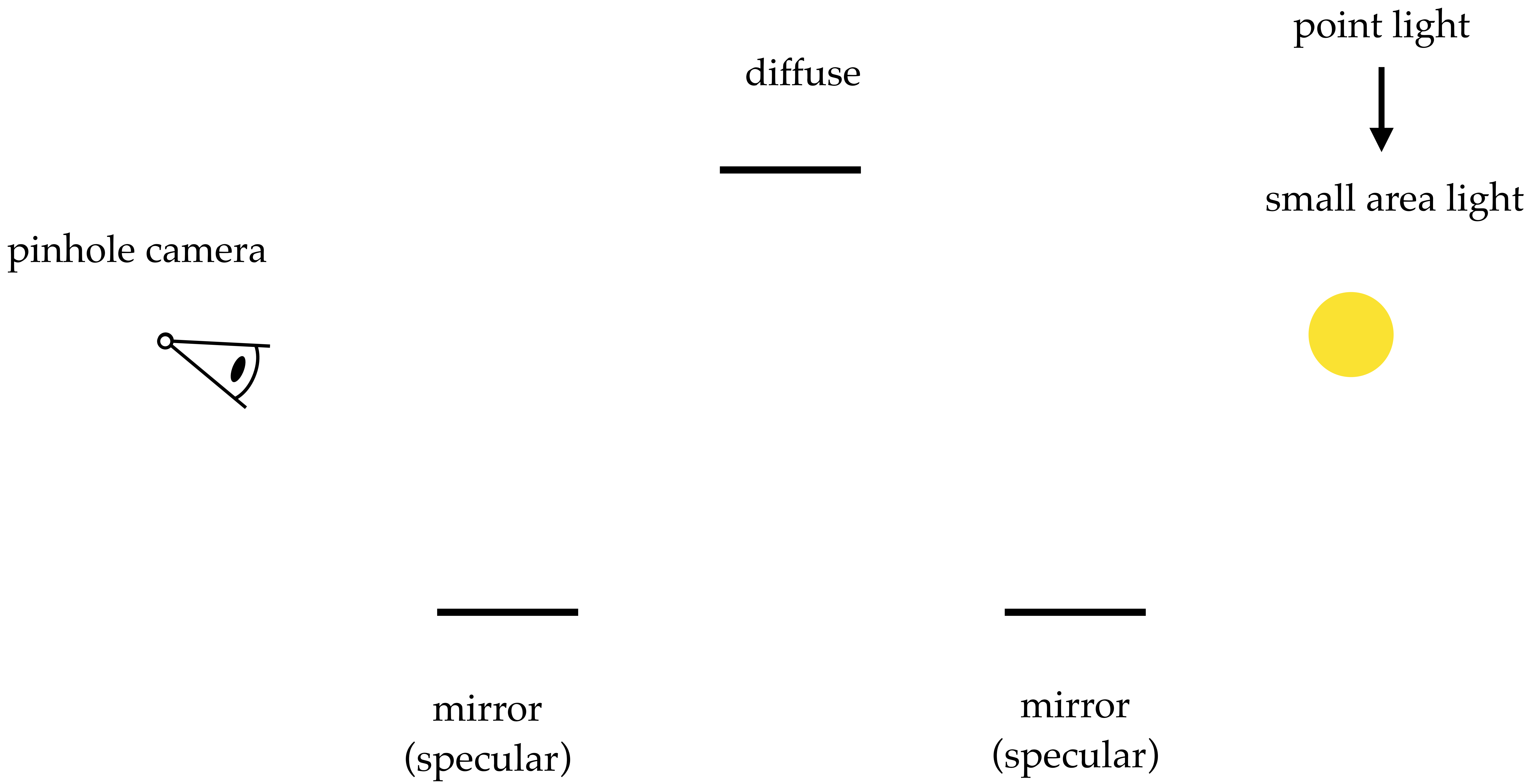
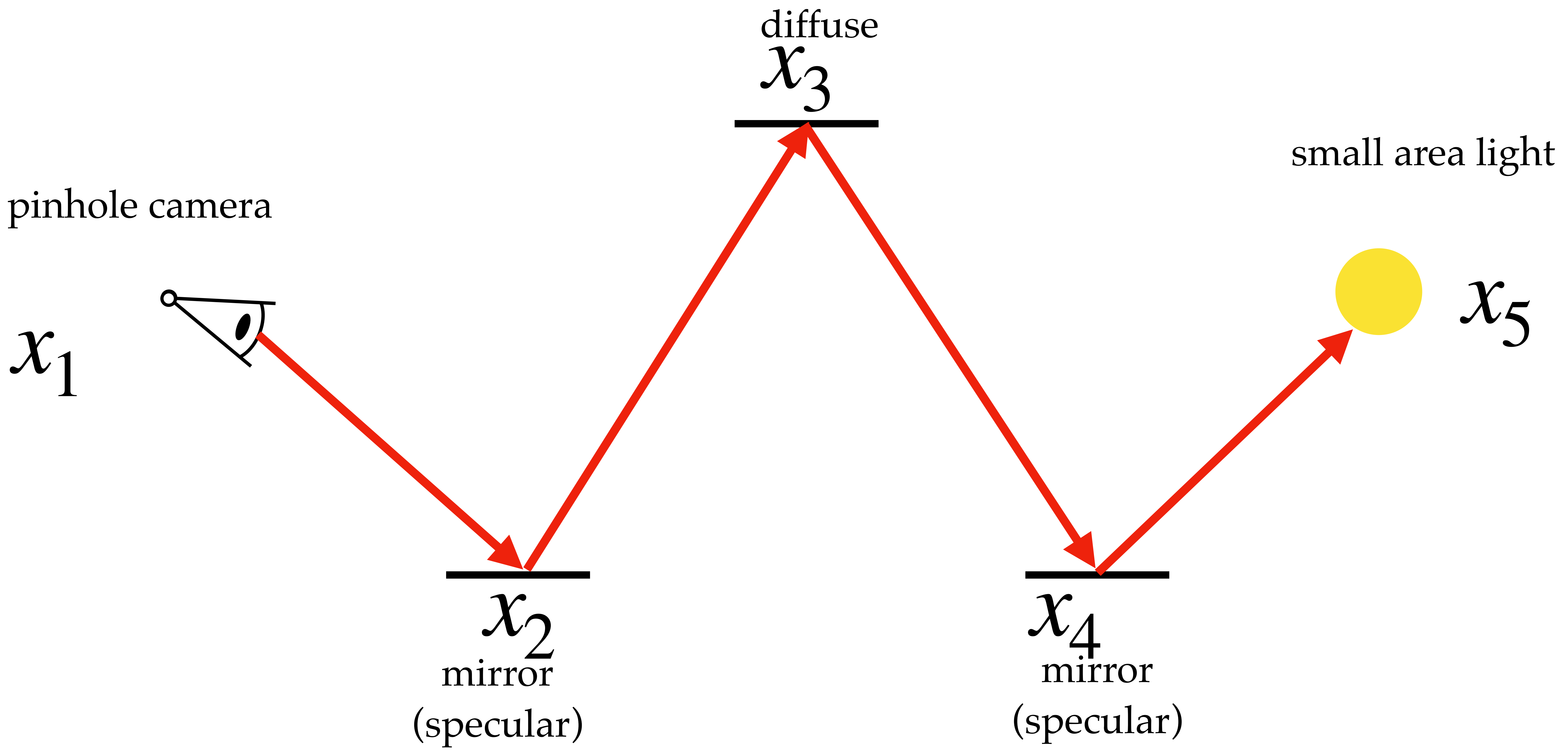
randomized initialization using Monte Carlo sampling

Let’s solve a slightly relaxed problem



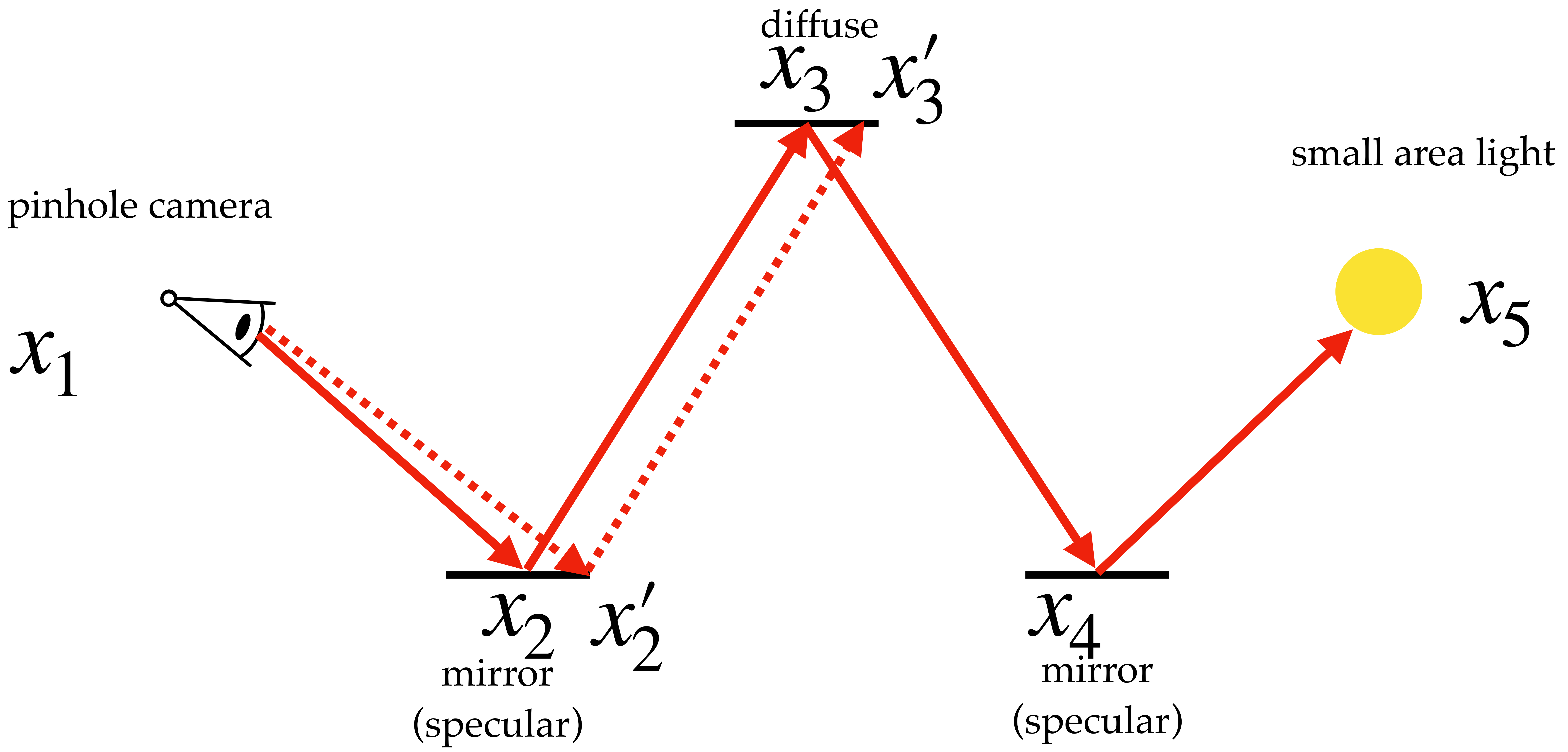
Manifold exploration with Metropolis sampling

- use bidirectional path tracing to find an initial path



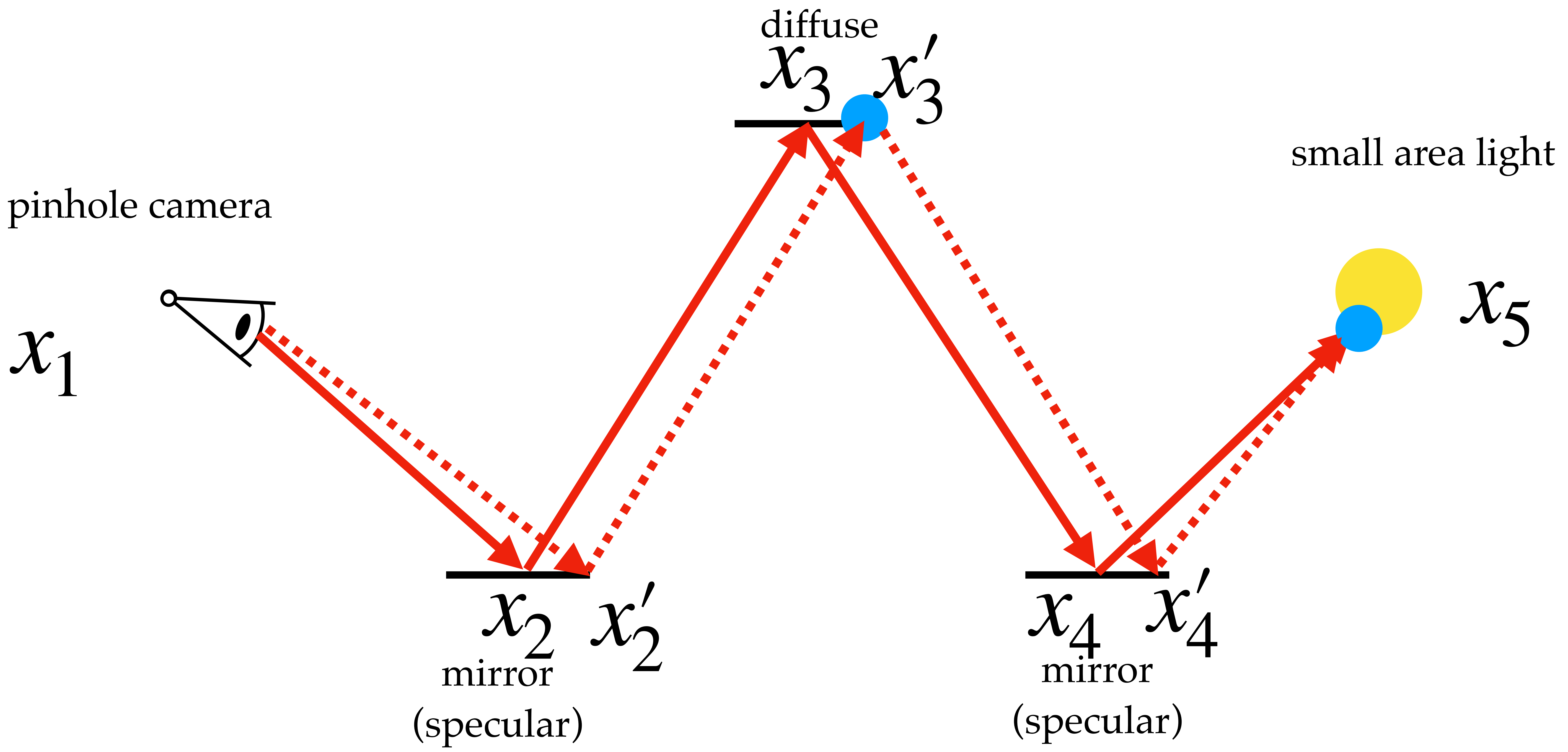
Manifold exploration with Metropolis sampling

- mutate the camera subpath until a diffuse hit



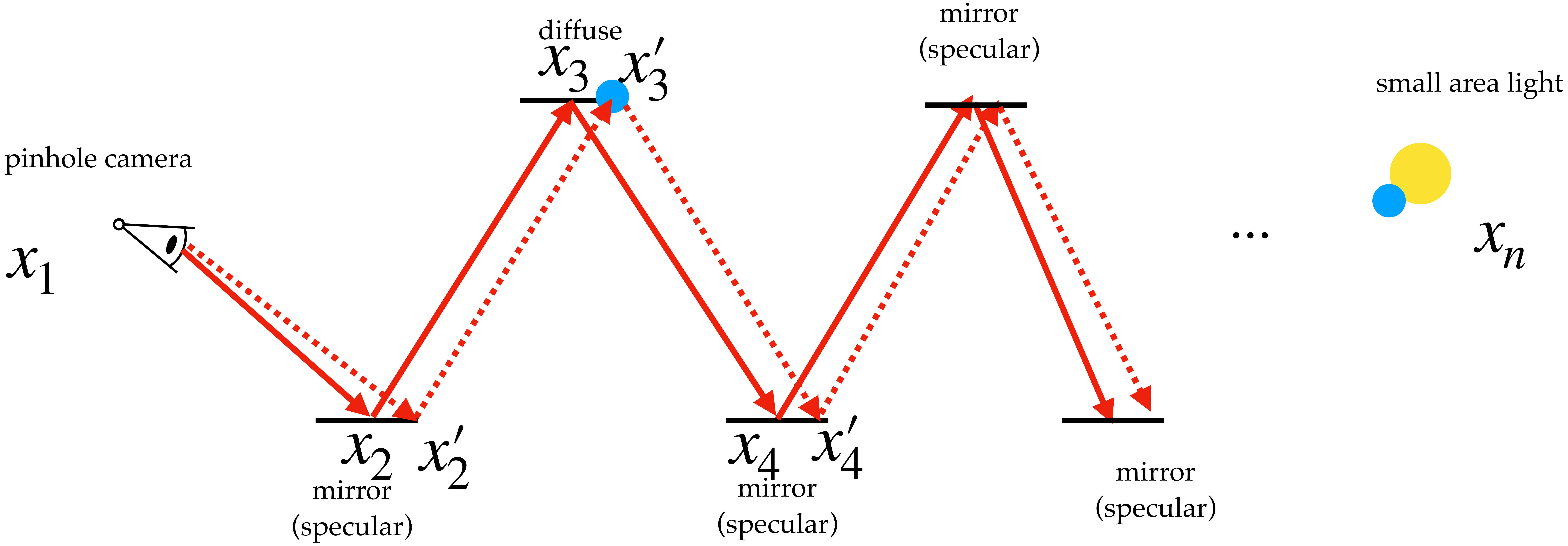
Manifold exploration with Metropolis sampling

- given x_3' and x_5, perturb x_4 using Newton's method to satisfy the constraint



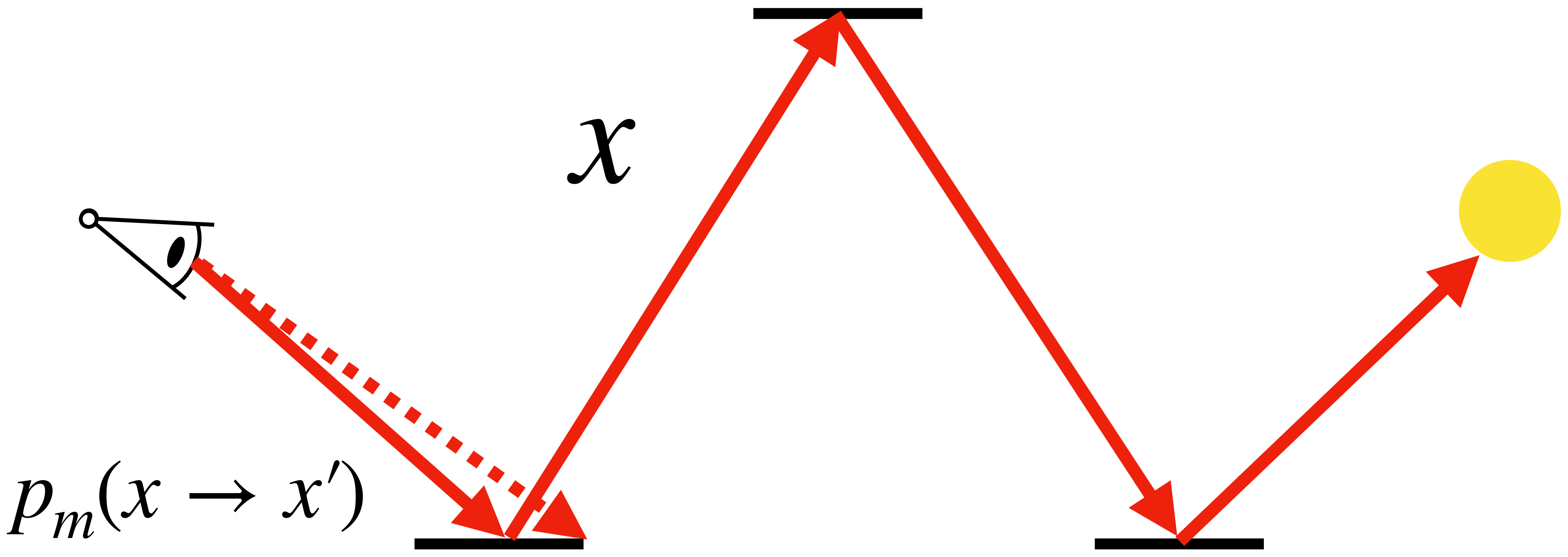
Manifold exploration with Metropolis sampling

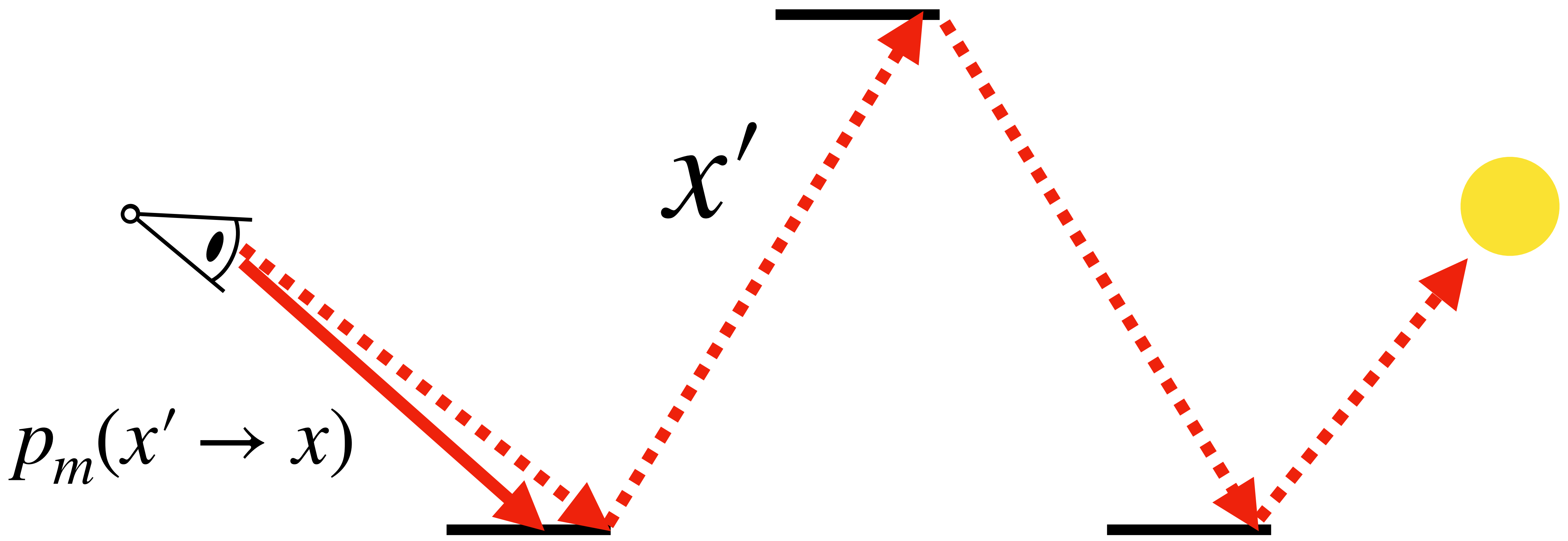
- works for arbitrary number of specular vertices



Satisfying detailed balance

- in Metropolis, we only need to compute the ratio of PDFs, making PDF calculation much easier

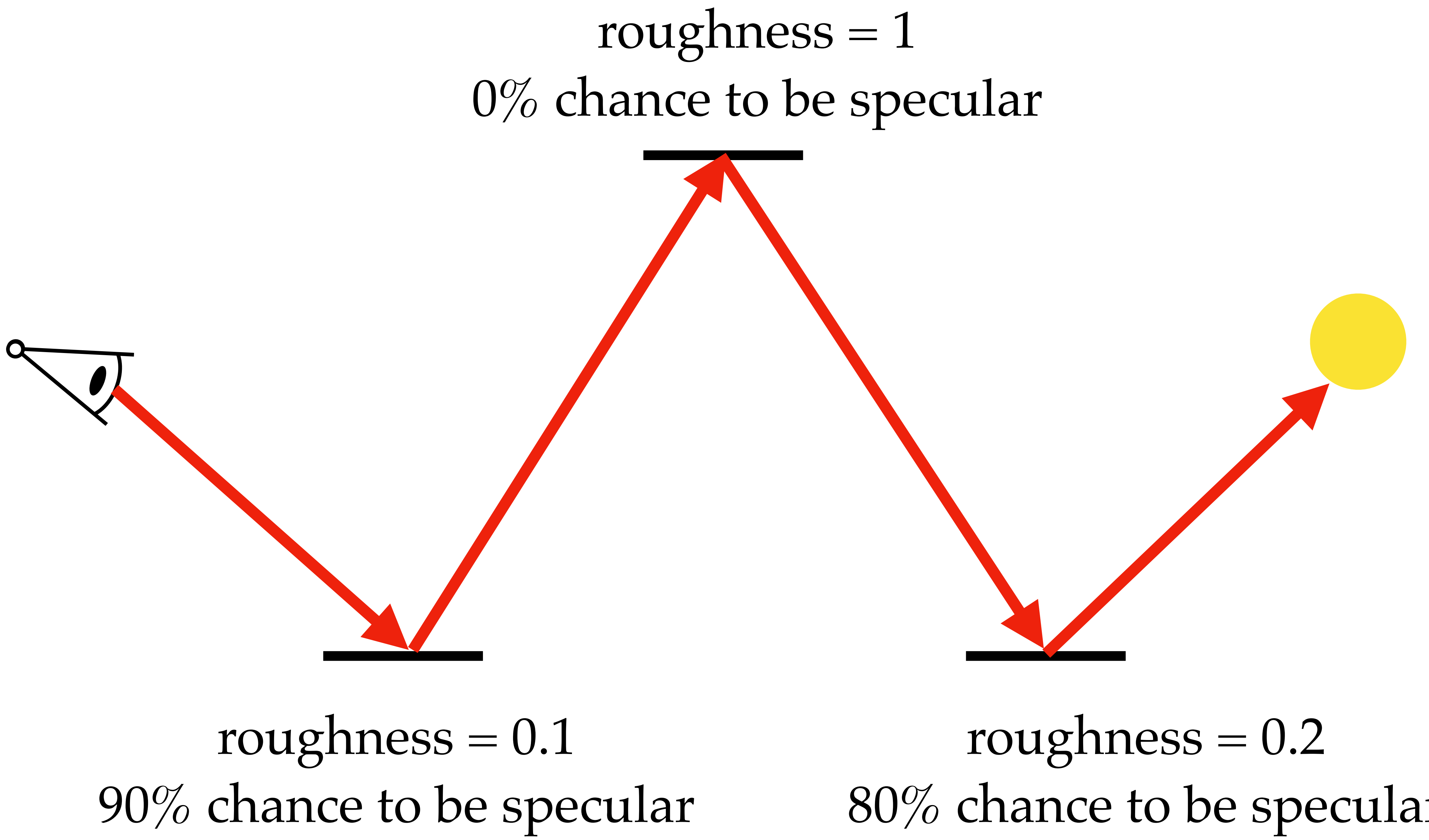


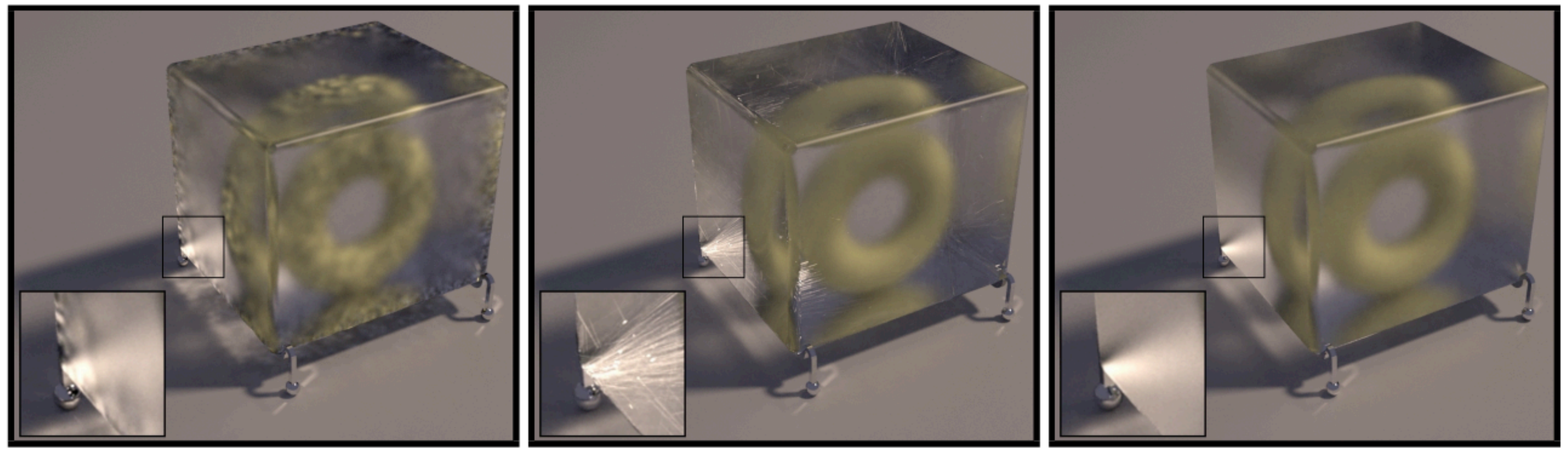



```
x = x0
for i in range(n):
  x' = mutate(x)
  a = min((f(x')/f(x)) *
          (p_m(x'->x)/p_m(x->x')), 1)
  if random() < a:
    x = x'
  record(image, x')
```

Extension to glossy surfaces

- probabilistically determine whether a surface is specular or not based on roughness
  - use the sampled micro-normal as the specular normal

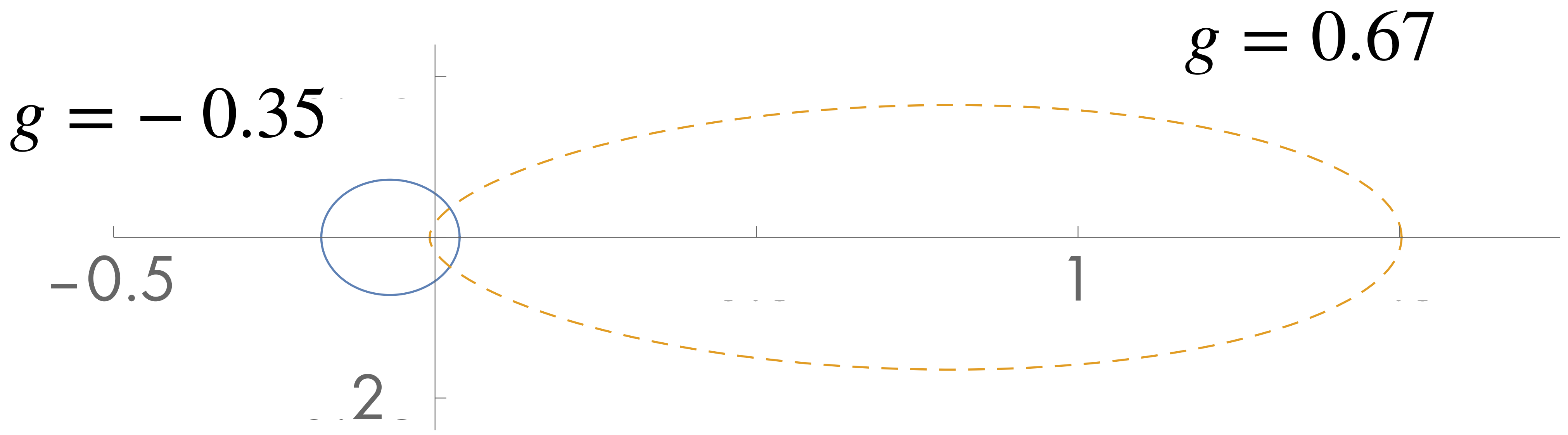


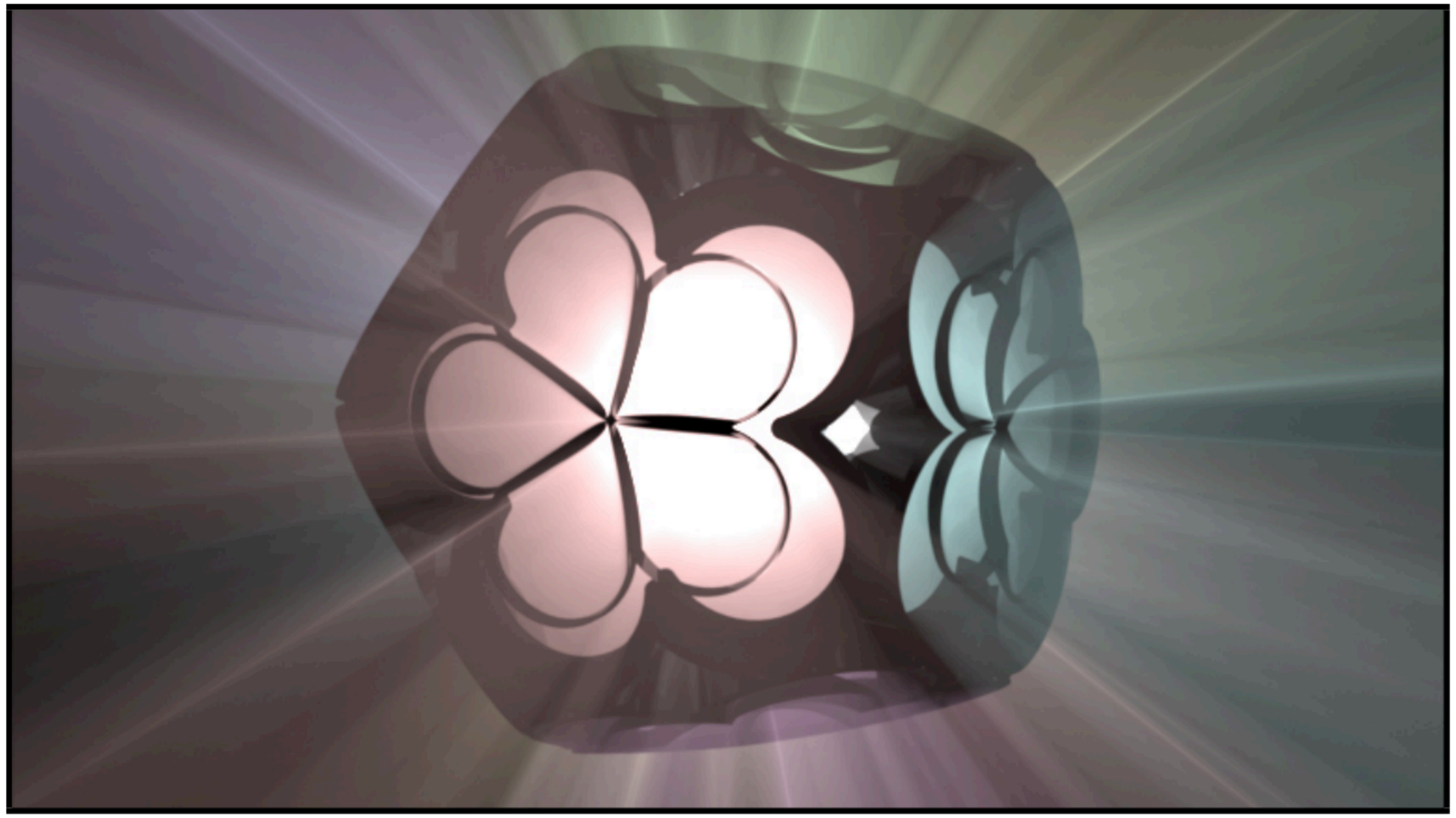


(a) *always non-specular* **(b)** *always specular* **(c)** *probabilistic*

Extension to volumetric light transport

- Henyey-Greenstein with high g can be seen as near-specular phase functions





https://www.pbr-book.org/3ed-2018/Volume_Scattering/Phase_Functions

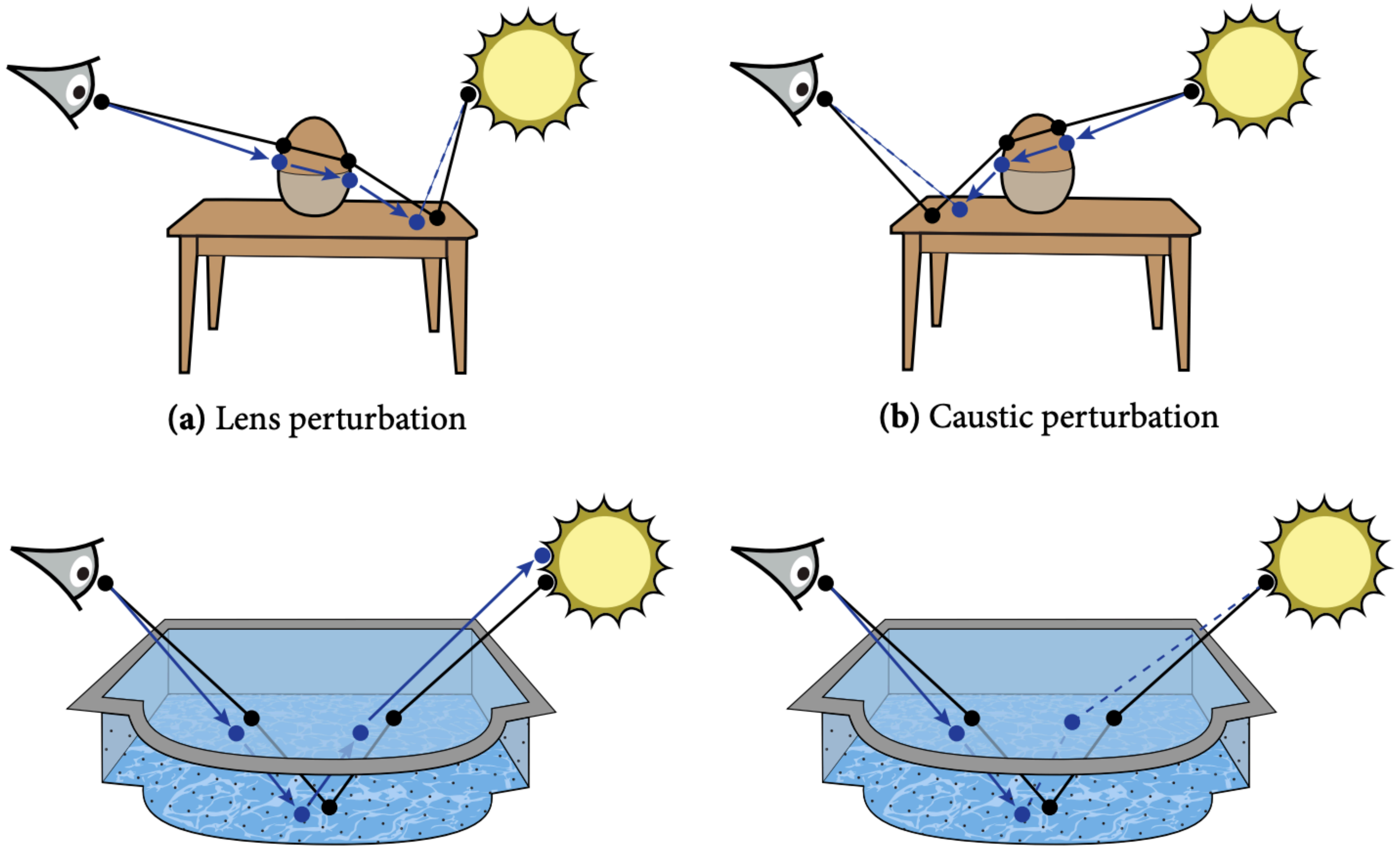
Metropolis light transport in Mitsuba

- first open source implementation of Veach-style MLT 15 years after Veach's publication!

8. PLUGIN REFERENCE 8.10. INTEGRATORS

8.10.11. Path Space Metropolis Light Transport (`mlt`)

Parameter	Type	Description
`maxDepth`	`integer`	Specifies the longest path depth in the generated output image (where -1 corresponds to ∞). A value of 1 will only render directly visible light sources. 2 will lead to single-bounce (direct-only) illumination, and so on. (Default: -1)
`directSamples`	`integer`	By default, the implementation renders direct illumination component separately using the `direct` plugin, which uses low-discrepancy number sequences for superior performance (in other words, it is *not* handled by MLT). This parameter specifies the number of samples allocated to that method. To force MLT to be responsible for the direct illumination component as well, set this to -1. (Default: 16)
`luminanceSamples`	`integer`	MLT-type algorithms create output images that are only *relative*. The algorithm can e.g. determine that a certain pixel is approximately twice as bright as another one, but the absolute scale is unknown. To recover it, this plugin computes the average luminance arriving at the sensor by generating a number of samples. (Default: `100000` samples)
`twoStage`	`boolean`	Use two-stage MLT? See `pssmlt` for details. (Default: `false`)
`bidirectional↙ Mutation, [lens,multiChain, caustic,manifold]↙ Perturbation`	`boolean`	These parameters can be used to pick the individual mutation and perturbation strategies that will be used to explore path space. By default, the original set by Veach and Guibas is enabled (i.e. everything except the manifold perturbation). It is possible to extend this integrator with additional custom perturbations strategies if needed.
`lambda`	`float`	Jump size of the manifold perturbation (Default: 50)



(a) Lens perturbation

(b) Caustic perturbation

(c) Multi-chain perturbation

(d) Manifold perturbation

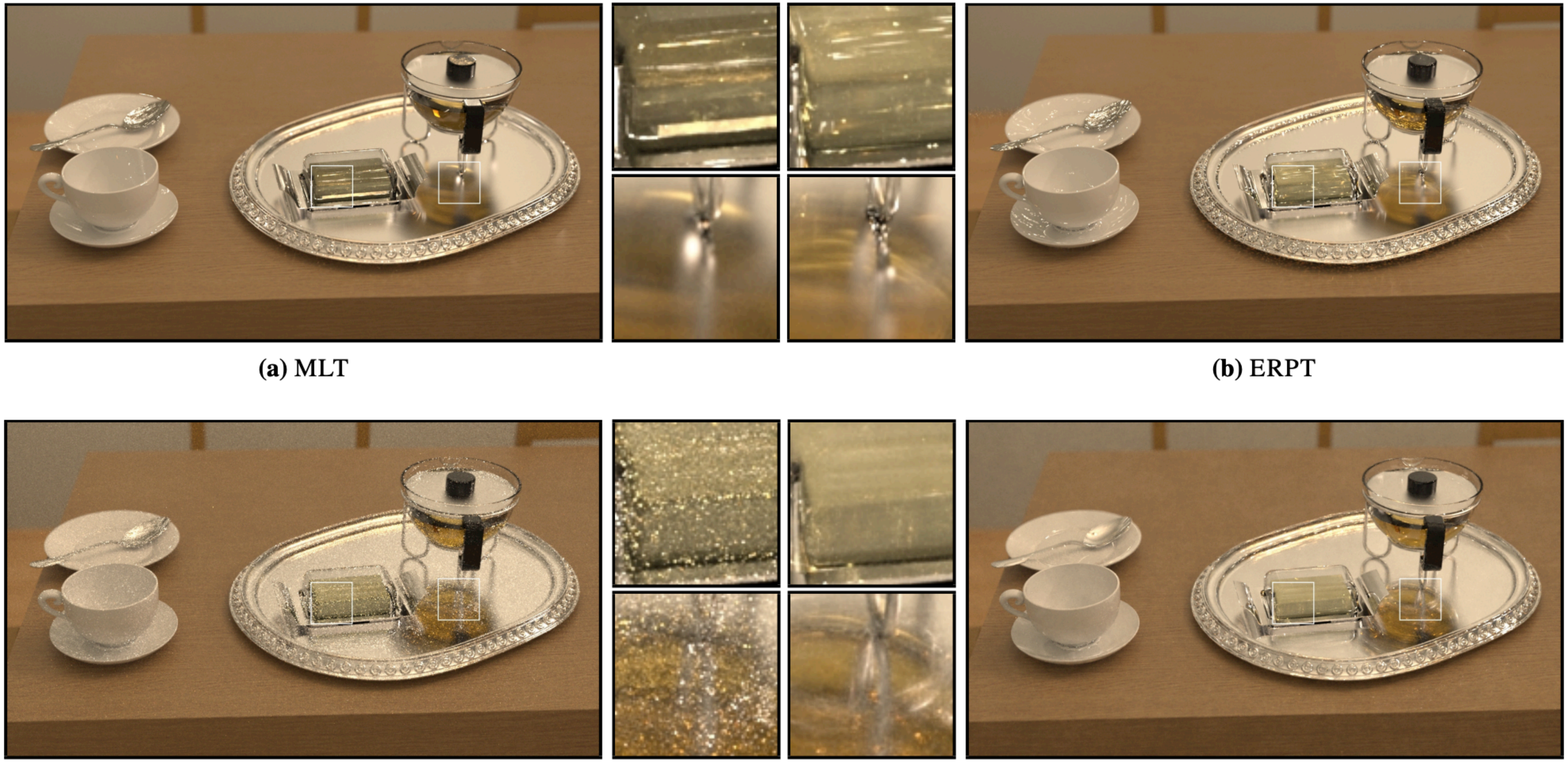
https://www.mitsuba-renderer.org/releases/current/documentation.pdf

Fancy images

(a) MLT (b) ERPT

(c) PSSMLT (d) MEPT

Fancy images



(a) MLT

(b) ERPT

(c) PSSMLT

(d) MEPT

Fancy images

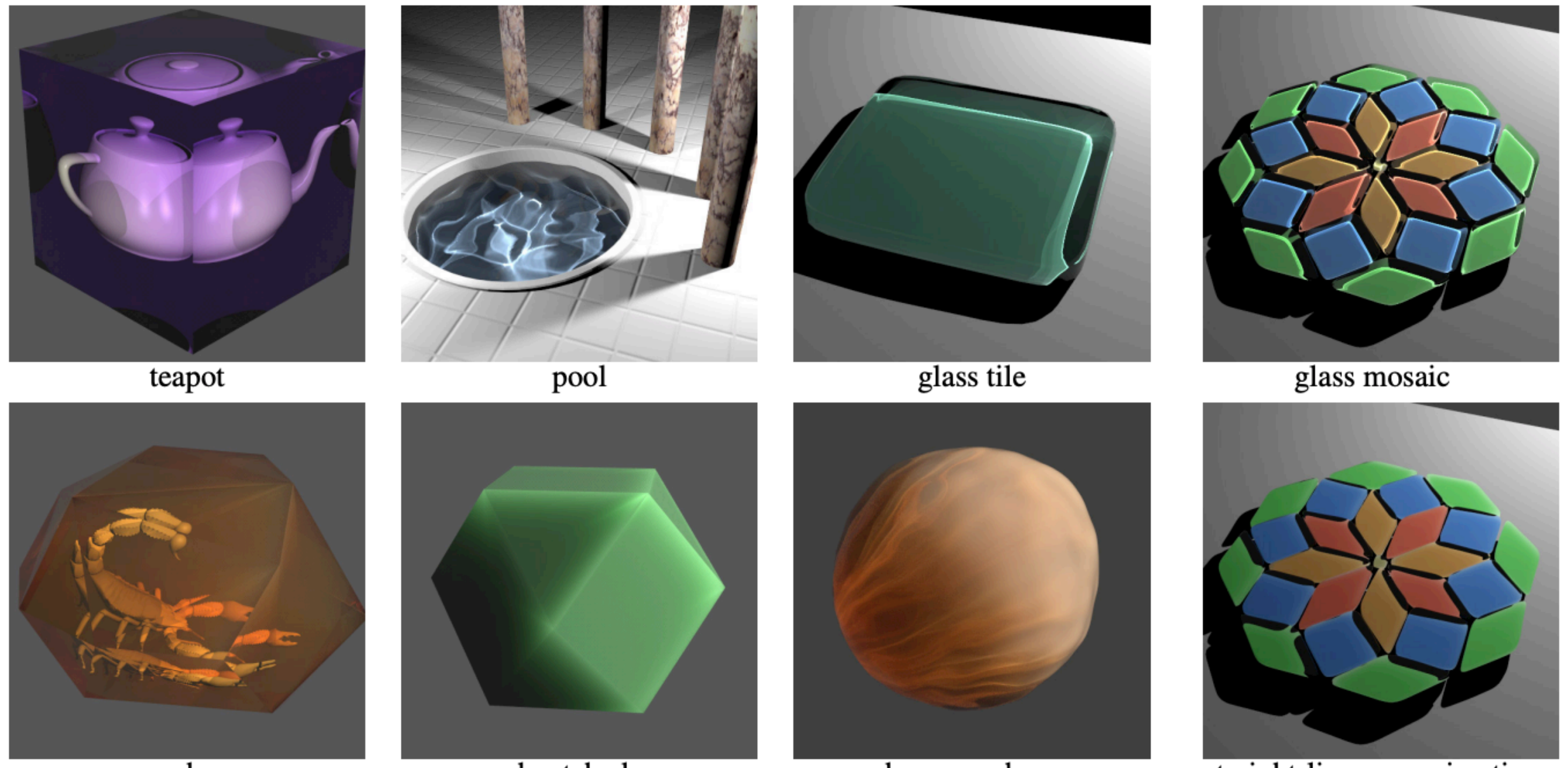
(a) MLT (b) ERPT (c) PSSMLT (d) **MEPT**

Three strategies to incorporate Newton's method into a renderer

use new data structure to enumerate roots

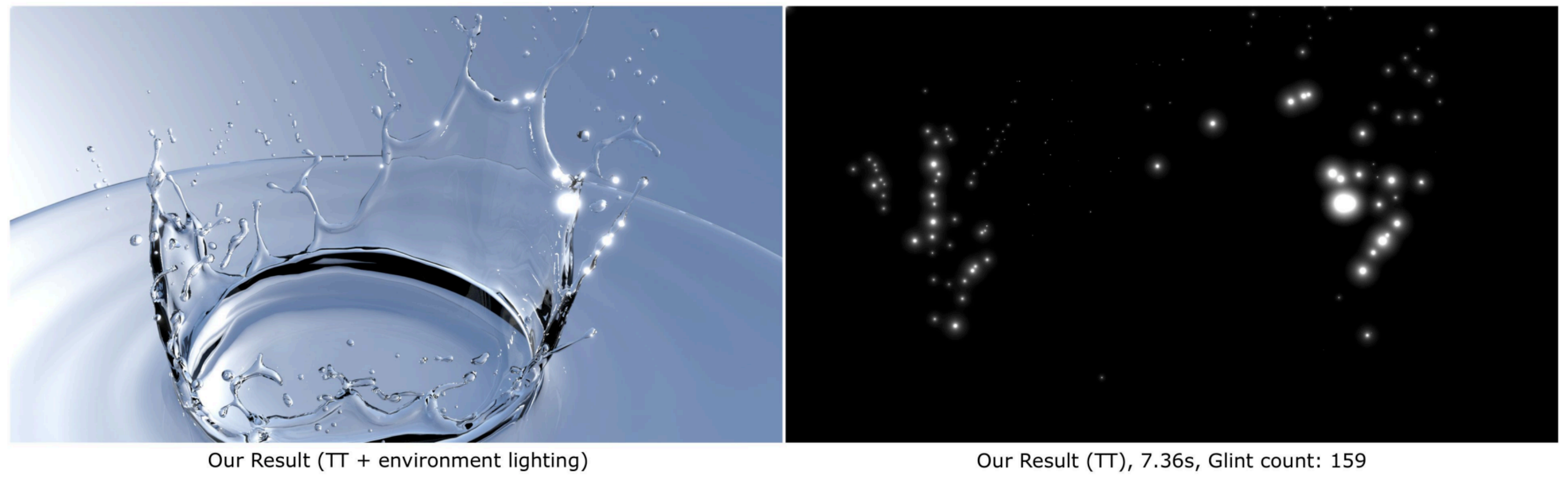
Single Scattering in Refractive Media with Triangle Mesh Boundaries

Bruce Walter, *Cornell University*
Shuang Zhao, *Cornell University*
Nicolas Holzschuch, *INRIA – LJK*
Kavita Bala, *Cornell University*



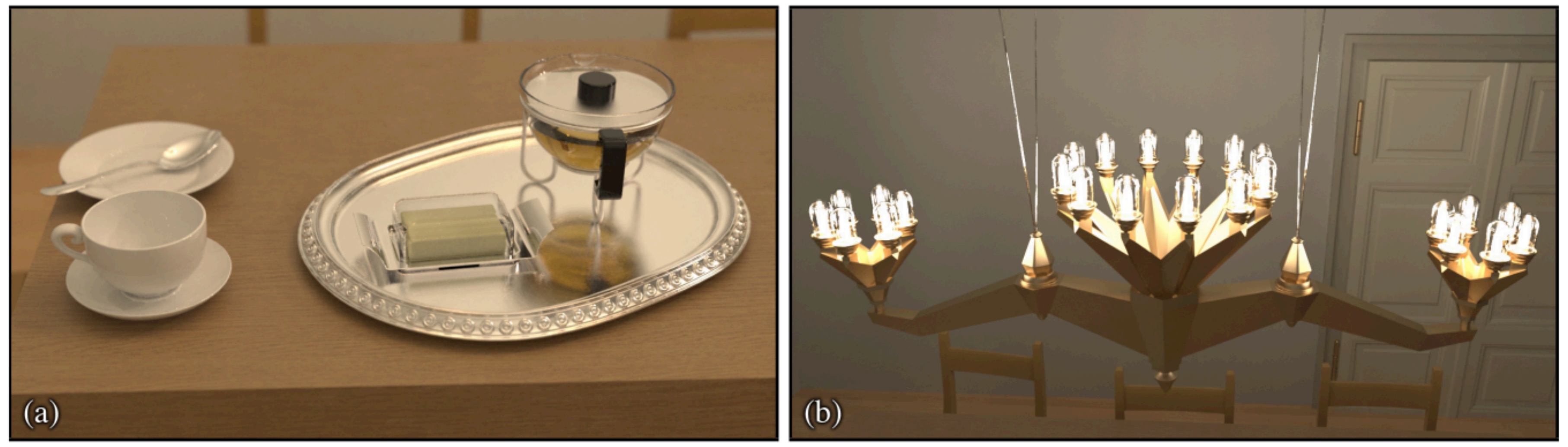
Path Cuts: Efficient Rendering of Pure Specular Light Transport

BEIBEI WANG, School of Computer Science and Engineering, Nanjing University of Science and Technology
MILOŠ HAŠAN, Adobe Research
LING-QI YAN, University of California, Santa Barbara



Manifold Exploration: A Markov Chain Monte Carlo technique for rendering scenes with difficult specular transport

Wenzel Jakob Steve Marschner
Cornell University



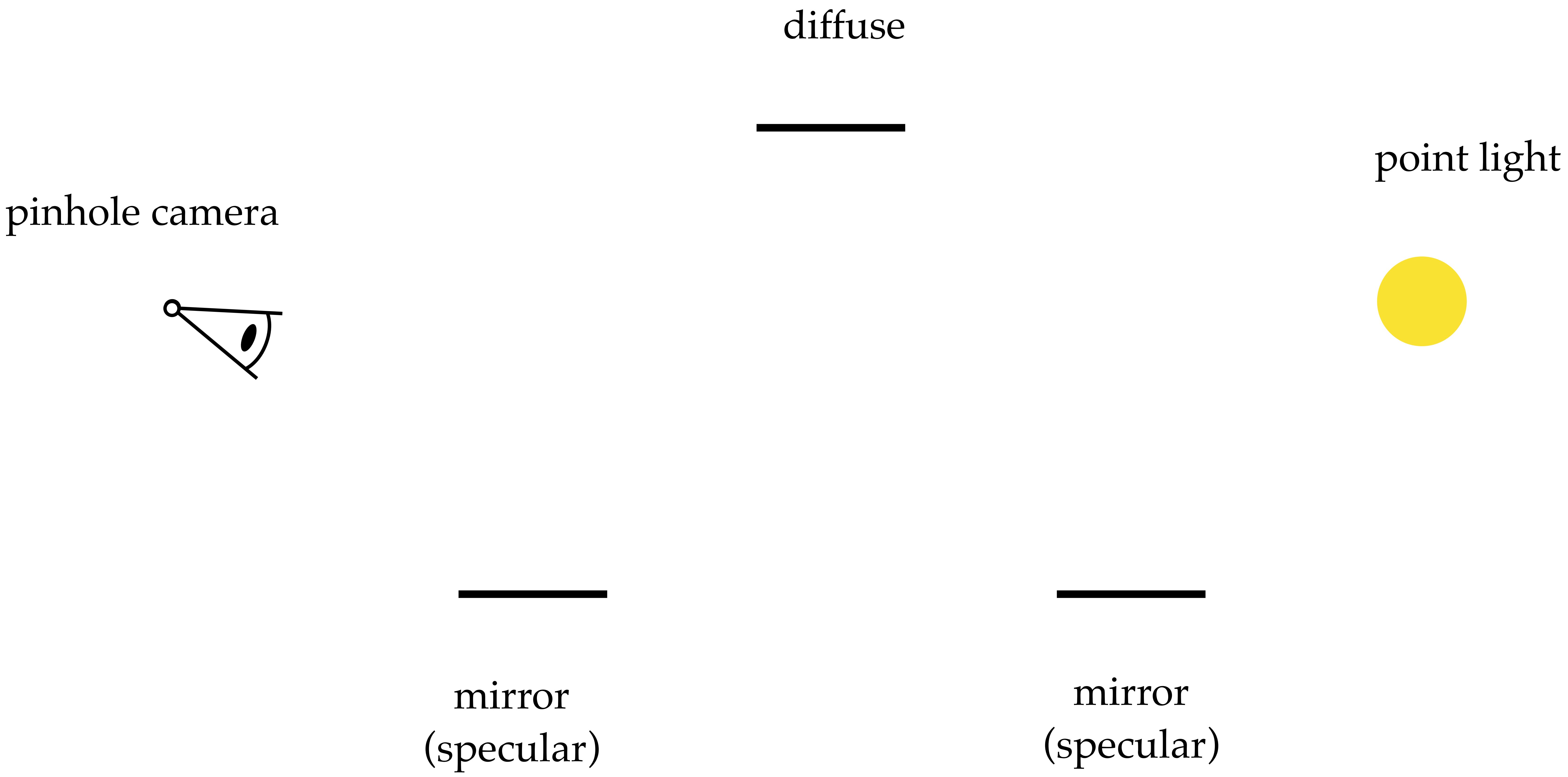
Metropolis light transport

Specular Manifold Sampling for Rendering High-Frequency Caustics and Glints

TIZIAN ZELTNER, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland
ILIYAN GEORGIEV, Autodesk, United Kingdom
WENZEL JAKOB, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland

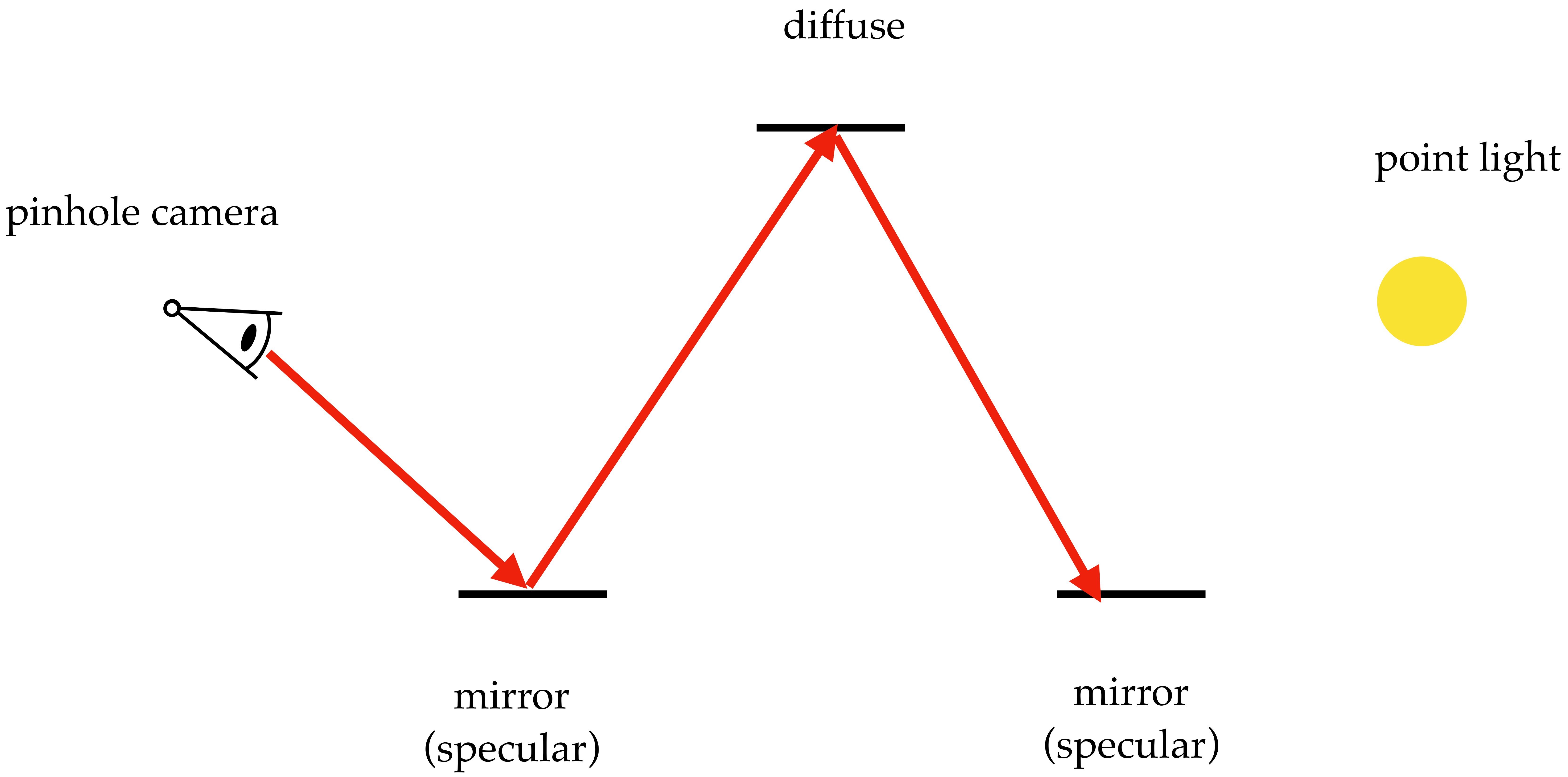
randomized initialization using Monte Carlo sampling

Back to point lights



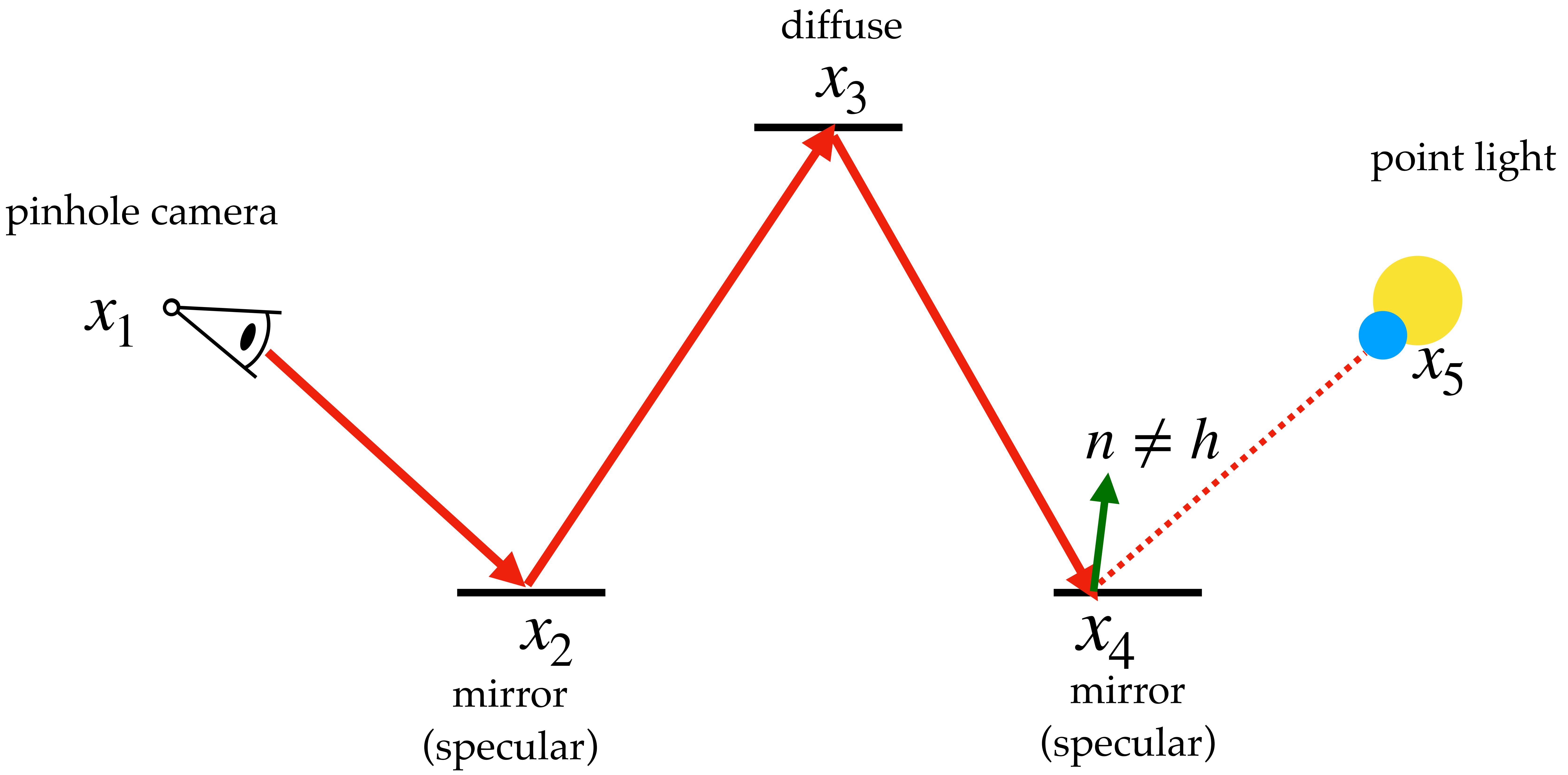
Manifold exploration with normal Monte Carlo sampling

- do normal path tracing before we hit the light



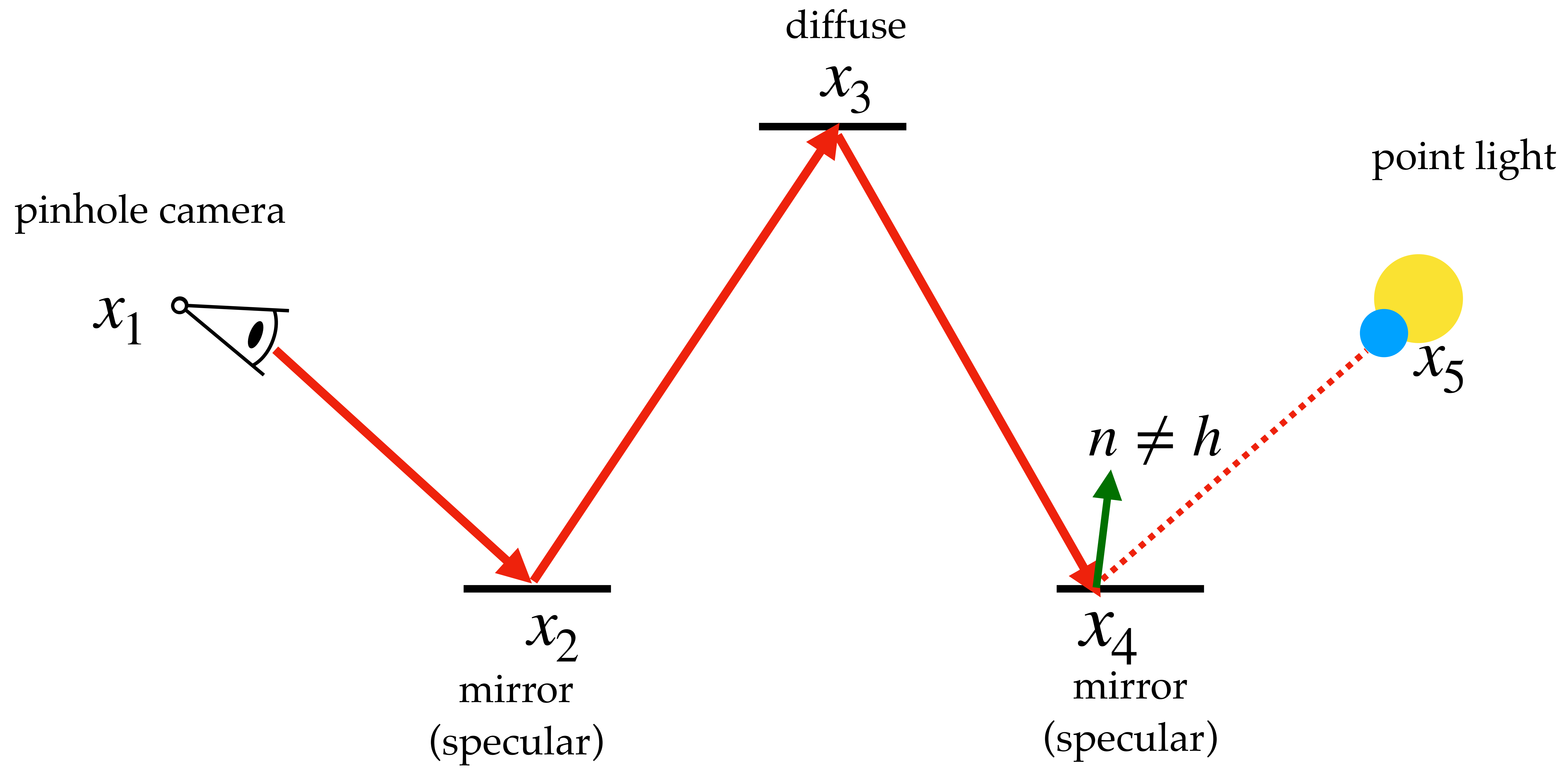
Manifold exploration with normal Monte Carlo sampling

- connect to the light source — **quiz**: what is the contribution of this light path?



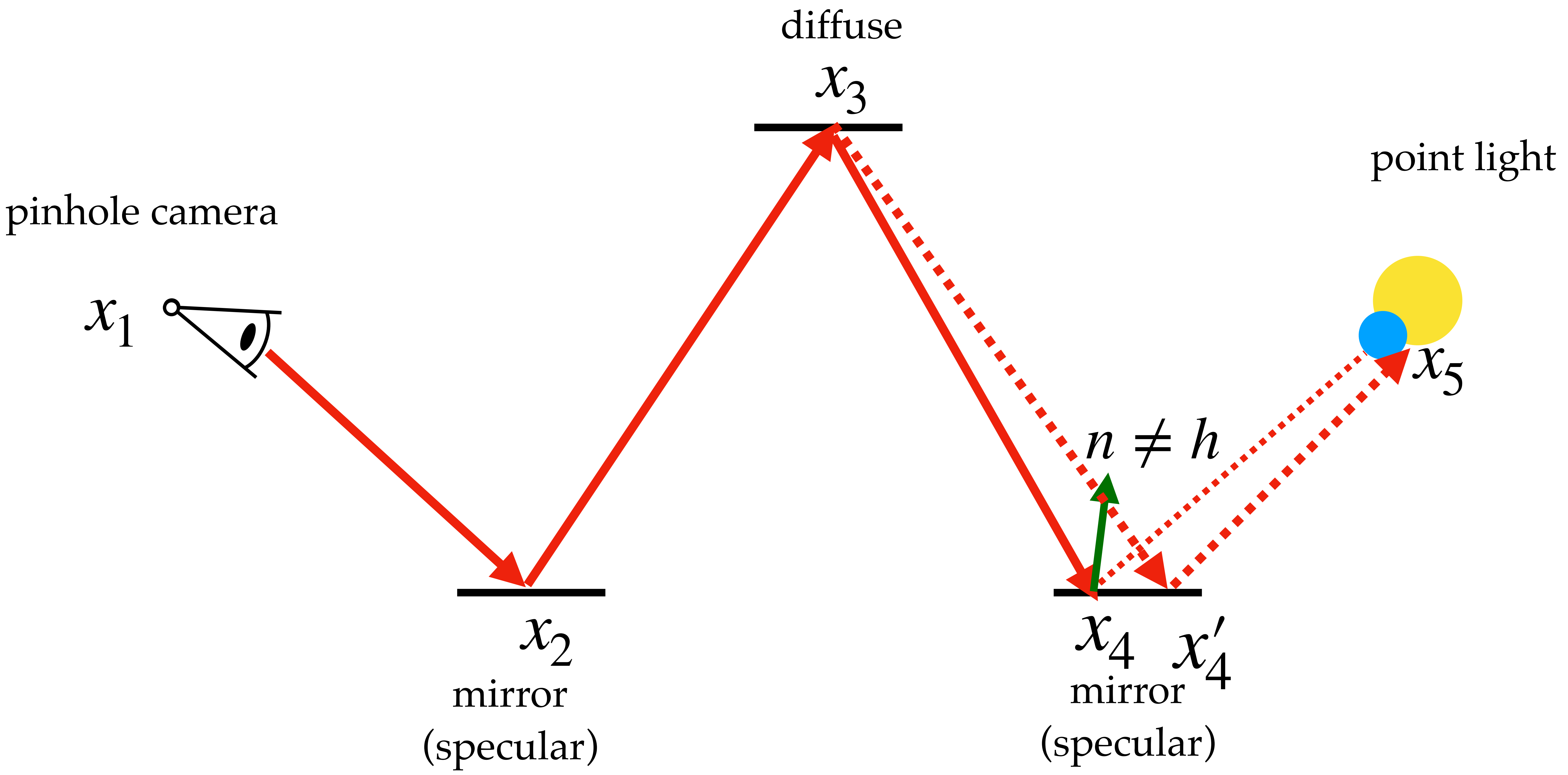
Manifold exploration with normal Monte Carlo sampling

- connect to the light source — contribution is zero since we are on a specular surface



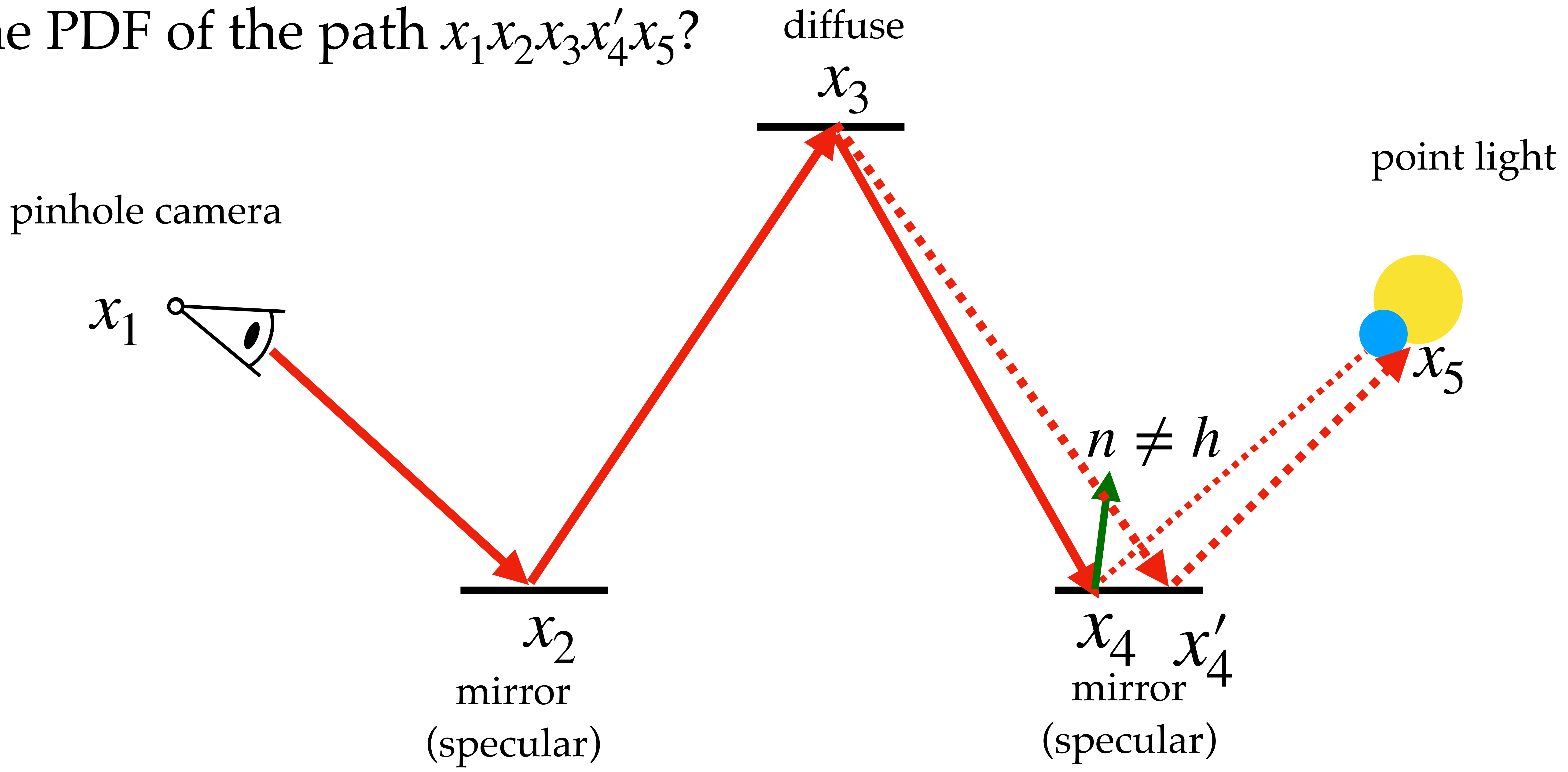
Manifold exploration with normal Monte Carlo sampling

- perturb x_4 to satisfy the specular constraint



Manifold exploration with normal Monte Carlo sampling

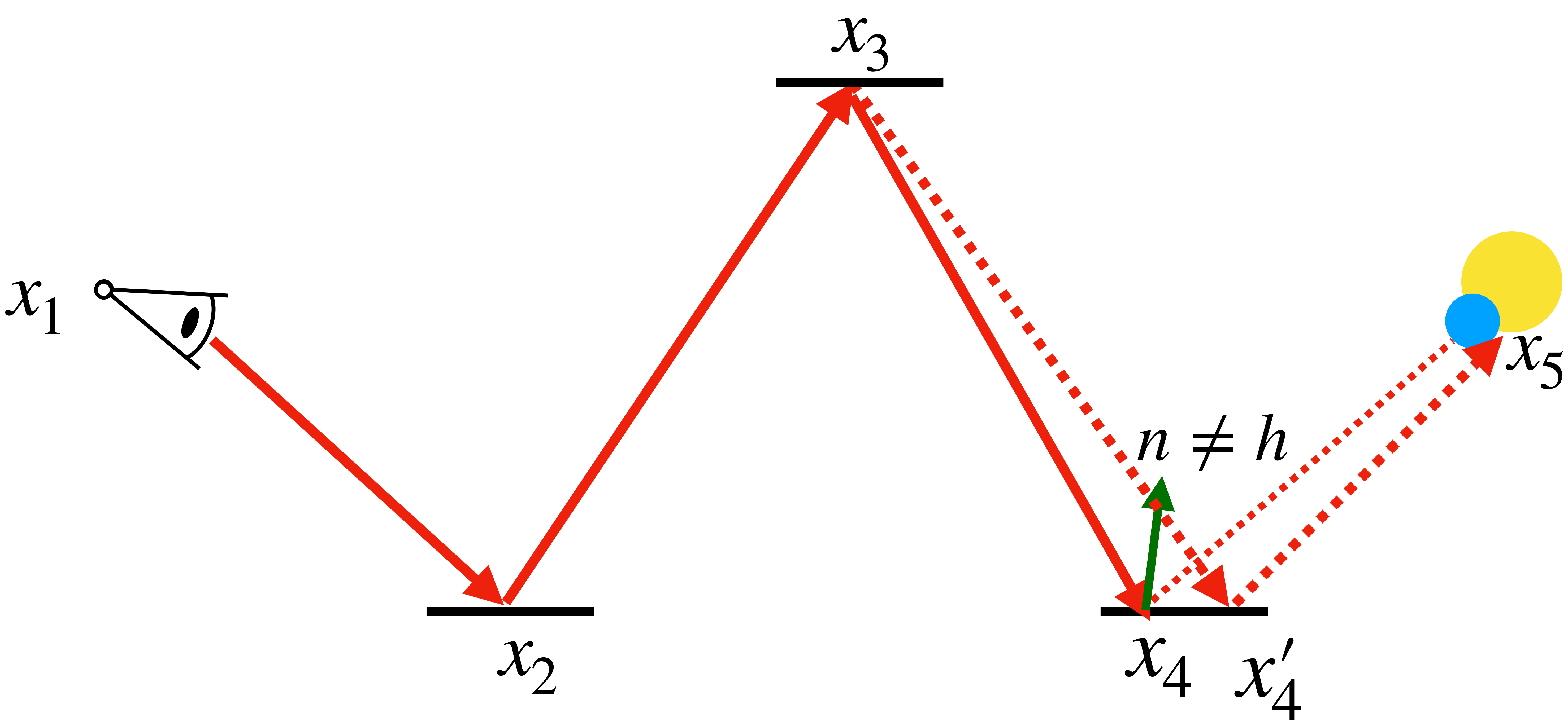
- perturb x_4 to satisfy the specular constraint
  - what is the PDF of the path $x_1 x_2 x_3 x_4' x_5$?



PDF of a specular path is an integral

- the probability density of sampling $x_1 x_2 x_3 x_4' x_5$ is the sum of all probability densities of path that will perturb to it

$$p(x') = \int p(x)p(x' | x)\mathrm{d}x$$



PDF of a specular path is an integral

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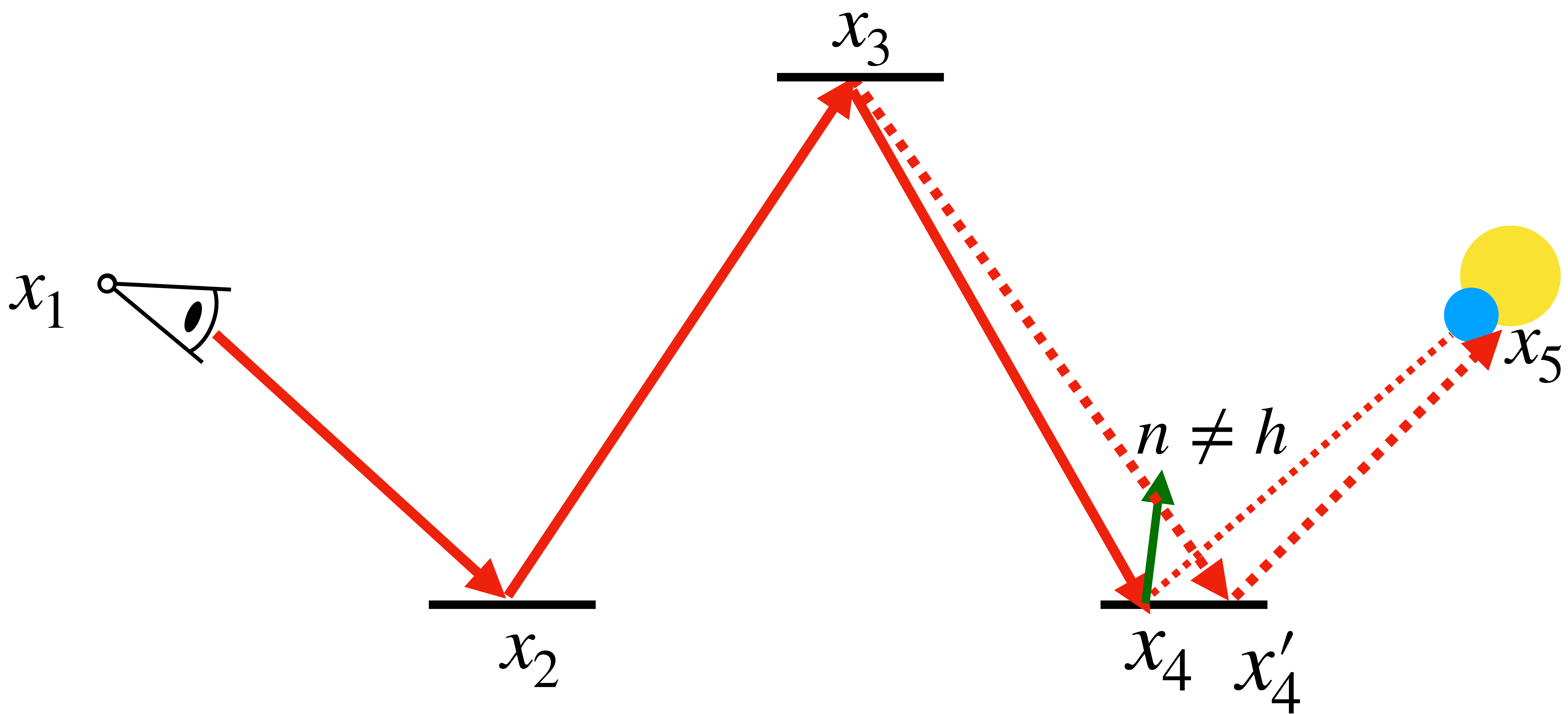
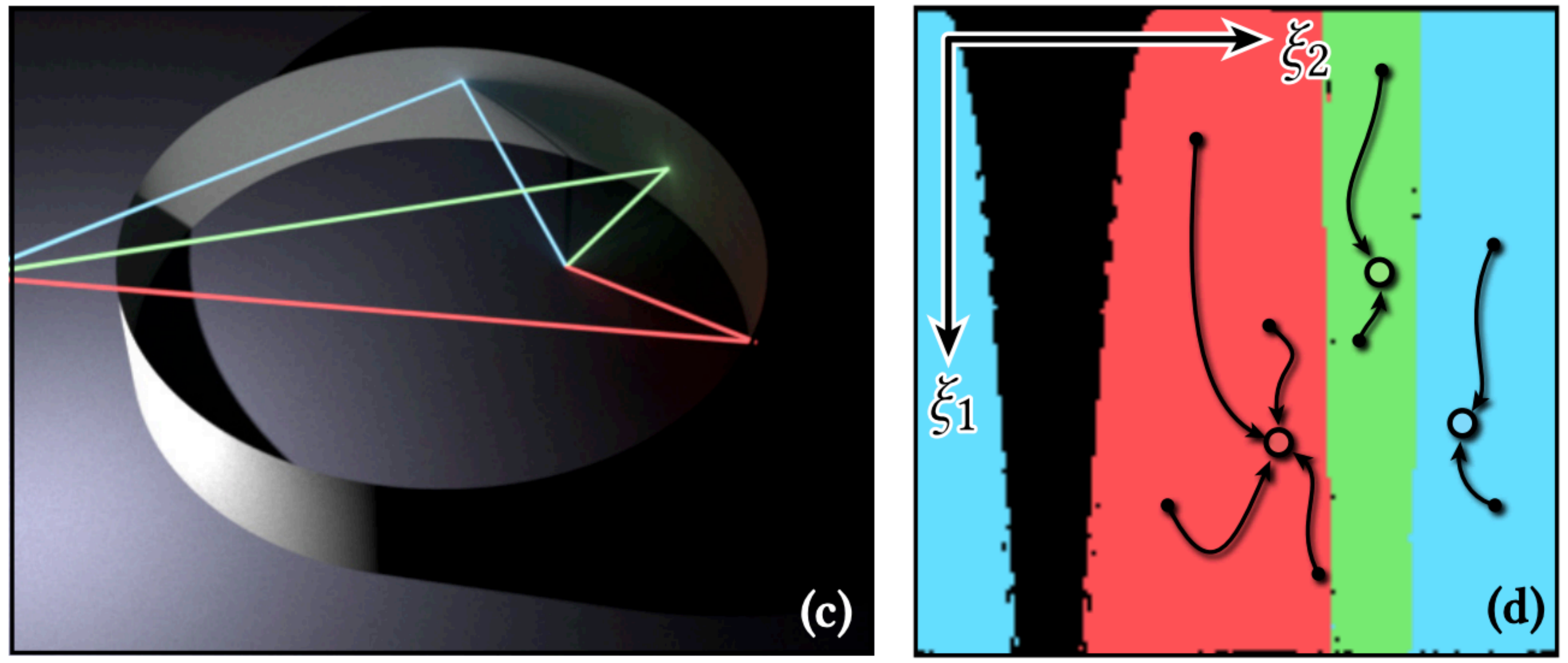
$$p(x') = \int p(x) p(x' | x) \mathrm{d}x$$

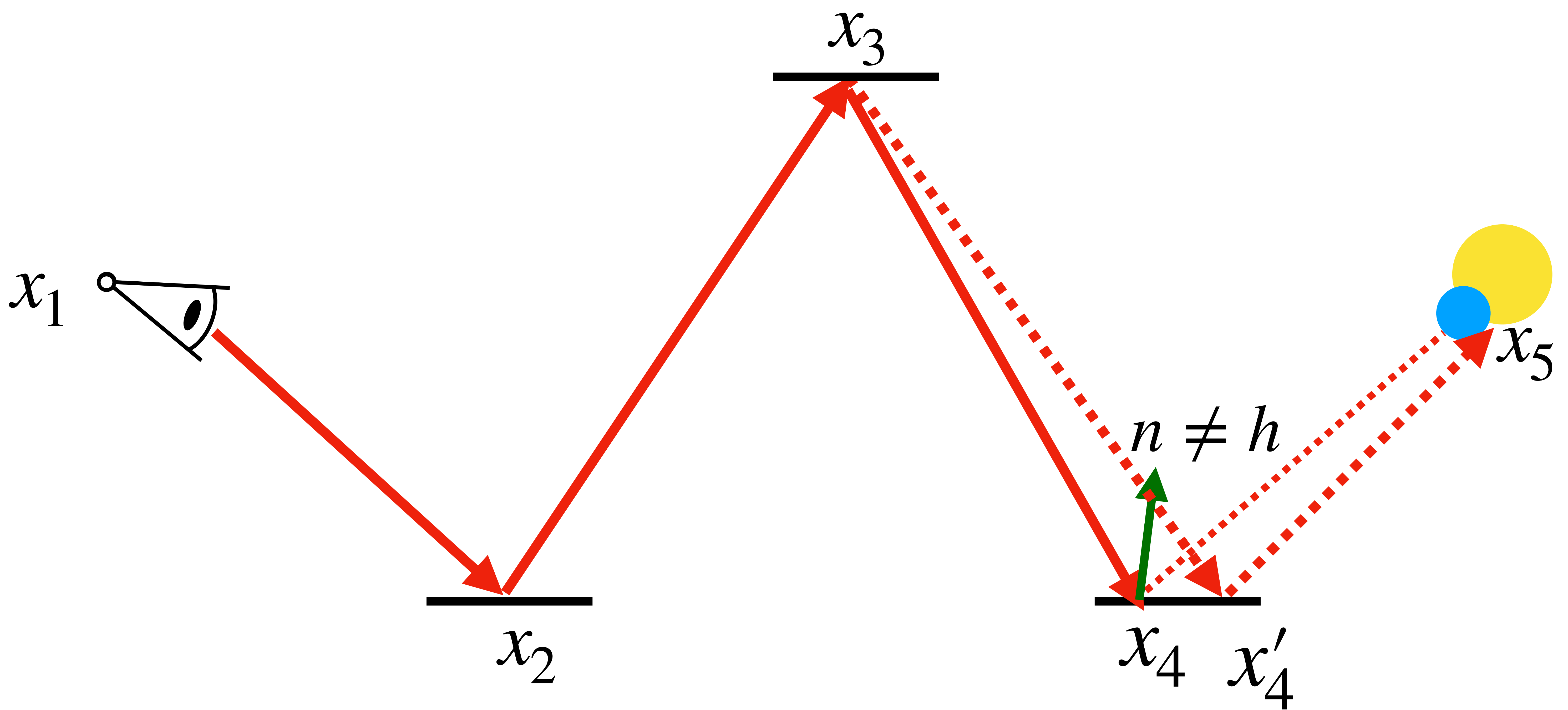




Evaluating contribution

- need to use Monte Carlo sampling to estimate the PDF $p(x')$ itself

$$\frac{f(x')}{p(x')} = \frac{f(x')}{\int p(x)p(x' \mid x)\mathrm{d}x}$$



$$p(x') = \int p(x)p(x' \mid x)\mathrm{d}x$$

Unbiased evaluation of reciprocal of integral

- same as the unbiased photon mapping paper

$$\frac{1}{\int f(x)\mathrm{d}x} = \frac{1}{1-F} = 1 + F + F^2 + \cdots$$

can estimate using Russian roulette

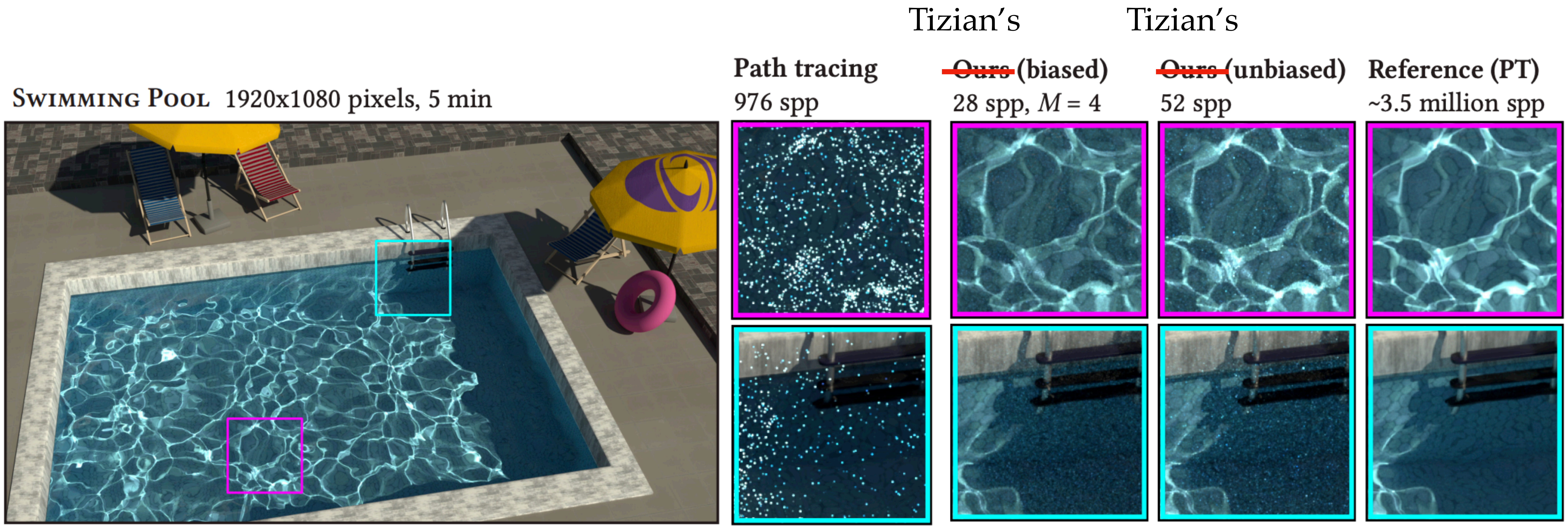
Pseudocode

ALGORITHM 2: Unbiased specular manifold sampling

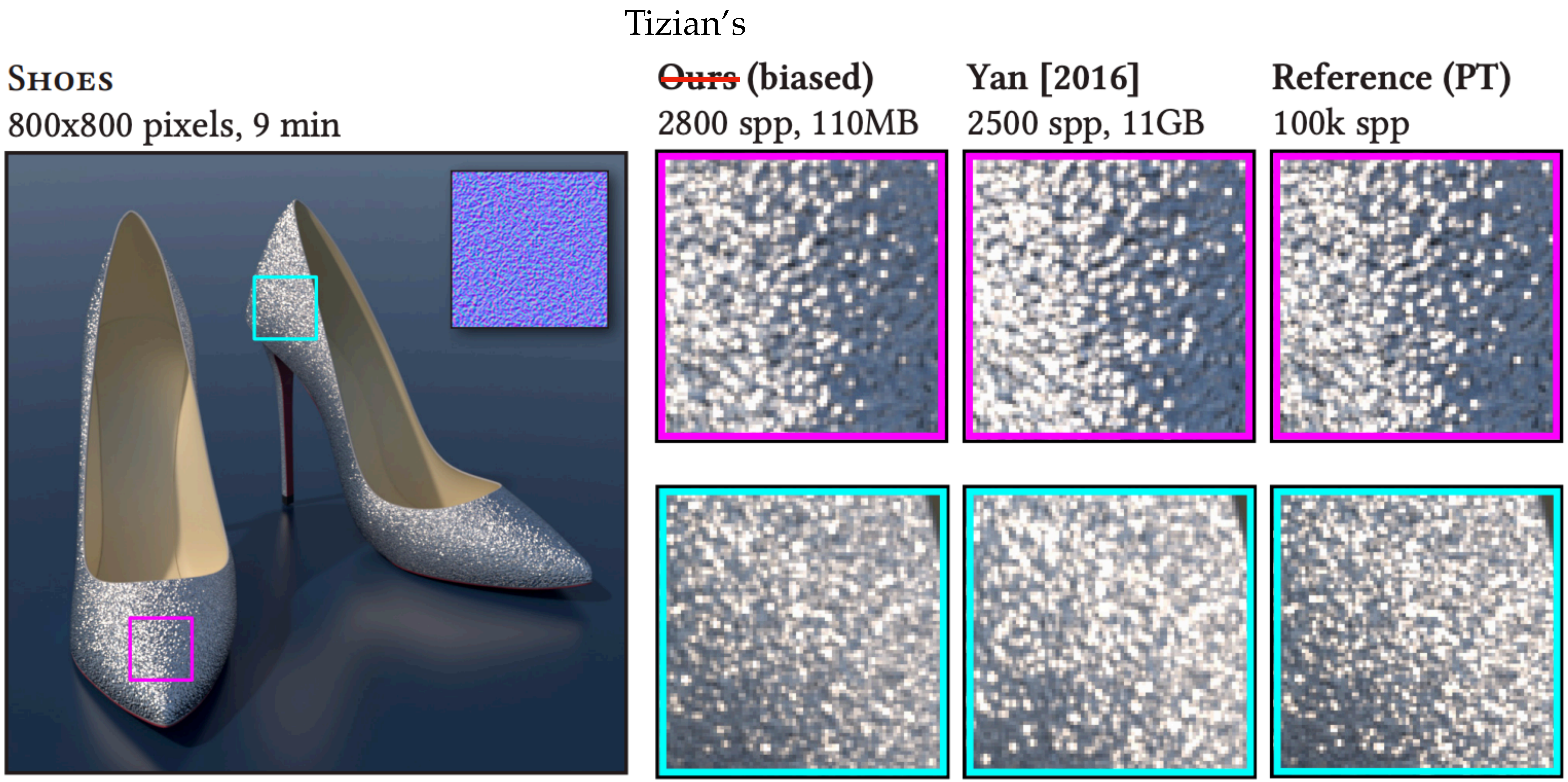
Input: Shading point $\mathbf{x}_1$ and emitter position $\mathbf{x}_3$ with density $p(\mathbf{x}_3)$
Output: Estimate of radiance traveling from $\mathbf{x}_3$ to $\mathbf{x}_1$

$\mathbf{x}_2 \leftarrow$ sample a specular vertex as initial position
$\mathbf{x}_2^* \leftarrow \texttt{manifold\_walk}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$
$\langle 1/p_k \rangle \leftarrow 1$ ▷ Estimate inverse probability of sampling $\mathbf{x}_2^*$
while *true* **do**
    $\mathbf{x}_2 \leftarrow$ sample specular vertex as above
    $\mathbf{x}_2' \leftarrow \texttt{manifold\_walk}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$
    if $\|\mathbf{x}_2' - \mathbf{x}_2^*\| < \varepsilon$ **then**
        break
    $\langle 1/p_k \rangle \leftarrow \langle 1/p_k \rangle + 1$
return $f_s(\mathbf{x}_2^*) \cdot G(\mathbf{x}_1 \leftrightarrow \mathbf{x}_2 \leftrightarrow \mathbf{x}_3) \cdot \langle 1/p_k \rangle \cdot L_e(\mathbf{x}_3) \,/\, p(\mathbf{x}_3)$

Fancy images



Fancy images

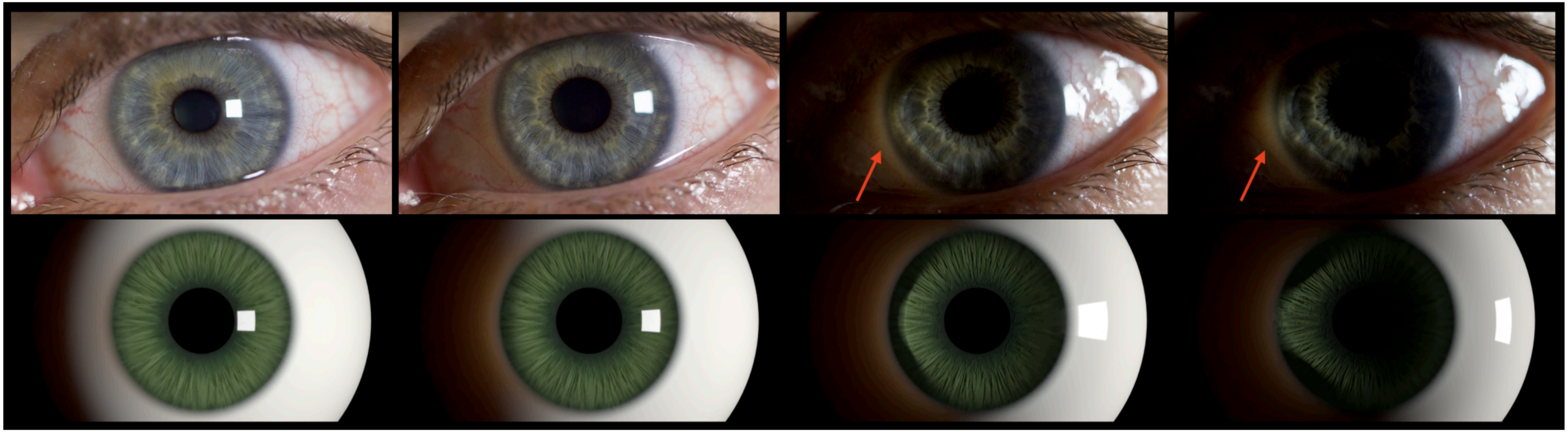


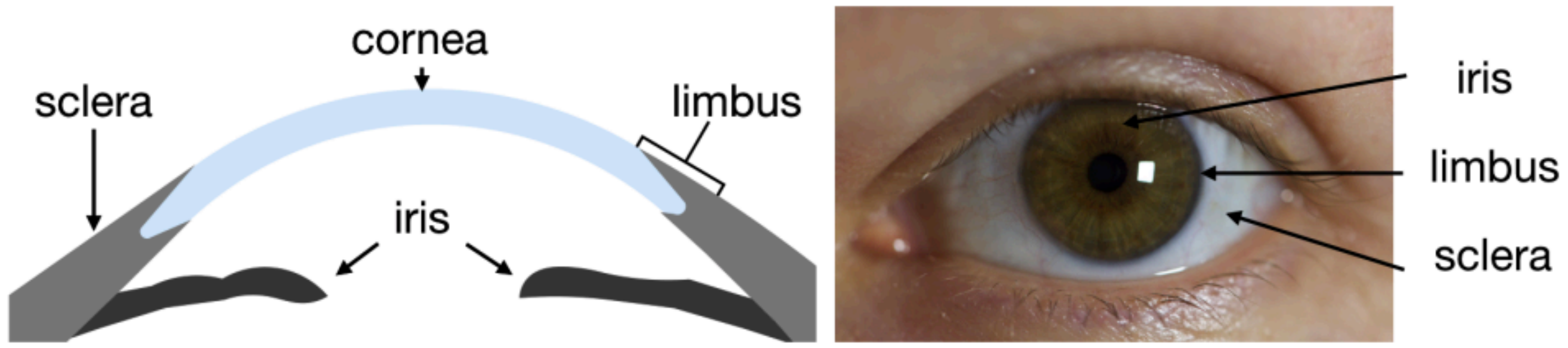
Manifold exploration is used in practice

Manifold Next Event Estimation

Johannes Hanika, Marc Droske and Luca Fascione

Manifold exploration is used in practice





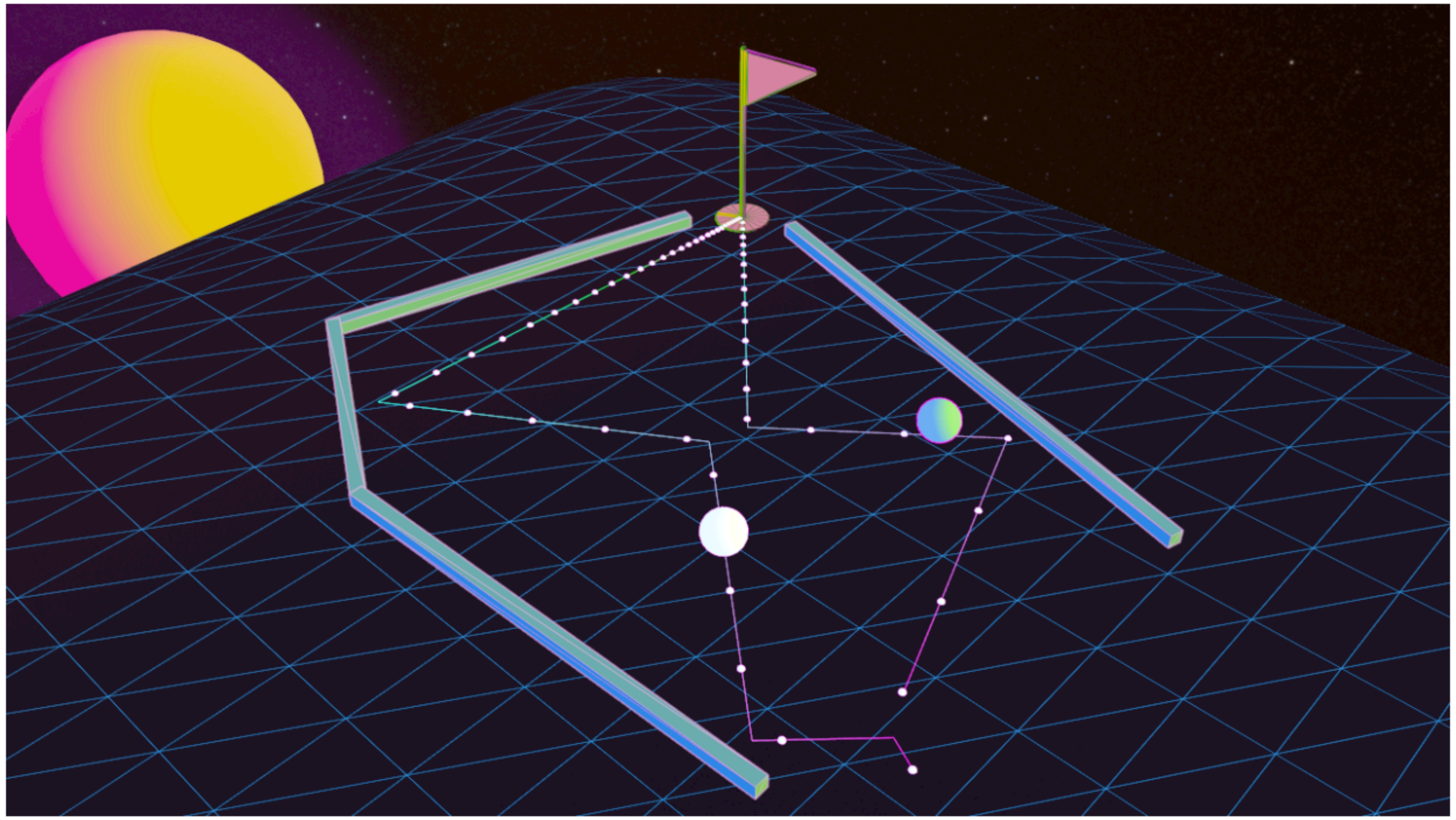
Plausible Iris Caustics and Limbal Arc Rendering

Matt Jen-Yuan Chiang
Walt Disney Animation Studios
matt.chiang@disneyanimation.com

Brent Burley
Walt Disney Animation Studios
brent.burley@disneyanimation.com

Connection to physical simulation

- Lagrangian mechanics/Hamilton's least action principle = finding shortest paths towards target
  - a generalization of Fermat's principle
- specular light path rendering is a physical trajectory finding problem!



Next: multiple importance sampling++

$$\sum w_i \frac{f_i}{p_i}$$