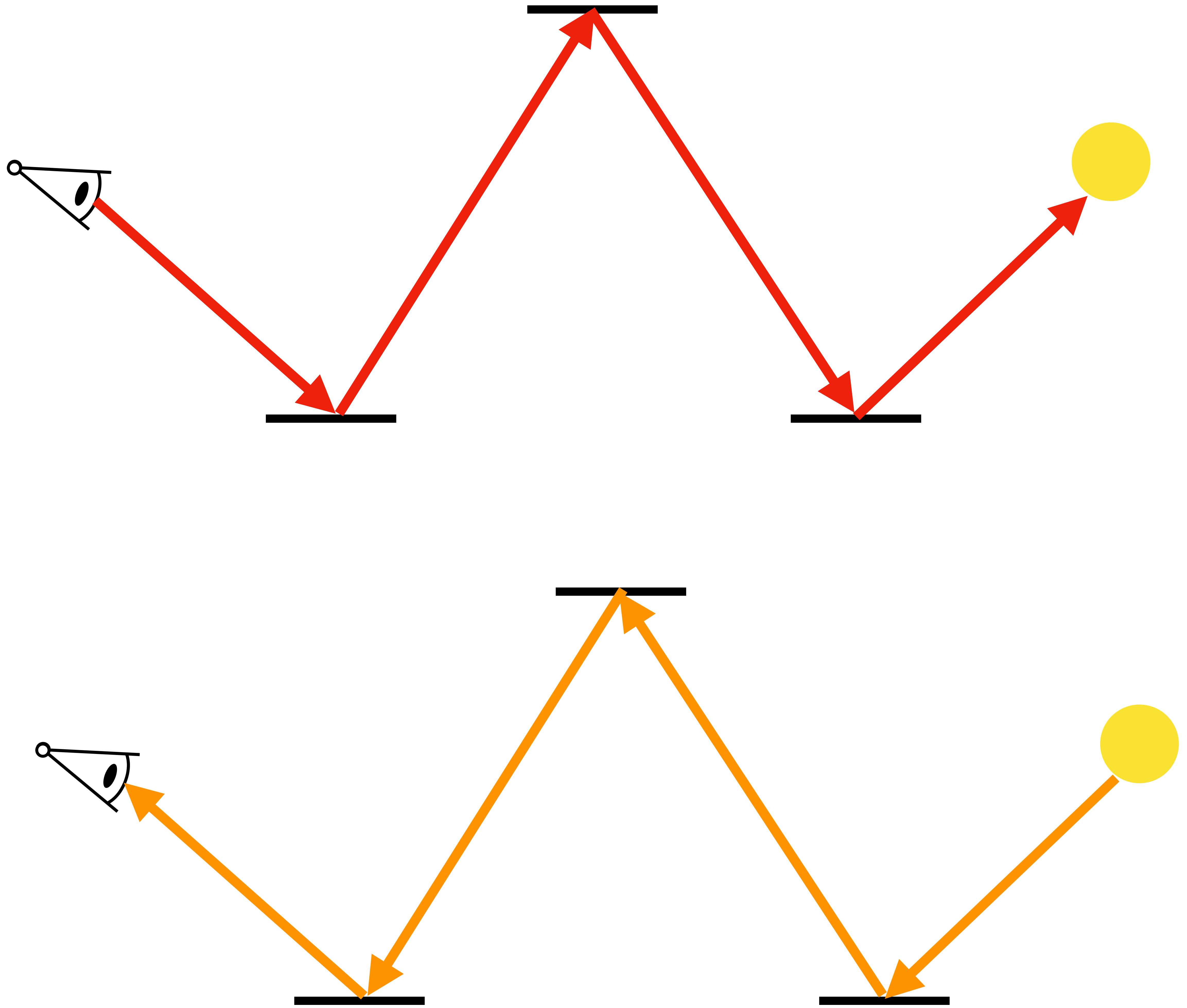
Path-space & bidirectional path tracing

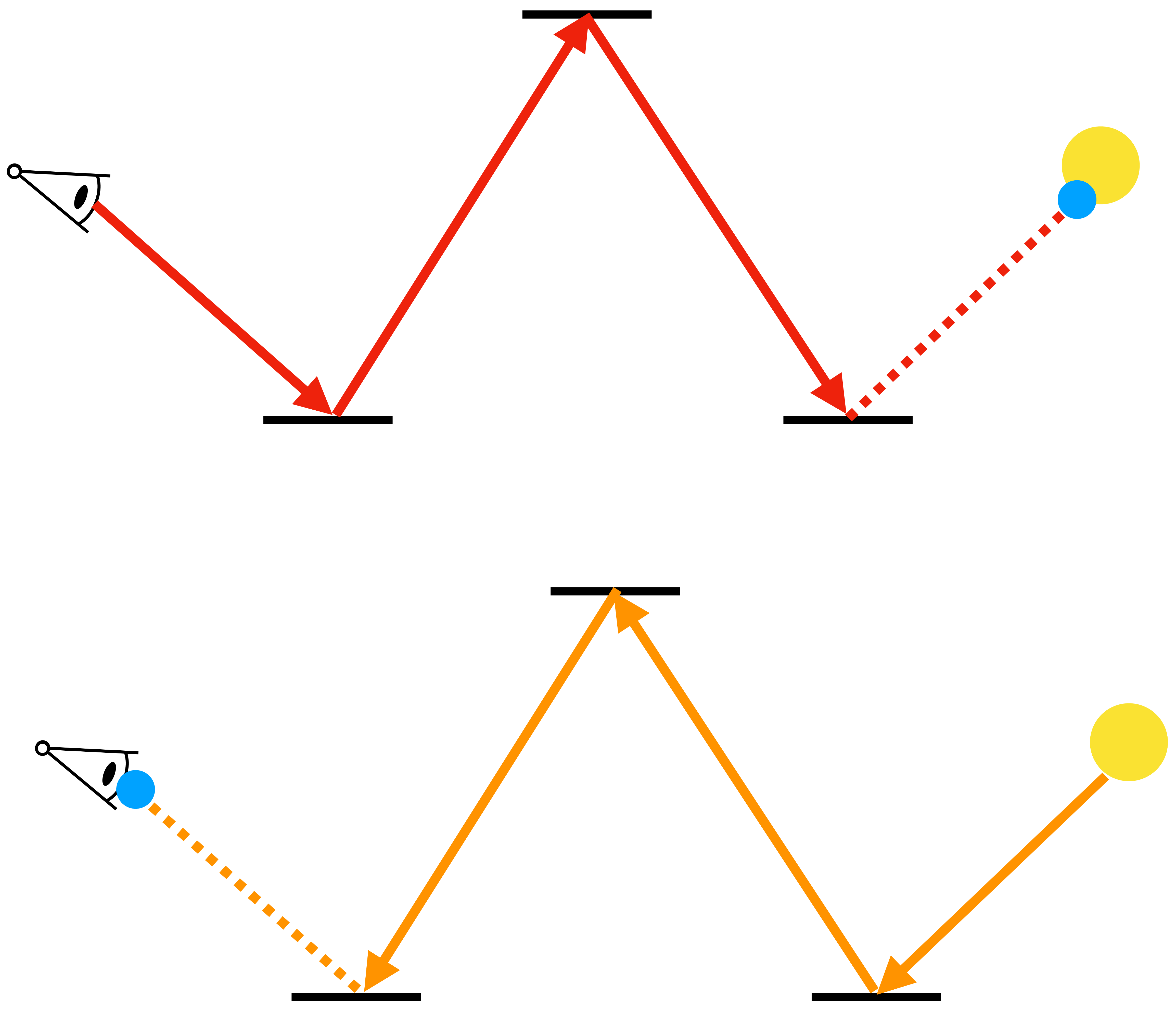
UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li

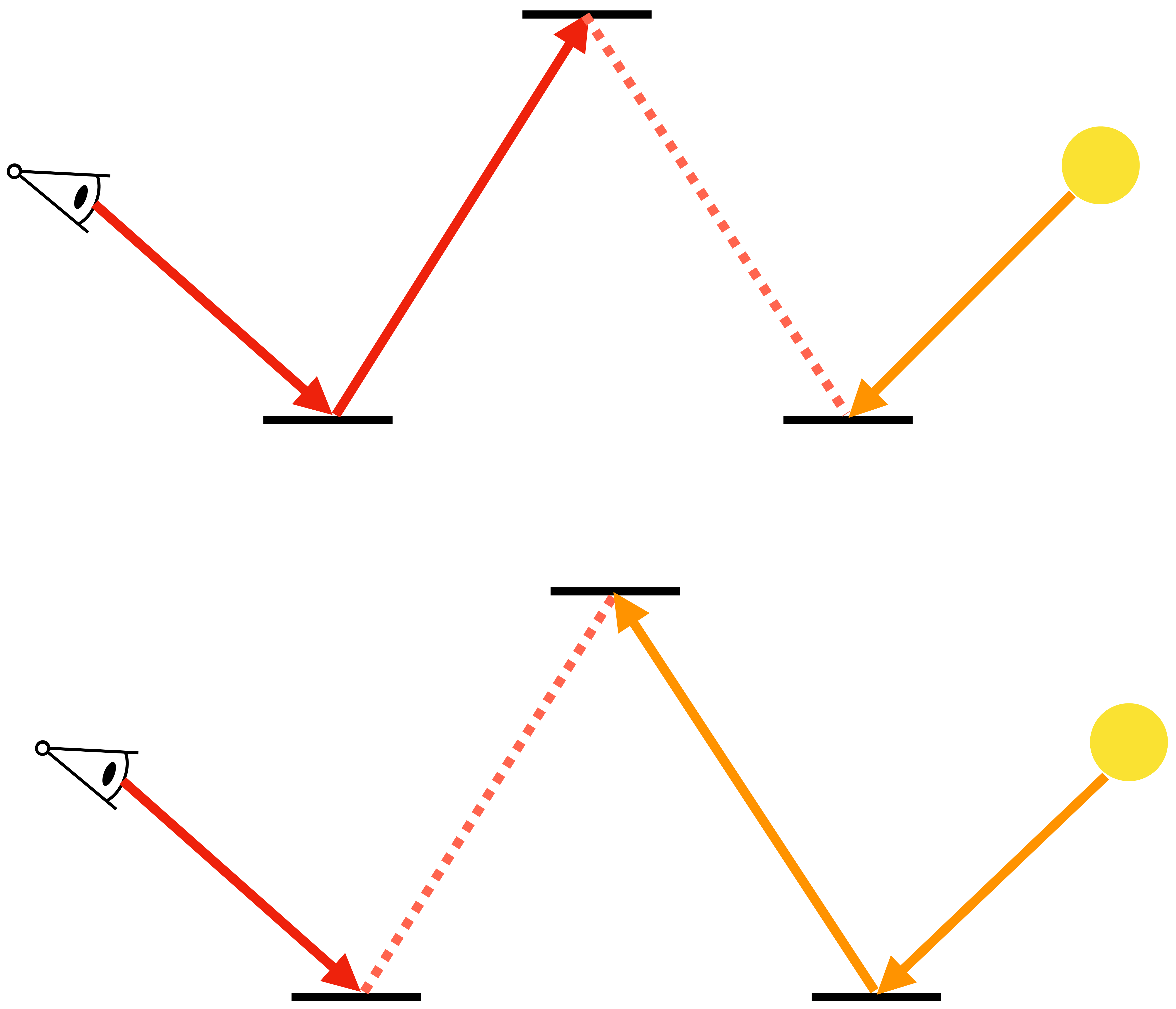
Path tracing vs light tracing



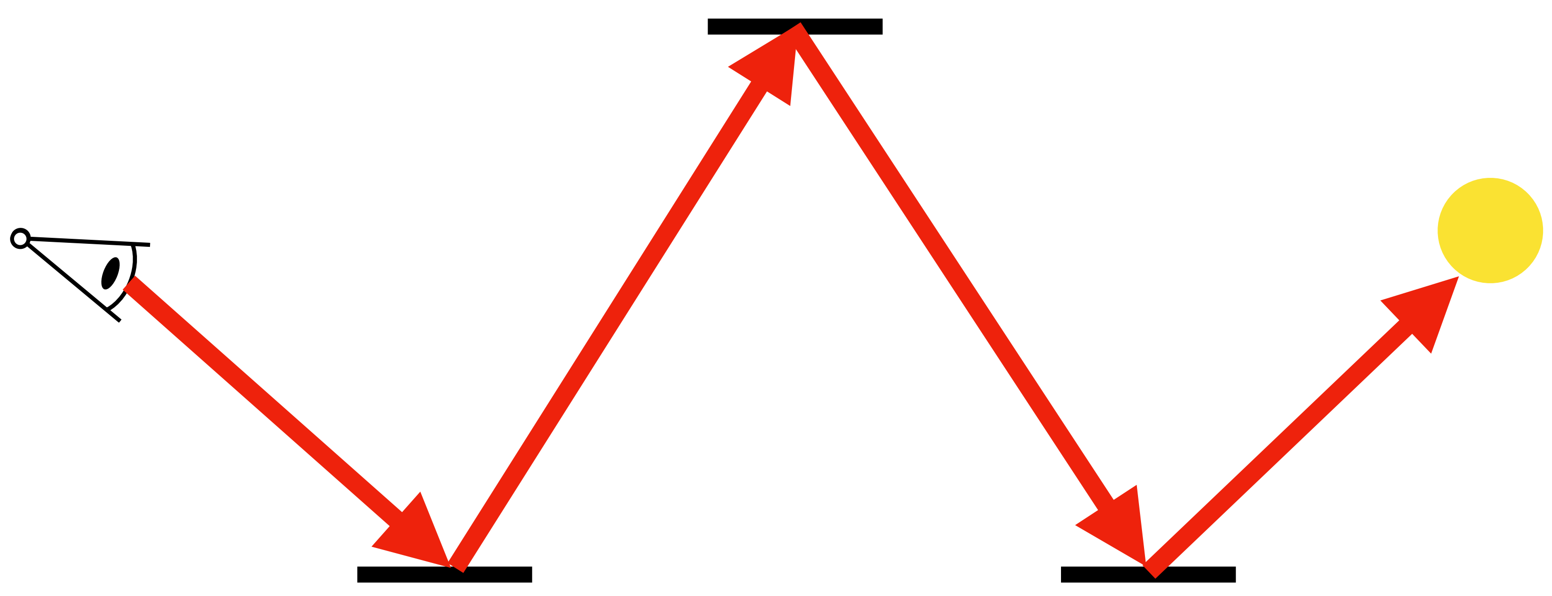
Next event estimation



Combining path tracing & light tracing



How many ways we can sample a light path with 5 vertices?



How many ways we can sample a light path with 5 vertices?

Path-space formulation for rendering

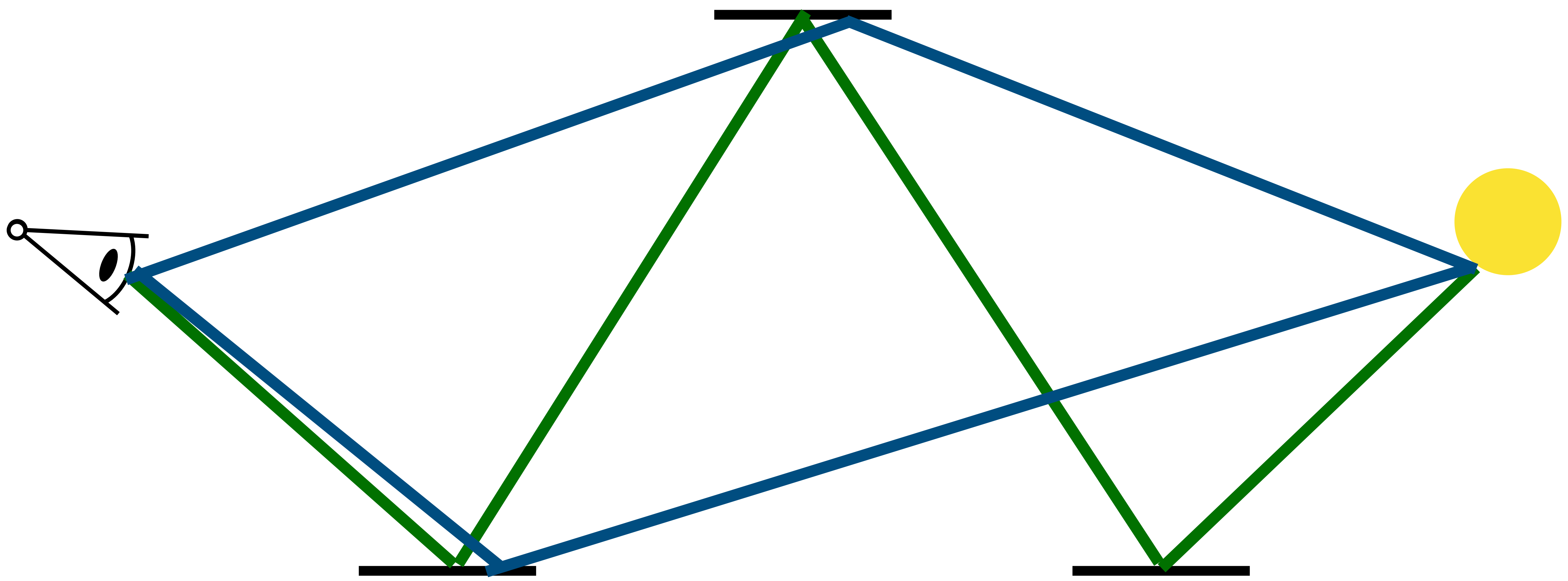
ROBUST MONTE CARLO METHODS
FOR LIGHT TRANSPORT SIMULATION

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF COMPUTER SCIENCE
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

by
Eric Veach
December 1997

Idea: rewrite rendering equation as an integral over **paths**

$$\int_{\text{light paths}} f(\bar{x})\mathrm{d}\bar{x}$$

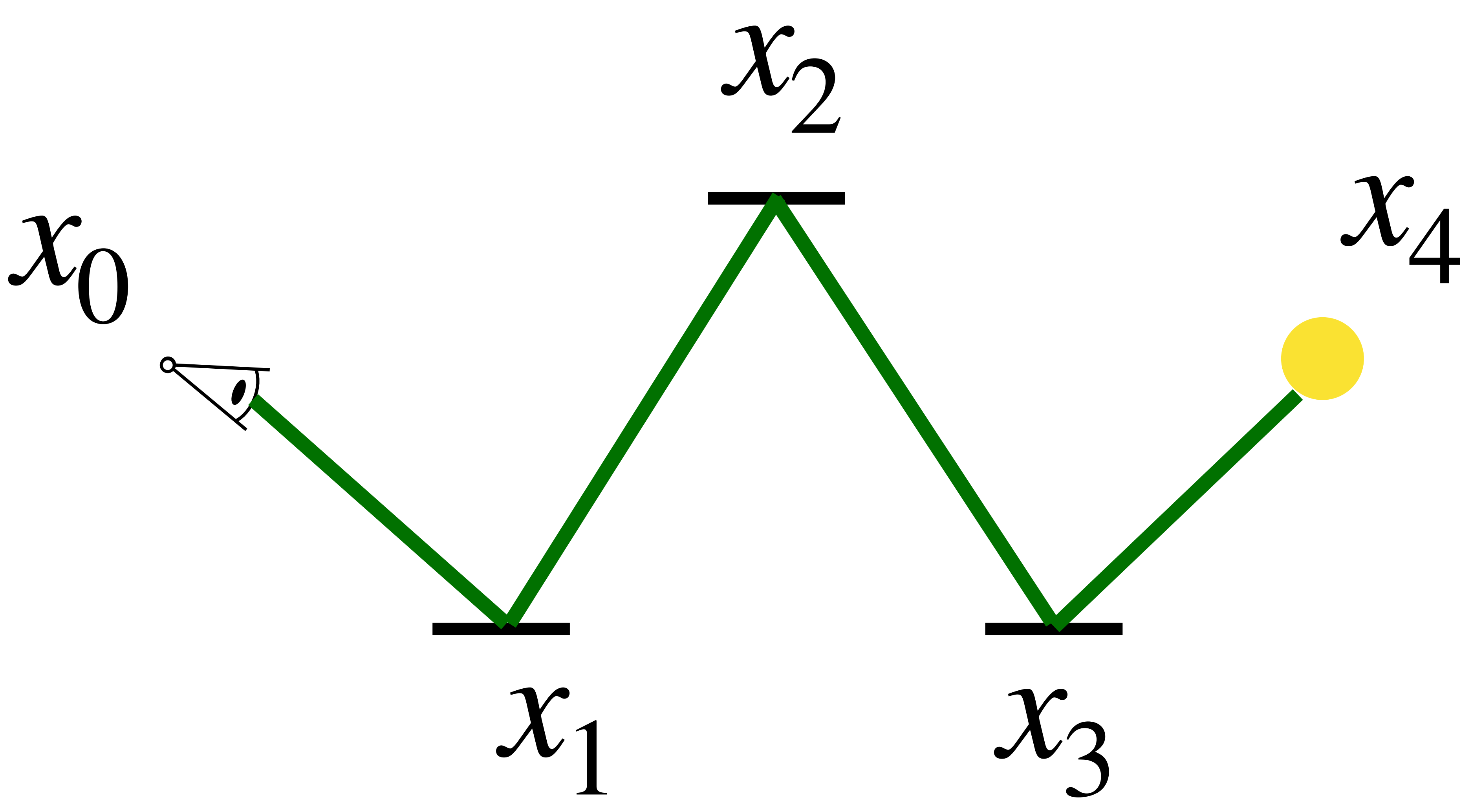


- what is a path $\bar{x}$?
- what is the path contribution f?
- how do we importance sample light paths?

Path: a sequence of vertices

$$\bar{x} = x_0 x_1 x_2 x_3 x_4$$

$$\mathrm{d}\bar{x} = \mathrm{d}x_0 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \mathrm{d}x_4$$



x_i: a 3D position
$\mathrm{d}x_i$: a small 2D surface area around the vertex

$$\int_{\text{light paths}} f(\bar{x})\mathrm{d}\bar{x}$$

Path contribution

geometry term

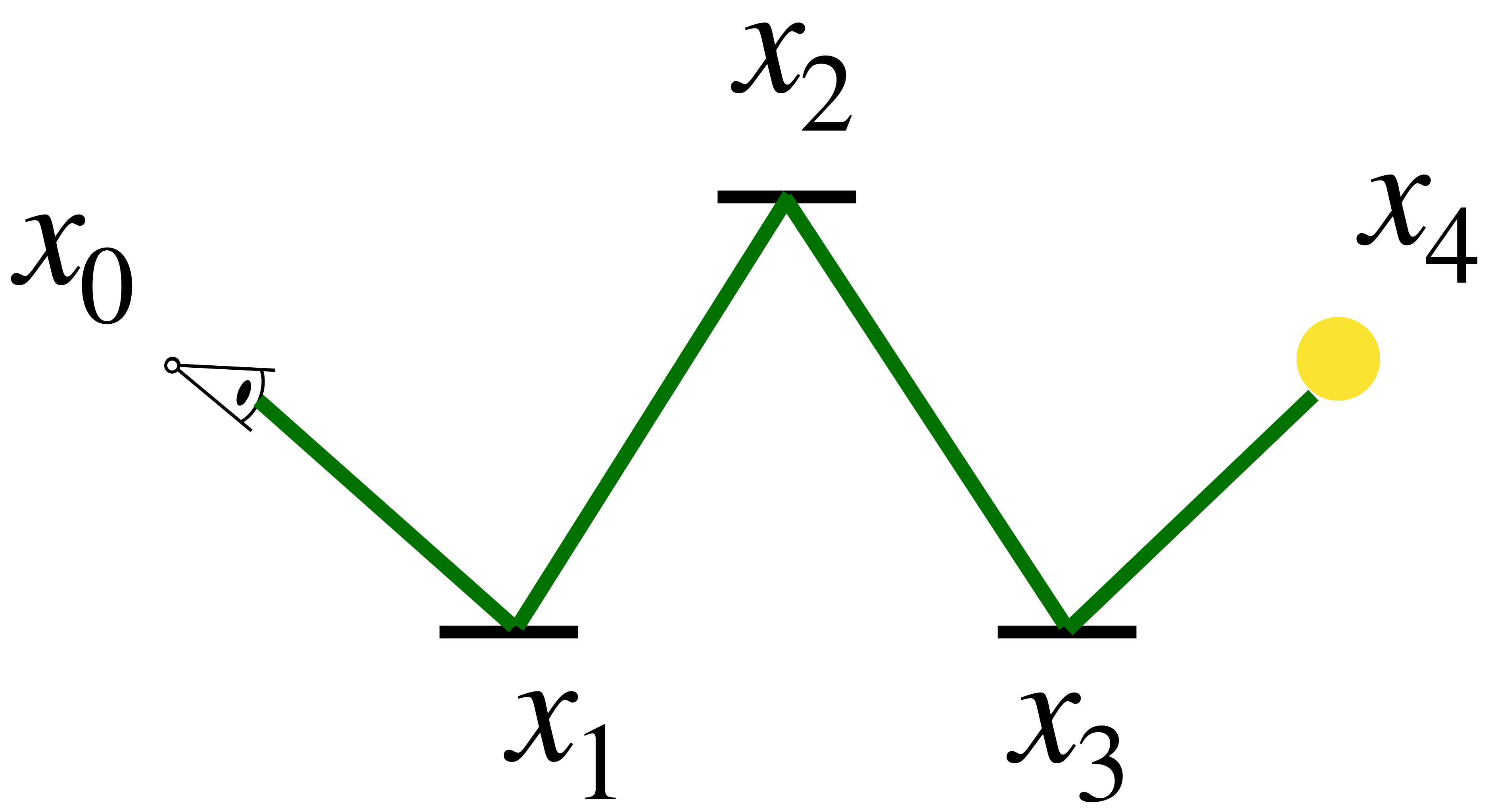
$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\rho(x_1 \to x_2 \to x_3)G(x_2 \leftrightarrow x_3)\rho(x_2 \to x_3 \to x_4)G(x_3 \leftrightarrow x_4)L_e(x_3 \to x_4)$$

pixel filter & camera sensitivity

BSDF & cosine

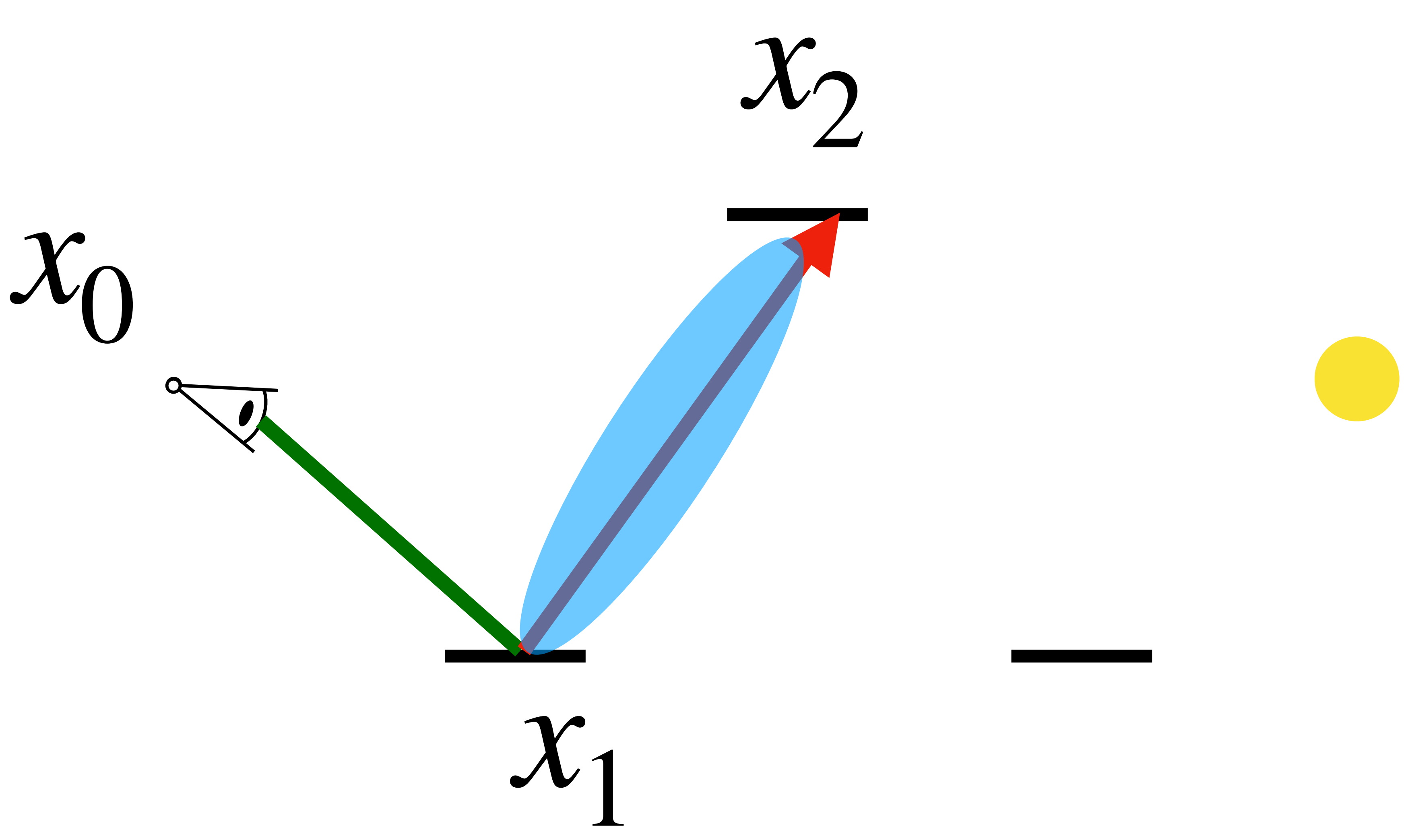
light emission

$$G(x \leftrightarrow y) = \frac{\left| \frac{x-y}{\| x-y \|} \cdot n_y \right|}{\| x-y \|^2} \text{visible}(x, y)$$



Observation: BSDF importance sampling cancels out ρ and G

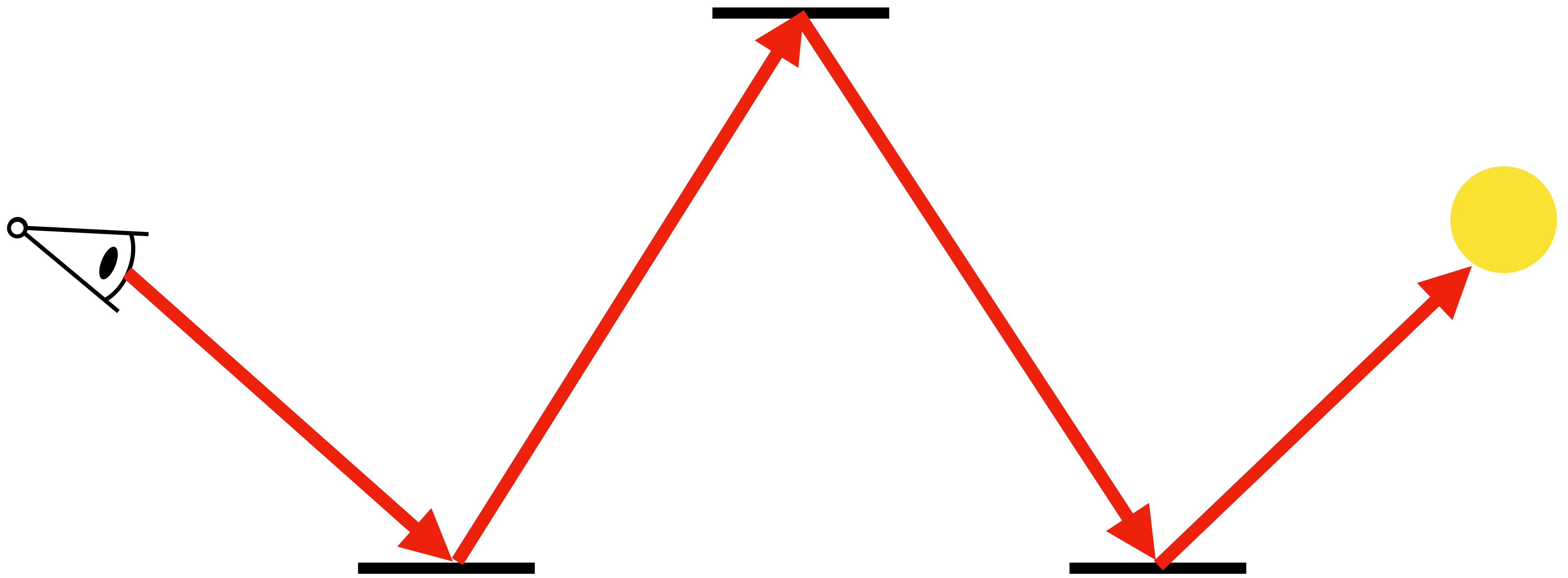
$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\rho(x_1 \to x_2 \to x_3)G(x_2 \leftrightarrow x_3)\rho(x_2 \to x_3 \to x_4)G(x_3 \leftrightarrow x_4)L_e(x_3 \to x_4)$$



Path tracing without next event estimation: importance sampling everything except Le

$$f(\bar{x}) = \boxed{W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\rho(x_1 \to x_2 \to x_3)G(x_2 \leftrightarrow x_3)\rho(x_2 \to x_3 \to x_4)G(x_3 \leftrightarrow x_4)} L_e(x_3 \to x_4)$$

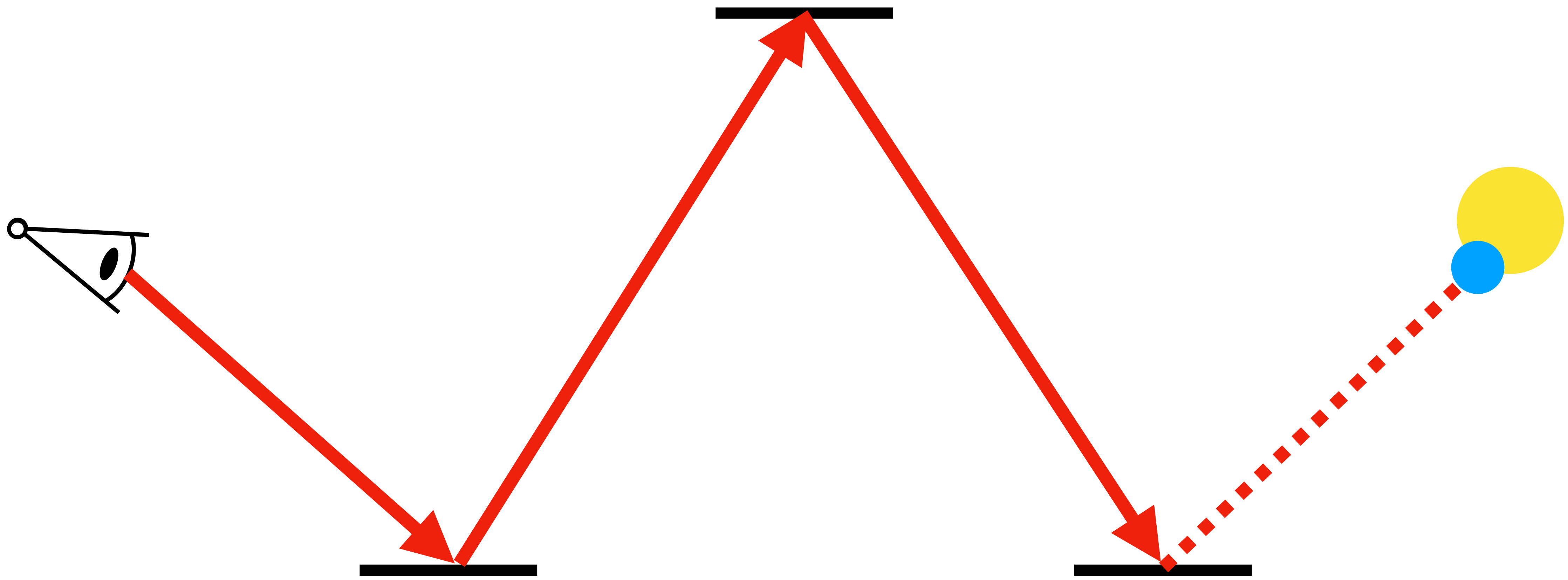
quiz: when will this be a good / bad strategy?



Path tracing with next event estimation

$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\rho(x_1 \to x_2 \to x_3)G(x_2 \leftrightarrow x_3)\rho(x_2 \to x_3 \to x_4)G(x_3 \leftrightarrow x_4)L_e(x_3 \to x_4)$$

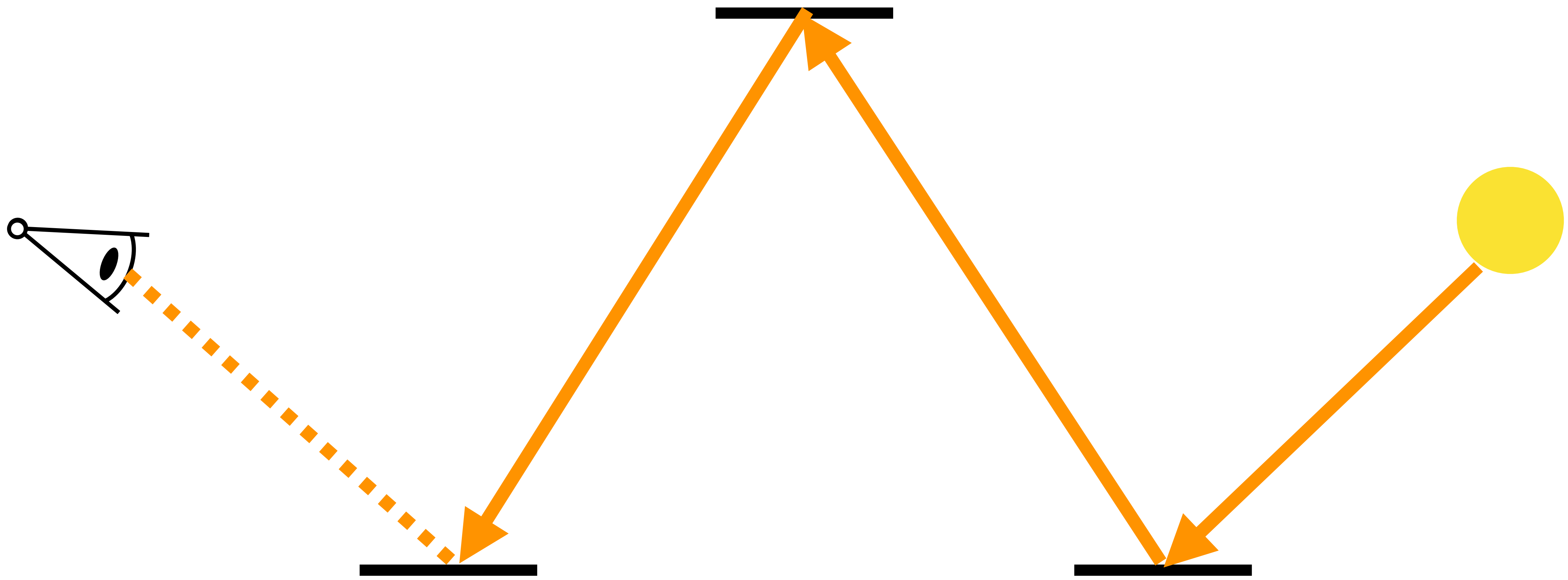
quiz: when will this be a good / bad strategy?



Light tracing with next event estimation

$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\rho(x_1 \to x_2 \to x_3)G(x_2 \leftrightarrow x_3)\rho(x_2 \to x_3 \to x_4)G(x_3 \leftrightarrow x_4)L_e(x_3 \to x_4)$$

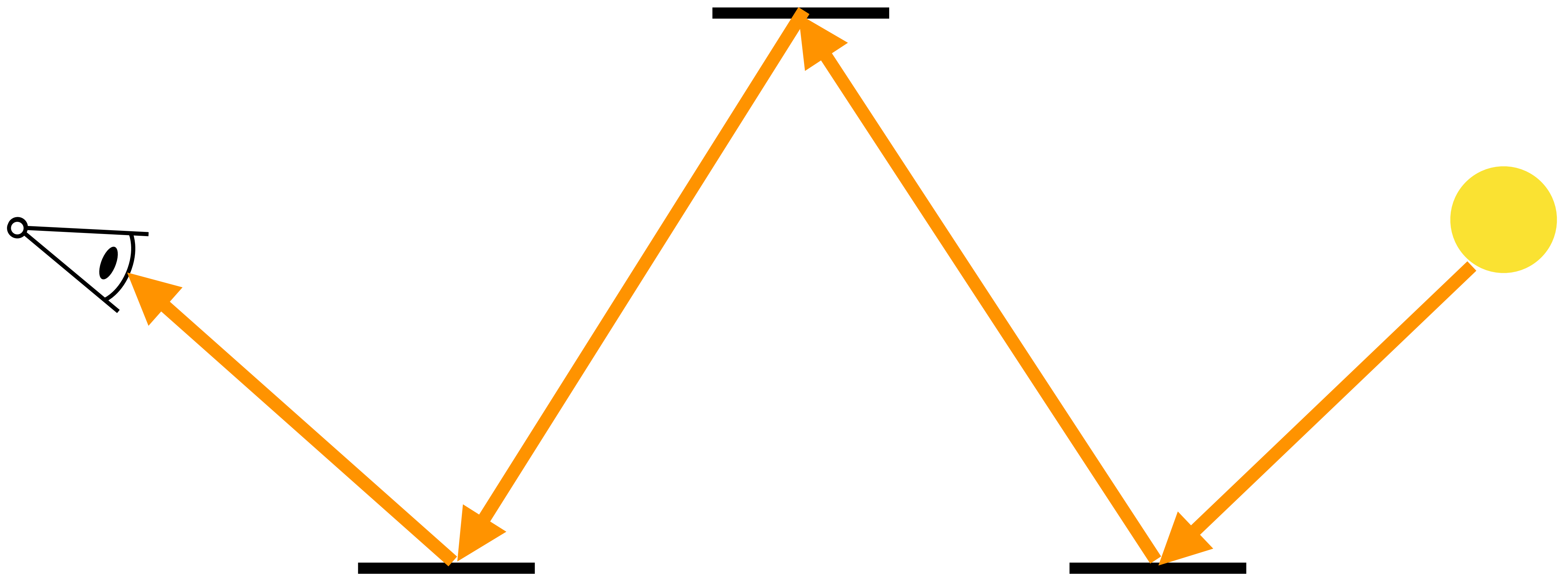
quiz: when will this be a good / bad strategy?



Light tracing without next event estimation

$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\rho(x_1 \to x_2 \to x_3)G(x_2 \leftrightarrow x_3)\rho(x_2 \to x_3 \to x_4)G(x_3 \leftrightarrow x_4)L_e(x_3 \to x_4)$$

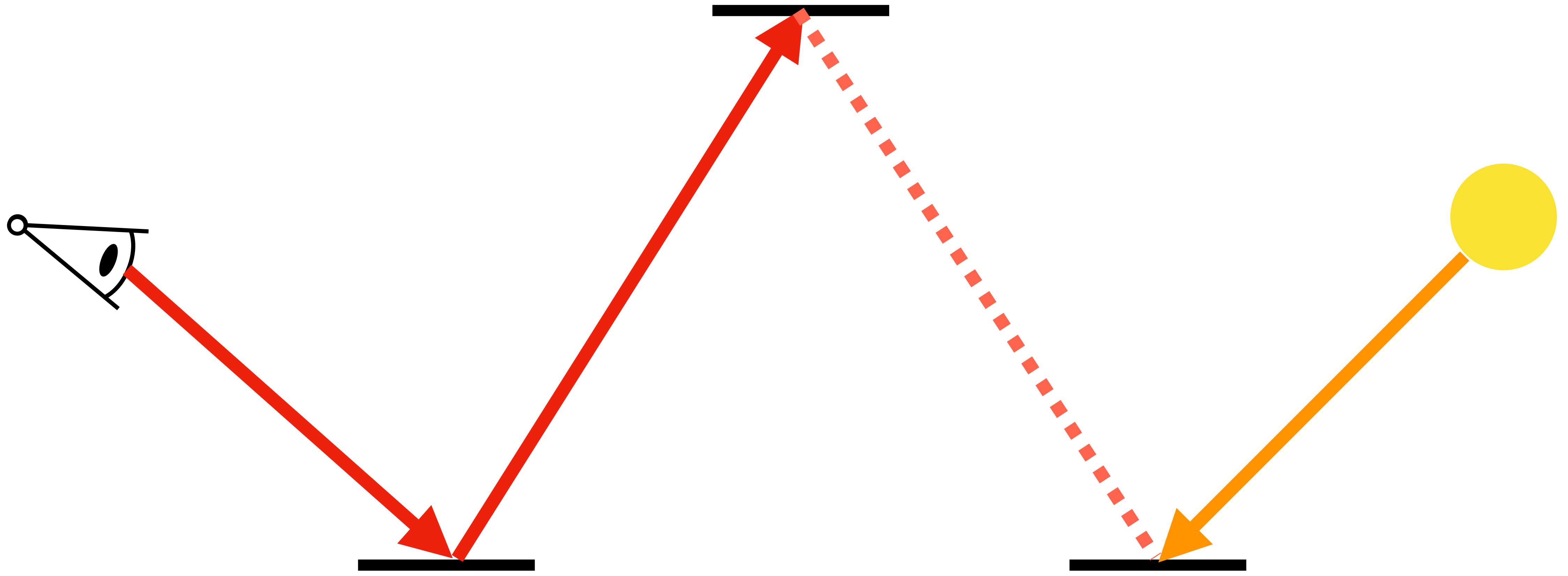
quiz: when will this be a good / bad strategy?



Hybrid path tracing & light tracing

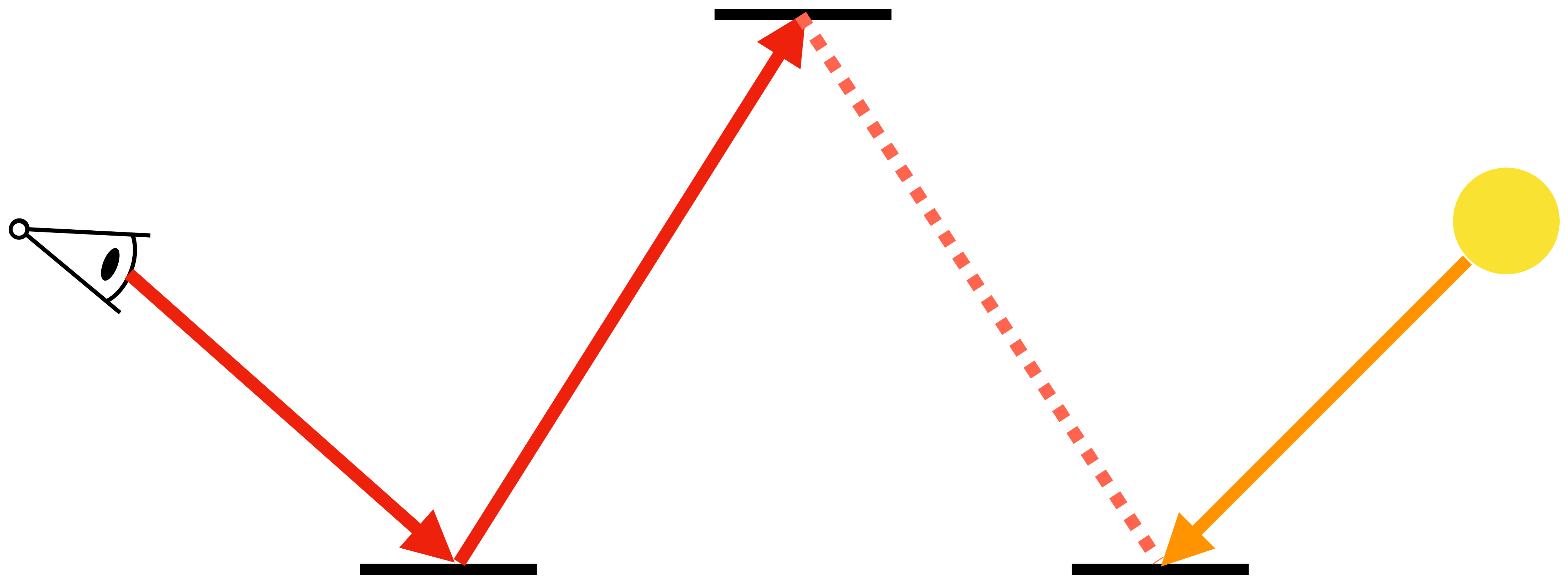
$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\rho(x_1 \to x_2 \to x_3)G(x_2 \leftrightarrow x_3)\rho(x_2 \to x_3 \to x_4)G(x_3 \leftrightarrow x_4)L_e(x_3 \to x_4)$$

quiz: when will this be a good / bad strategy?



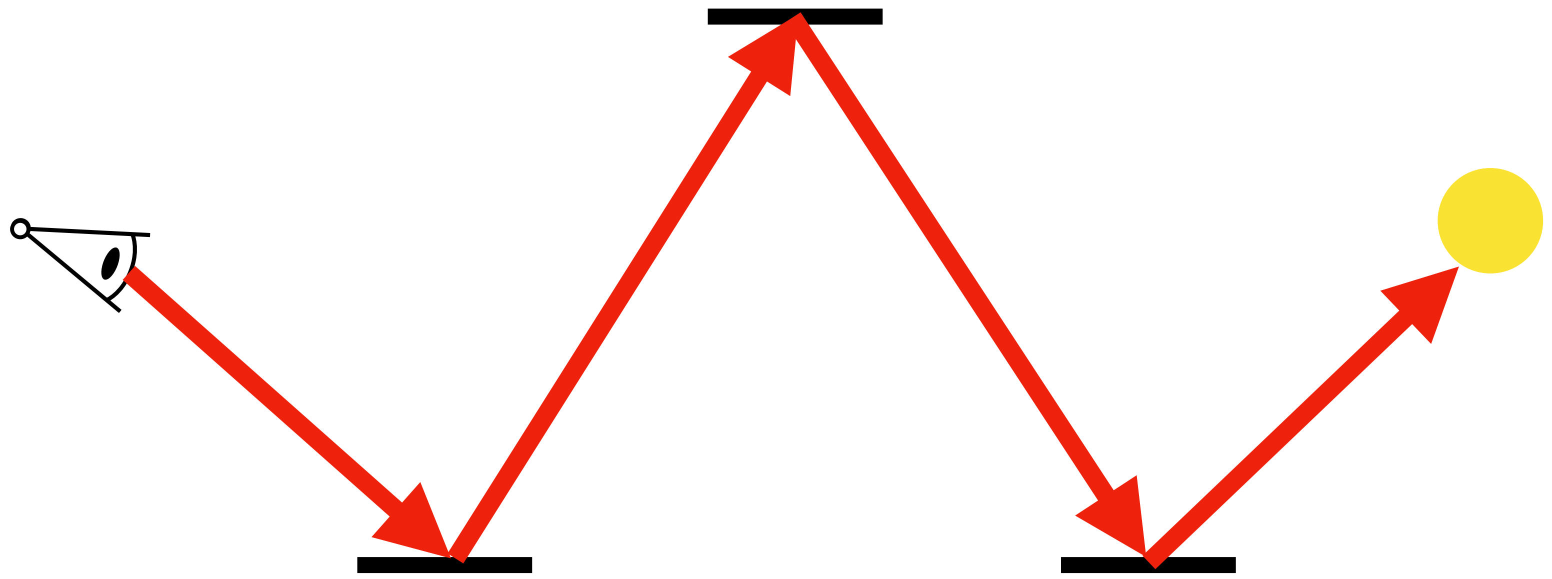
Local path sampling strategy

- two sampling operations
  - sample a point on camera or light
  - BSDF importance sampling

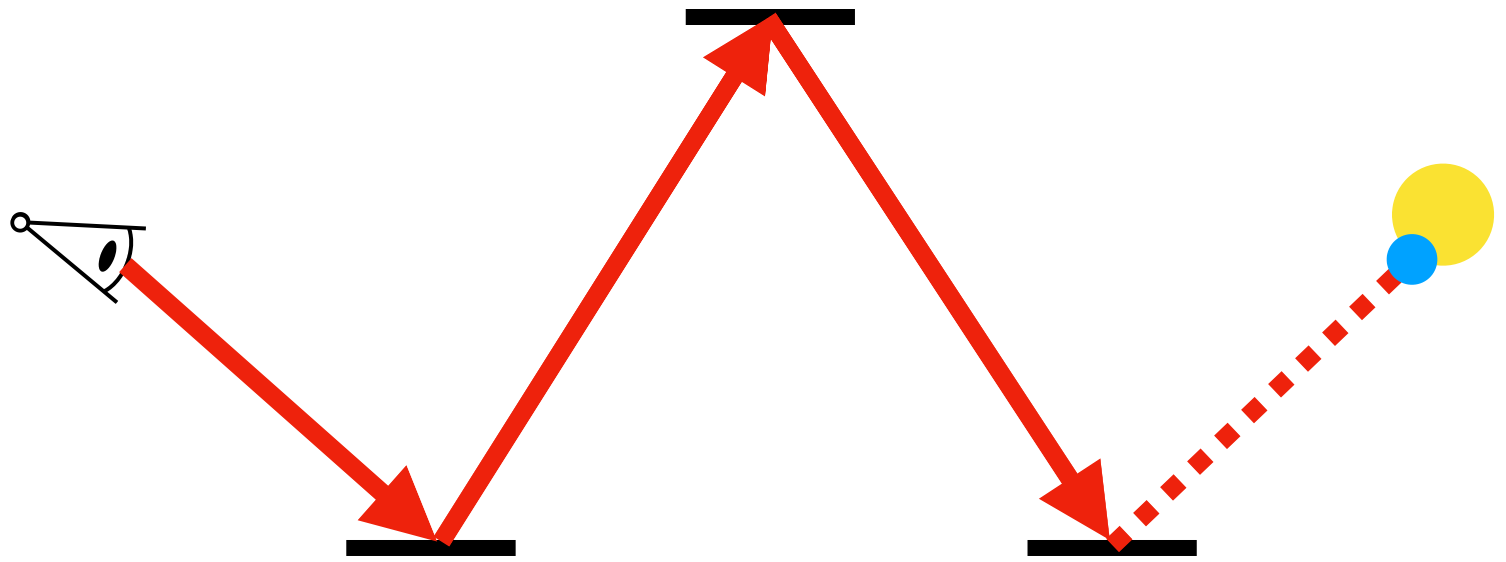


6 ways to sample path length = 4

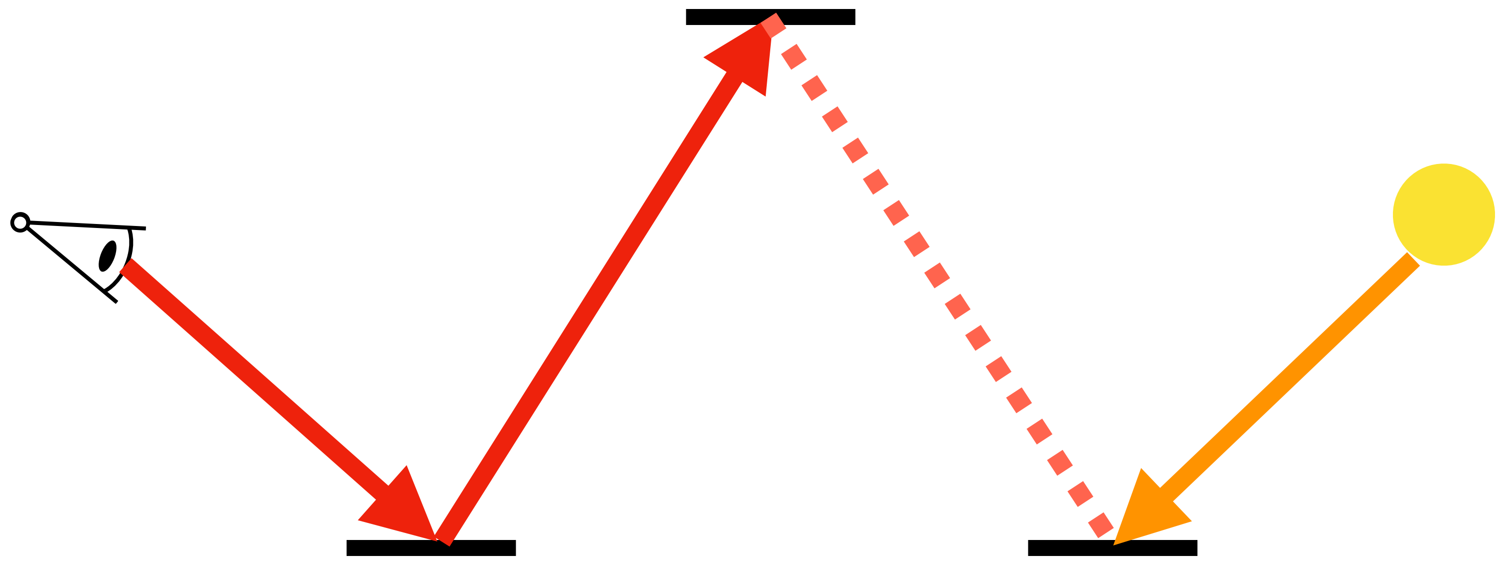
- combine all of them using multiple importance sampling!



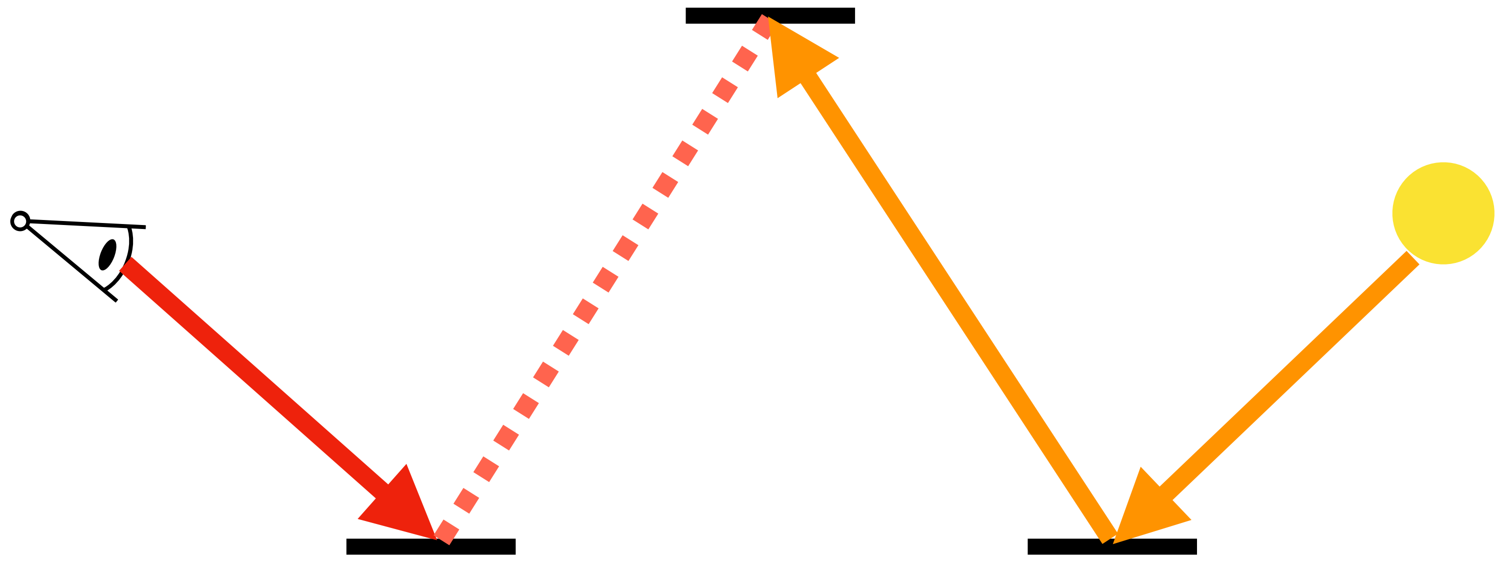
camera vertices count = 5
light vertices count = 0



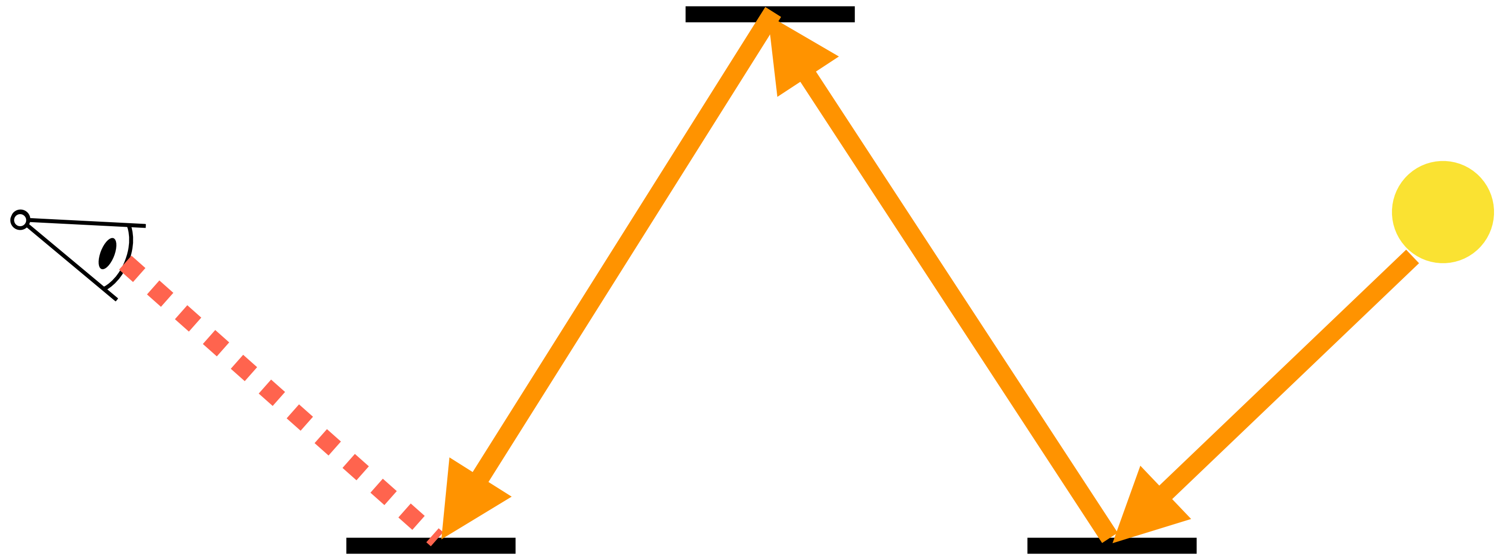
camera vertices count = 4
light vertices count = 1



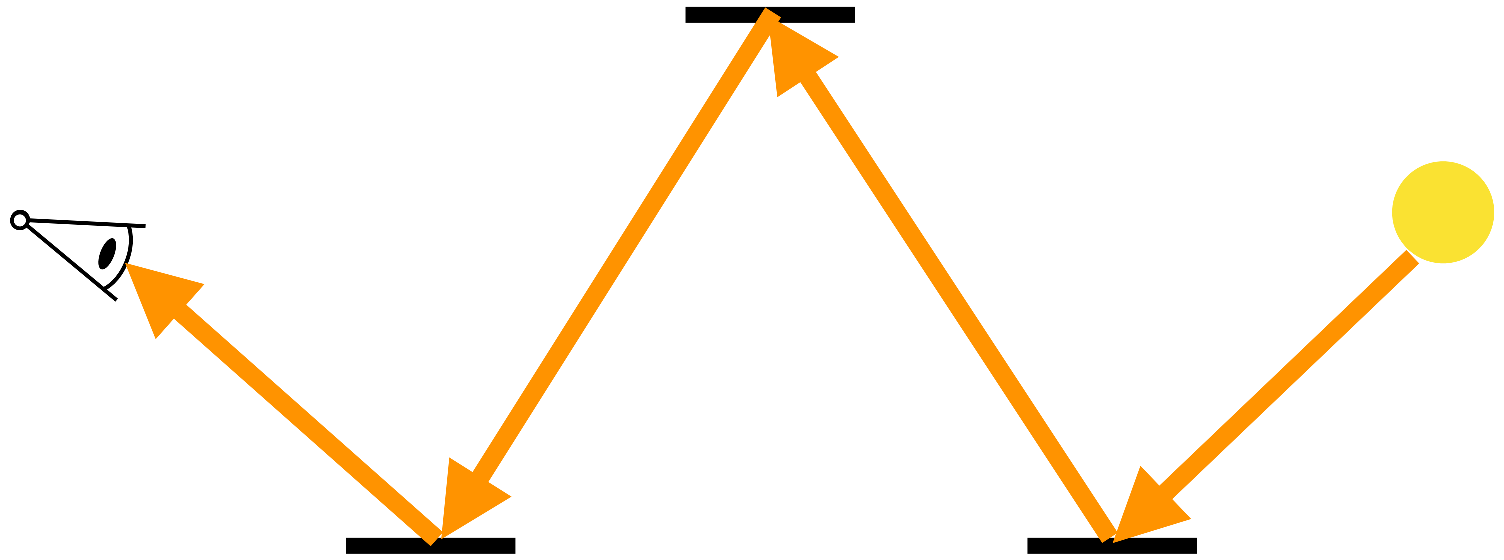
camera vertices count = 3
light vertices count = 2



camera vertices count = 2
light vertices count = 3



camera vertices count = 1
light vertices count = 4



camera vertices count = 0
light vertices count = 5

Veach's bidirectional path tracing scene

quiz: which strategy is good at which region?

https://benedikt-bitterli.me/resources/images/veach-bidir.png

Bidirectional path tracing: combine all sampling

Walkthrough of a bidirectional path tracer

https://cs.uwaterloo.ca/~thachisu/smallpssmlt.cpp

Handling non-symmetric BSDFs

$$f(\omega, \omega') \neq f(\omega', \omega)$$

path tracing integral

$$L = L_e + \int_{S^2} L f(\omega, \omega') \mathrm{d}\omega'$$

light tracing integral

$$W = W_e + \int_{S^2} W f(\omega', \omega) \mathrm{d}\omega'$$

Handling non-symmetric BSDFs

$$f(\omega, \omega') \neq f(\omega', \omega)$$

define the ***adjoint*** of a BSDF

$$f^*(\omega, \omega') = f(\omega', \omega)$$

path tracing integral

$$L = L_e + \int_{S^2} L f(\omega, \omega') \mathrm{d}\omega'$$

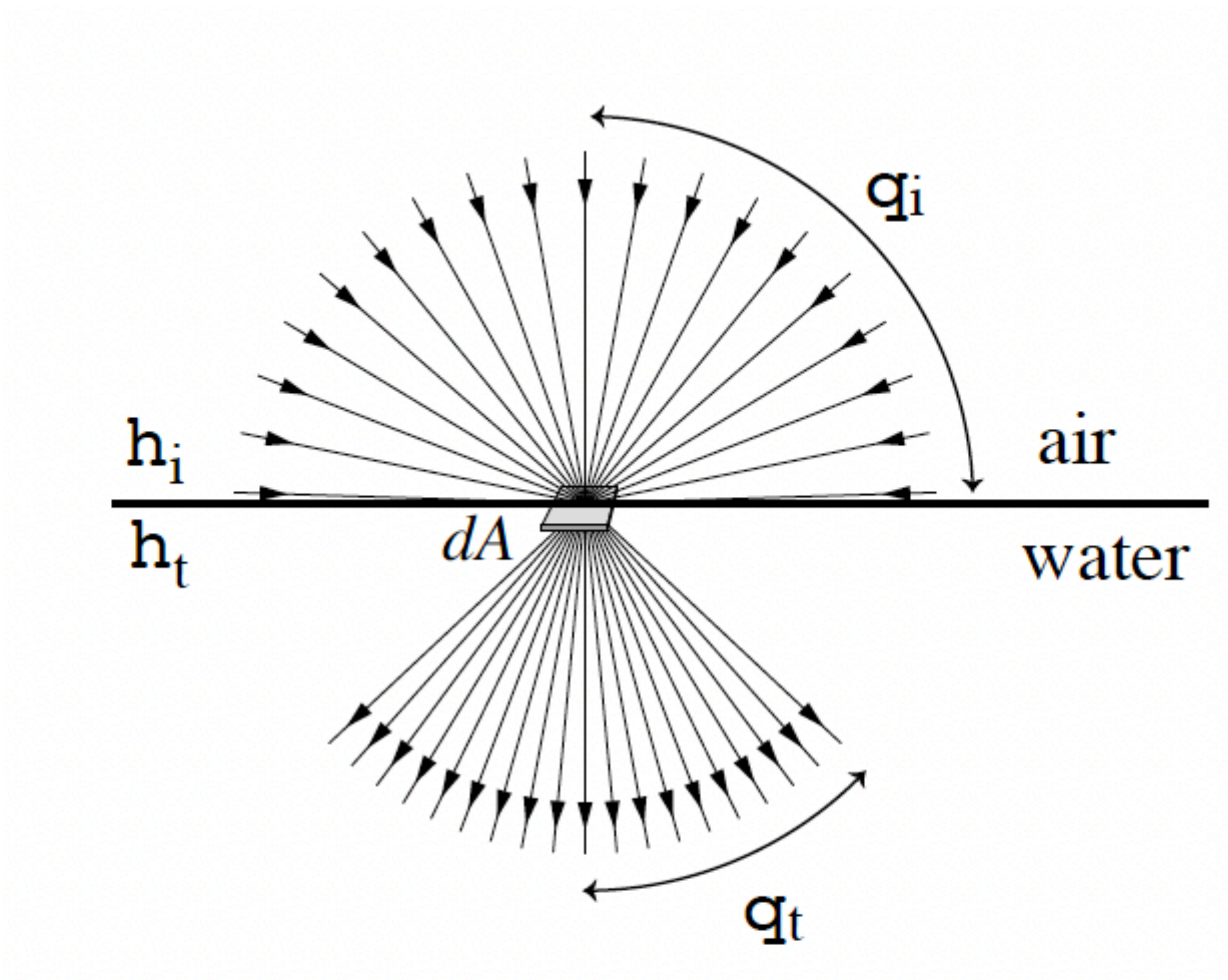
light tracing integral

$$W = W_e + \int_{S^2} W f^*(\omega, \omega') \mathrm{d}\omega'$$

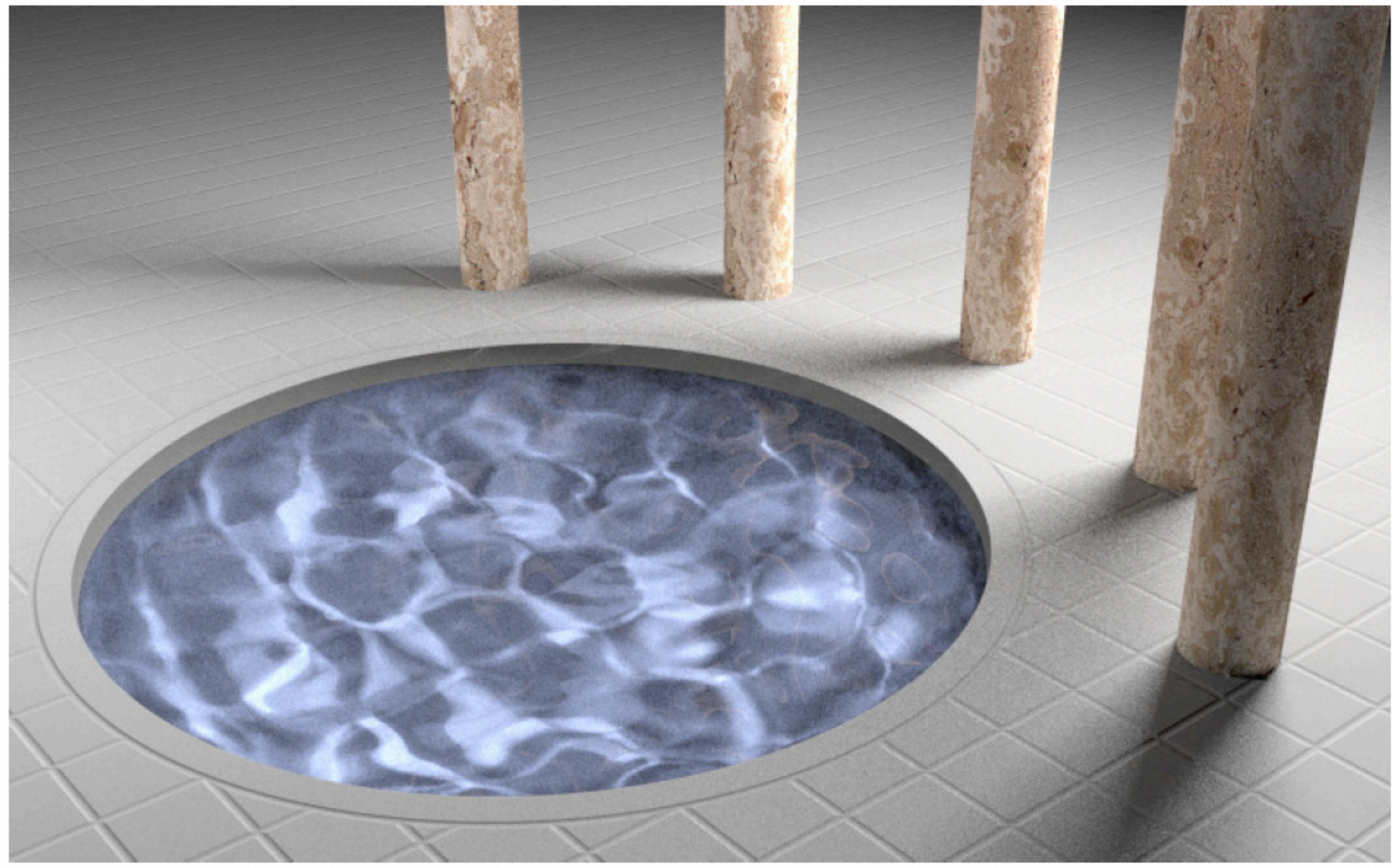
When is a BSDF non-symmetric?

- refraction

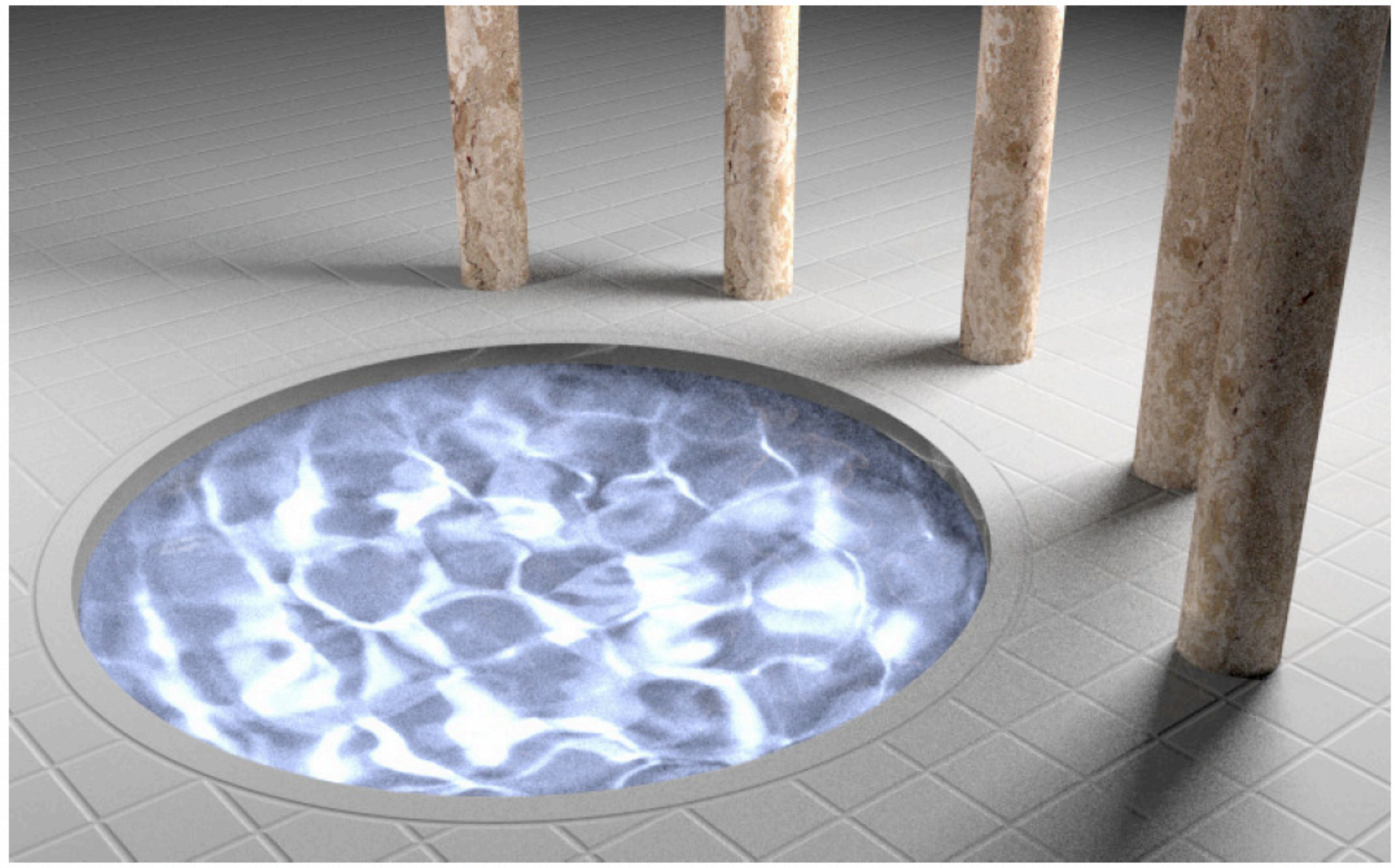
$$\frac{f(\omega_{\text{in}}, \omega_{\text{out}})}{\eta_{\text{out}}} = \frac{f(\omega_{\text{out}}, \omega_{\text{in}})}{\eta_{\text{in}}} = \frac{f^*(\omega_{\text{in}}, \omega_{\text{out}})}{\eta_{\text{in}}}$$



Using wrong BSDFs lead to wrong results



(a)



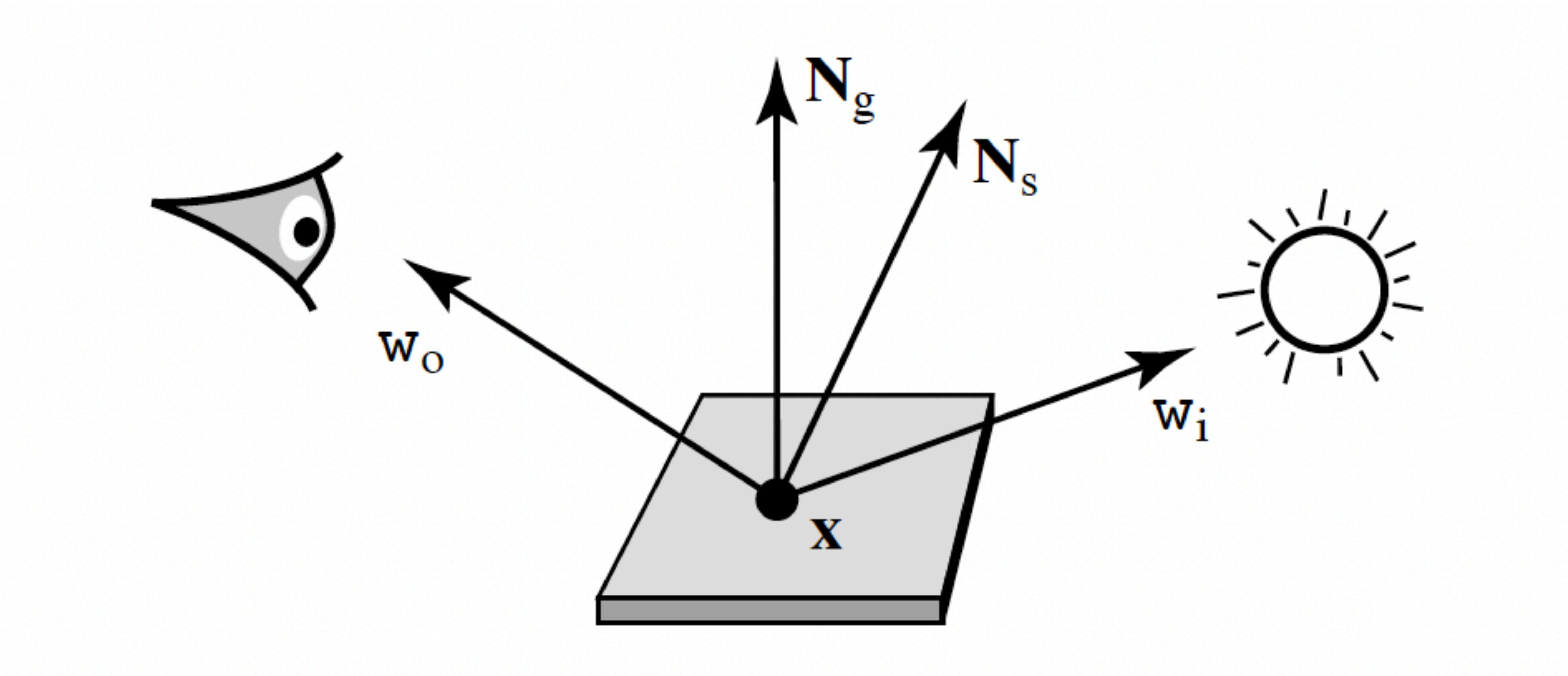
(b)

When is a BSDF non-symmetric?

- shading normal

$$\rho(\omega_i, \omega_o) = f_s(\omega_i, \omega_o)\frac{|N_s \cdot \omega_o|}{|N_g \cdot \omega_o|}$$

$$\rho(\omega_o, \omega_i) = f_s(\omega_i, \omega_o)\frac{|N_s \cdot \omega_i|}{|N_g \cdot \omega_i|}$$



$$\rho * (\omega_i, \omega_o) = \rho(\omega_o, \omega_i)\frac{|N_s \cdot \omega_i|\ |N_g \cdot \omega_o|}{|N_g \cdot \omega_i|\ |N_s \cdot \omega_o|}$$

Shading normals violate conservation of energy

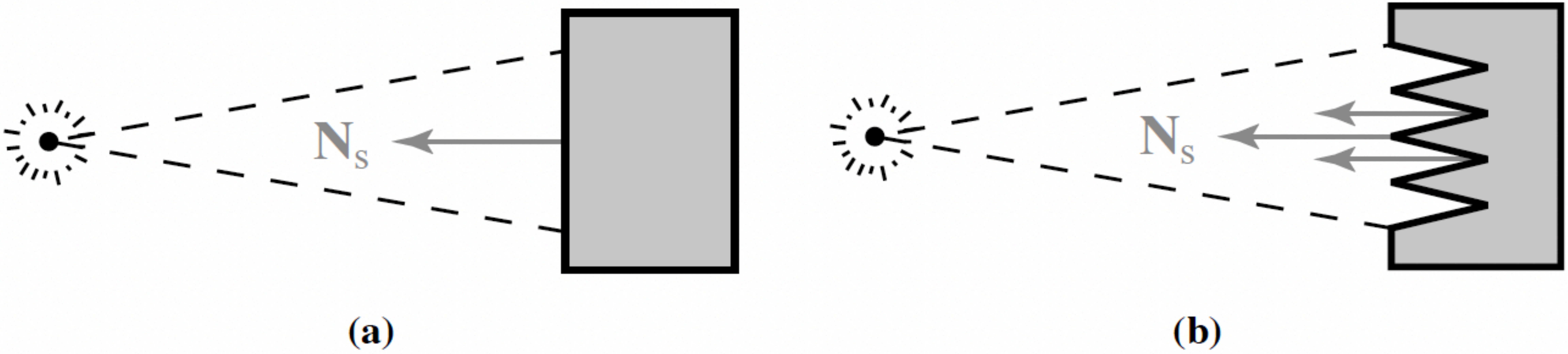
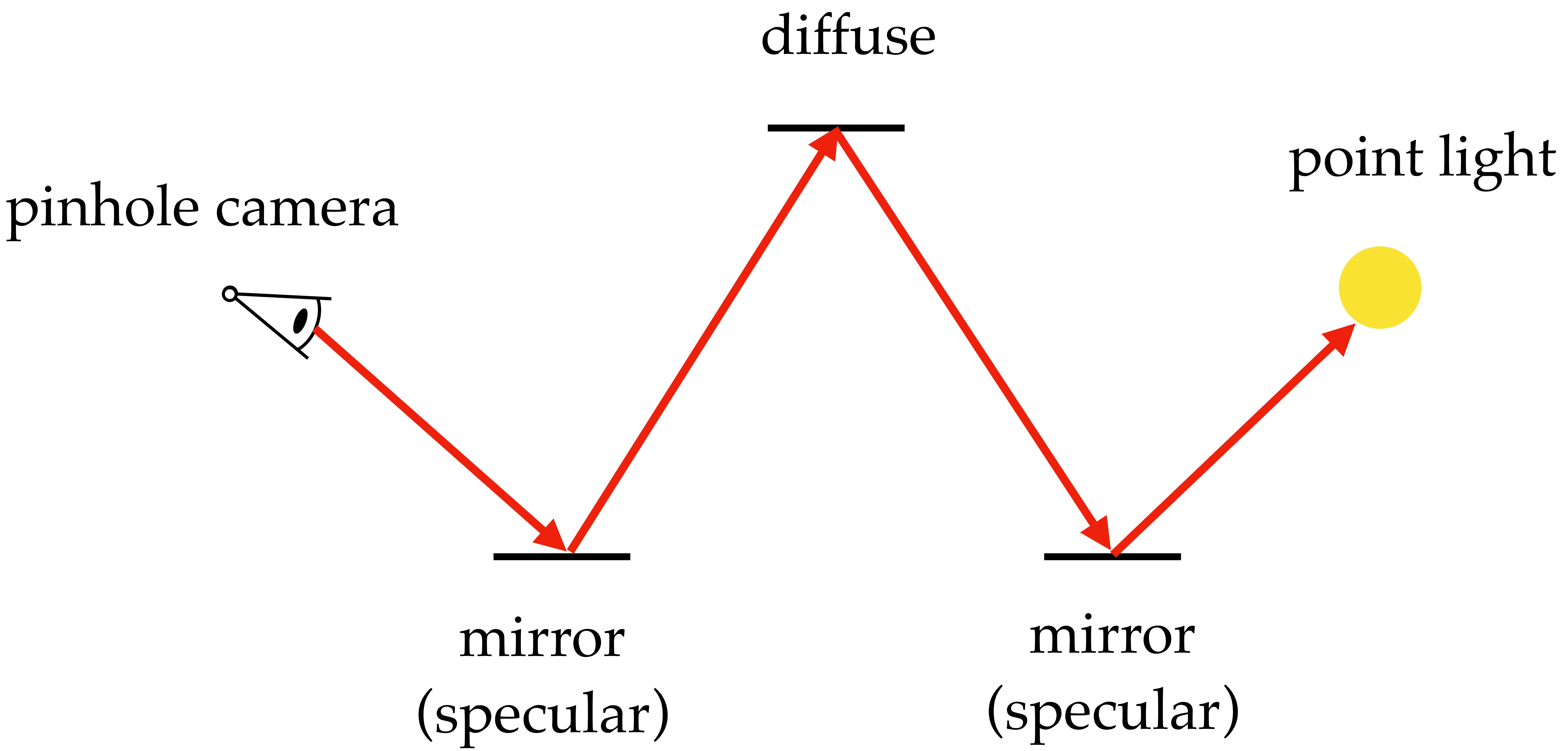
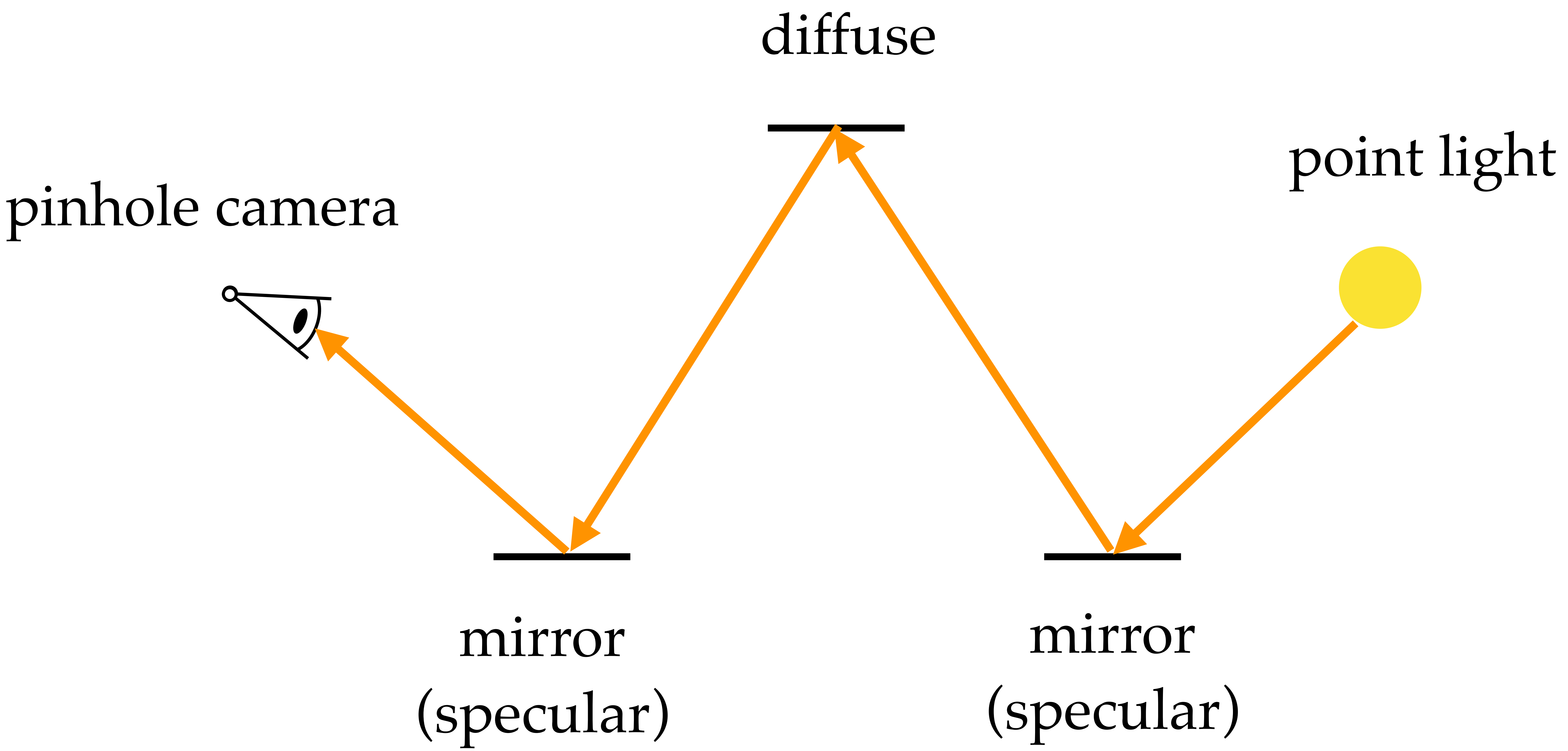


Figure 5.9: **(a)** A flat, diffuse surface facing toward a point light source, with $\mathbf{N}_s = \mathbf{N}_g$. The surface is assumed to not to absorb any light, so that the incident and reflected power is the same. **(b)** A ridged surface with shading normals that point toward the light. It receives the same power as (a), but reflects far more due to its larger surface area.

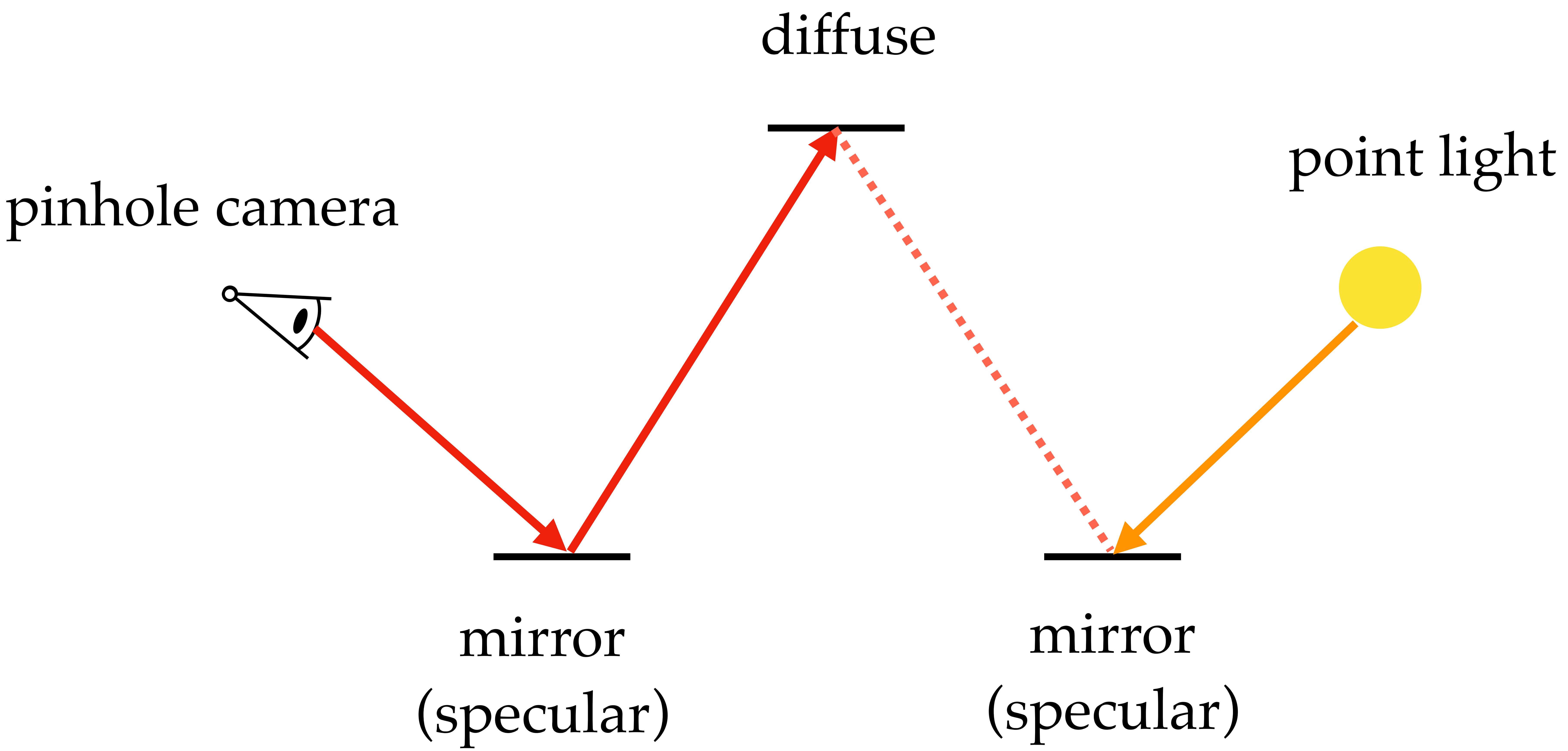
Limitations of local sampling strategies



Limitations of local sampling strategies

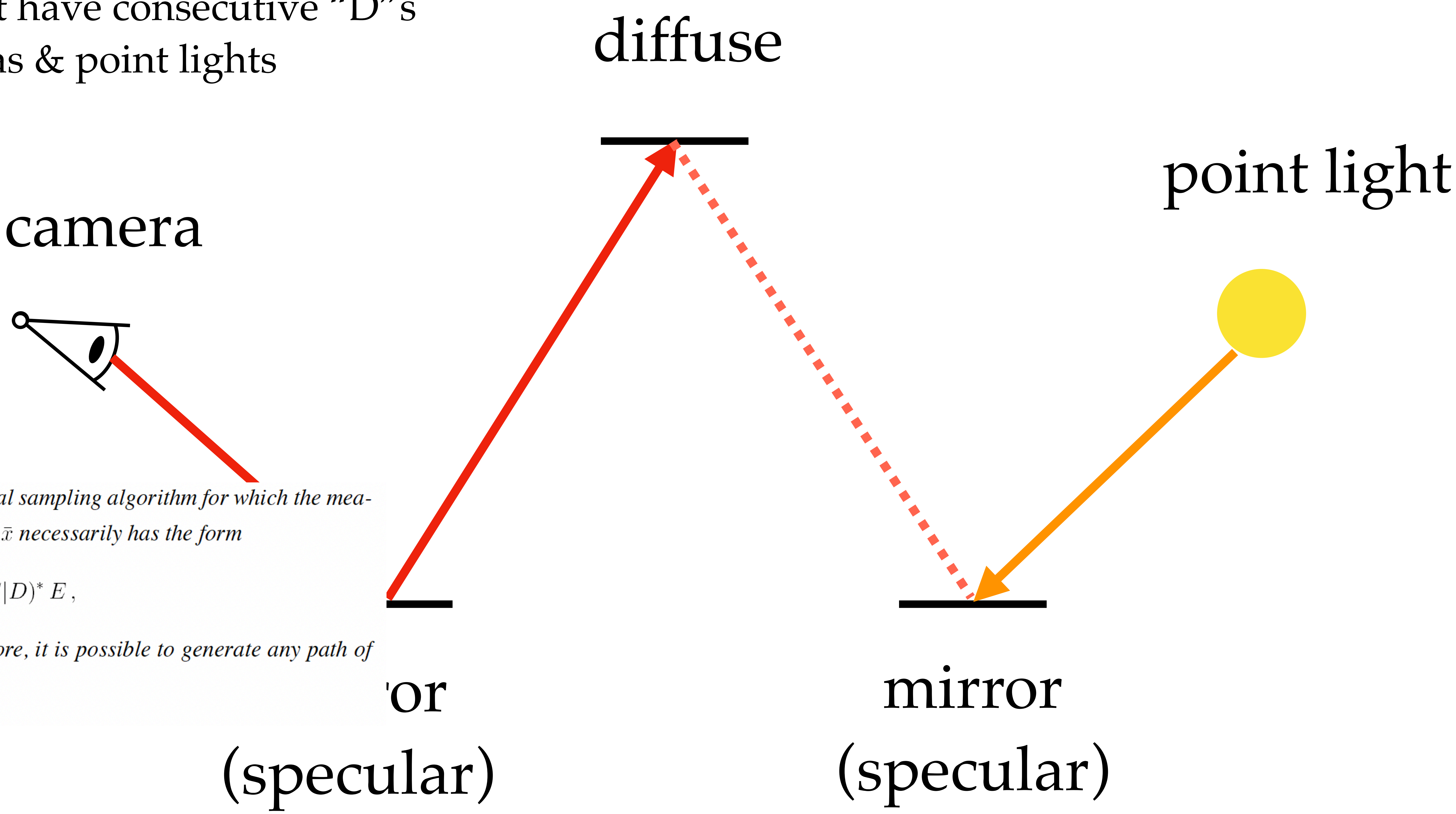


Limitations of local sampling strategies



Limitations of local sampling strategies

Theorem: no local sampling strategy can handle light paths that don't have consecutive "D"s with pinhole cameras & point lights



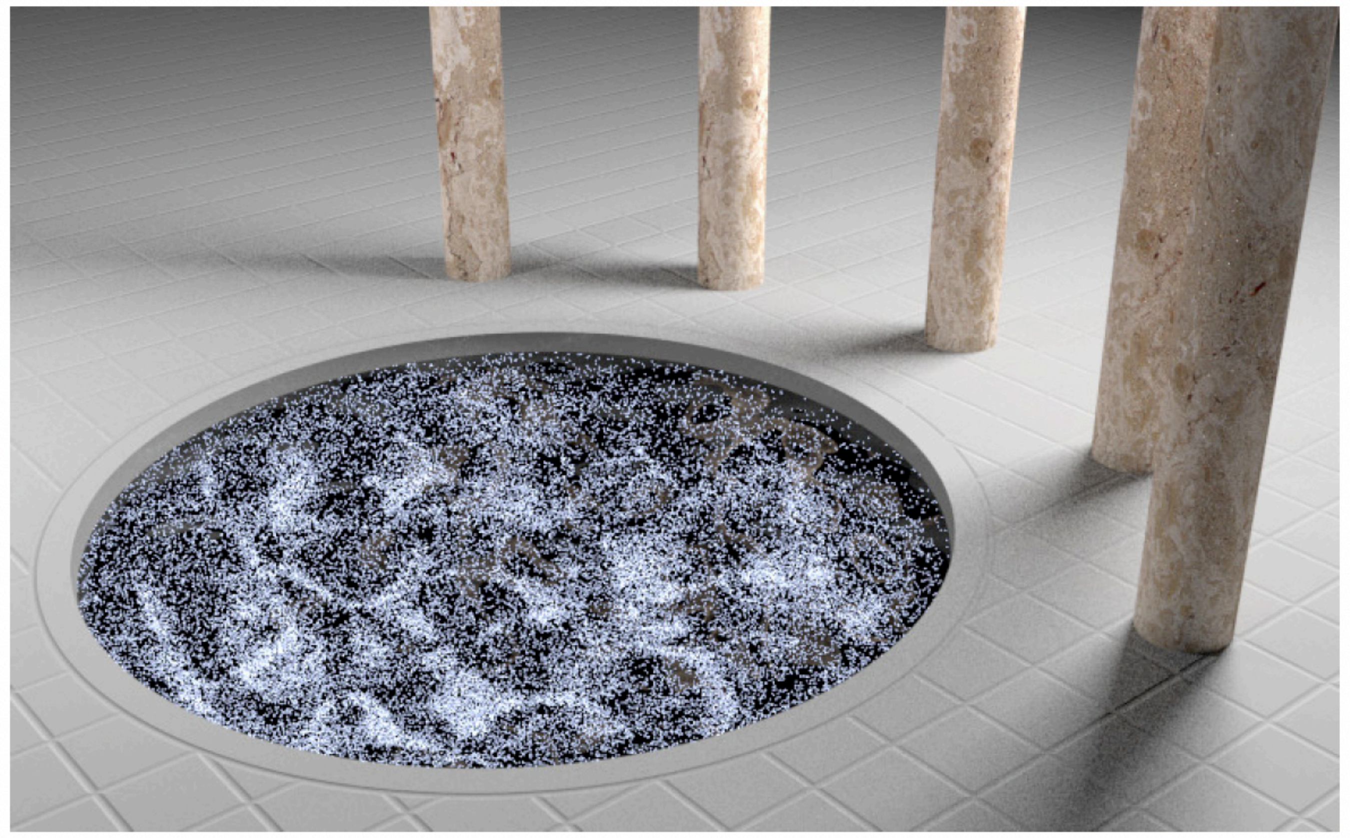
Theorem 8.3. *Let $\bar{x}$ be a path generated by a local sampling algorithm for which the measurement contribution function is non-zero. Then $\bar{x}$ necessarily has the form*

$$L\,(S|D)^*\,D\,D\,(S|D)^*\,E\,,$$

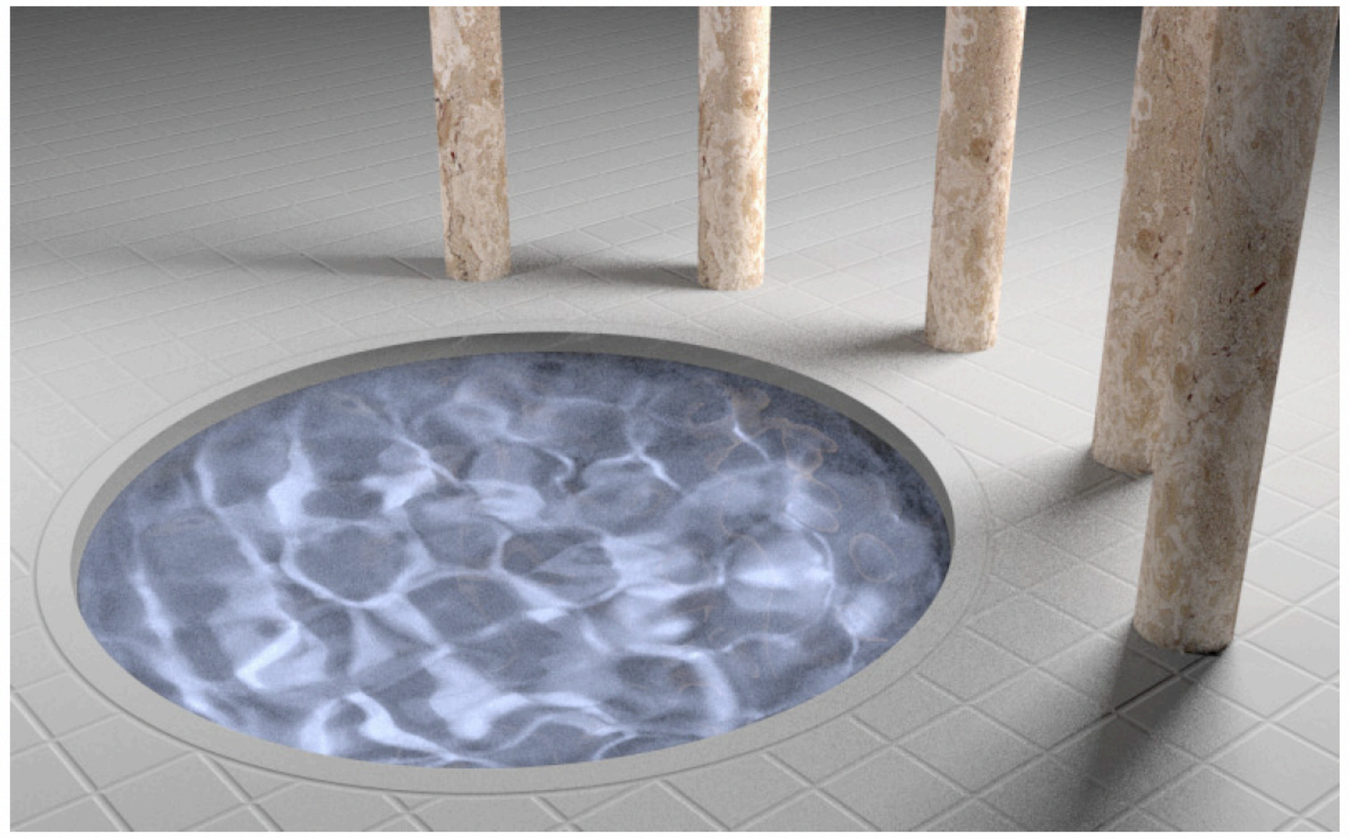
i.e. it must contain the substring DD. Furthermore, it is possible to generate any path of this form using local sampling strategies.

SDS light paths

- caustics seen through a mirror or glass



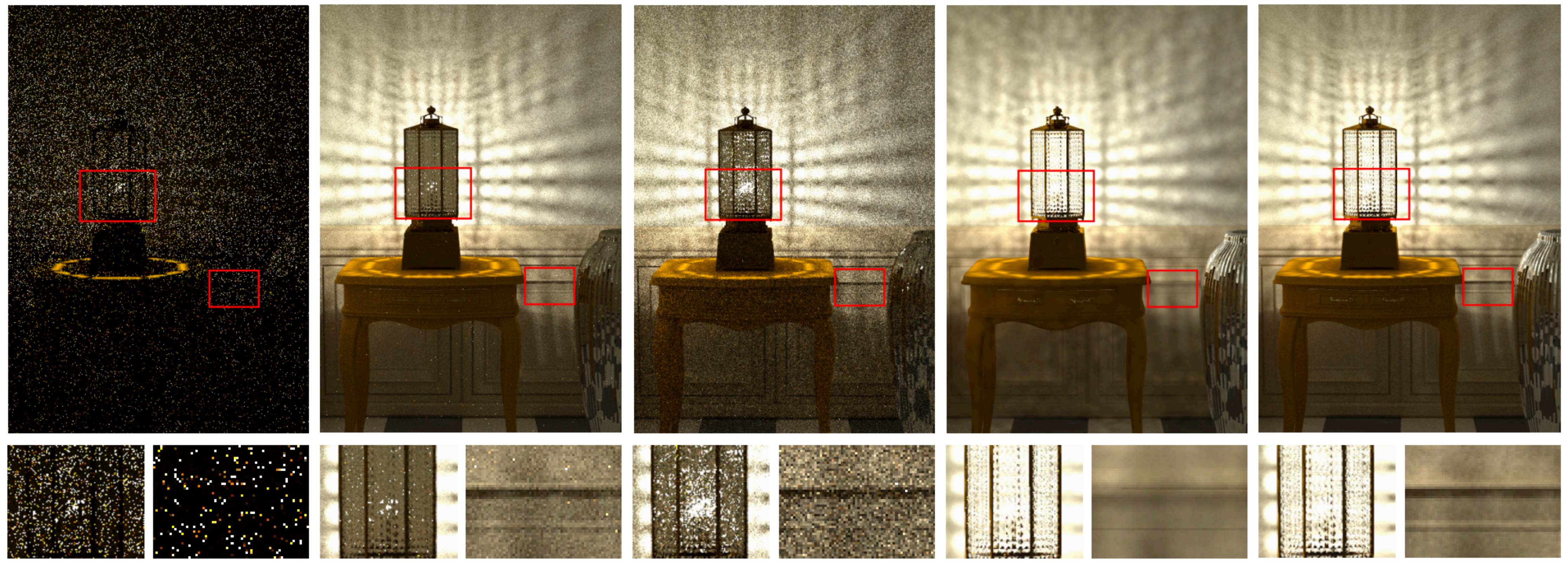
(a) Path tracing with 210 samples per pixel.



(b) Metropolis light transport with 100 mutations per pixel [the same computation time as (a)].

Progressive Photon Mapping

Toshiya Hachisuka, UC San Diego
Shinji Ogaki, The University of Nottingham
Henrik Wann Jensen, UC San Diego



Path tracing | Bidirectional path tracing | Metropolis light transport | Photon mapping | Progressive photon mapping

more about photon mapping next time

“Non-local” path sampling

- more about them in the future lectures

Illumination from Curved Reflectors

Don Mitchell †
Pat Hanrahan ‡

† AT&T Bell Laboratories
‡ † Princeton University

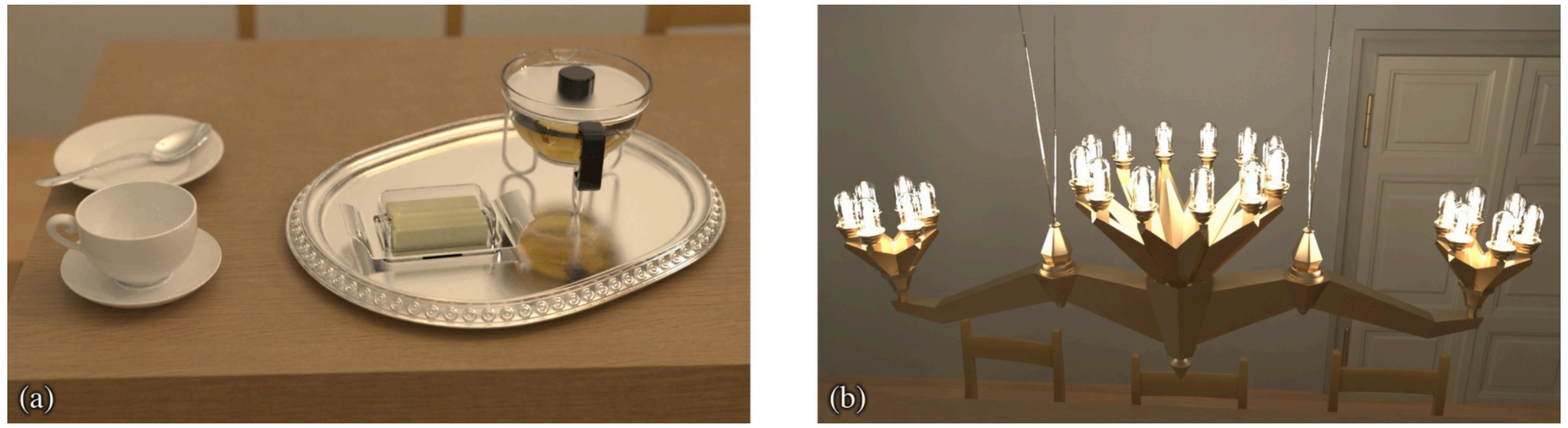
Single Scattering in Refractive Media with Triangle Mesh Boundaries

Bruce Walter, *Cornell University*
Shuang Zhao, *Cornell University*
Nicolas Holzschuch, *INRIA – LJK*
Kavita Bala, *Cornell University*

Manifold Exploration: A Markov Chain Monte Carlo Technique for Rendering Scenes with Difficult Specular Transport

Wenzel Jakob Steve Marschner

In *ACM Transactions on Graphics (Proceedings of SIGGRAPH 2012)*

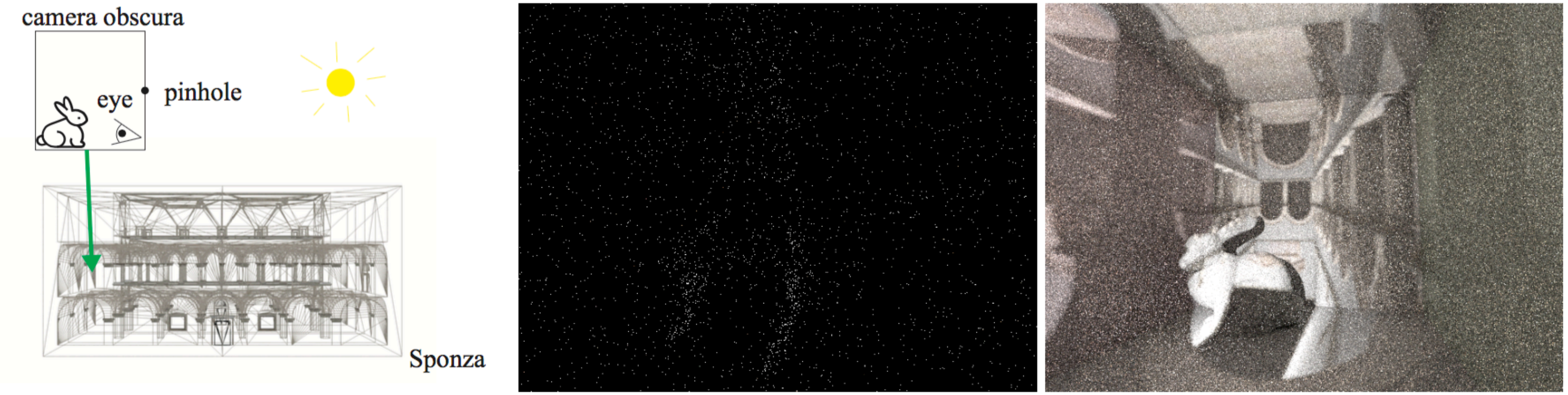


Specular Manifold Sampling for Rendering High-Frequency Caustics and Glints

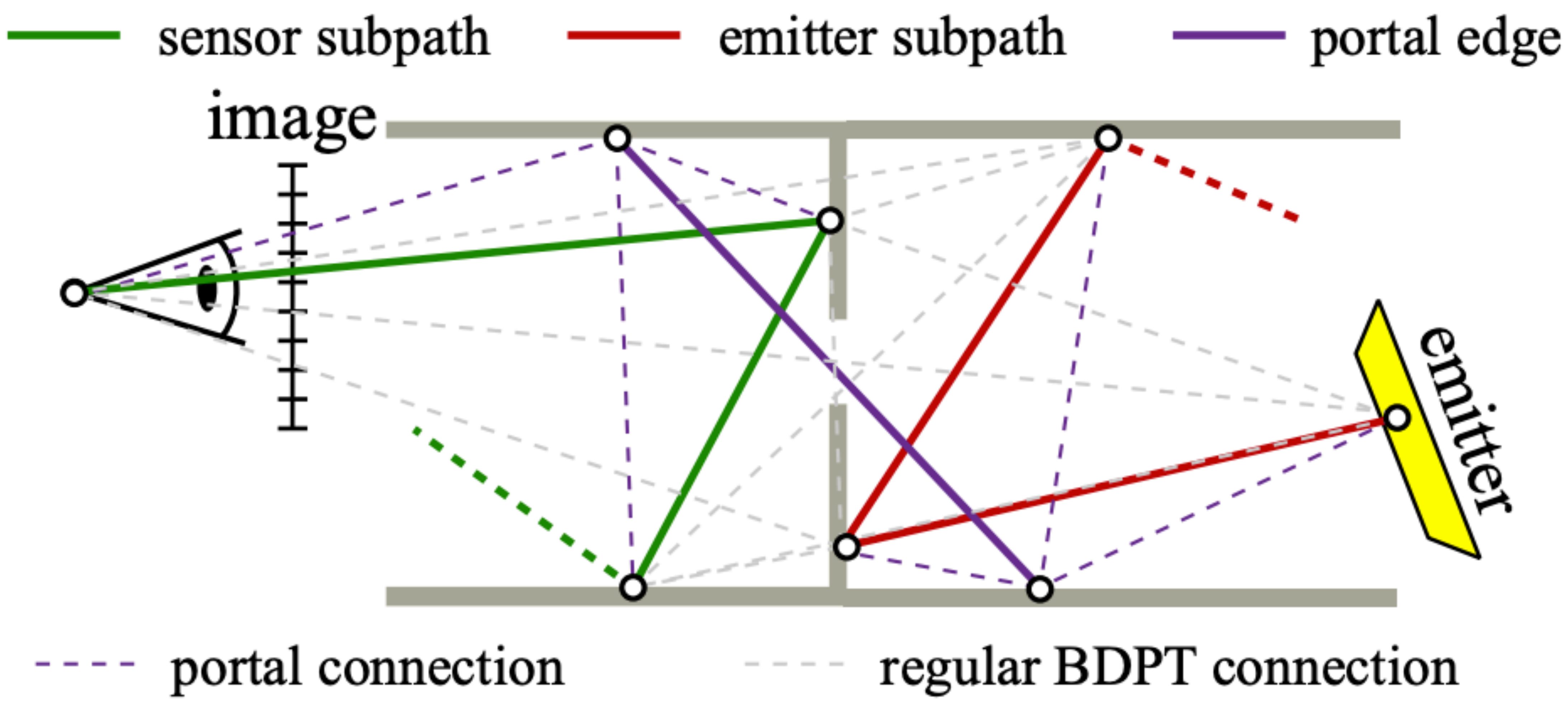
Tizian Zeltner, EPFL
Iliyan Georgiev, Autodesk
Wenzel Jakob, EPFL

In *Transactions on Graphics (Proceedings of SIGGRAPH 2020)*

Tri-directional path tracing



Camera Obscura Scene | Bidirectional Path Tracing, 64 spp | Our Tridirectional Path Tracing, 64 spp



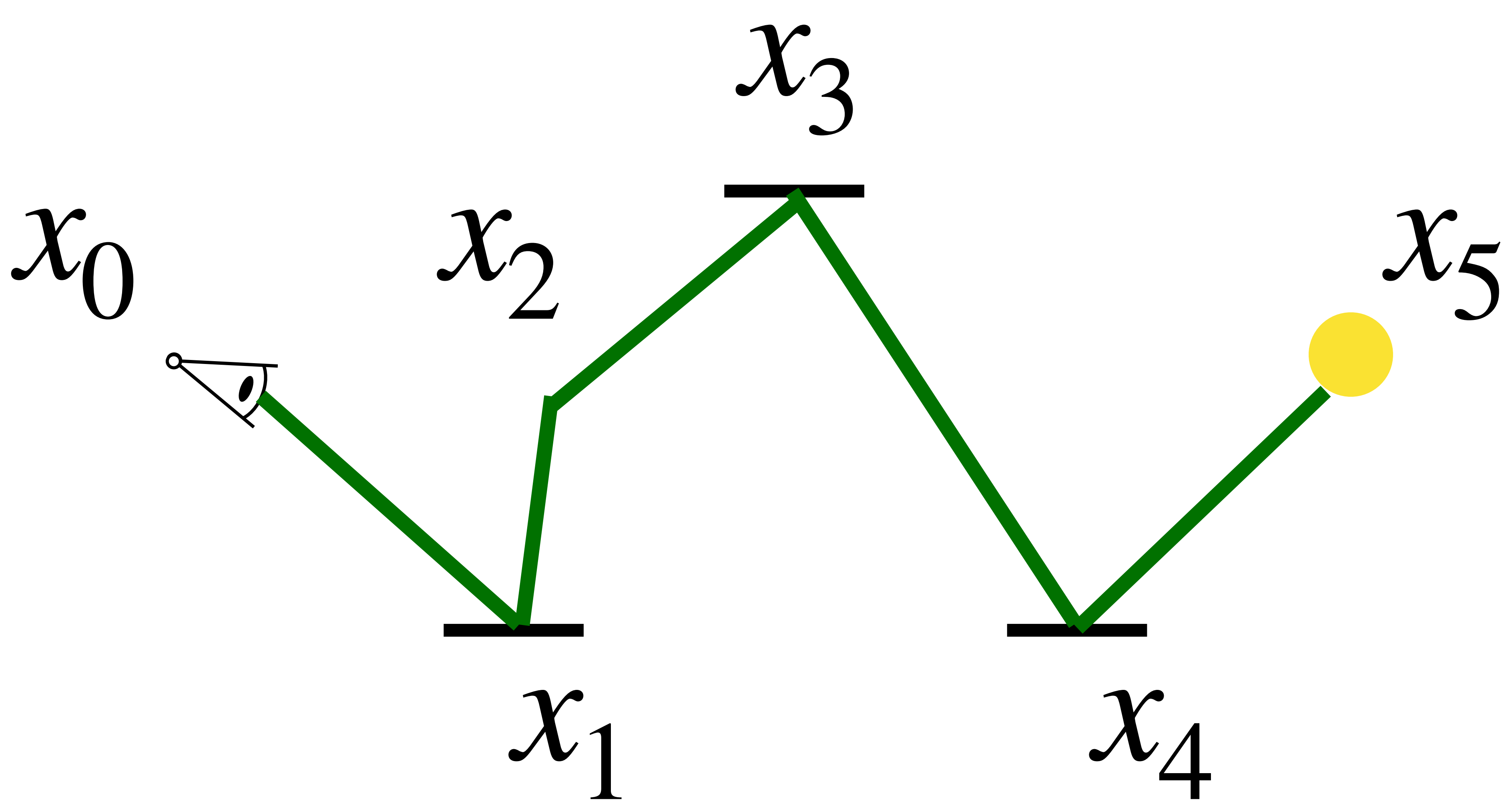
Aether: An Embedded Domain Specific Sampling Language for Monte Carlo Rendering

LUKE ANDERSON, MIT CSAIL
TZU-MAO LI, MIT CSAIL
JAAKKO LEHTINEN, Aalto University and NVIDIA
FRÉDO DURAND, MIT CSAIL and Inria, Université Côte d'Azur

Path integral for volumetric rendering

quiz: how do we modify the following surface path contribution to take volumes into consideration?

$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)\cdots$$



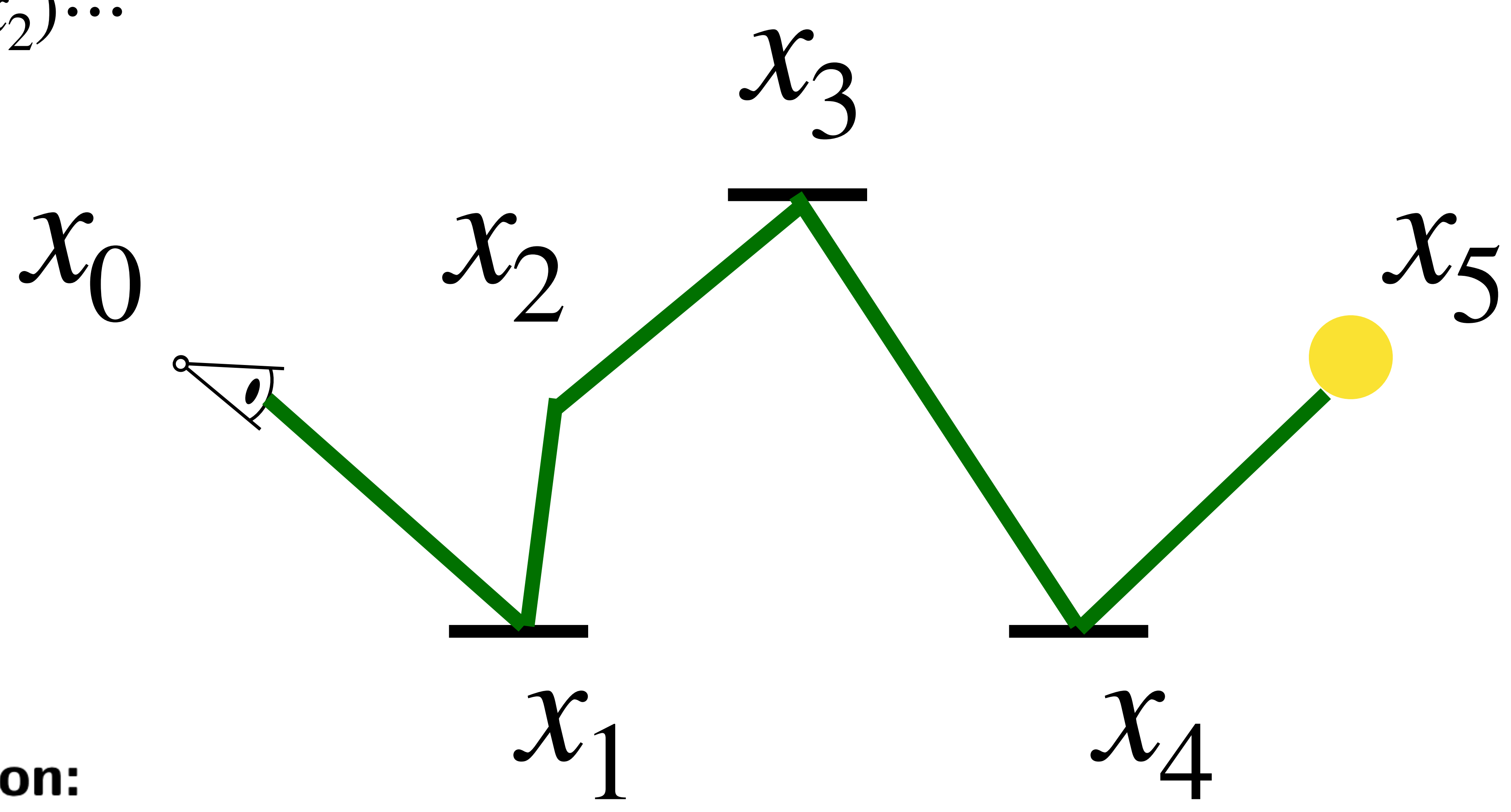
Path integral for volumetric rendering

transmittance

$$f(\bar{x}) = W(x_0 \to x_1)G(x_0 \leftrightarrow x_1)T(x_0 \leftrightarrow x_1)\rho(x_0 \to x_1 \to x_2)G(x_1 \leftrightarrow x_2)T(x_1 \leftrightarrow x_2)\cdots$$

BSDF or
phase function

G(x, y) doesn't have the cosine term if y is in volume



Metropolis Light Transport for Participating Media

Mark Pauly — ETH Zürich — pauly@inf.ethz.ch
Thomas Kollig, Alexander Keller — University of Kaiserslautern — {kollig, keller}@informatik.uni-kl.de

Manifold Exploration:
A Markov Chain Monte Carlo technique for rendering scenes with difficult specular transport

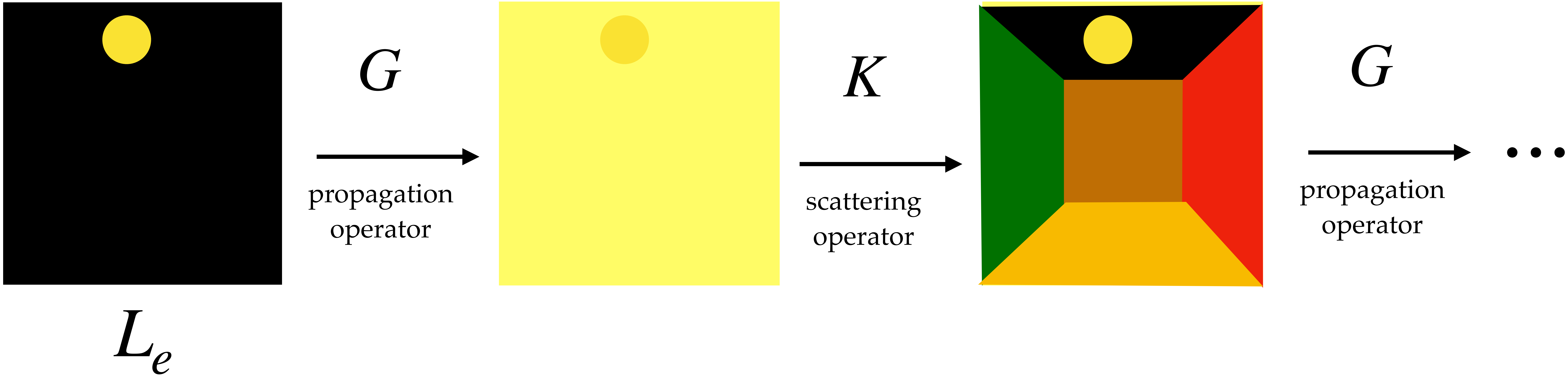
Expanded Technical Report

Wenzel Jakob, Steve Marschner
Cornell University

May 7, 2012

Operator formulation of rendering

- rendering = linear functions operate on 4D surface light fields



Rendering = solving equilibrium of a linear operator

$$L = L_e + \mathbf{T}L$$

$$L = (\mathbf{I} - \mathbf{T})^{-1} L_e$$

Rendering = solving equilibrium of a linear operator

$$L = L_e + \mathbf{T}L$$

$$L = (\mathbf{I} - \mathbf{T})^{-1}L_e$$

$$L = L_e + TL_e + T^2L_e + \cdots$$

Neumann series

(doesn't converge if BSDF is not energy conserving)

https://en.wikipedia.org/wiki/Neumann_series

Operator formulation can be used for studying inverse rendering

$$L = L_e + \mathbf{T}L$$

goal: given L and L_e, solve $\mathbf{T}$

A Theory of Inverse Light Transport

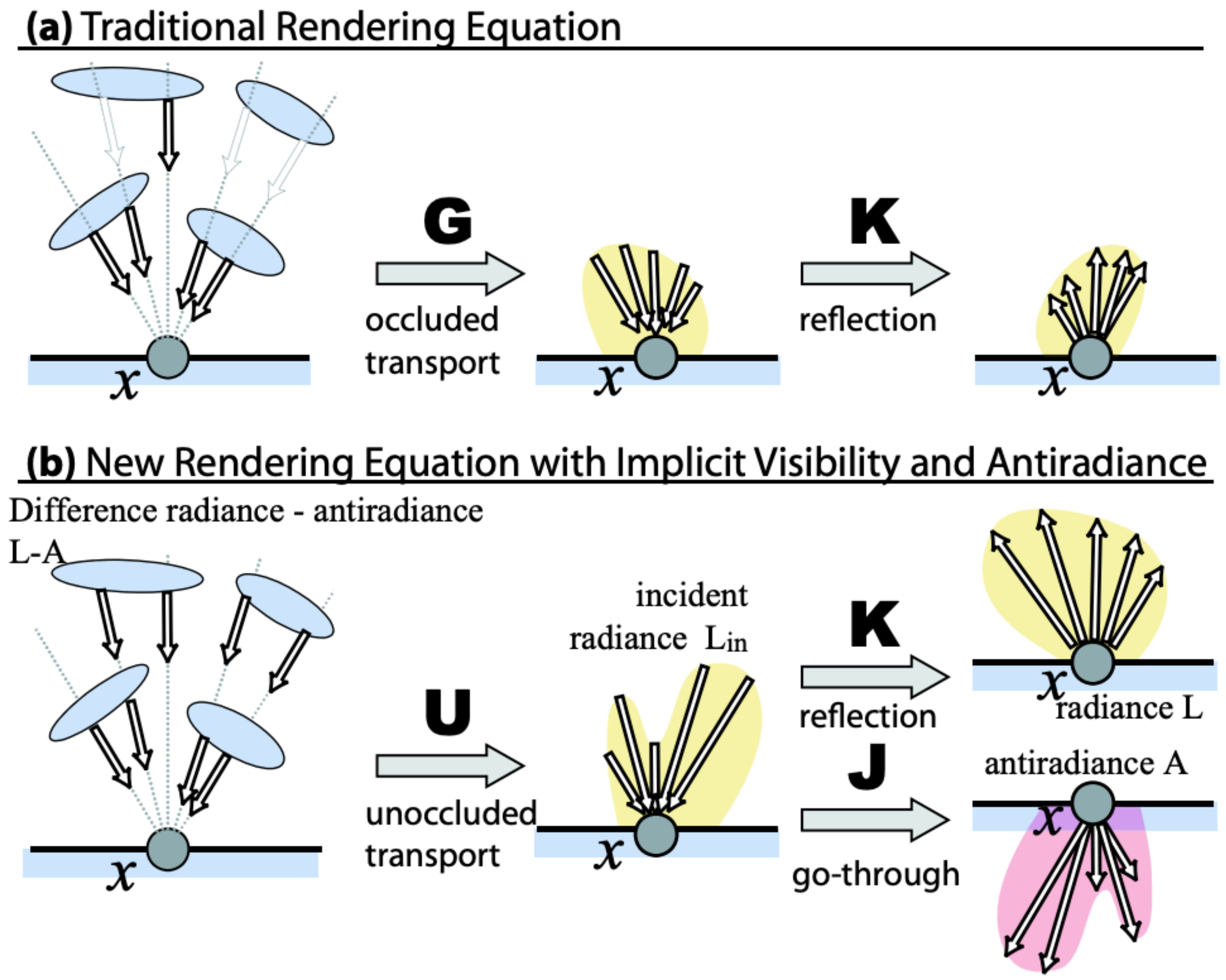
Steven M. Seitz, University of Washington
Yasuyuki Matsushita, Microsoft Research Asia
Kiriakos N. Kutulakos*, University of Toronto

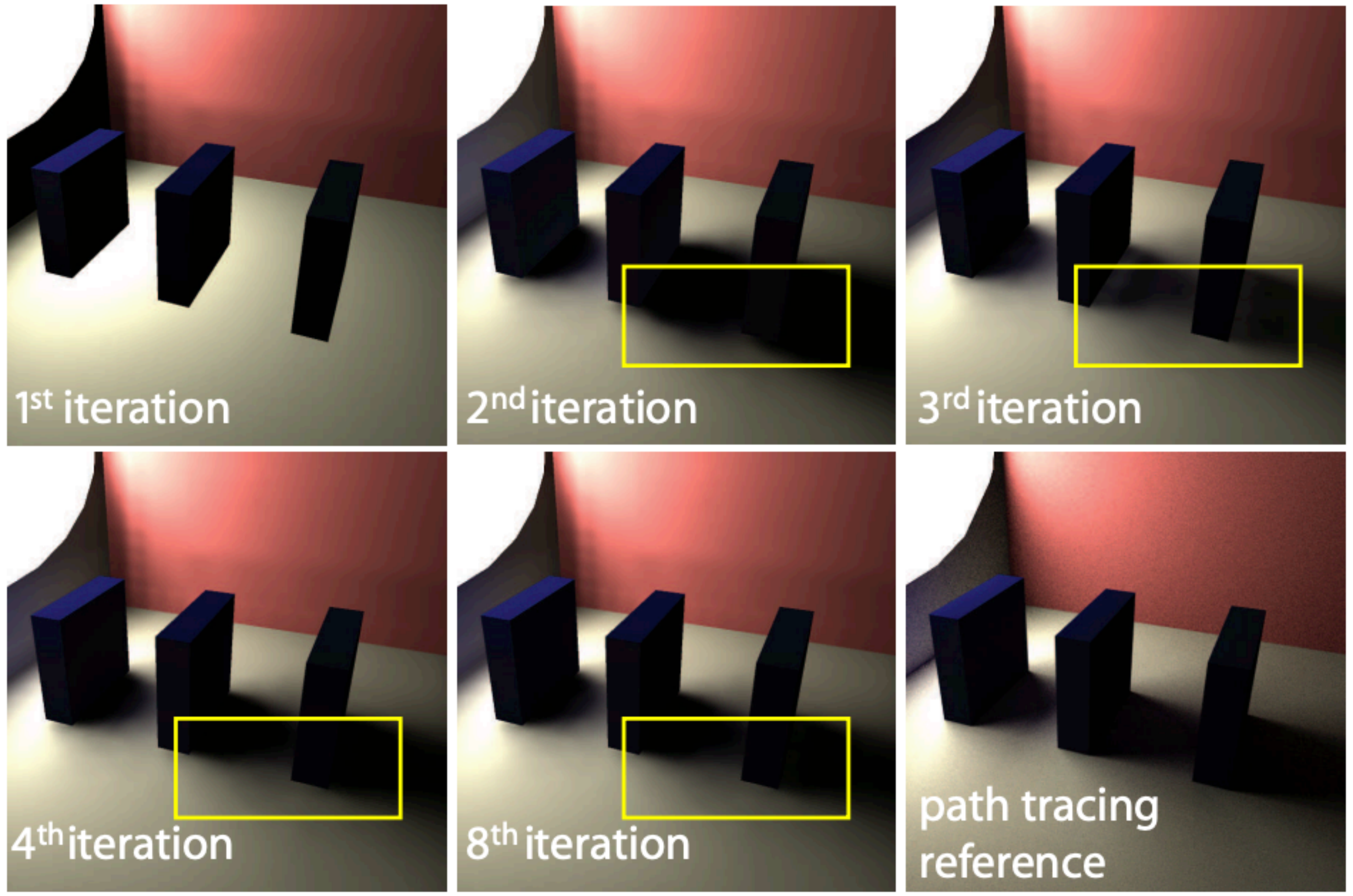
On the Duality of Forward and Inverse Light Transport

Manmohan Chandraker, Jiamin Bai, Tian-Tsong Ng and Ravi Ramamoorthi

Antiradiance: adding a "pass through" operator can speedup rendering

- warning: Neumann series may not converge under this formulation





Implicit Visibility and Antiradiance for Interactive Global Illumination

Carsten Dachsbacher
REVES/INRIA Sophia-Antipolis

Marc Stamminger
University of Erlangen

George Drettakis
REVES/INRIA Sophia-Antipolis

Frédo Durand
MIT CSAIL

Functional analysis of the light transport operator

uncountably infinite dimensional

$$L = L_e + \mathbf{T}L$$

$\longrightarrow$

finite dimensional

$$\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{bmatrix} = \begin{bmatrix} L_{e_1} \\ L_{e_2} \\ \vdots \\ L_{e_n} \end{bmatrix} + \begin{bmatrix} T_{1,1} & T_{1,1} & \cdots & T_{1,n} \\ T_{2,1} & T_{2,1} & \cdots & T_{2,n} \\ & & \vdots & \\ T_{n,1} & T_{1,1} & \cdots & T_{n,n} \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{bmatrix}$$

Soler et al.: the error can be unbounded over arbitrary L_e because T is not "compact"

The Role of Functional Analysis in Global Illumination

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A Theoretical Analysis of Compactness of the Light Transport Operator

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Next: photon mapping

Global Illumination using Photon Maps

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The Technical University of Denmark

A simple museum scene rendered with photon mapping

Note the caustic below the glass sphere, the glossy reflections,
and the overall quality of the global illumination.