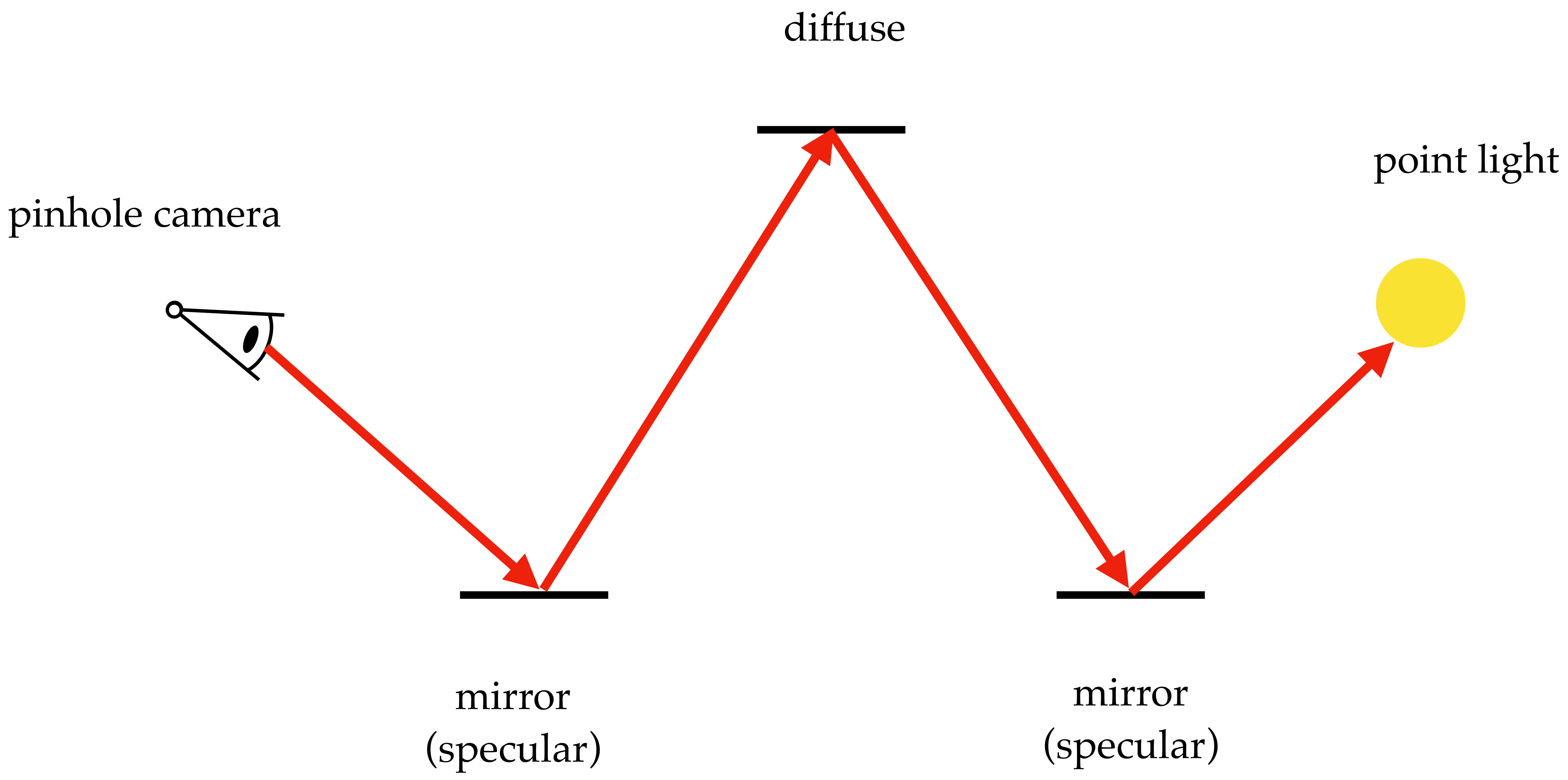
Photon Mapping

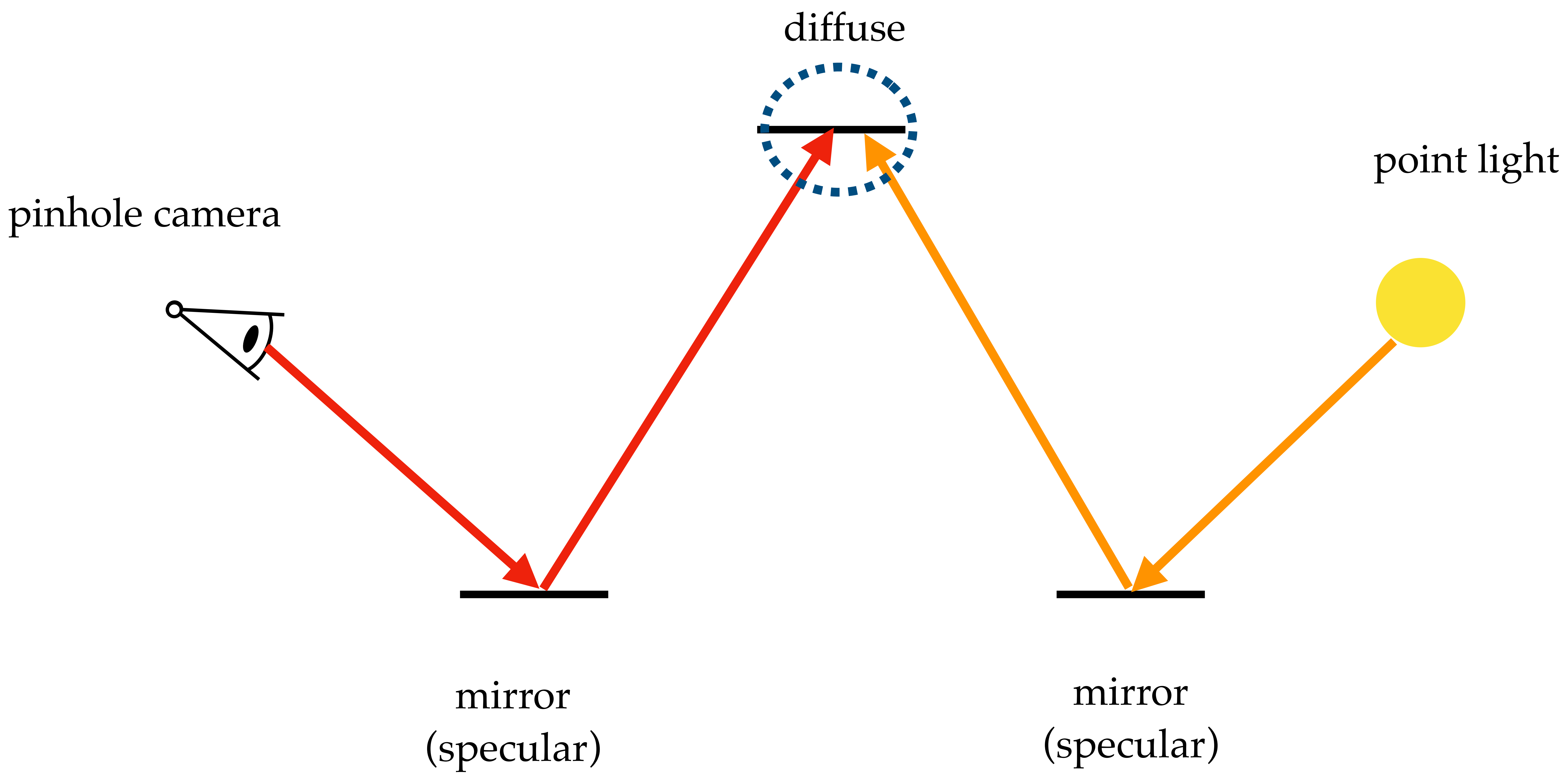
UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li

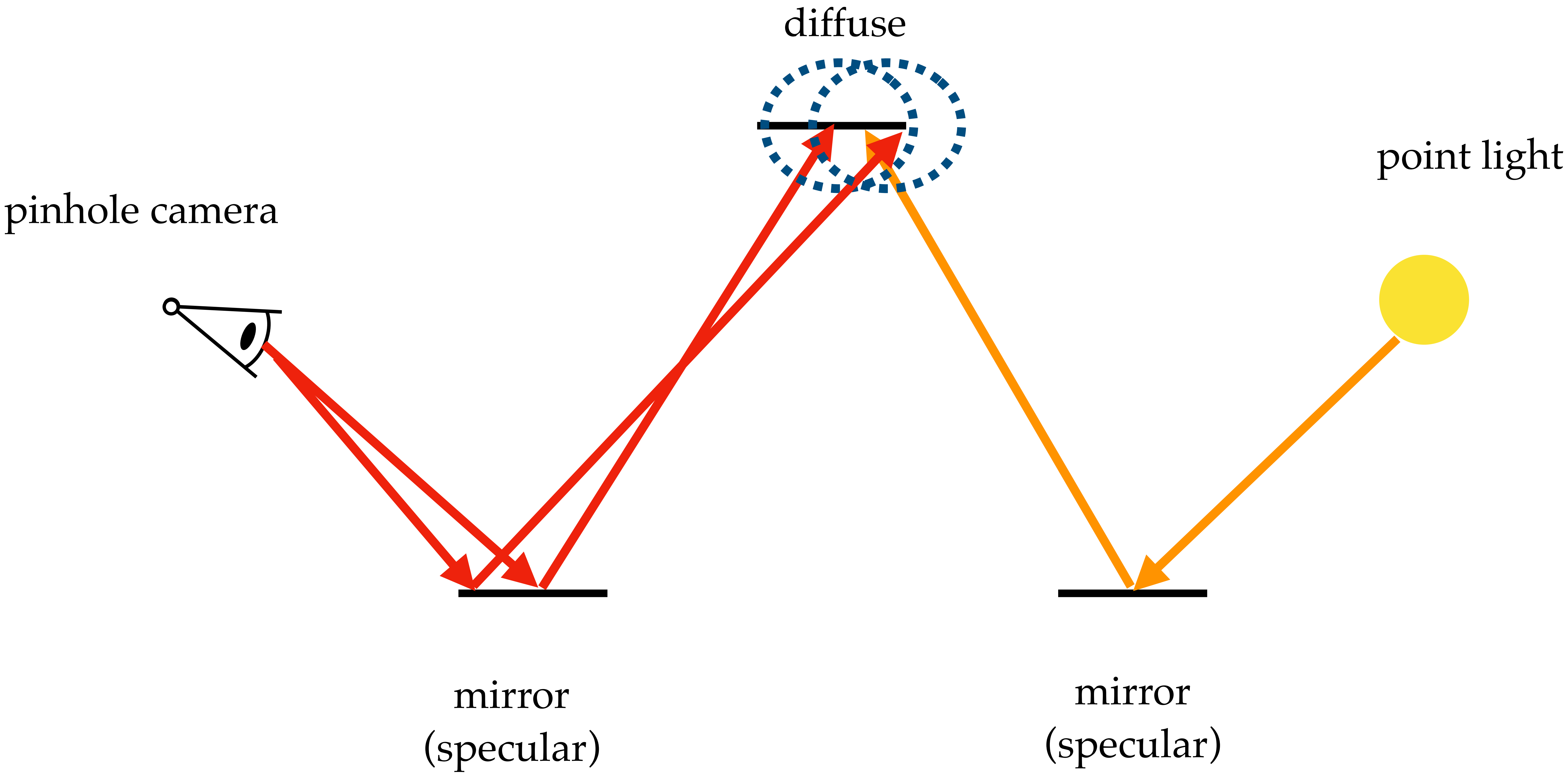
SDS light paths



Idea 1: allow “near miss”

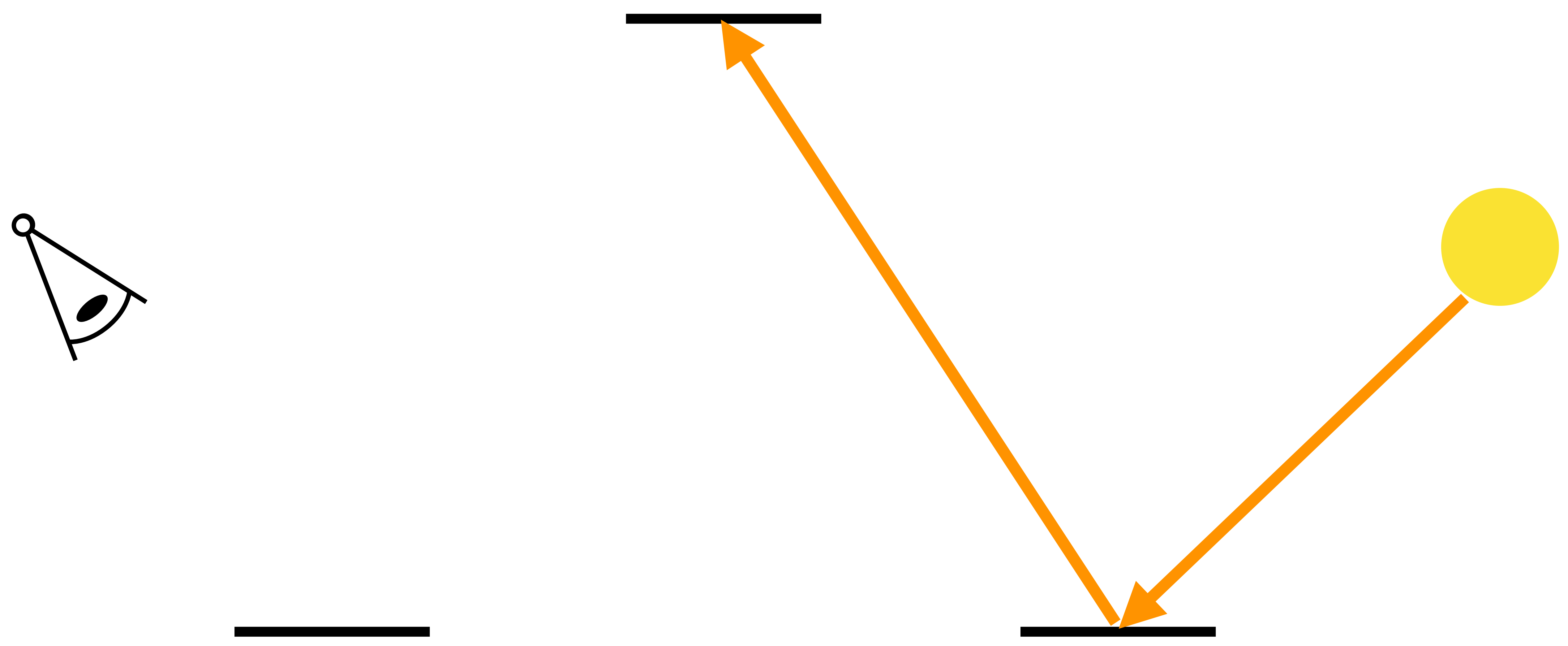


Idea 2: share light subpaths among different pixels



Photon mapping

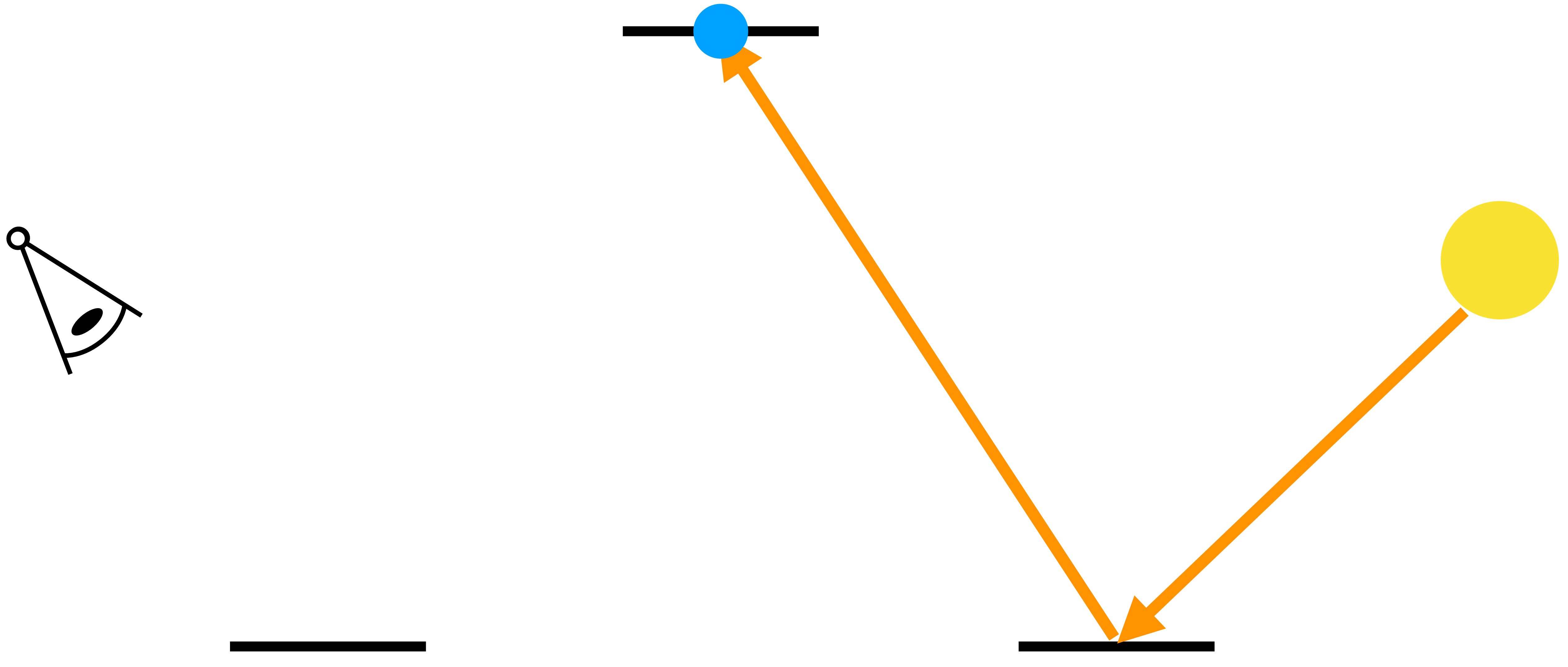
1. trace random light subpaths



Photon mapping

1. trace random light subpaths

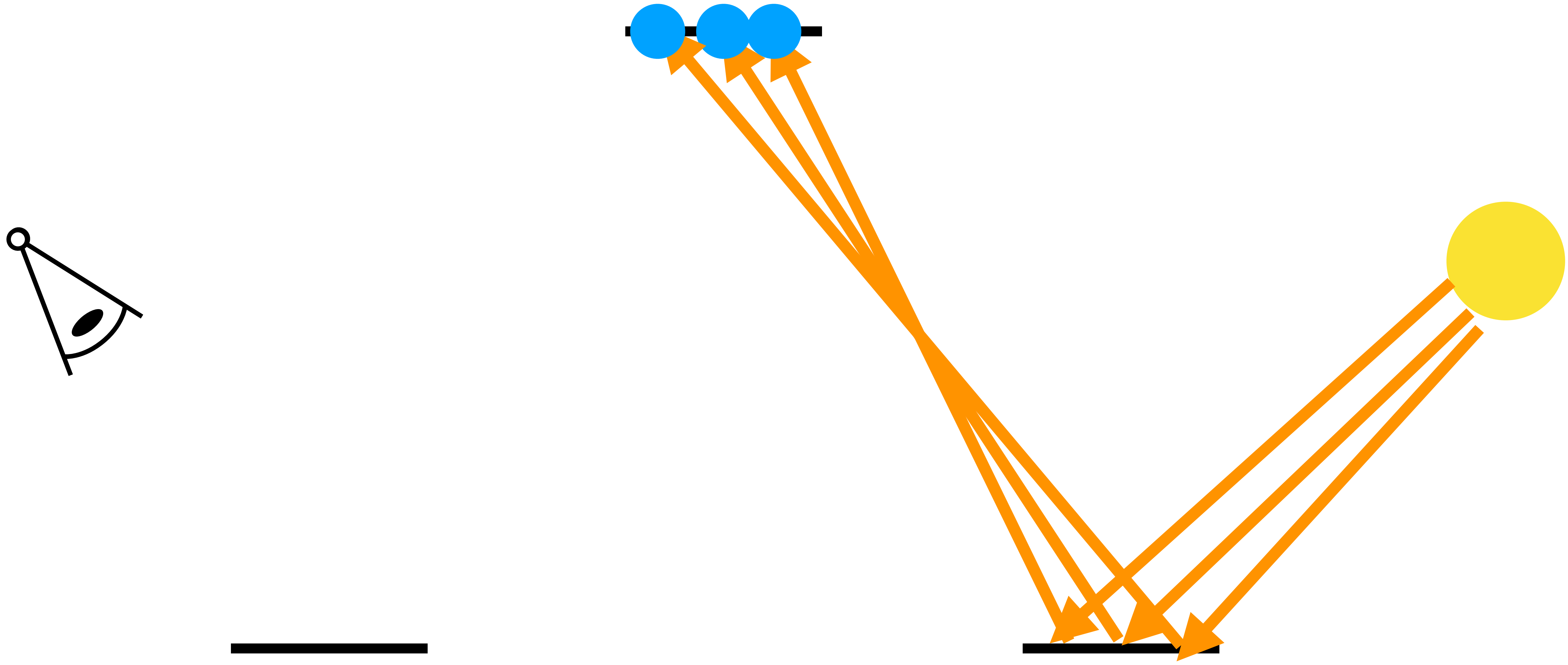
2. store **photons** on diffuse surfaces



Photon mapping

1. trace random light subpaths

2. store **photons** on diffuse surfaces

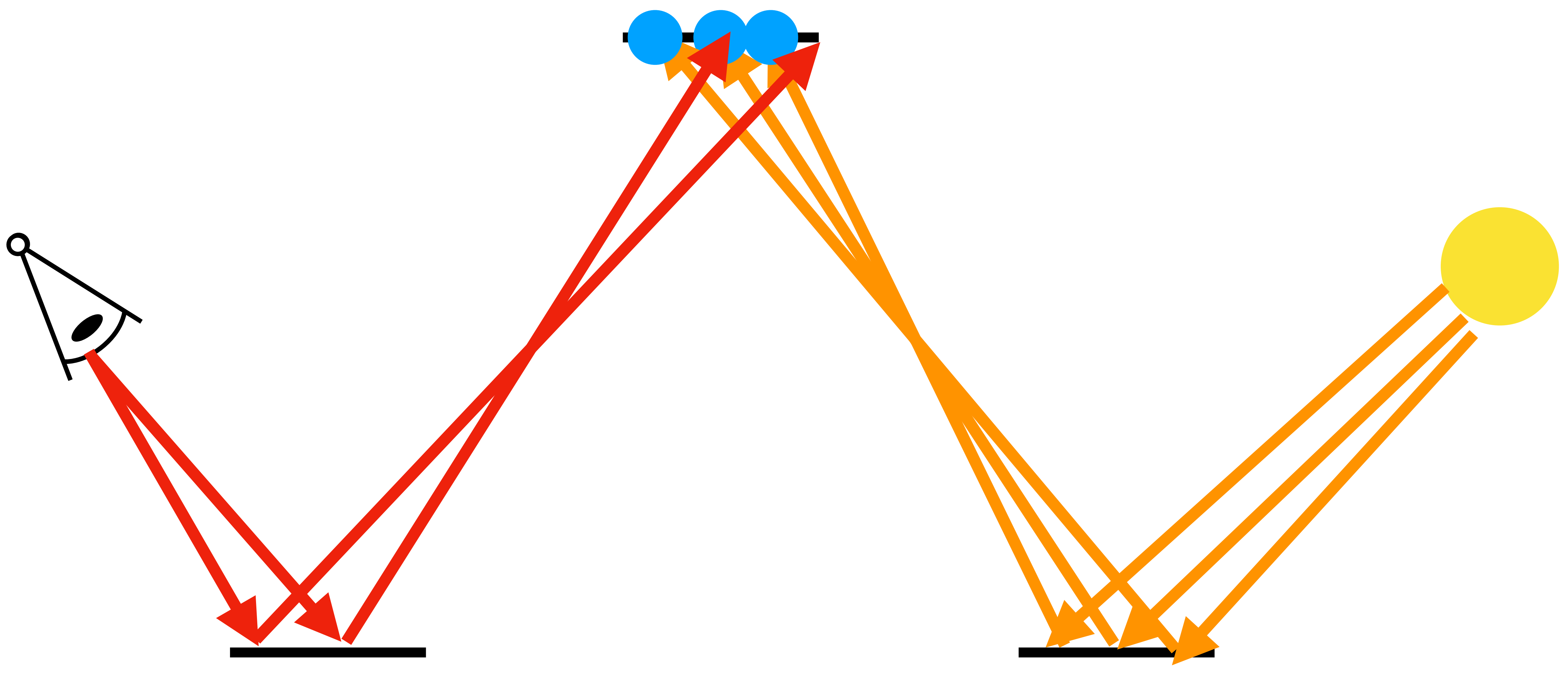


Photon mapping

1. trace random light subpaths

2. store **photons** on diffuse surfaces

3. trace random camera subpaths



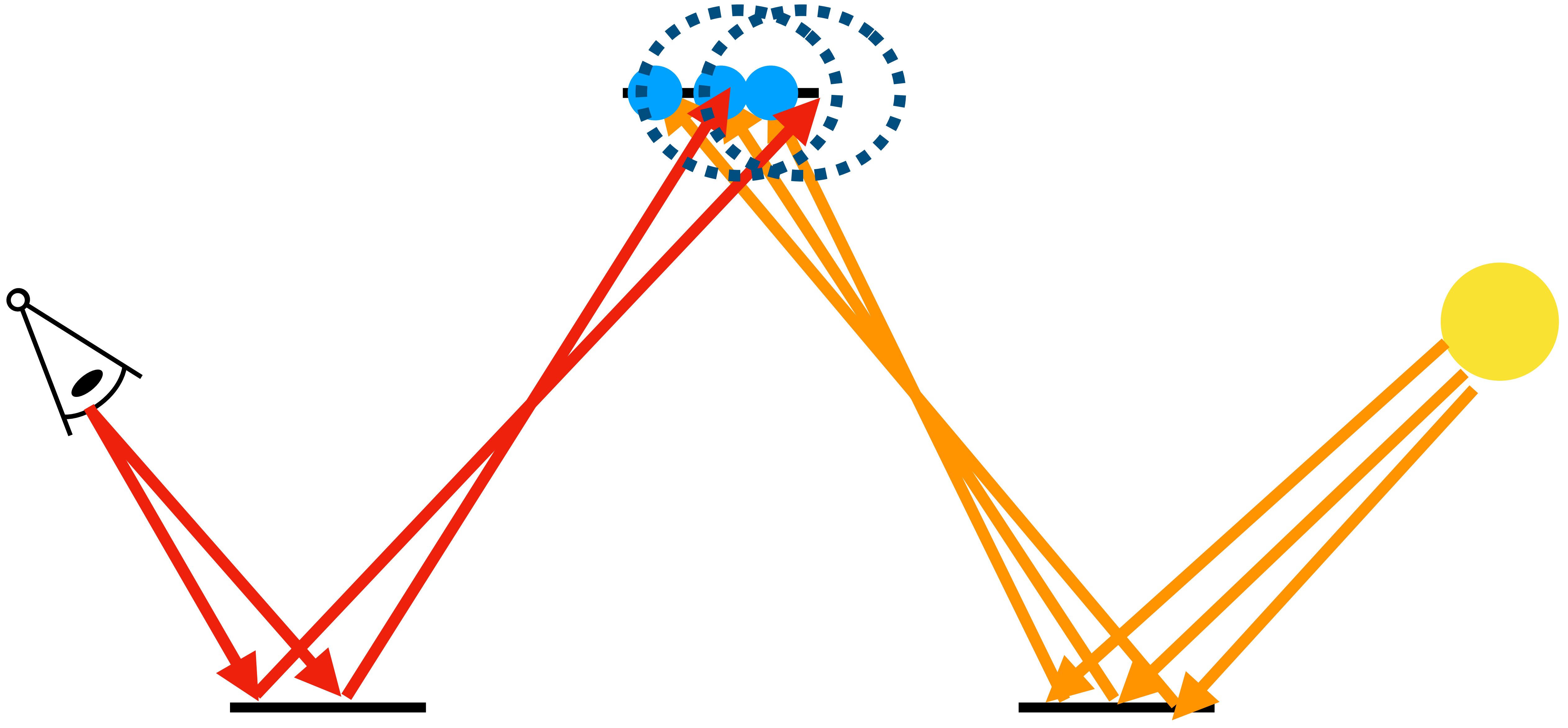
Photon mapping

1. trace random light subpaths

2. store **photons** on diffuse surfaces

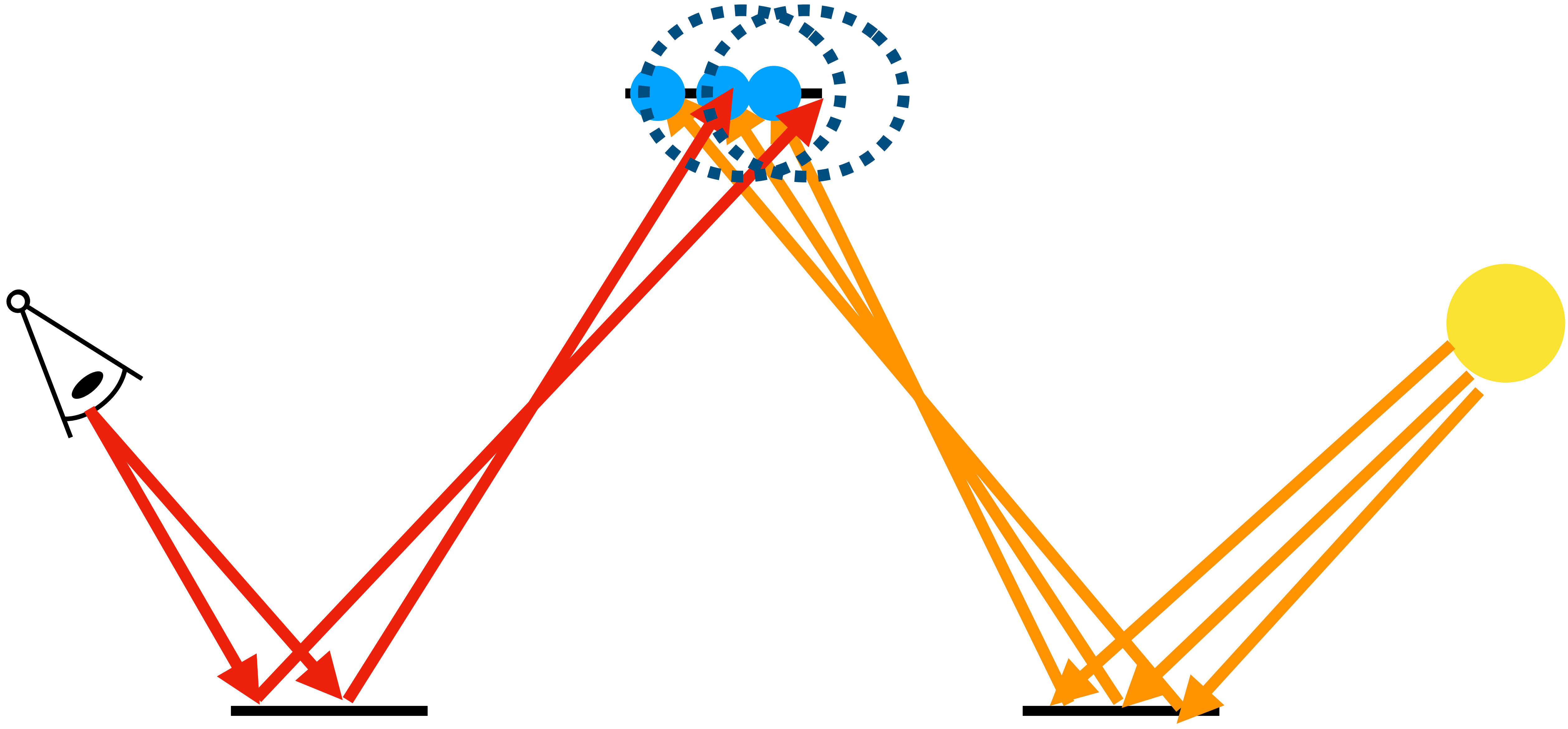
3. trace random camera subpaths

4. reconstruct path contribution from photons



Photon mapping

1. trace random light subpaths
2. store **photons** on diffuse surfaces
3. trace random camera subpaths
4. reconstruct path contribution from photons



Bidirectional Photon Mapping

Jiří Vorba
Supervised by: Jaroslav Křivánek

Charles University, Prague

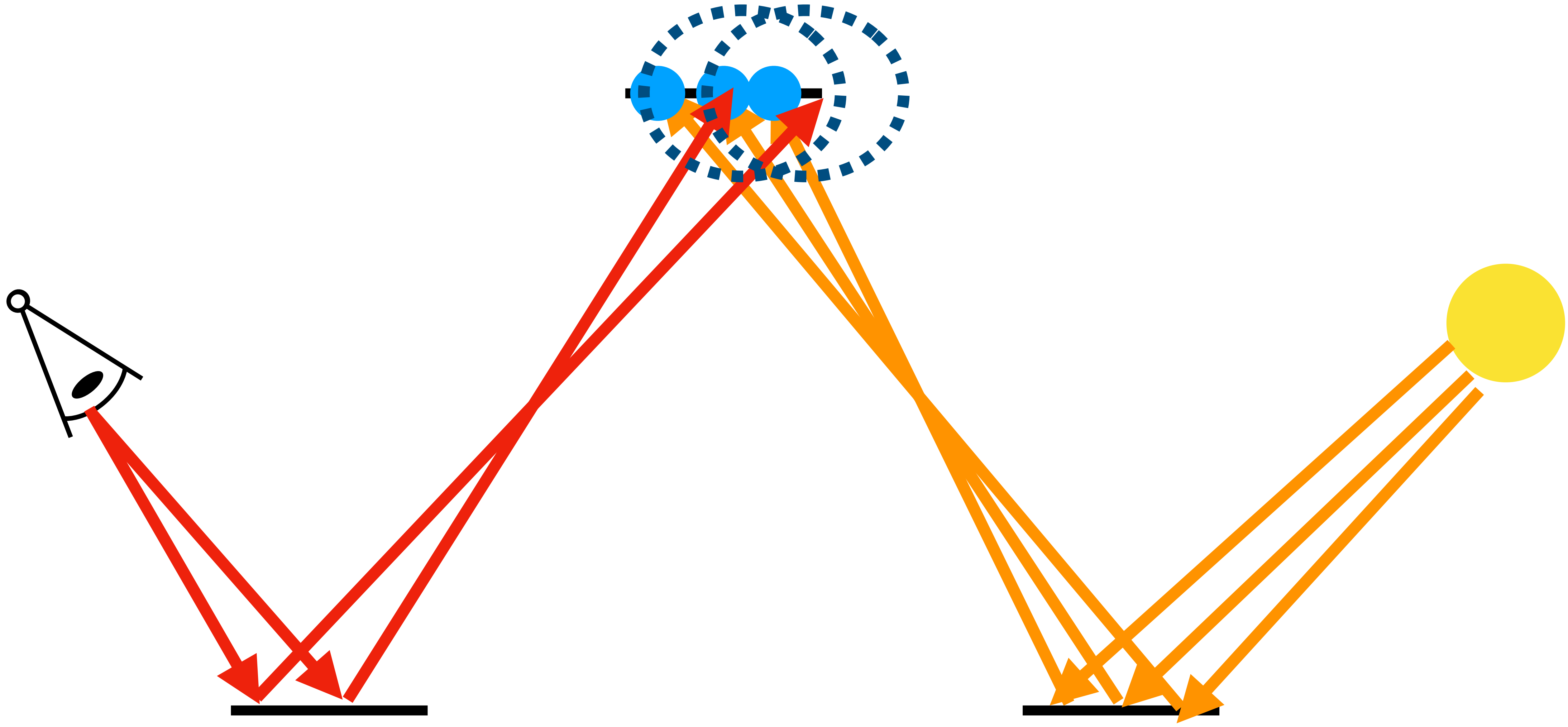
Photon mapping

1. trace random light subpaths

2. store **photons** on diffuse surfaces

3. trace random camera subpaths

4. reconstruct path contribution from photons

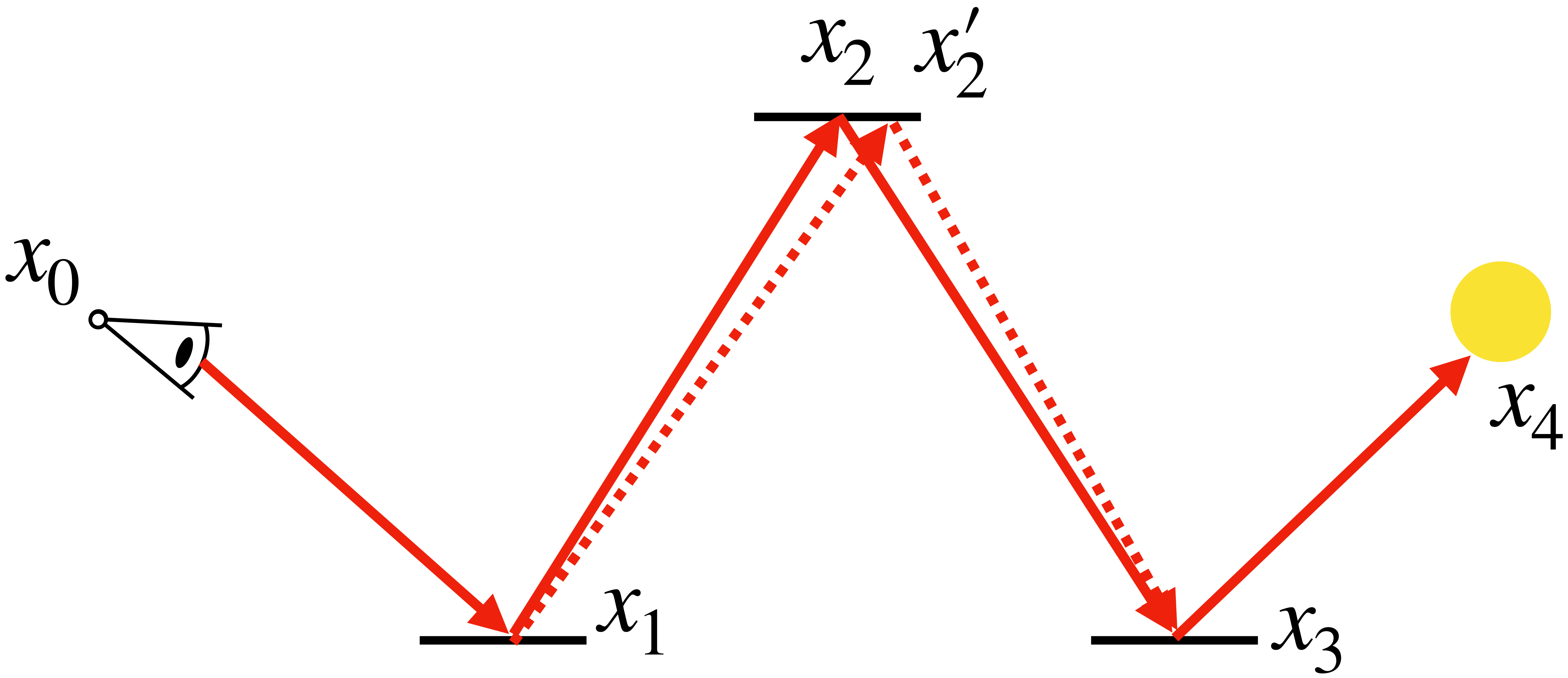


Math formulation: blurring path contribution

$$\int_{\text{light paths}} f(\bar{x})\,\mathrm{d}\bar{x} \longrightarrow \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x_2')f(\bar{x}')\,\mathrm{d}\bar{x}\,\mathrm{d}x_2'$$

k: convolution kernel

e.g. a disk kernel $\frac{1}{\pi r^2}$

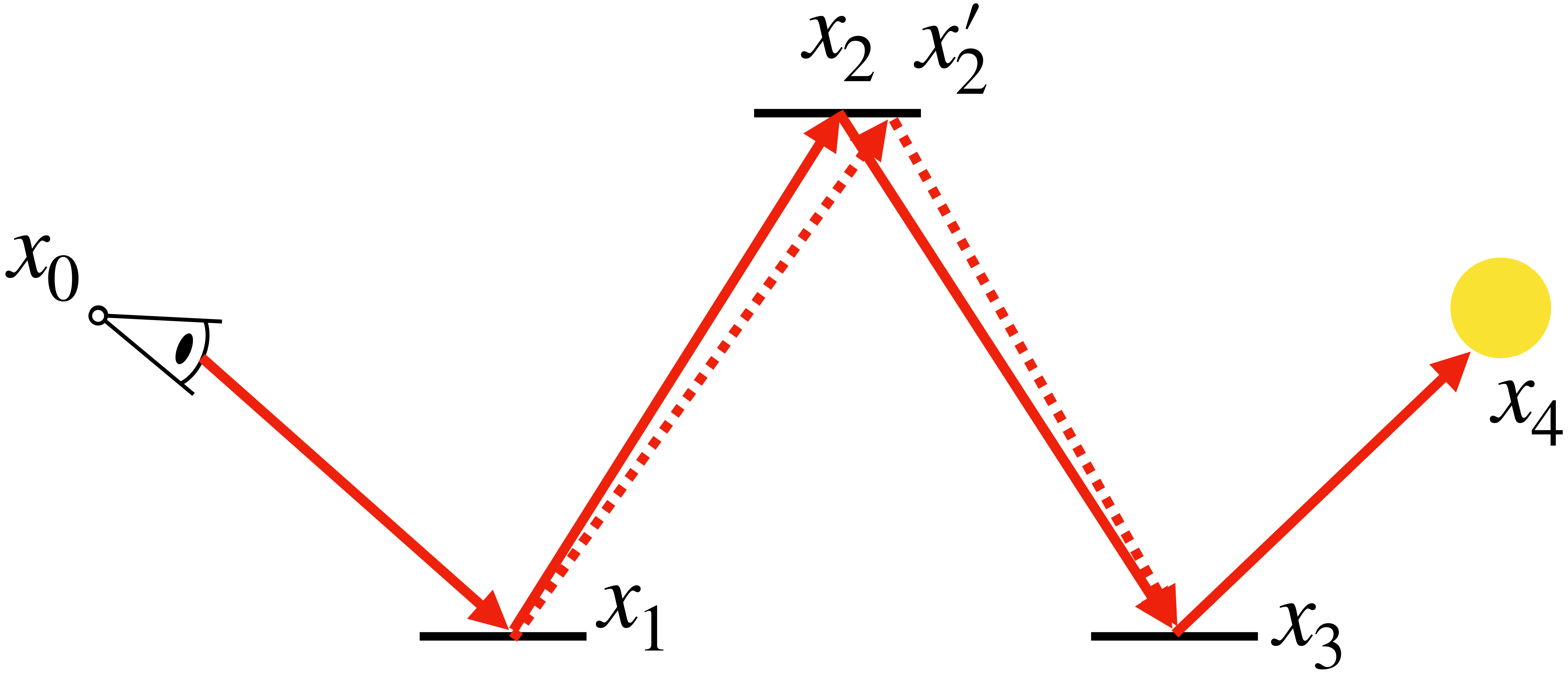


Math formulation: blurring path contribution

$$\int_{\text{light paths}} f(\bar{x})\mathrm{d}\bar{x} \longrightarrow \int_{\text{surface}}\int_{\text{light paths}} k(x_2, x_2')f(\bar{x}')\mathrm{d}\bar{x}\mathrm{d}x_2'$$

k: convolution kernel

e.g. a disk kernel $\frac{1}{\pi r^2}$



Light Transport Simulation with Vertex Connection and Merging

Iliyan Georgiev* Saarland University Intel VCI, Saarbrücken
Jaroslav Křivánek† Charles University, Prague
Tomaś Davidovič‡ Saarland University Intel VCI, Saarbrücken
Philipp Slusallek§ Saarland University Intel VCI & DFKI, Saarbrücken

A Path Space Extension for Robust Light Transport Simulation

Toshiya Hachisuka[1,3] [1]Aarhus University
Jacopo Pantaleoni[2] [2]NVIDIA Research
Henrik Wann Jensen[3] [3]UC San Diego

Sidetrack: blurring an integrand does *not* necessarily change its integral!

recall: integration = taking DC in frequency domain

$$\int f(x)\mathrm{d}x = \hat{f}(0)$$

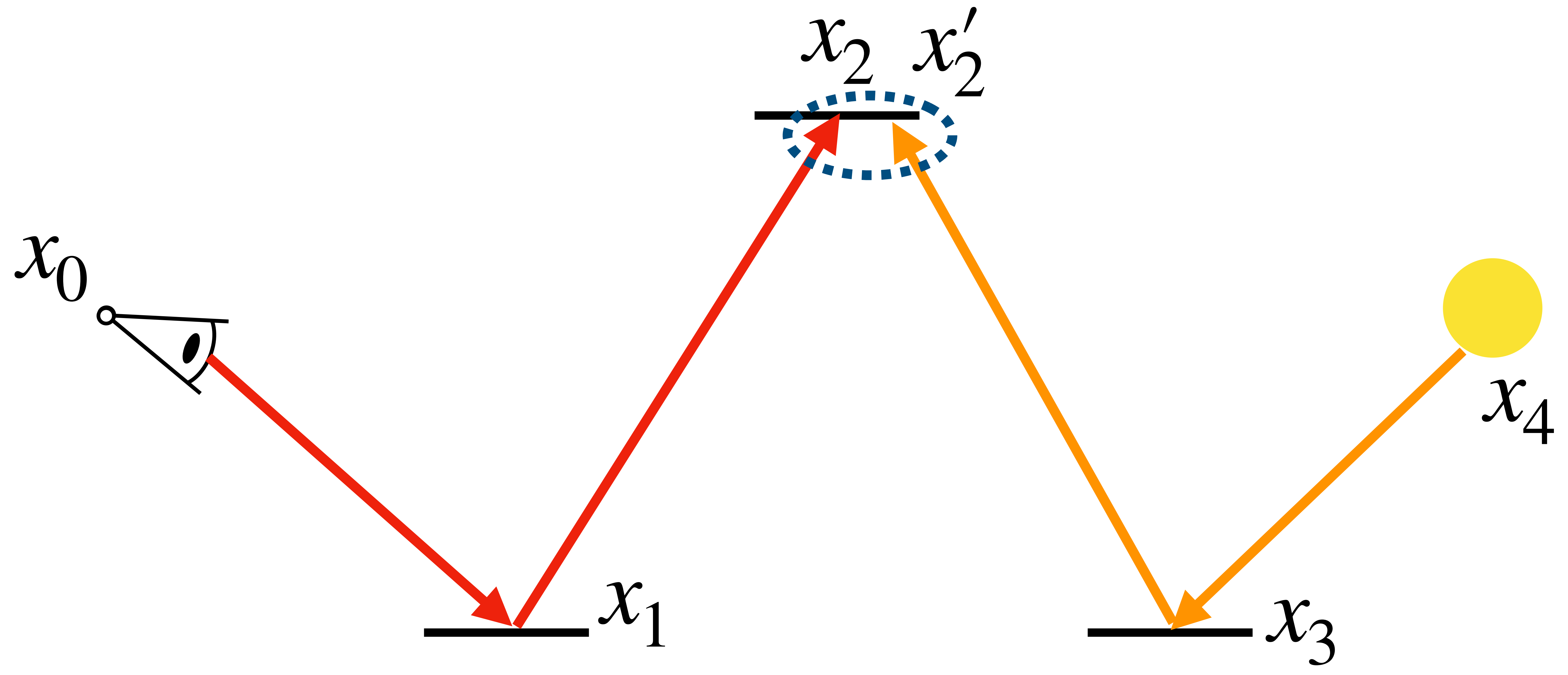
blurring = multiply the DCs in frequency domain

$$\int\int k(x, y)f(x)\mathrm{d}x\mathrm{d}y = \hat{f}(0)\hat{k}(0)$$

as long as $\hat{k}(0) = 1$, the integral is preserved!

Photon mapping: estimating the blurring integral using camera subpaths & light subpaths

$$\int_{\text{surface}} \int_{\text{light paths}} k(x_2, x_2') f(\bar{x}') \mathrm{d}\bar{x} \mathrm{d}x_2' \quad \approx \frac{k(x_2, x_2') f(\bar{x}')}{p(x_0 \to x_1 \to x_2) p(x_4 \to x_3 \to x_2')}$$

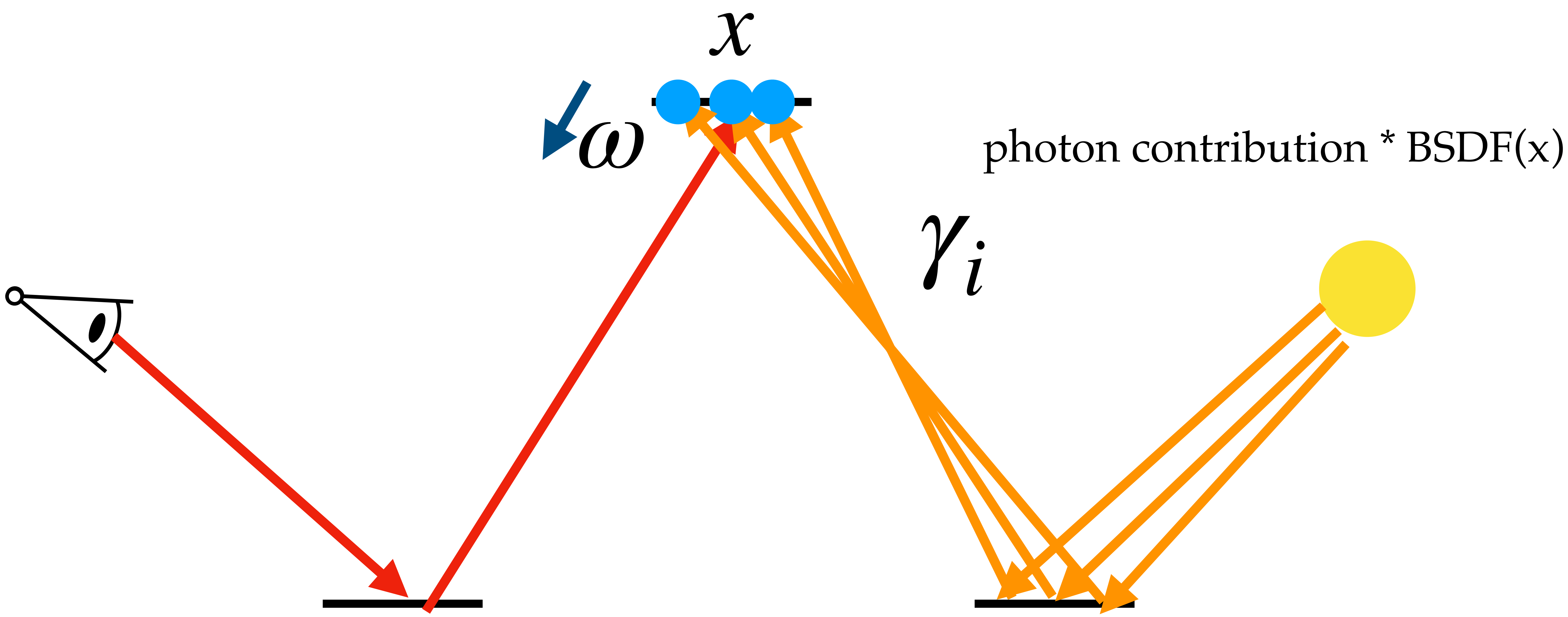


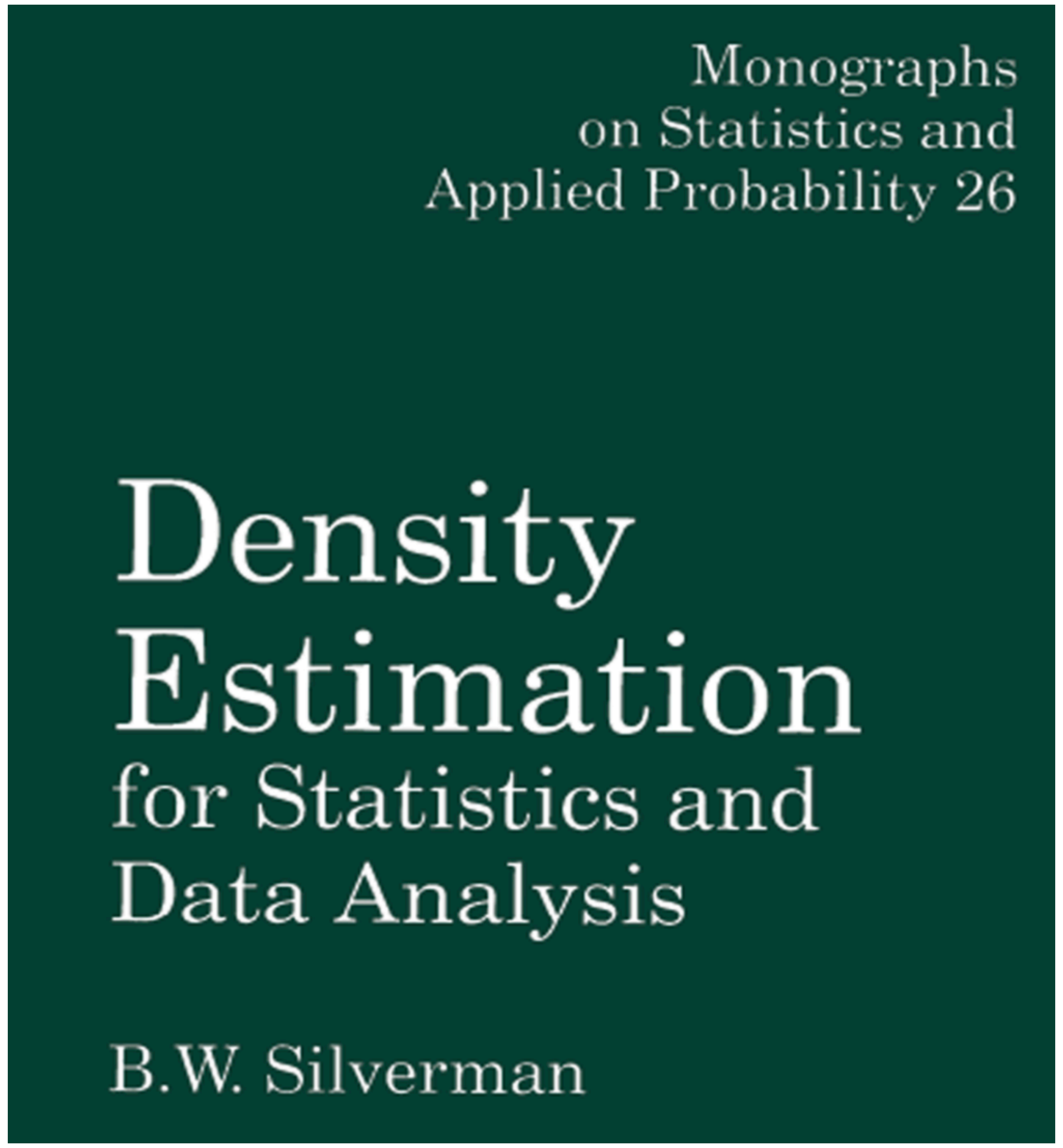
Density estimation interpretation of photon mapping

- reconstructing radiance at position x using randomly sampled photons at position x_i

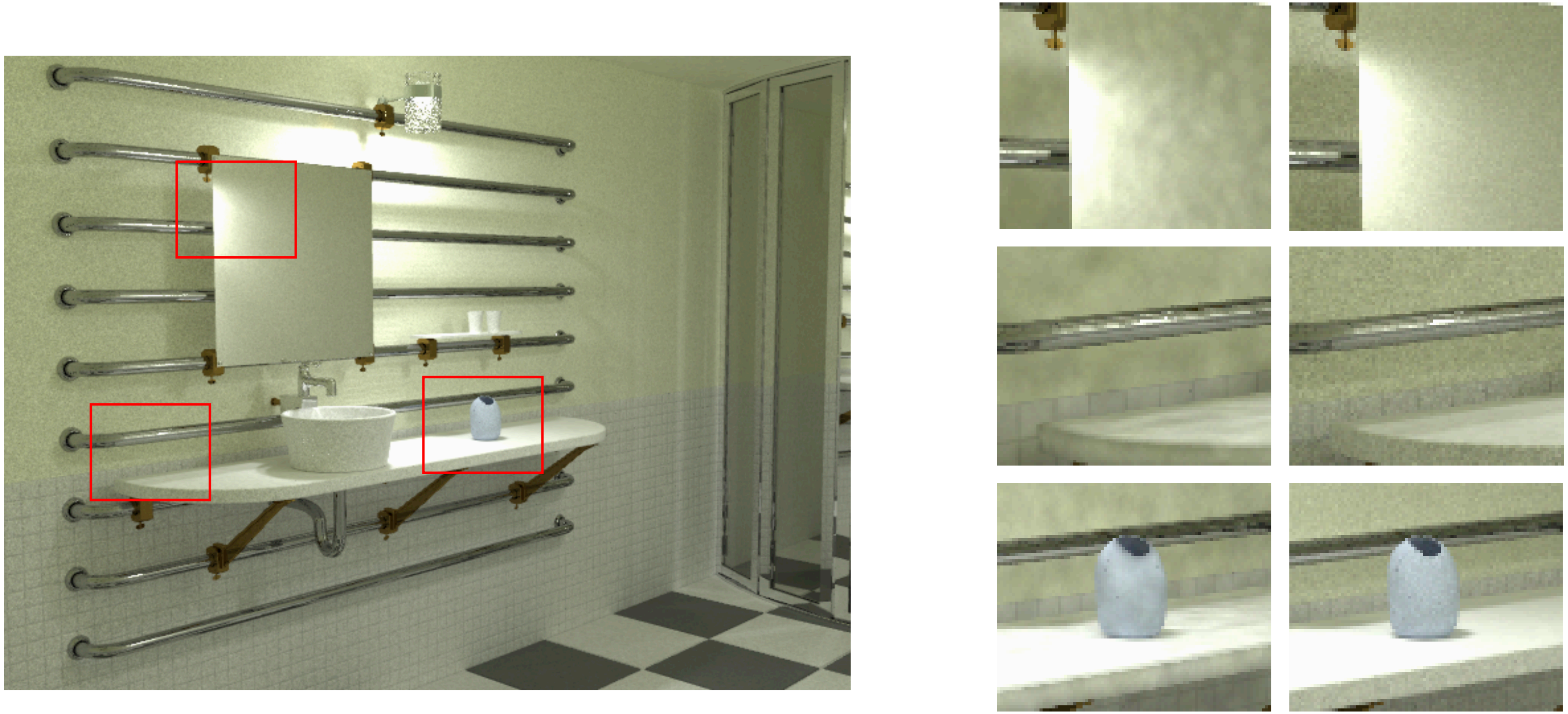
$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} k(x_i, x)\gamma_i$$

important:
N = all photons, not just photon nearby to x!

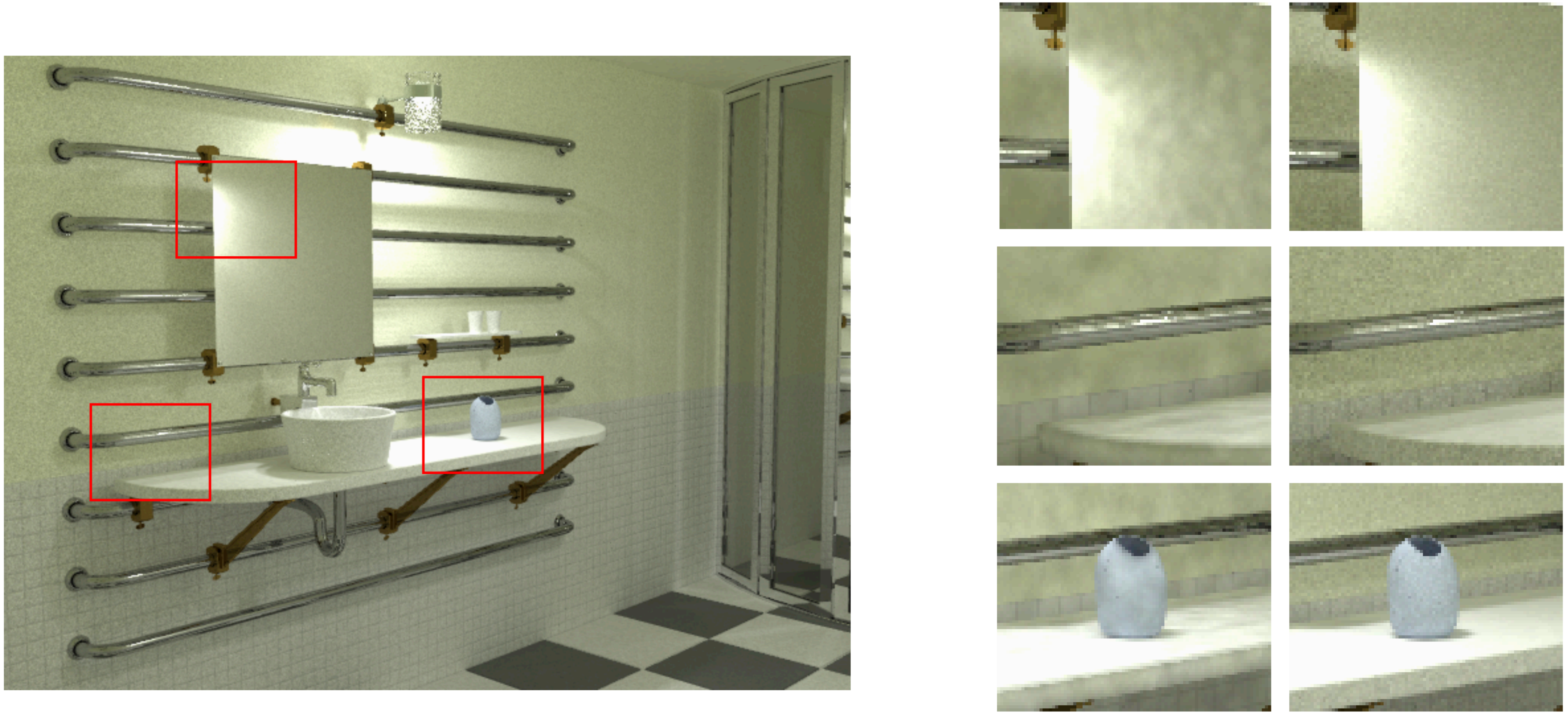




Bias-variance trade-off in photon mapping



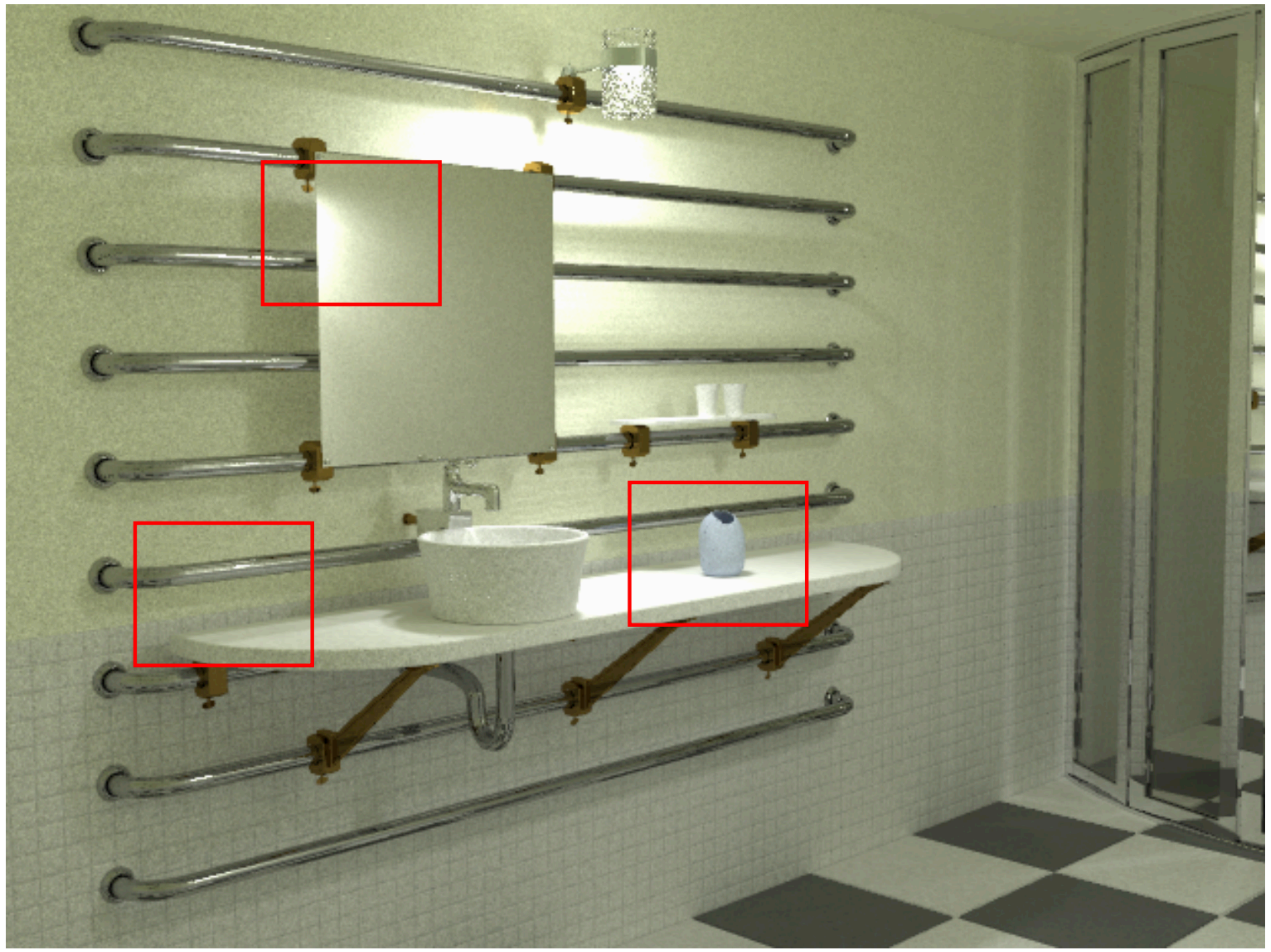
Bias-variance trade-off in photon mapping



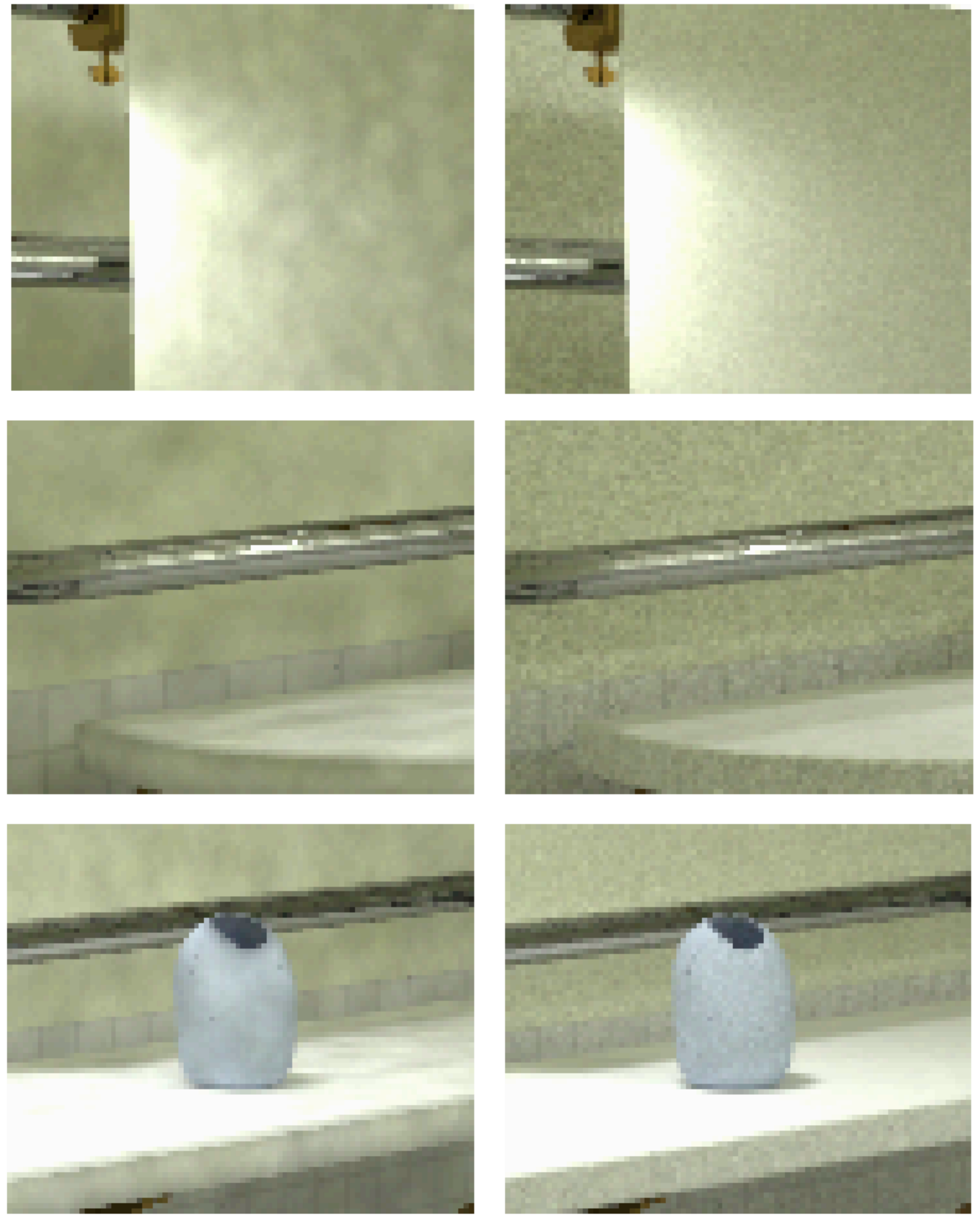
large radius
high bias,
low variance

small radius
low bias,
higher variance

Bias-variance trade-off in photon mapping



how do we analyze the effect of the interpolation radius?



large radius
high bias,
low variance

small radius
low bias,
higher variance

Bias-variance analysis of photon mapping

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int tk(t)\mathrm{d}t = 0$$

Progressive Photon Mapping: A Probabilistic Approach

Claude Knaus and Matthias Zwicker
University of Bern, Switzerland

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i\right] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int tk(t)\mathrm{d}t = 0$$

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right]E[\gamma] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int tk(t)\mathrm{d}t = 0$$

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right]E[\gamma] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

$$E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right] = \frac{1}{r^2}\int k\left(\frac{X-x}{r}\right)p(X)\mathrm{d}X$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int t k(t)\mathrm{d}t = 0$$

p(X): PDF of a photon landing at location X

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right]E[\gamma] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

$$E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right] = \frac{1}{r^2}\int k\left(t\right)p(x+rt)\mathrm{d}t$$

$$t = \frac{X-x}{r}$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int tk(t)\mathrm{d}t = 0$$

p(X): PDF of a photon landing at location X

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right]E[\gamma] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

$$E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right] = \frac{1}{r^2}\int k\,(t)\,p(x+rt)\mathrm{d}t \qquad t = \frac{X-x}{r}$$

$$p(x+rt) \approx p(x) + rt\,\nabla p(x) + r^2 t^T H_p(x)t$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int tk(t)\mathrm{d}t = 0$$

p(X): PDF of a photon landing at location X

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right]E[\gamma] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i-x}{r}\right)\gamma_i$$

$$E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right] = \frac{1}{r^2}\int k\left(t\right)p(x+rt)\mathrm{d}t \qquad t = \frac{X-x}{r}$$

$$p(x+rt) \approx p(x) + rt\nabla p(x) + r^2 t^T H_p(x) t$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int tk(t)\mathrm{d}t = 0$$

$$\int k\left(t\right)p(x+rt)\mathrm{d}t \approx p(x) + r^2 \cdot \int t^T H_p(x) t \mathrm{d}t$$

p(X): PDF of a photon landing at location X

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right]E[\gamma] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

$$E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right] \approx p(x) + r^2 \cdot \int t^T H_p(x) t \, \mathrm{d}t$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int t k(t)\mathrm{d}t = 0$$

p(X): PDF of a photon landing at location X

Bias-variance analysis of photon mapping

$$\text{bias} = E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right]E[\gamma] - L(x,\omega)$$

$$L(x,\omega) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{r^2}k\left(\frac{x_i - x}{r}\right)\gamma_i$$

$$E\left[\frac{1}{r^2}k\left(\frac{X-x}{r}\right)\right] \approx p(x) + r^2 \cdot \int t^T H_p(x) t \mathrm{d}t$$

normalize kernel s.t. $x' - x$
is constrained to a unit circle

$$L(x,\omega) = E[\gamma]E[\delta(X-x)] = E[\gamma]p(x)$$

$$\frac{1}{r^2}\int k(t)\mathrm{d}t = 1 \qquad \int tk(t)\mathrm{d}t = 0$$

p(X): PDF of a photon landing at location X

Bias-variance analysis of photon mapping

$$\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t \mathrm{d}t$$

$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k\left(\frac{x_i - x}{r}\right) \gamma_i$$

Bias-variance analysis of photon mapping

$$\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t \, \mathrm{d}t$$

$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k\left(\frac{x_i - x}{r}\right) \gamma_i$$

$$\text{variance} \approx \left(\mathrm{Var}[\gamma] + E[\gamma]^2\right) \frac{p(x)}{Nr^2} \int k(t)^2 \mathrm{d}t$$

Bias-variance analysis of photon mapping

$$\text{bias} \approx \boxed{r^2} E[\gamma] \int t^T H_p(x) t \, \mathrm{d}t$$

$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k\left(\frac{x_i - x}{r}\right) \gamma_i$$

$$\text{variance} \approx \left(\mathrm{Var}[\gamma] + E[\gamma]^2\right) \frac{p(x)}{\boxed{N r^2}} \int k(t)^2 \mathrm{d}t$$

Observation:

- variance reduces with N, bias does not
- bias increases with r, but variance reduces with r

Bias-variance analysis of photon mapping

$$\text{bias} \propto r^2 \qquad\qquad \text{variance} \propto \frac{1}{Nr^2}$$

Observation:

- variance reduces with N, bias does not
- bias increases with r, but variance reduces with r

Bias-variance analysis of photon mapping

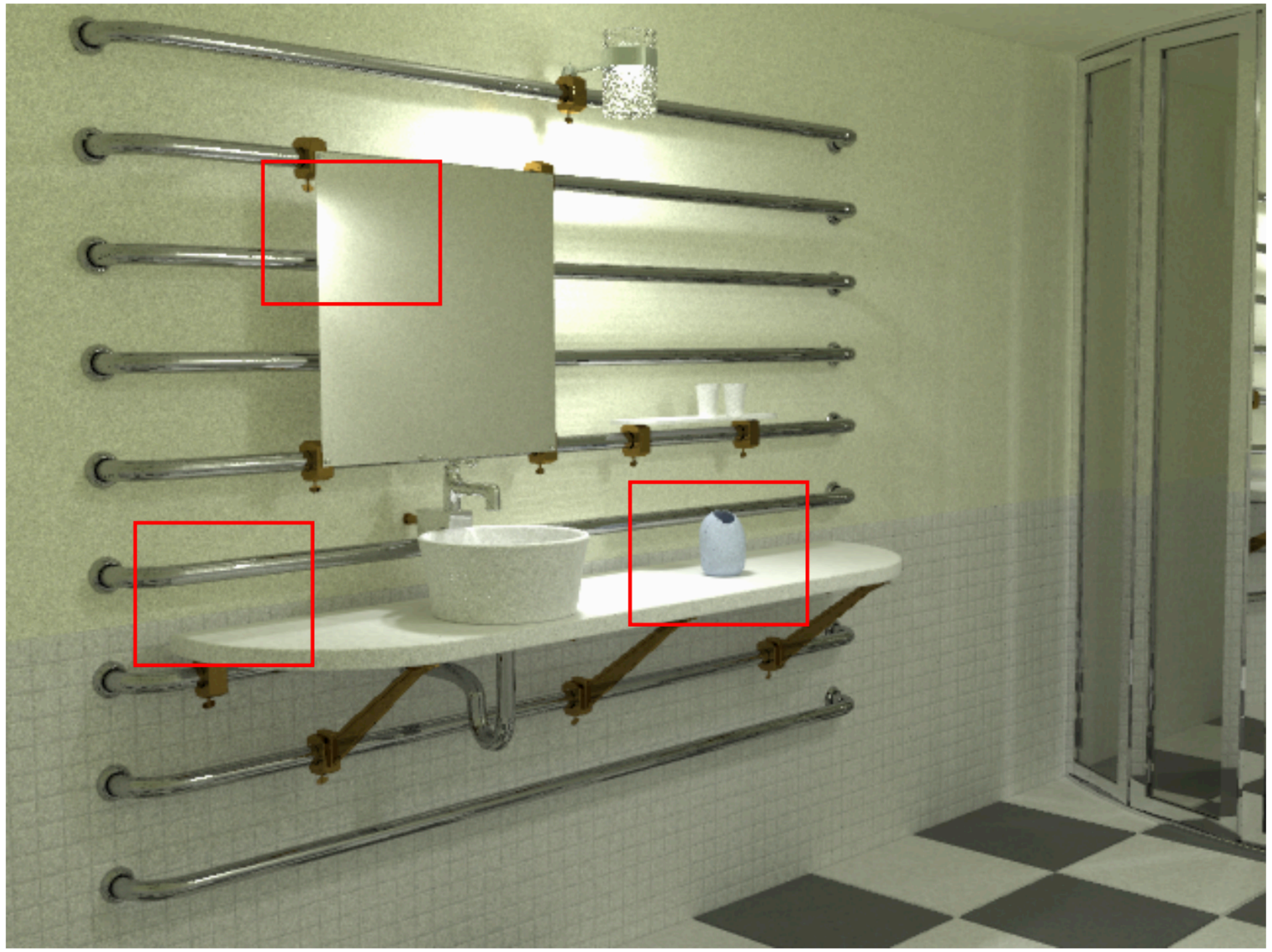
quiz: is photon mapping a consistent estimator?

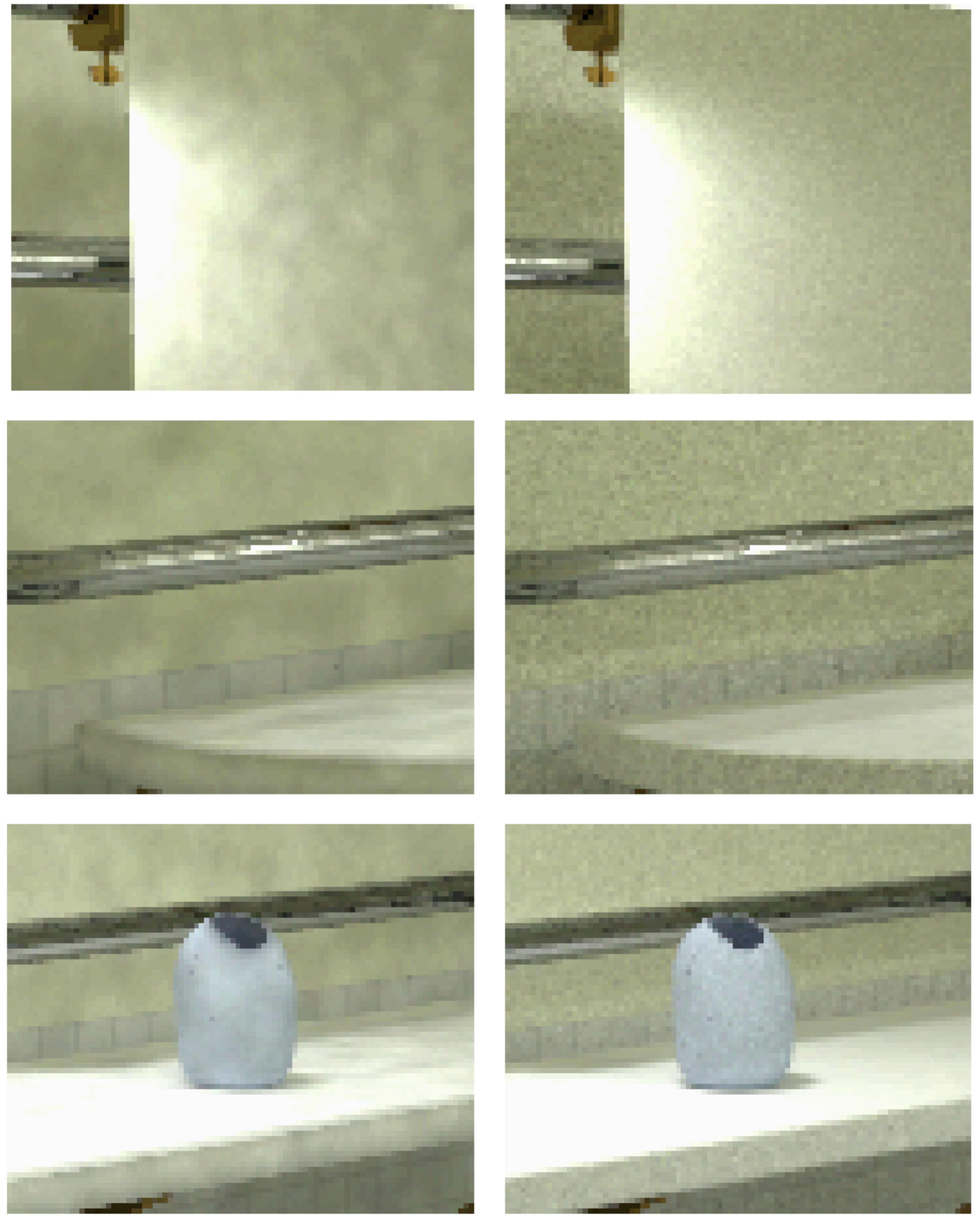
$$\text{bias} \propto r^2 \qquad\qquad \text{variance} \propto \frac{1}{Nr^2}$$

Observation:

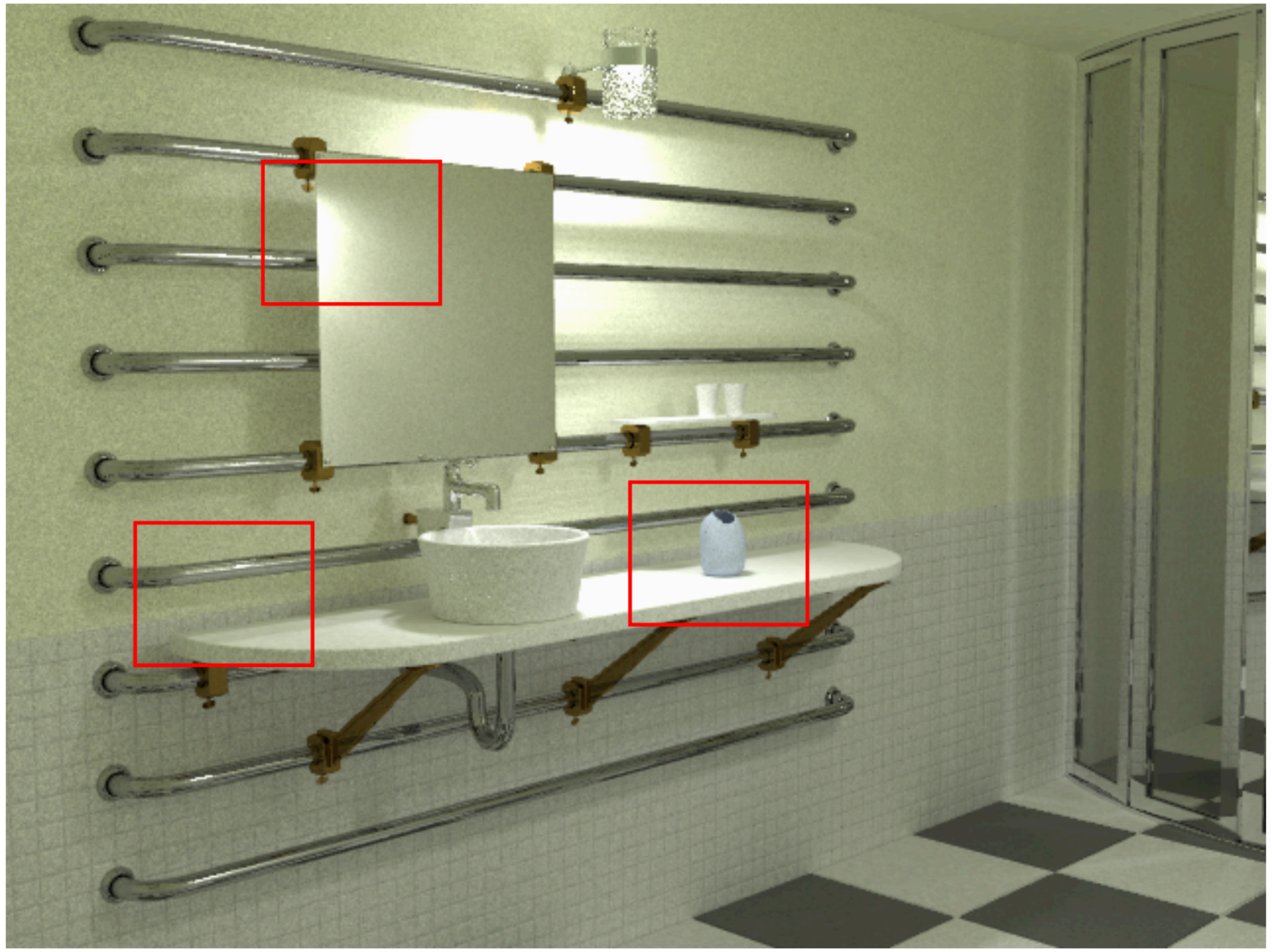
- variance reduces with N, bias does not
- bias increases with r, but variance reduces with r

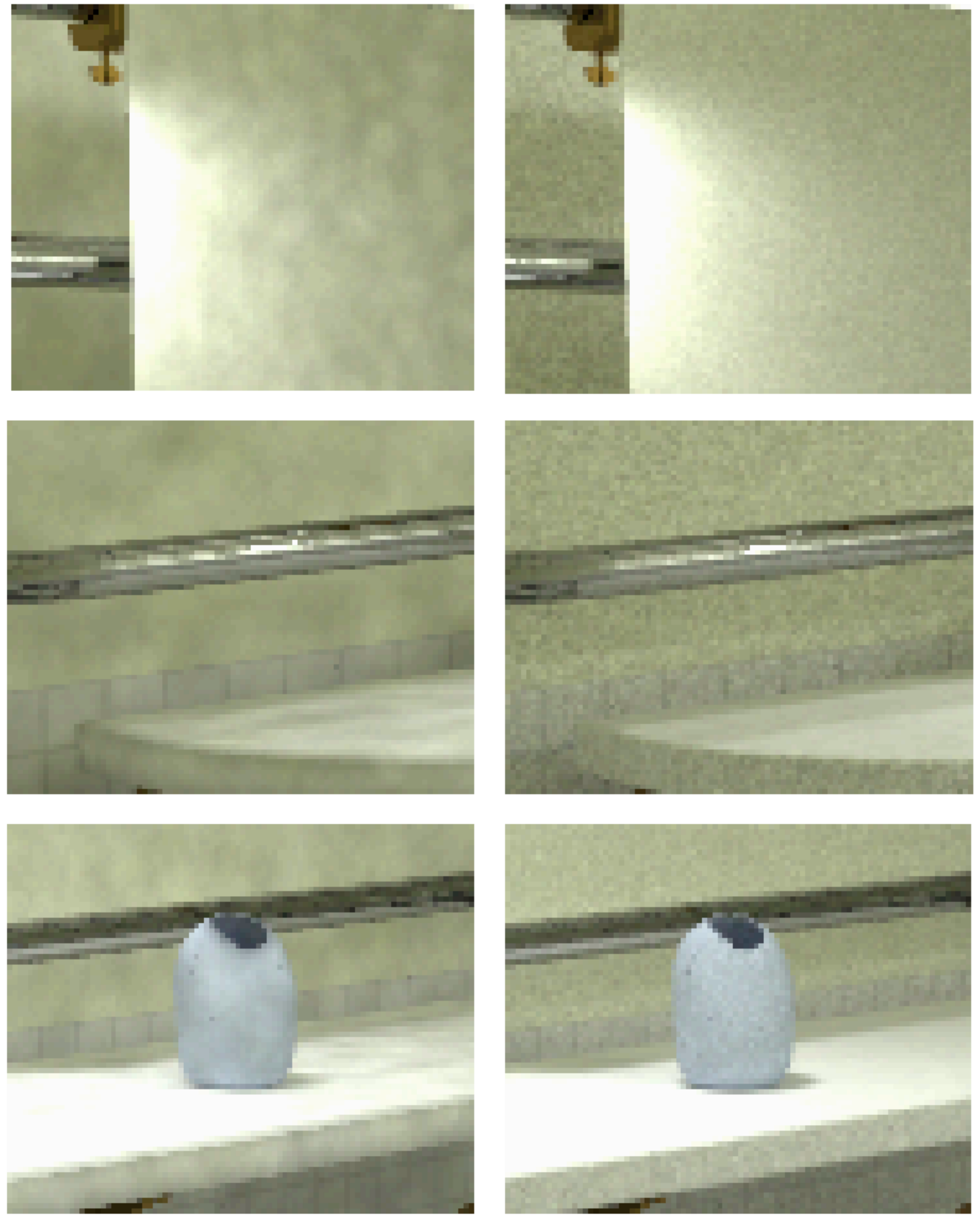
Bias-variance trade-off in photon mapping





Bias-variance trade-off in photon mapping





large radius
high bias,
low variance

small radius
low bias,
higher variance

Epanechnikov kernel minimizes the variance

$$k(t) = \begin{cases} \frac{3}{4\sqrt{5}}\left(1 - \frac{1}{5}t^2\right) & -\sqrt{5} \le t \le \sqrt{5} \\ 0 & \text{otherwise} \end{cases}$$

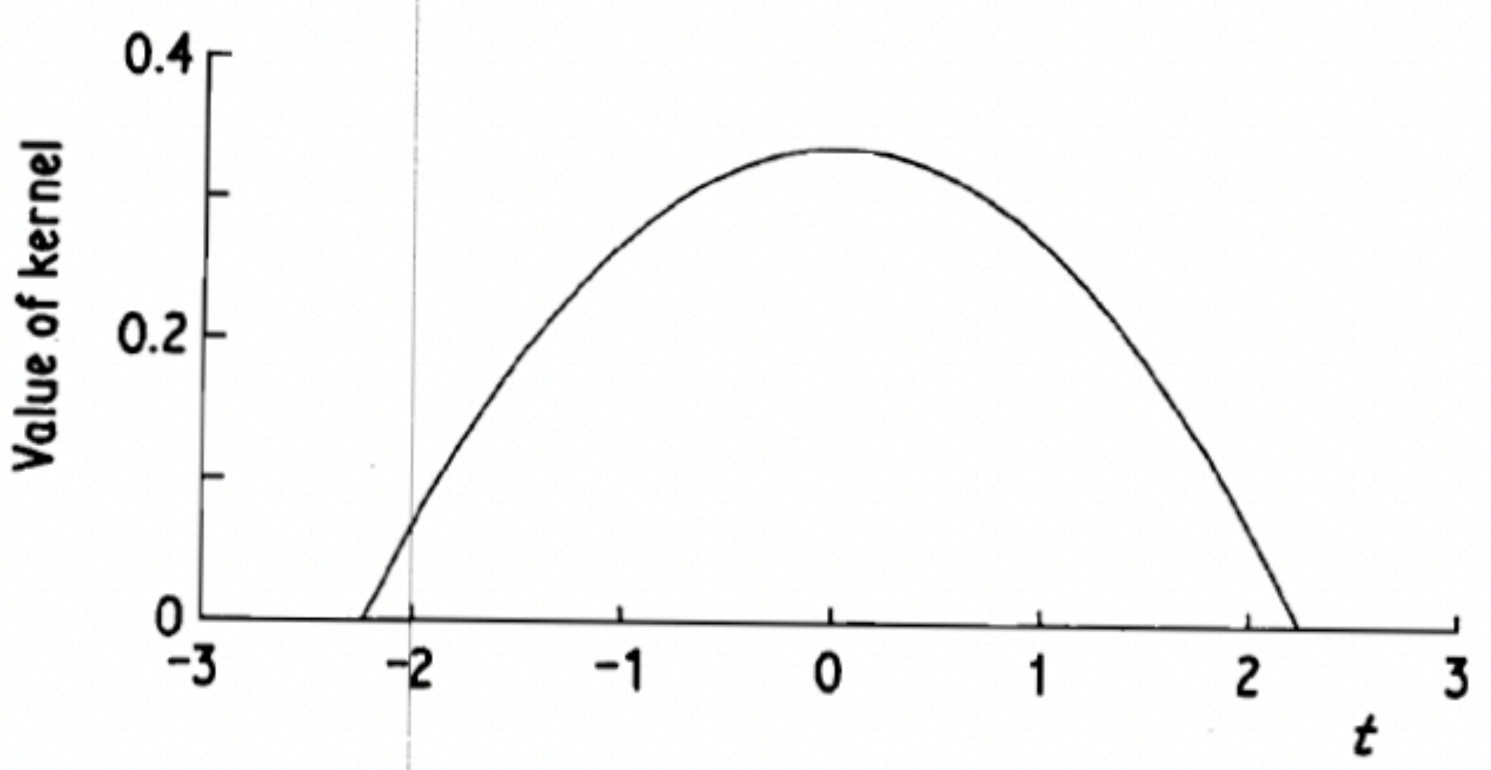


Fig. 3.1 *The Epanechnikov kernel.*

Silverman 1986

$$\text{variance} \approx \left(\text{Var}[\gamma] + E[\gamma]^2\right) \frac{p(x)}{Nr^2} \int k(t)^2 \mathrm{d}t$$

$$\text{minimize} \int k(t)^2 \mathrm{d}t$$

$$\text{s.t.} \quad \frac{1}{r^2} \int k(t)\mathrm{d}t = 1 \qquad \int t k(t) \mathrm{d}t = 0$$

Progressive photon mapping: a consistent photon mapping estimator

$$L \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k\left(\frac{x_i - x}{r}\right) \gamma_i$$

$$\text{bias} \propto r^2$$

$$\text{variance} \propto \frac{1}{Nr^2}$$

can we eliminate bias when N goes to infinity?

Toshiya Hachisuka, UC San Diego
Shinji Ogaki, The University of Nottingham
Henrik Wann Jensen, UC San Diego

Progressive photon mapping: a consistent photon mapping estimator

- key idea: select a sequence r_i with gradually reduced radius to remove bias

- can't reduce too fast, can't reduce too slow

$$\text{bias} \propto r^2$$

$$\text{variance} \propto \frac{1}{Nr^2}$$

$$L \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r_i^2} k\left(\frac{x_i - x}{r_i}\right) \gamma_i$$

Progressive Photon Mapping

Toshiya Hachisuka, UC San Diego — Shinji Ogaki, The University of Nottingham — Henrik Wann Jensen, UC San Diego

Progressive photon mapping: a consistent photon mapping estimator

- key idea: select a sequence r_i with gradually reduced radius to remove bias

- can't reduce too fast, can't reduce too slow

$$L \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r_i^2} k\left(\frac{x_i - x}{r_i}\right) \gamma_i$$

for each iteration i

$$\text{bias} \propto r_i^2$$

$$\text{variance} \propto \frac{1}{r_i^2}$$

Progressive Photon Mapping

Toshiya Hachisuka, UC San Diego
Shinji Ogaki, The University of Nottingham
Henrik Wann Jensen, UC San Diego

Progressive photon mapping: a consistent photon mapping estimator

goal: decrease *r* so that bias goes to 0,
but variance does not go to infinity

for each iteration i

$$\text{bias} \propto r_i^2$$

$$\text{variance} \propto \frac{1}{r_i^2}$$

Progressive photon mapping: a consistent photon mapping estimator

goal: decrease r so that bias goes to 0, but variance does not go to infinity

idea: set r_i such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i+\alpha}{i+1}$ $(\alpha \in (0,1))$

for each iteration i

$$\text{bias} \propto r_i^2$$

$$\text{variance} \propto \frac{1}{r_i^2}$$

Progressive photon mapping: a consistent photon mapping estimator

for each iteration i

$$\text{bias} \propto r_i^2$$

$$\text{variance} \propto \frac{1}{r_i^2}$$

goal: decrease r so that bias goes to 0, but variance does not go to infinity

idea: set r_i such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i+\alpha}{i+1}$ $\quad (\alpha \in (0,1))$

$$\frac{\text{Var}_{i+1}}{\text{Var}_i} = \frac{i+1}{i+\alpha} \qquad \frac{\text{Bias}_{i+1}}{\text{Bias}_i} = \frac{i+\alpha}{i+1}$$

Progressive photon mapping: a consistent photon mapping estimator

goal: decrease r so that bias goes to 0, but variance does not go to infinity

idea: set r_i such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i+\alpha}{i+1}$ $(\alpha \in (0,1))$

for each iteration i

$$\text{bias} \propto r_i^2$$

$$\text{variance} \propto \frac{1}{r_i^2}$$

$$\frac{\text{Var}_{i+1}}{\text{Var}_i} = \frac{i+1}{i+\alpha} \qquad \frac{\text{Bias}_{i+1}}{\text{Bias}_i} = \frac{i+\alpha}{i+1}$$

$$\text{Var} = \frac{1}{N^2}\sum_i \text{Var}_i = O\left(N^{-\alpha}\right) \qquad \text{Bias} = \frac{1}{N}\sum_i \text{Bias}_i = O\left(N^{1-\alpha}\right)$$

Progressive photon mapping: a consistent photon mapping estimator

goal: decrease r so that bias goes to 0, but variance does not go to infinity

idea: set r_i such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i+\alpha}{i+1}$ $(\alpha \in (0,1))$

$$\text{Var} = \frac{1}{N^2} \sum_i \text{Var}_i = O\left(N^{-\alpha}\right)$$

$$\text{Bias} = \frac{1}{N} \sum_i \text{Bias}_i = O\left(N^{1-\alpha}\right)$$

Progressive photon mapping: a consistent photon mapping estimator

goal: decrease r so that bias goes to 0,
but variance does not go to infinity

idea: set r_i such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i+\alpha}{i+1}$ $(\alpha \in (0,1))$

$$\text{Var} = \frac{1}{N^2}\sum_i \text{Var}_i = O\left(N^{-\alpha}\right)$$

$$\text{Bias} = \frac{1}{N}\sum_i \text{Bias}_i = O\left(N^{1-\alpha}\right)$$

quiz: what is the asymptotically optimal α?

Progressive photon mapping: a consistent photon mapping estimator

goal: decrease r so that bias goes to 0,
but variance does not go to infinity

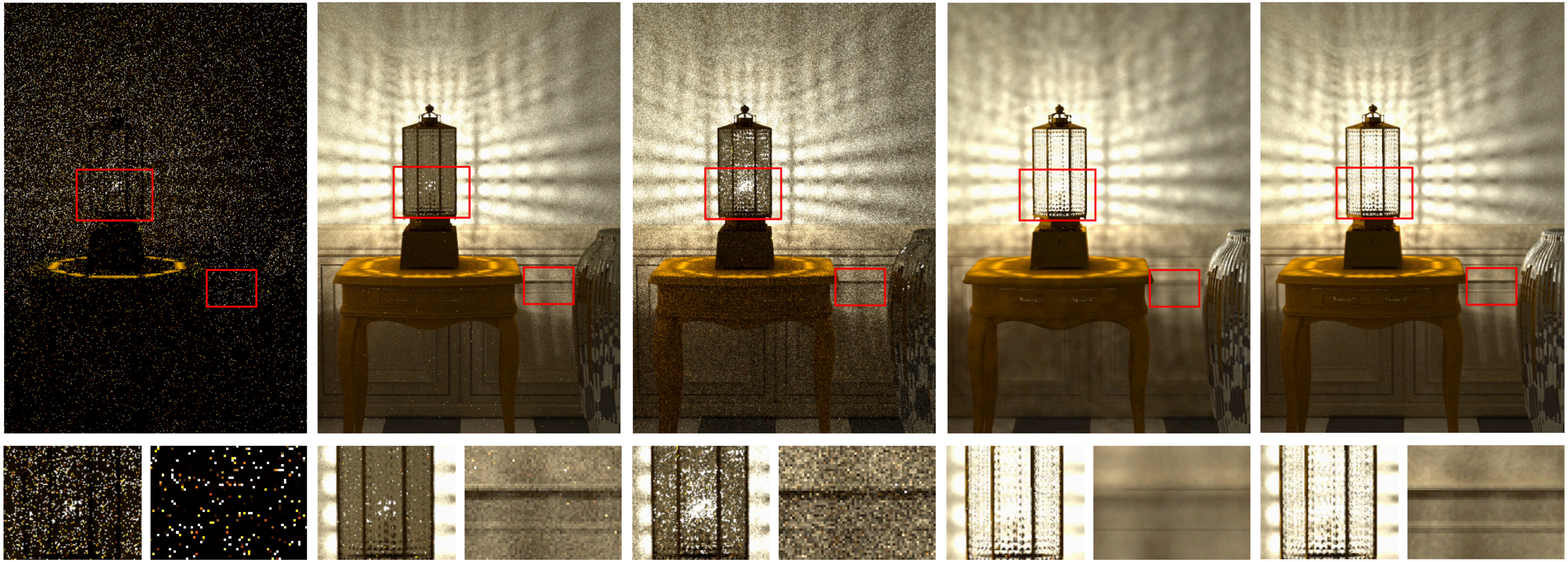
idea: set r_i such that $\frac{r_{i+1}^2}{r_i^2} = \frac{i+\alpha}{i+1}$ $(\alpha \in (0,1))$

$$\text{Var} = \frac{1}{N^2} \sum_i \text{Var}_i = O\left(N^{-\alpha}\right)$$

$$\text{Bias} = \frac{1}{N} \sum_i \text{Bias}_i = O\left(N^{1-\alpha}\right)$$

$\alpha = \frac{2}{3}$ gives optimal mean square error = bias^2 + variance

Photon mapping is good at SDS light paths



Path tracing | Bidirectional path tracing | Metropolis light transport | Photon mapping | Progressive photon mapping

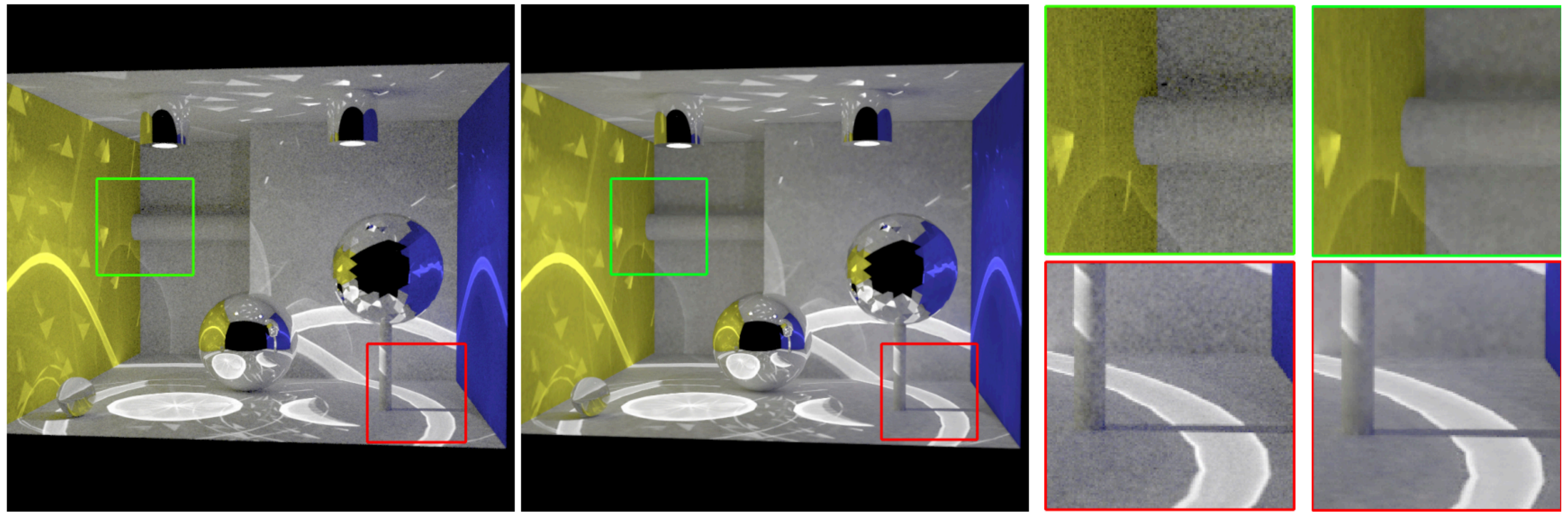
Photon mapping is good at SDS light paths

PT BDPT MLT PPM Reference

Alternative: directly set *r* to minimize mean square error

Adaptive Progressive Photon Mapping

ANTON S. KAPLANYAN and CARSTEN DACHSBACHER
Karlsruhe Institute of Technology



(a) progressive photon mapping (b) adaptive PPM ~~(our method)~~ (c) PPM (d) ~~our method~~

Anton's method Anton's method

$$\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t \, \mathrm{d}t$$

$$\text{variance} \approx \left(\mathrm{Var}[\gamma] + E[\gamma]^2\right) \frac{p(x)}{Nr^2} \int k(t)^2 \mathrm{d}t$$

mean square error = bias^2 + variance

Combining with bidirectional path tracing (VCM/UPS)

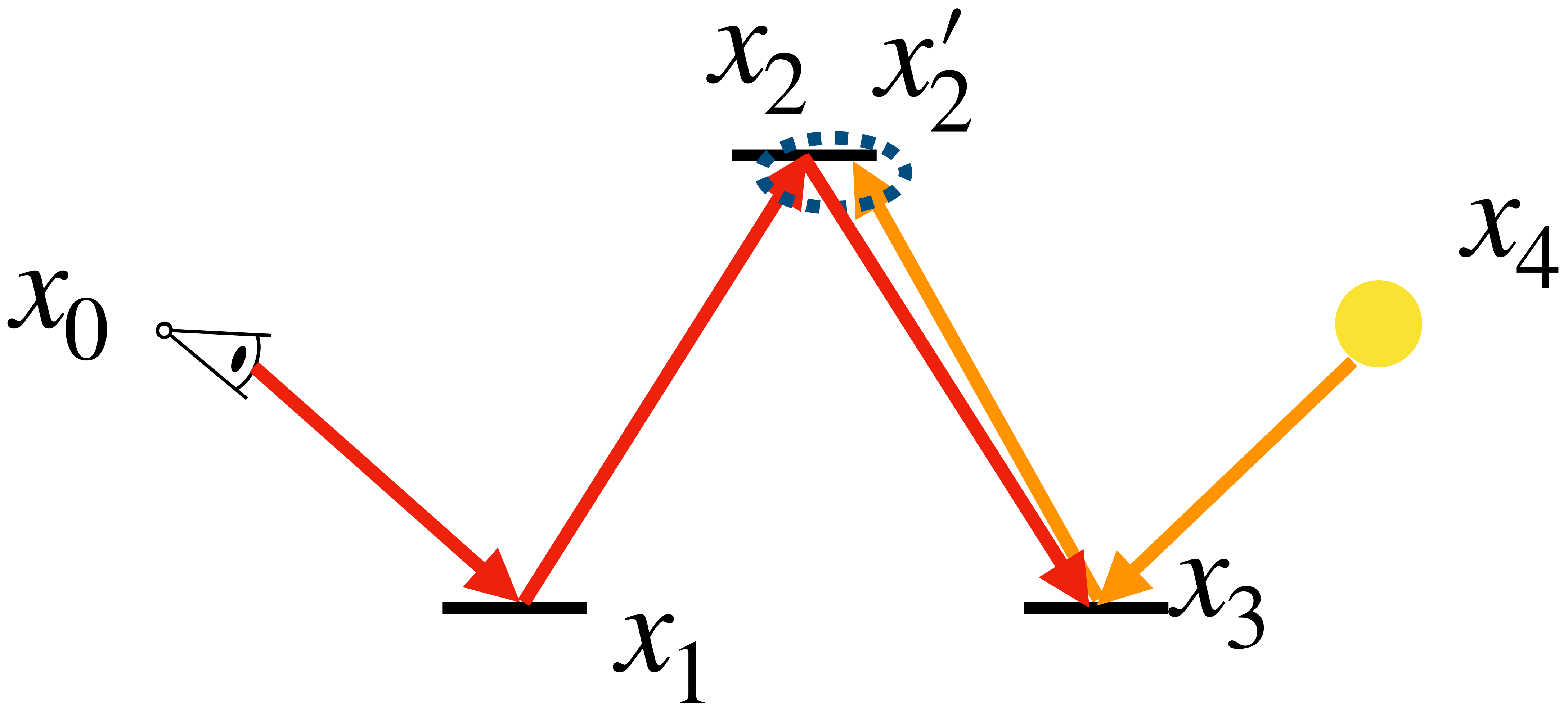
- apply multiple importance sampling

Light Transport Simulation with Vertex Connection and Merging

Iliyan Georgiev* Saarland University Intel VCI, Saarbrücken
Jaroslav Křivánek† Charles University, Prague
Tomaś Davidovič‡ Saarland University Intel VCI, Saarbrücken
Philipp Slusallek§ Saarland University Intel VCI & DFKI, Saarbrücken

A Path Space Extension for Robust Light Transport Simulation

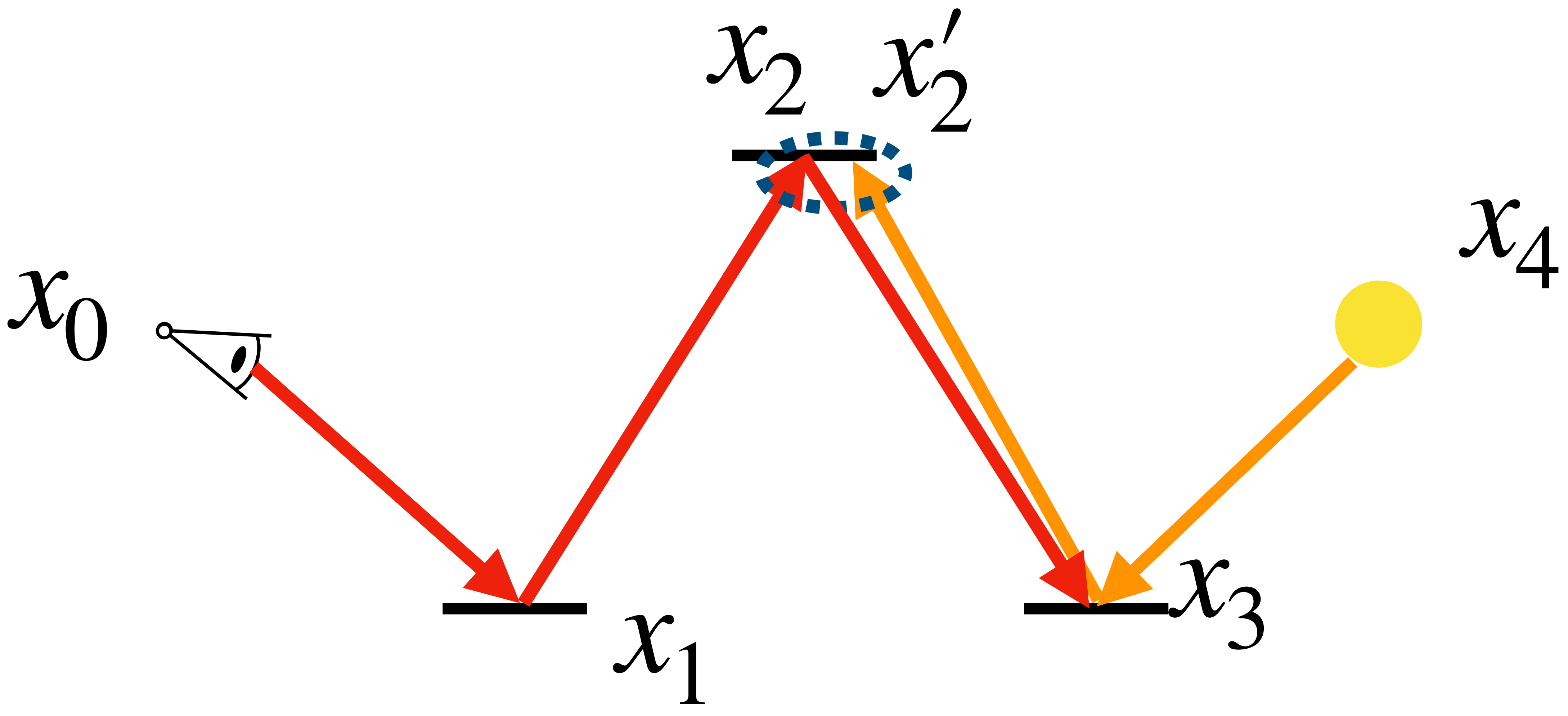
Toshiya Hachisuka[1,3] Jacopo Pantaleoni[2] Henrik Wann Jensen[3]
[1]Aarhus University [2]NVIDIA Research [3]UC San Diego



Combining with bidirectional path tracing (VCM/UPS)

- apply multiple importance sampling
- challenge: photon mapping has one more vertex (x_2' in this case), can't compare PDFs

path tracing: $x_0x_1x_2x_3x_4$
photon mapping: $x_0x_1x_2x_2'x_3x_4$



Light Transport Simulation with Vertex Connection and Merging

Iliyan Georgiev* Saarland University Intel VCI, Saarbrücken
Jaroslav Křivánek† Charles University, Prague
Tomaś Davidovič‡ Saarland University Intel VCI, Saarbrücken
Philipp Slusallek§ Saarland University Intel VCI & DFKI, Saarbrücken

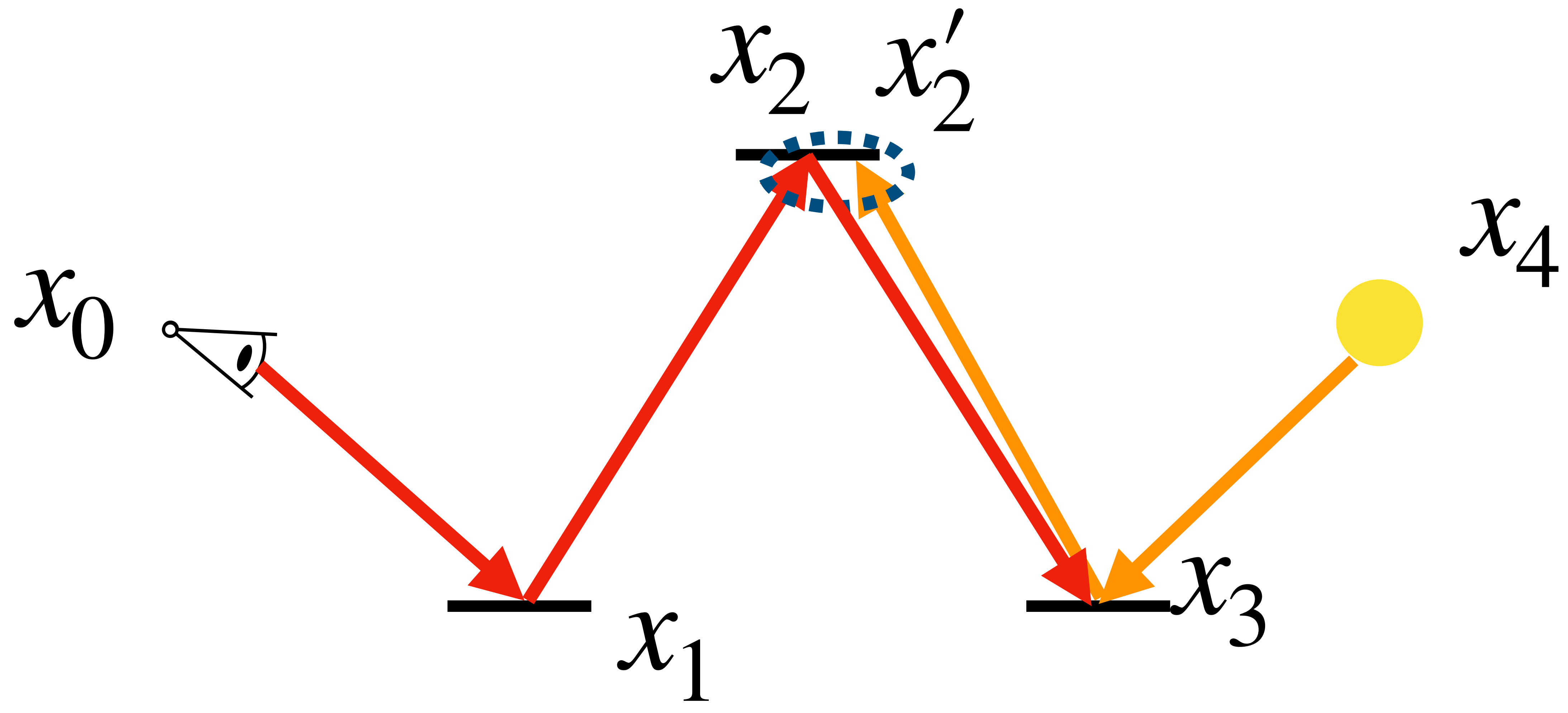
A Path Space Extension for Robust Light Transport Simulation

Toshiya Hachisuka[1,3] Jacopo Pantaleoni[2] Henrik Wann Jensen[3]
[1]Aarhus University [2]NVIDIA Research [3]UC San Diego

Combining with bidirectional path tracing (VCM/UPS)

- apply multiple importance sampling
- challenge: photon mapping has one more vertex (x_2' in this case), can't compare PDFs
- idea: perturb the bidirectional path tracing vertex to match, approximate perturbation probability as $\frac{1}{\pi r^2}$

path tracing: $x_0 x_1 x_2 x_3 x_4$
photon mapping: $x_0 x_1 x_2 x_2' x_3 x_4$



Light Transport Simulation with Vertex Connection and Merging

Iliyan Georgiev* Saarland University Intel VCI, Saarbrücken
Jaroslav Křivánek† Charles University, Prague
Tomaś Davidovič‡ Saarland University Intel VCI, Saarbrücken
Philipp Slusallek§ Saarland University Intel VCI & DFKI, Saarbrücken

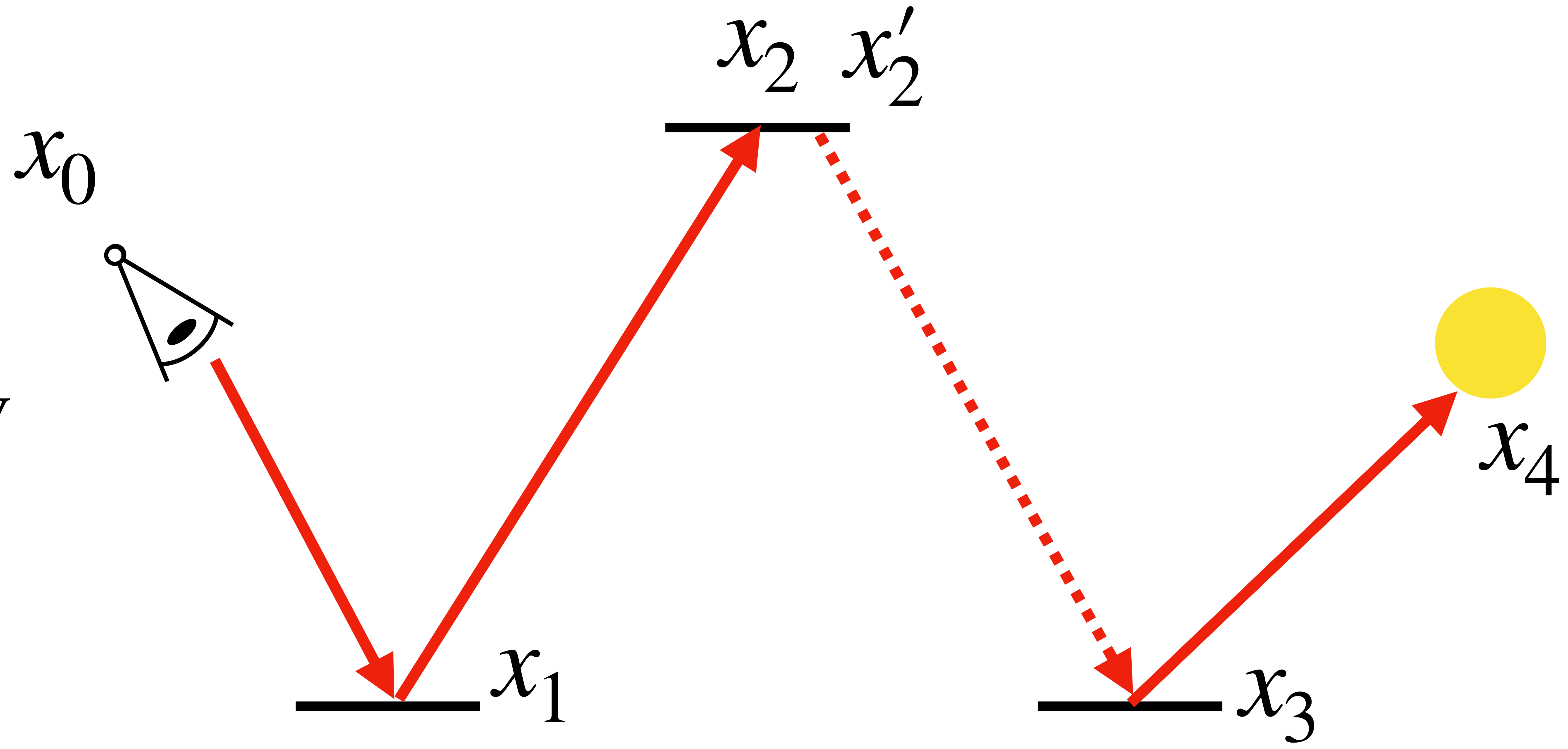
A Path Space Extension for Robust Light Transport Simulation

Toshiya Hachisuka[1,3] [1]Aarhus University
Jacopo Pantaleoni[2] [2]NVIDIA Research
Henrik Wann Jensen[3] [3]UC San Diego

Photon mapping is good at SDS paths
BPT is better at non SDS paths

Can we make photon mapping unbiased?

- surprisingly — yes!
  - recall: blurring the integrand doesn’t change the integral if the kernel is properly normalized
- why is photon mapping biased?
  - it usually uses fake BSDF & visibility
  - kernel is not normalized w.r.t. visibility

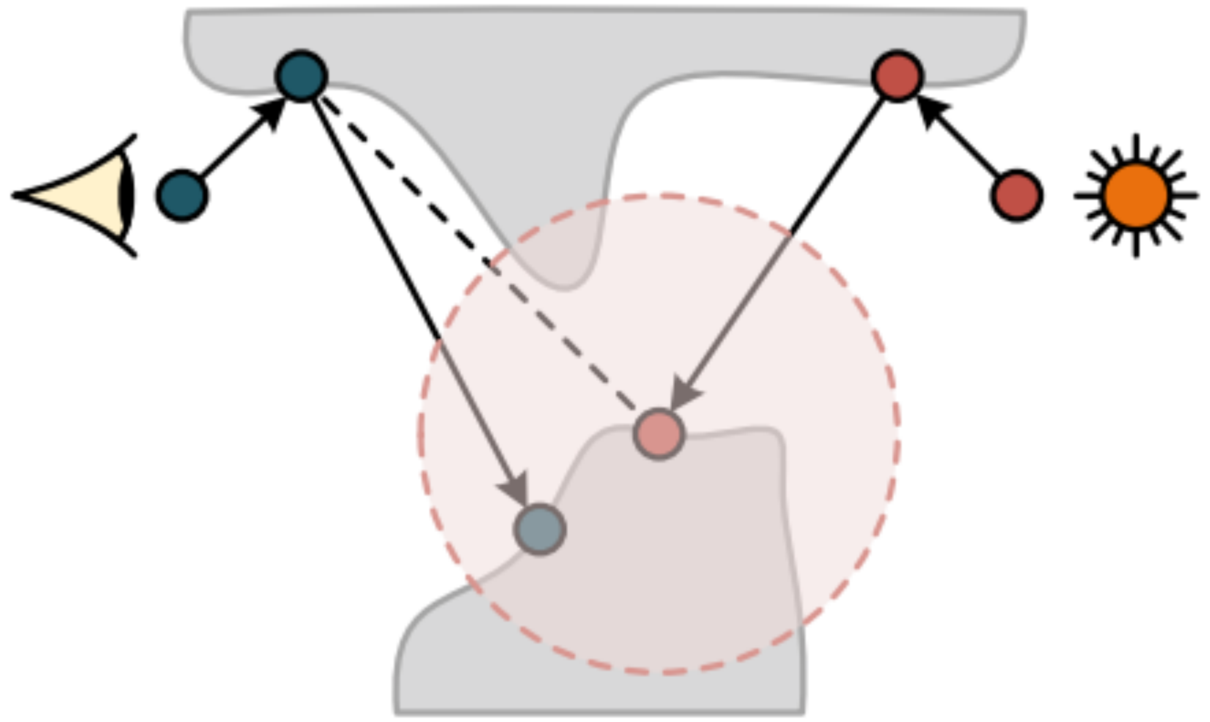


Unbiased Photon Gathering for Light Transport Simulation

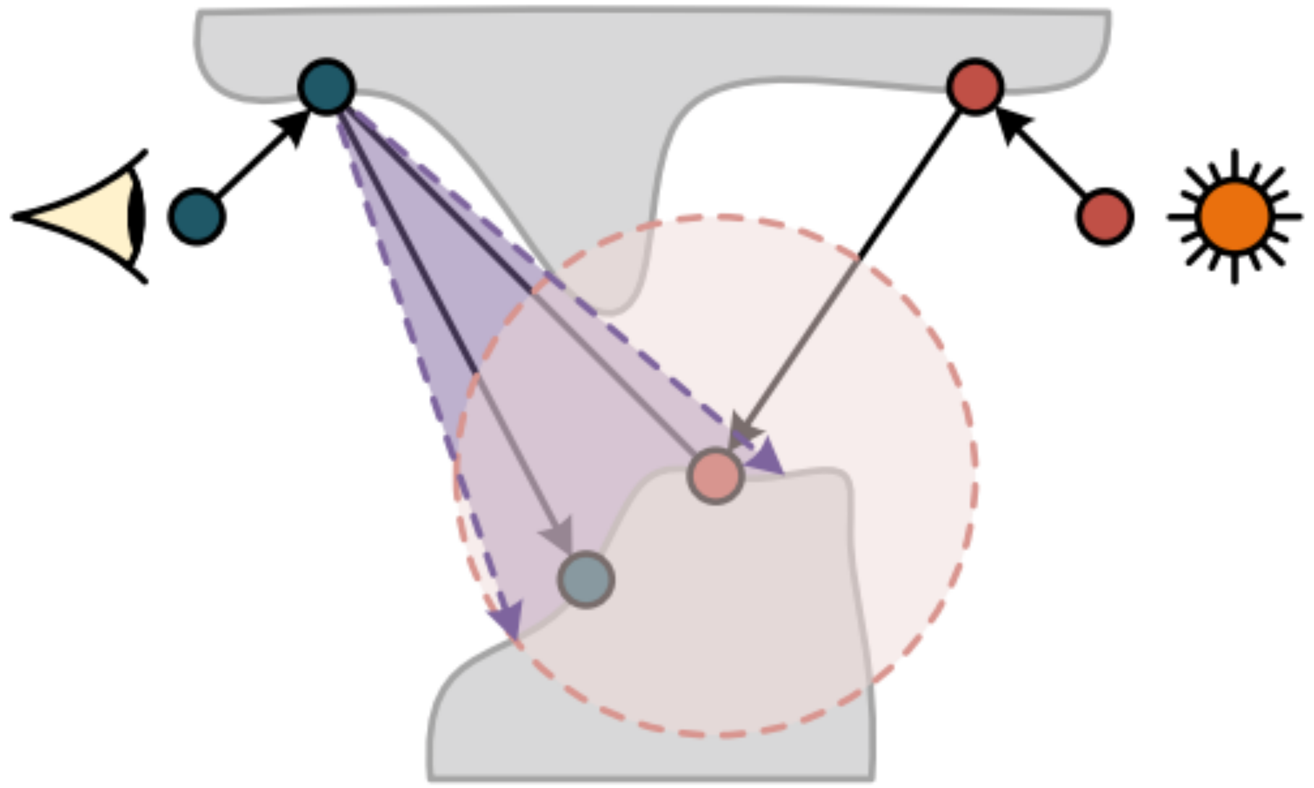
Hao Qin* Xin Sun† Qiming Hou*‡ Baining Guo† Kun Zhou*

*State Key Lab of CAD&CG, Zhejiang University †Microsoft Research Asia

Unbiased photon mapping: trace rays to the photon to debias



photon mapping

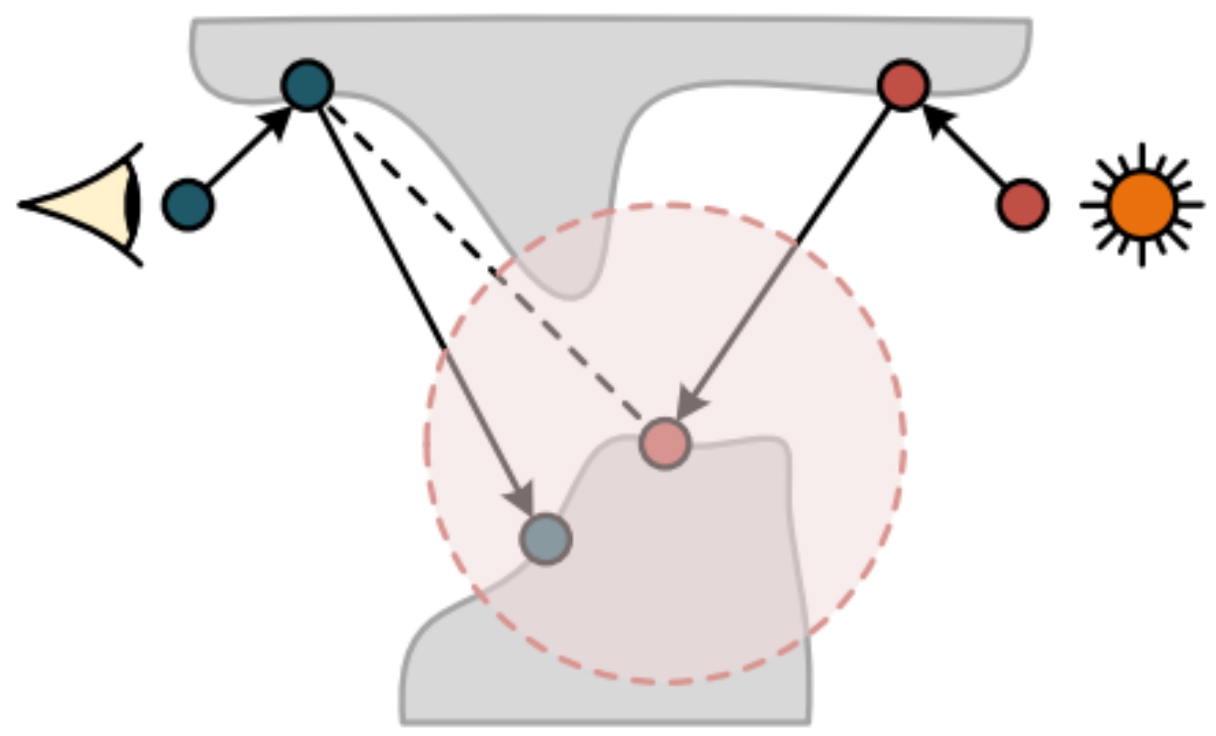


unbiased photon mapping

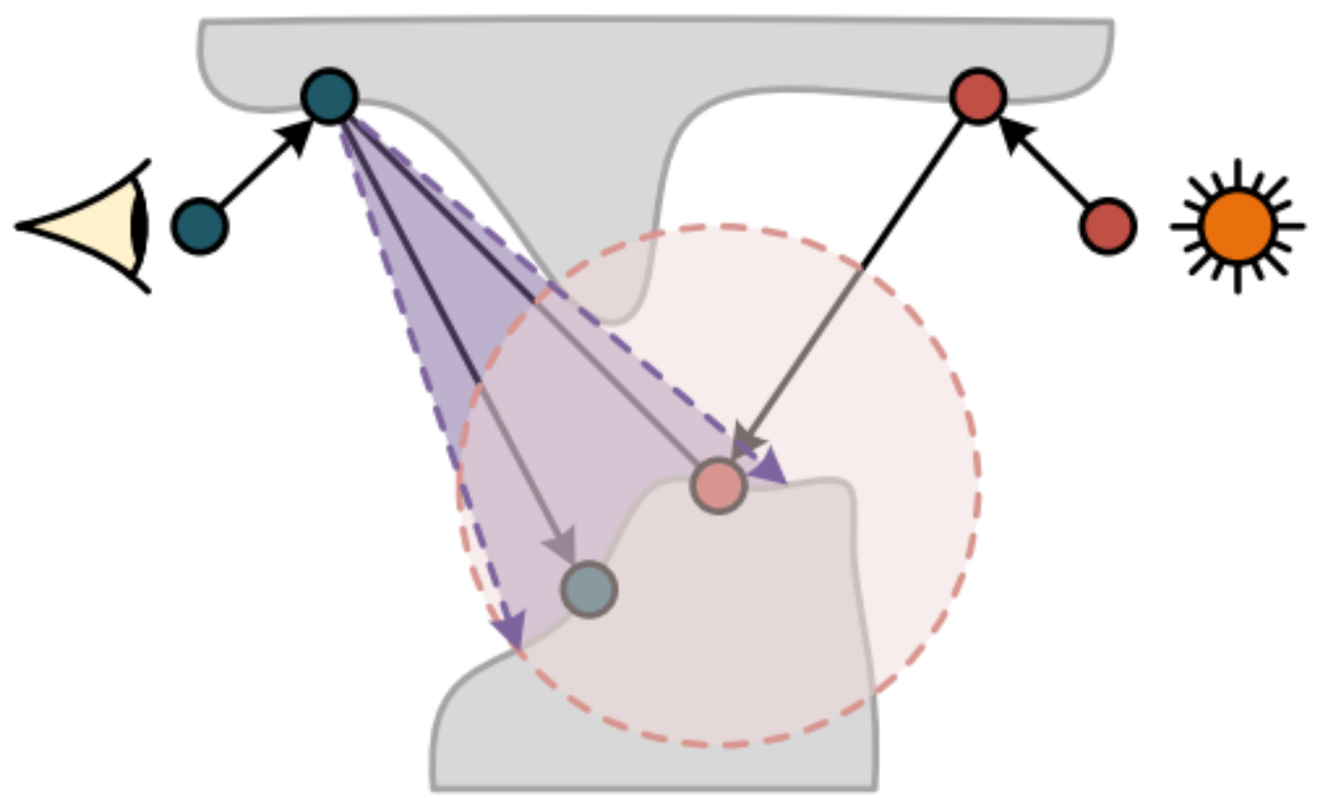
$$\int_{\text{surface}} \int_{\text{light paths}} k(x_2, x_2') f(\bar{x}') \mathrm{d}\bar{x} \mathrm{d}x_2'$$

$$\approx \frac{k(x_2, x_2') f(\bar{x}')}{p(x_0 \to x_1 \to x_2) p(x_4 \to x_3 \to x_2') \int k(x_2, x_2') \mathrm{d}x_2'}$$

Unbiased photon mapping: trace rays to the photon to debias



photon mapping



unbiased photon mapping

$$\int_{\text{surface}} \int_{\text{light paths}} k(x_2, x_2') f(\bar{x}') \mathrm{d}\bar{x} \mathrm{d}x_2'$$

$$\approx \frac{k(x_2, x_2') f(\bar{x}')}{p(x_0 \to x_1 \to x_2) p(x_4 \to x_3 \to x_2') \int k(x_2, x_2') \mathrm{d}x_2'}$$

challenge: taking reciprocal of a Monte Carlo estimator leads to bias!

Unbiased estimation of a reciprocal integral

$$\frac{1}{\int g(x)\mathrm{d}x} \neq E\left[\frac{1}{\frac{1}{N}\sum_{i=1}^{N} g(x_i)}\right]$$

similar to the problem we faced when estimating transmittance

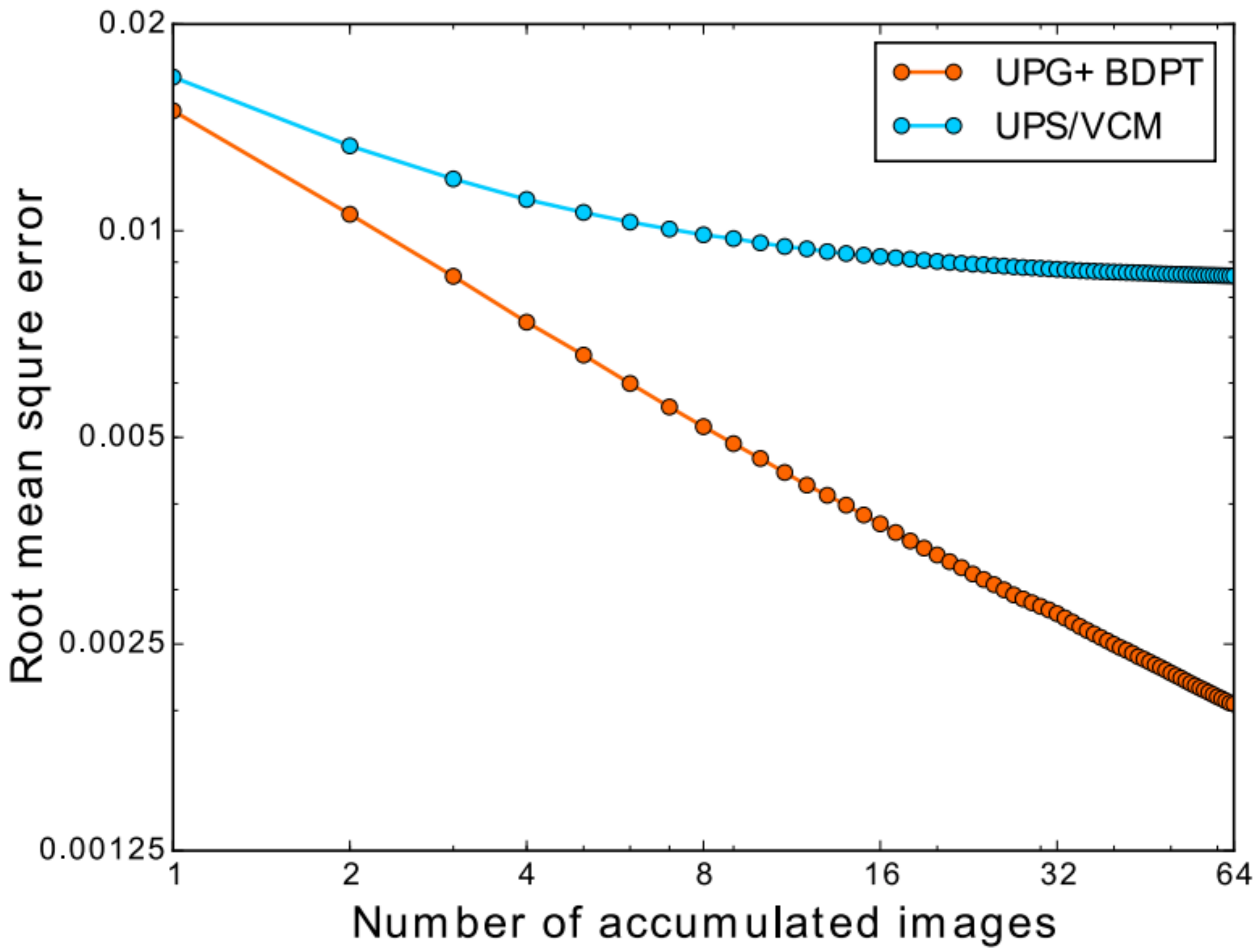
Unbiased estimation of a reciprocal integral

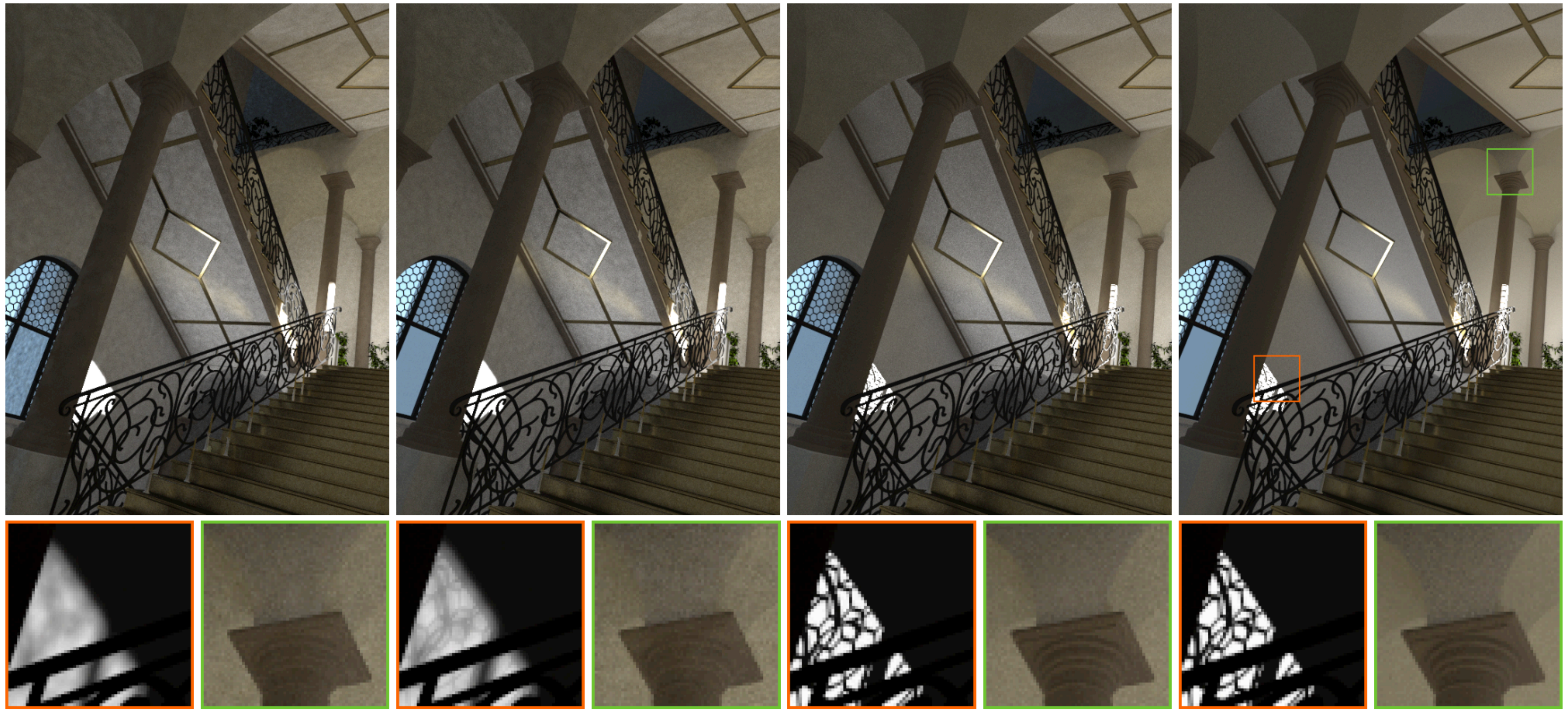
idea: rewrite the reciprocal using an infinite series

$$\frac{1}{\int g(x)\mathrm{d}x} = \frac{1}{1-G} = 1 + G + G^2 + \cdots$$

can be estimated using Russian roulette

Unbiased photon mapping converges faster, but can't do pure specular paths

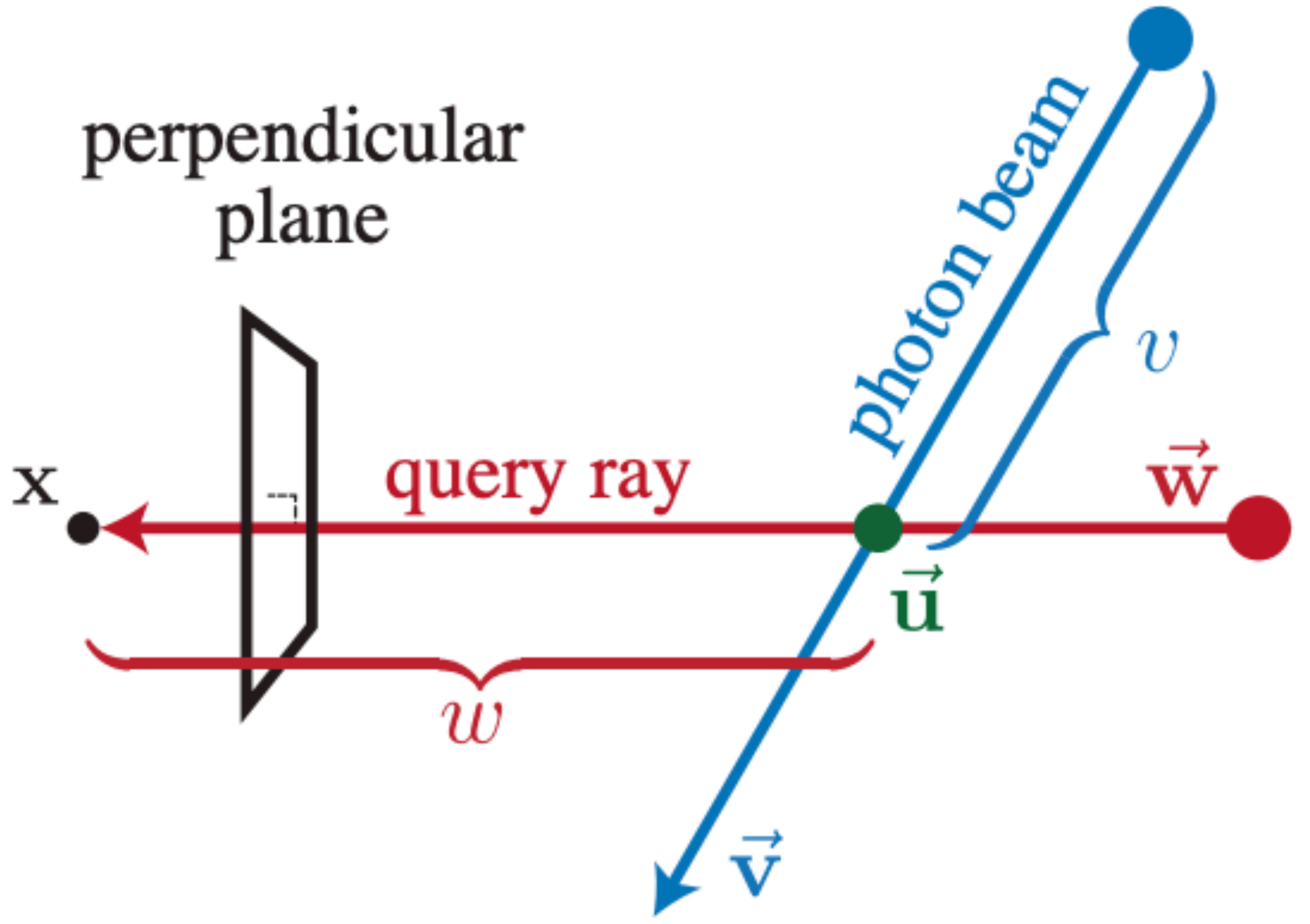


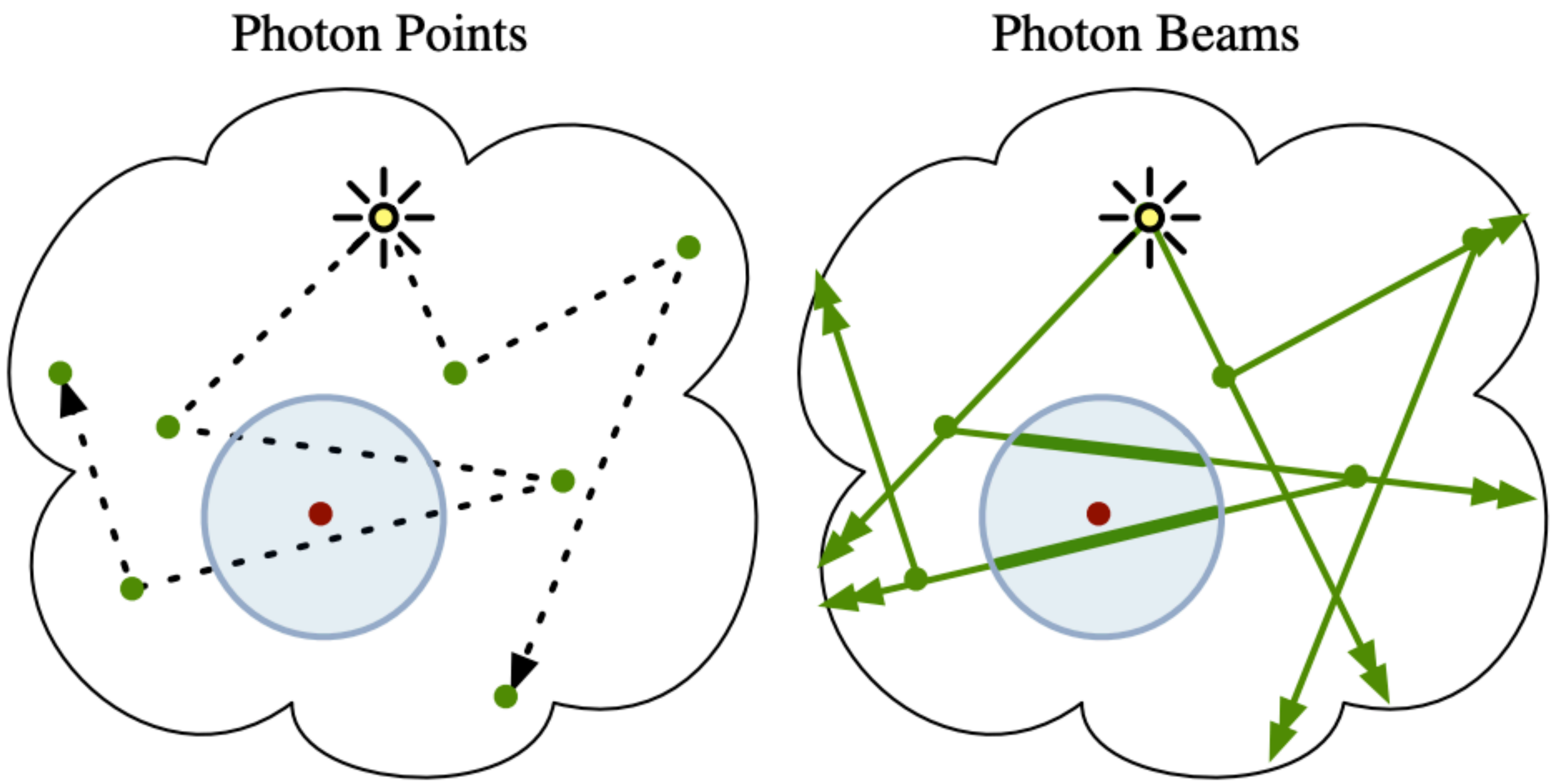


(a) SPPM, 1 hour (b) UPS/VCM, 1 hour (c) Our UPG+BDPT, 1 hour (d) Reference, BDPT, 2 days

Photon beams for volumetric rendering

- treat a light subpath as infinitely many photons
- treat a camera subpath as infinitely many query points





Progressive Photon Beams

Wojciech Jarosz[1] Derek Nowrouzezahrai[1] Robert Thomas[1] Peter-Pike Sloan[2] Matthias Zwicker[3]
[1]Disney Research Zürich [2]Disney Interactive Studios [3]University of Bern

Combining photon beams, points, and bidirectional path tracing

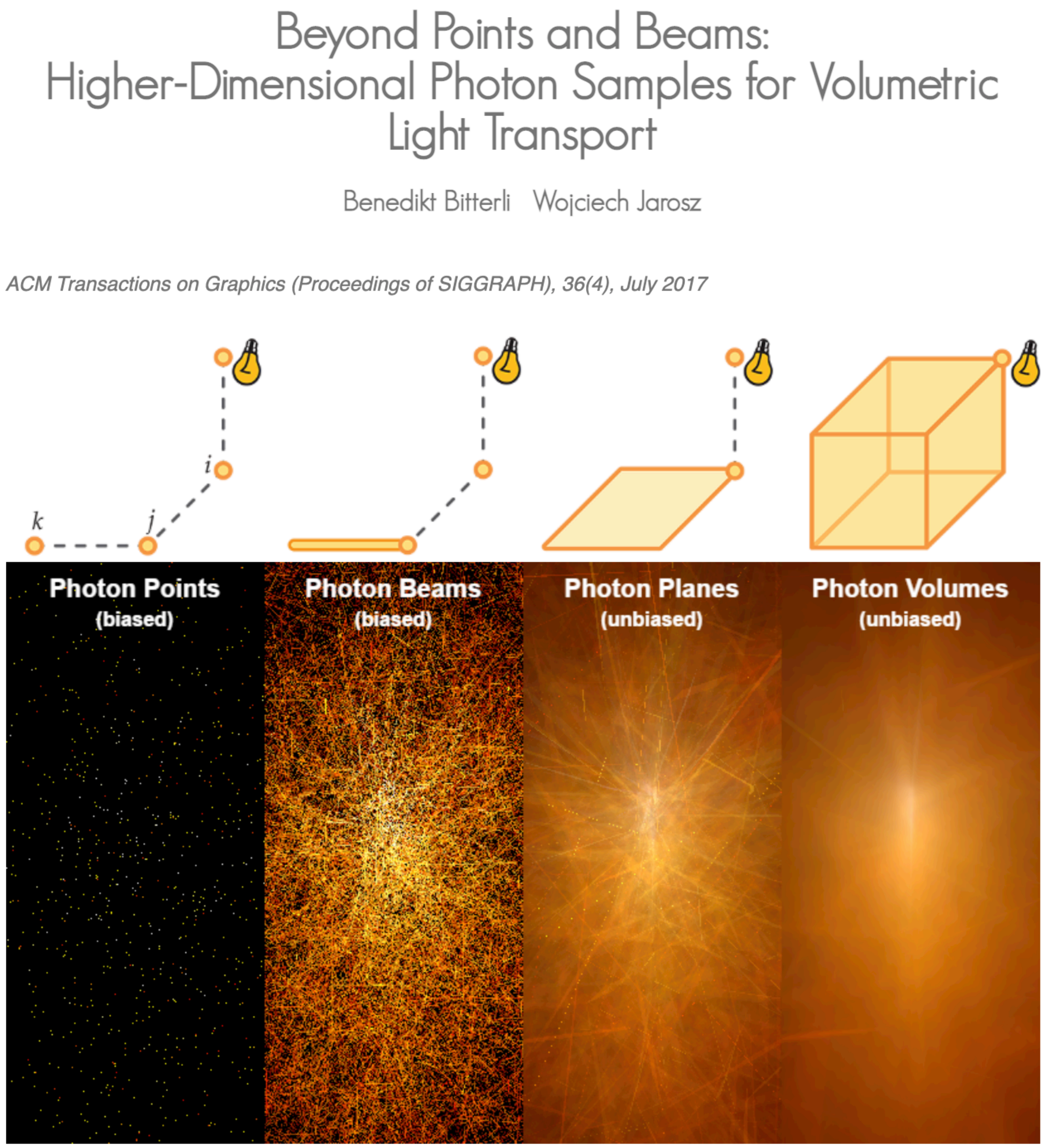
Unifying Points, Beams, and Paths in Volumetric Light Transport Simulation

Jaroslav Křivánek[1] Iliyan Georgiev[2] Toshiya Hachisuka[3] Petr Vévoda[1]
Martin Šik[1] Derek Nowrouzezahrai[4] Wojciech Jarosz[5]

[1]Charles University in Prague [2]Light Transportation Ltd. [3]Aarhus University [4]Université de Montréal [5]Disney Research Zürich

Photon planes and photon volumes

- infinitely many photons in planes & volumes



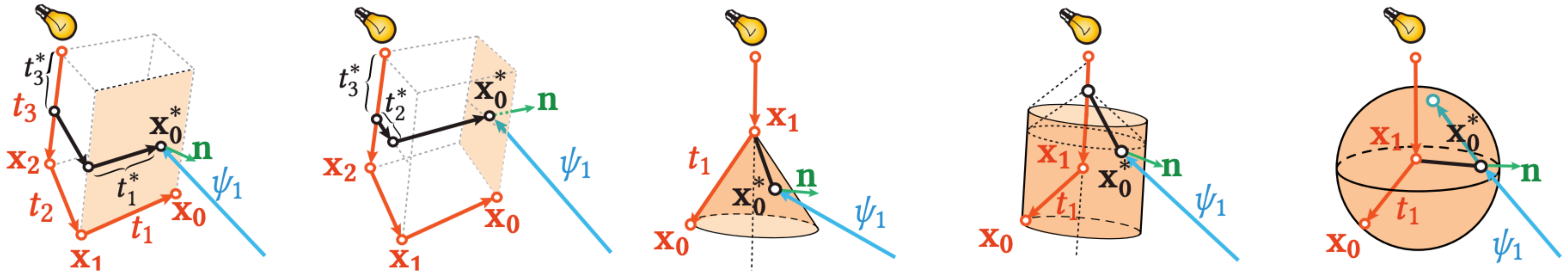
Photon cones / cylinders / spheres and photon bunnies

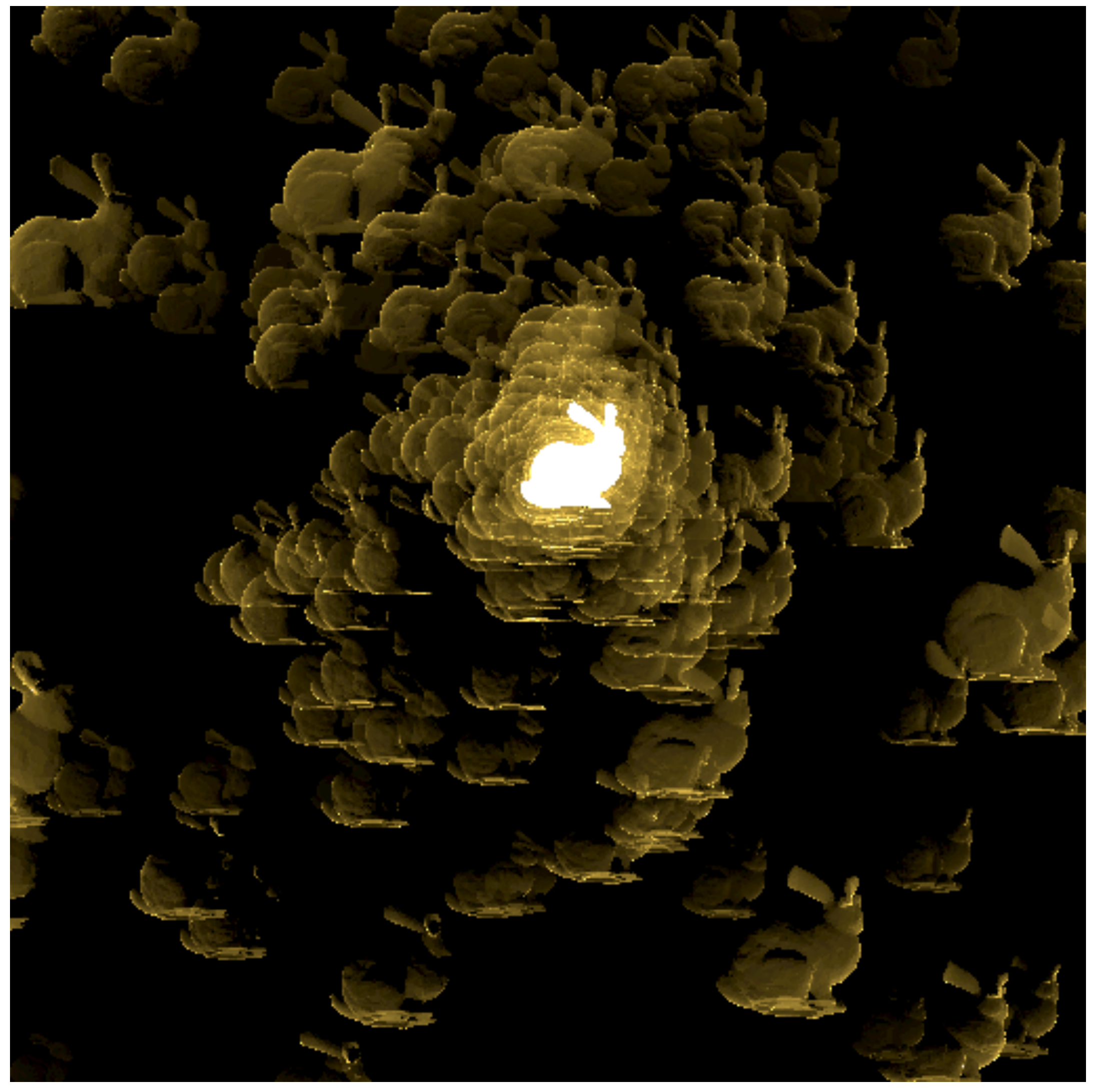
Photon surfaces for robust, unbiased volumetric density estimation

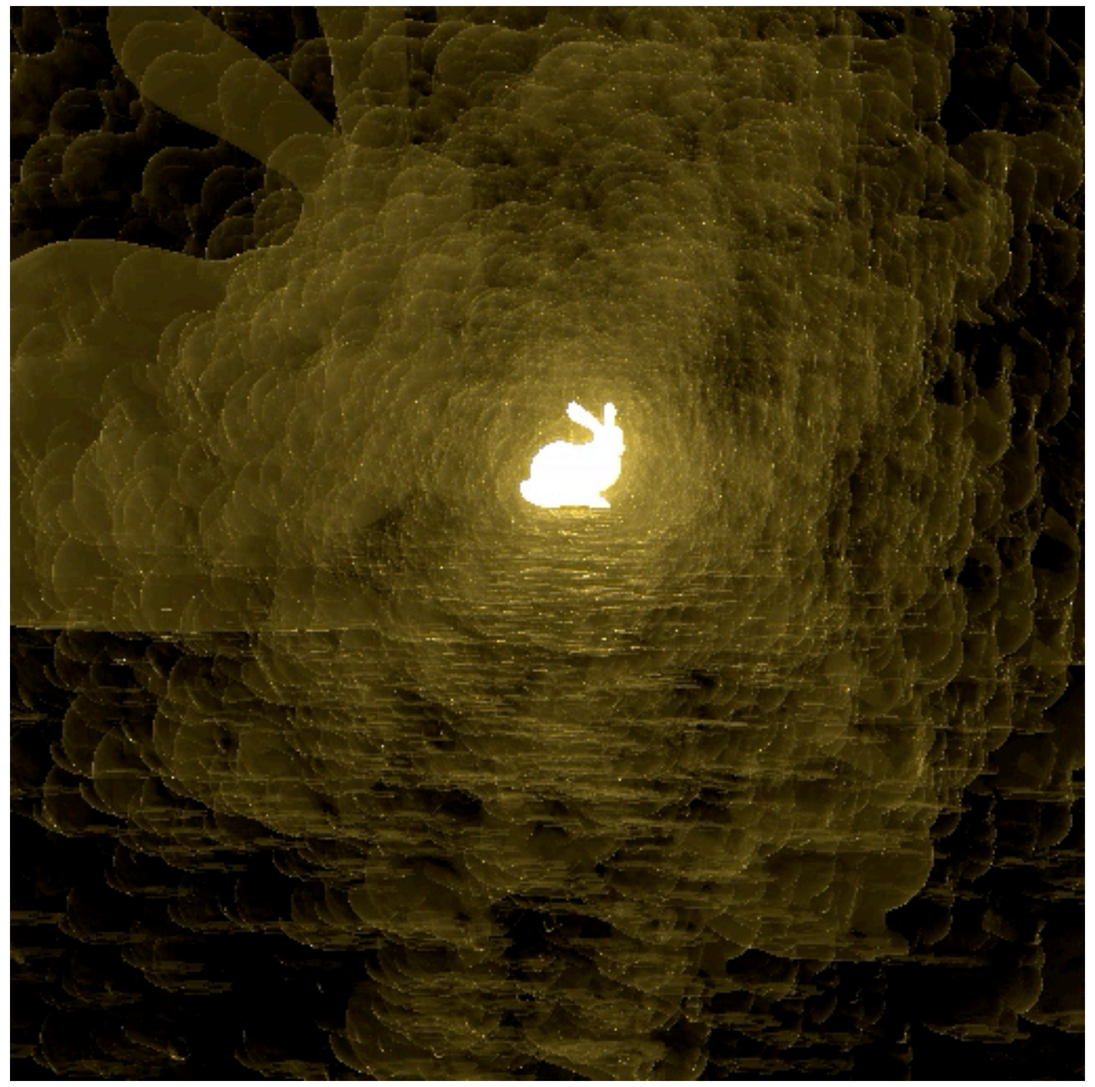
Xi Deng[1] Shaojie Jiao[1] Benedikt Bitterli[1] Wojciech Jarosz[1]

[1]Dartmouth College

In *ACM Transactions on Graphics (Proceedings of SIGGRAPH)*, 2019







History / biblography

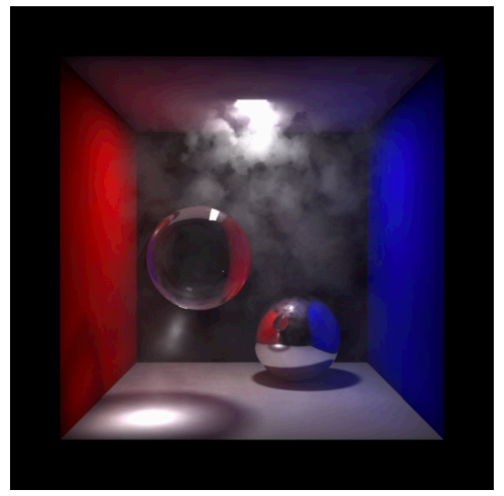
Global Illumination using Photon Maps

Henrik Wann Jensen
The Technical University of Denmark

1996

Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps

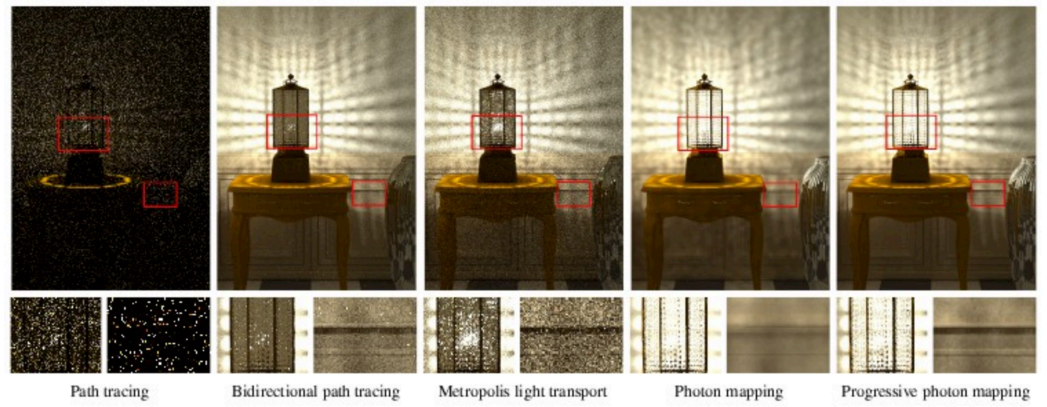
Henrik Wann Jensen *Per H. Christensen*
mental images*



1998

Progressive Photon Mapping

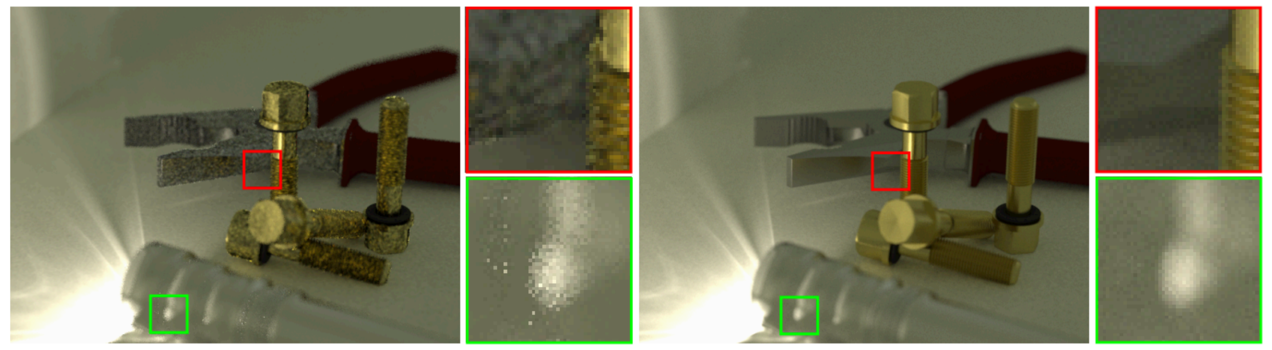
Toshiya Hachisuka
UC San Diego
Shinji Ogaki
The University of Nottingham
Henrik Wann Jensen
UC San Diego



2008

Stochastic Progressive Photon Mapping

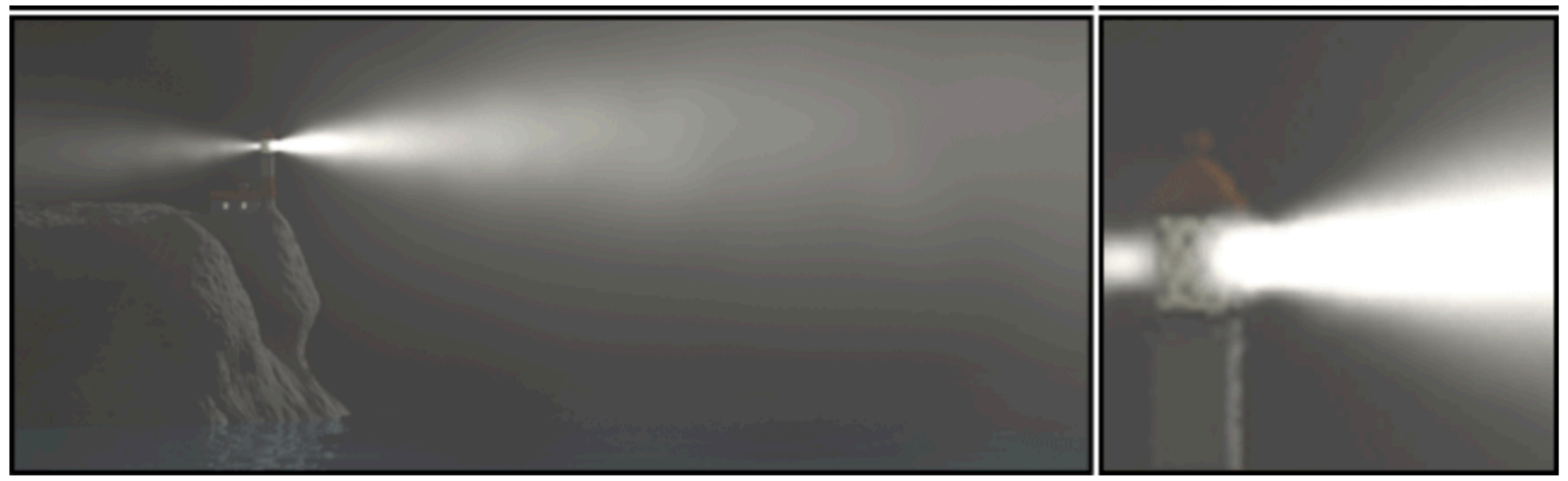
Toshiya Hachisuka Henrik Wann Jensen
UC San Diego



2009

The beam radiance estimate for volumetric photon mapping

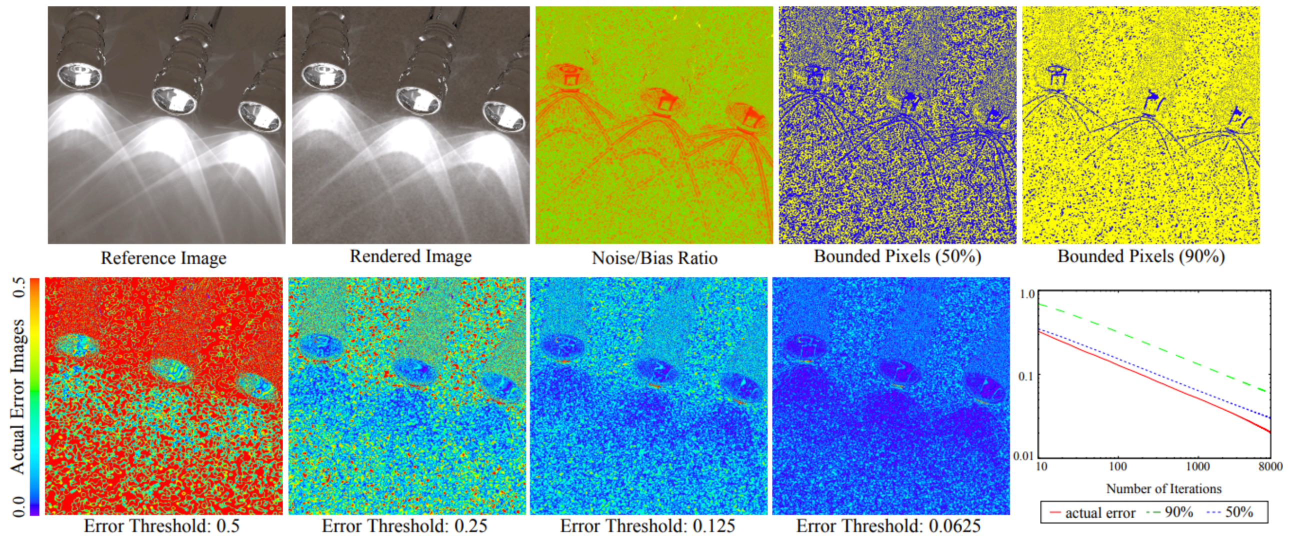
Wojciech Jarosz[1] Matthias Zwicker[1] Henrik Wann Jensen[1]
[1]UC San Diego



2008

A Progressive Error Estimation Framework for Photon Density Estimation

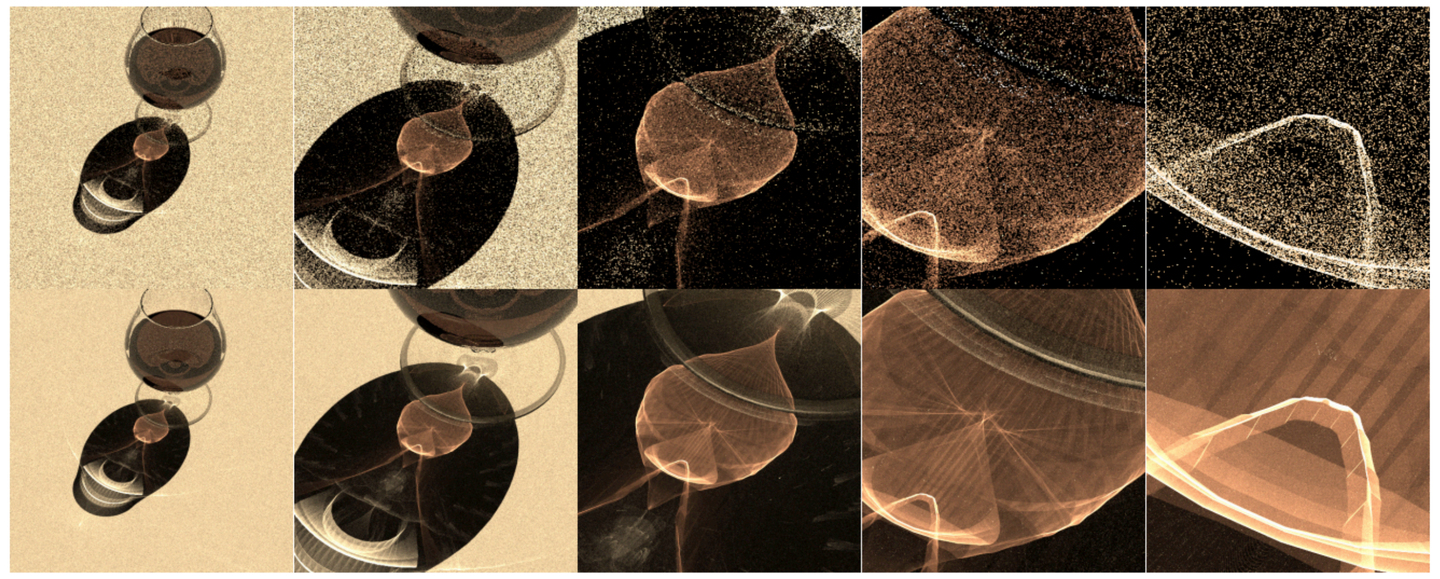
Toshiya Hachisuka* Wojciech Jarosz†,* Henrik Wann Jensen*
*UC San Diego †Disney Research Zürich



2010

Robust Adaptive Photon Tracing using Photon Path Visibility

TOSHIYA HACHISUKA and HENRIK WANN JENSEN
University of California, San Diego



2011

History / biblography

Into the Blue: Better Caustics through Photon Relaxation

2009

B. Spencer and M. W. Jones
roup, Swansea University, UK

Progressive Photon Relaxation

BEN SPENCER
and
MARK W. JONES
Visual and Interactive Computing Group, Swansea University

2013

A comprehensive theory of volumetric radiance estimation using photon points and beams

Wojciech Jarosz[1,2] Derek Nowrouzezahrai[1,3] Iman Sadeghi[2] Henrik Wann Jensen[2]
[1]Disney Research Zürich [2]UC San Diego [3]University of Toronto

In *ACM Transactions on Graphics (Presented at SIGGRAPH)*, 2011

Progressive Photon Mapping: A Probabilistic Approach

Claude Knaus and Matthias Zwicker
University of Bern, Switzerland

2011

Adaptive Progressive Photon Mapping

ANTON S. KAPLANYAN and CARSTEN DACHSBACHER
Karlsruhe Institute of Technology

2012

Improved Stochastic Progressive Photon Mapping with Metropolis Sampling

Jiating Chen[1,2,3,4], Bin Wang[1,3,4] and Jun-Hai Yong[1,3,4]

2011

Progressive Expectation–Maximization for hierarchical volumetric photon mapping

Wenzel Jakob[1,2] Christian Regg[1,3] Wojciech Jarosz[1]
[1]Disney Research Zürich [2]Cornell University [3]ETH Zürich

In *Computer Graphics Forum (Proceedings of EGSR)*, 2011

Line Space Gathering for Single Scattering in Large Scenes

Xin Sun* Kun Zhou† Stephen Lin* Baining Guo*
*Microsoft Research Asia †State Key Lab of CAD&CG, Zhejiang University

2010

Light Transport Simulation with Vertex Connection and Merging

Iliyan Georgiev* Saarland University Intel VCI, Saarbrücken
Jaroslav Křivánek† Charles University, Prague
Tomaš Davidovič‡ Saarland University Intel VCI, Saarbrücken
Philipp Slusallek§ Saarland University Intel VCI & DFKI, Saarbrücken

2012

A Path Space Extension for Robust Light Transport Simulation

Toshiya Hachisuka[1,3] Jacopo Pantaleoni[2] Henrik Wann Jensen[3]
[1]Aarhus University [2]NVIDIA Research [3]UC San Diego

2012

Path Space Regularization for Holistic and Robust Light Transport

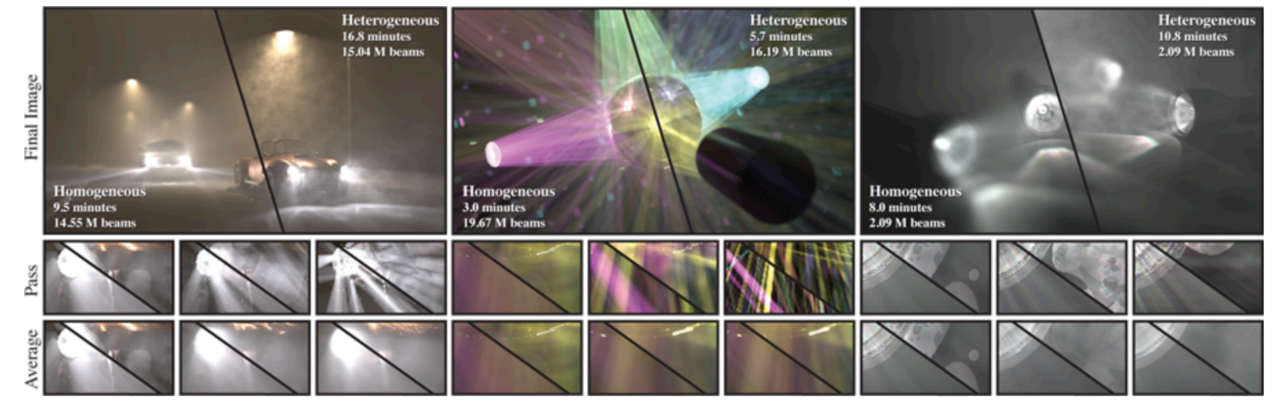
Anton S. Kaplanyan and Carsten Dachsbacher
Karlsruhe Institute of Technology, Germany

2013

Progressive photon beams

Wojciech Jarosz[1] Derek Nowrouzezahrai[1] Robert Thomas[1] Peter-Pike Sloan[2] Matthias Zwicke
[1]Disney Research Zürich [2]Disney Interactive Studios [3]University of Bern

In *ACM Transactions on Graphics (Proceedings of SIGGRAPH Asia)*, 2011
Featured in the proceedings inside cover and the papers fast forward video!



Unifying points, beams, and paths in volumetric light transport simulation

Jaroslav Křivánek[1] Iliyan Georgiev[2] Toshiya Hachisuka[3] Petr Vévoda[1] Martin Šik[1] Derek Nowrouzezahrai[4] Wojciech Jarosz[5]
[1]Charles University, Prague [2]Light Transportation Ltd. [3]Aarhus University [4]Université de Montréal [5]Disney Research Zürich

In *ACM Transactions on Graphics (Proceedings of SIGGRAPH)*, 2014

History / biblography

Unbiased Photon Gathering for Light Transport Simulation

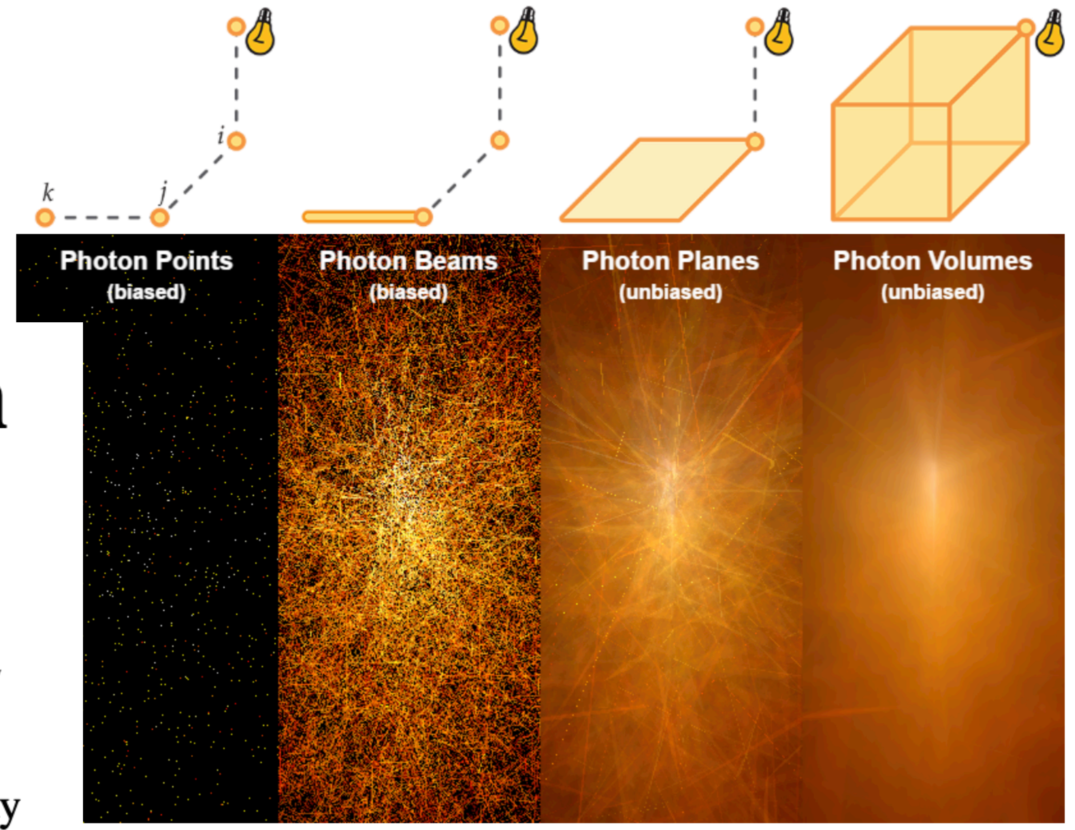
Hao Qin* Xin Sun† Qiming Hou*‡ Baining Guo† Kun Zhou*
*State Key Lab of CAD&CG, Zhejiang University †Microsoft Research Asia

2015

Beyond Points and Beams: Higher-Dimensional Photon Samples for Volumetric Light Transport

Benedikt Bitterli Wojciech Jarosz

ACM Transactions on Graphics (Proceedings of SIGGRAPH), 36(4), July 2017



A Spatial Target Function For Metropolis Photon Tracing

Adrien Gruson, IRISA, University of Rennes 1, France
Mickael Ribardiere, XLIM-SIC, University of Poitiers, France
Martin Sik, Charles University, Czech Republic
Jiri Vorba, Charles University, Czech Republic
Remi Cozot, IRISA, University of Rennes 1, France
Kadi Bouatouch, IRISA, University of Rennes 1, France
Jaroslav Krivanek, Charles University, Czech Republic

In *ACM Trans. Graph*, 2016 (Presented at Siggraph 2017).

Photon surfaces for robust, unbiased volumetric density estimation

Xi Deng[1] Shaojie Jiao[1] Benedikt Bitterli[1] Wojciech Jarosz[1]
[1]Dartmouth College

In *ACM Transactions on Graphics (Proceedings of SIGGRAPH)*, 2019

Gradient-Domain Photon Density Estimation

Binh-Son Hua[1] Adrien Gruson[2] Derek Nowrouzezahrai[3] Toshiya Hachisuka[2]
[1]Singapore University of Technology and Design [2]The University of Tokyo [3]McGill University

2017

Hierarchical Neural Reconstruction for Path Guiding Using Hybrid Path and Photon Samples

SHILIN ZHU, University of California San Diego, USA
ZEXIANG XU, Adobe Research, USA
TIANCHENG SUN, University of California San Diego, USA
ALEXANDR KUZNETSOV, University of California San Diego, USA
MARK MEYER, Pixar Animation Studios, USA
HENRIK WANN JENSEN, University of California San Diego and Luxion, USA
HAO SU, University of California San Diego, USA
RAVI RAMAMOORTHI, University of California San Diego, USA

2021

CPPM: Chi-squared Progressive Photon Mapping

ZEHUI LIN, SHENG LI*, XINLU ZENG, and CONGYI ZHANG, Dept. of Computer Science and Technology, Peking University
JINZHU JIA, Dept. of Biostatistics and Center for Statistical Science, Peking University
GUOPING WANG, Dept. of Computer Science and Technology, Peking University
DINESH MANOCHA, University of Maryland at College Park

2020

Next time: Metropolis light transport