UNBIASED WARPED-AREA SAMPLING FOR DIFFERENTIABLE RENDERING

UCSD CSE 272
Advanced Image Synthesis

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with slides from Sai Bangaru
CHALLENGES: EDGE SAMPLING IS HARD!
EDGE SAMPLING HAS TROUBLE WITH SPECULAR REFLECTIONS

(Near-)Perfect Mirror

Rendering Caustics

Manifold-Exploration
MLT
[Jakob 2012]

Natural Constraint Representation for MLT
[Kaplanyan 2014]
SILHOUETTE EXTRACTION IS DIFFICULT FOR IMPLICIT REPRESENTATIONS
CAN WE DESIGN AN UNBIASED AREA SAMPLING METHOD?

Reparameterizing Discontinuous Integrands for Differentiable Rendering [Loubet 2019]

Transform samples with $\theta$. Avoids discontinuities.

Heuristic Approximation! May not work for all samples.
OUR APPROACH
THE REYNOLDS TRANSPORT THEOREM

\[ \partial_D \int_D f = \int_D \partial_D f + \int_{\partial D} f \nabla \cdot \vec{n} \]

Interior term  
Edge term
CONVERTING EDGE SAMPLES TO AREA SAMPLES

Goal: Rewrite $\int_{\partial D} f \vec{v} \cdot \vec{n}$ into area integral $\int_D g$ is estimated through edge samples.
THE DIVERGENCE THEOREM

$\int_{\partial D} \vec{f} \cdot \vec{n} \quad \leftrightarrow \quad \int_{D} \nabla \cdot \vec{f}$

[Gauss 1813]
APPLYING THE DIVERGENCE THEOREM TO THE EDGE INTEGRAL

Goal: Rewrite $\int_{\partial D} f \vec{v} \cdot \vec{n}$ into area integral $\int_D g$

Solution: Rewrite $\int_{\partial D} f \vec{v} \cdot \vec{n}$ into $\int_D \nabla \cdot (\vec{v}_\theta f)$

$\int_D \nabla \cdot (\vec{v}_\theta f)$ can be estimated through area samples
QUICK RECAP

- Used *Reynolds transport theorem* to find the boundary integral
  \[ \int_{\partial D} f \vec{v} \cdot \hat{n} \]

- Rewrote \[ \int_{\partial D} f \vec{v} \cdot \hat{n} \] to \[ \int_{D} \nabla \cdot (\vec{V}_{\theta} f) \] using the *divergence theorem*.

- Have to define the *vector field* \[ \vec{V}_{\theta} \] over domain D
A 2D EXAMPLE SCENE

\( \omega \in \Omega \), the domain of integration

\( \omega_1^{(b)}, \omega_2^{(b)} \in \partial \Omega \), the discontinuous set
VELOCITY $\vec{V}$: THE BOUNDARY DERIVATIVE

$\partial_\theta \omega_i^{(b)}$: Derivative of boundary position w.r.t $\theta$
WARP FIELD $\mathcal{V}_\theta$: EXTENSION OF $\rightarrow\mathcal{V}$ TO ALL POINTS

$\mathcal{V}_\theta(\omega)$ defined over $D$

$\mathcal{V}_\theta$ defined over $\partial D$
VALIDITY OF $\vec{V}_\theta$

Rule 1: Continuous
Rule 2: Boundary Consistent
INTERPOLATION WITHOUT KNOWLEDGE OF BOUNDARIES

Available quantities
- Origin point
- Ray
- Intersection
- Primitive

No access to discontinuity points
CONSTRUCTING $\vec{V}_\theta$

Attempt 1: Find $\partial_\theta \omega$ through *implicit derivative*

$$y = \text{INTERSECT}(\omega, \theta) \implies \partial_\theta \omega = \frac{\partial \omega y}{\partial \theta y}$$

At all points (not just boundaries)

+ Boundary consistent
- Not continuous

(Incorrect)
CONSTRUCTING $\vec{v}_\theta$

Attempt 2  Filter Attempt 1 with a Gaussian filter

\[ \int_{\Omega'} k(\omega, \omega') \frac{\partial \omega y}{\partial \theta y} \]

$k(...)$ = Gaussian filter

+ Continuous
- Not boundary consistent
BOUNDARY-AWARE WEIGHTING

Goal: Find weights $k(\omega, \omega')$ s.t. $\vec{v}_\theta = \frac{\partial \omega y}{\partial \theta y}$ at boundaries.

Ideal weighting function

Approach Dirac delta near boundaries
BOUNDARY-AWARE WEIGHTING

Implicit Boundary through geometric normals

\[ \langle \omega, n \rangle = 0 \]

at boundaries
CONSTRUCTING $\vec{V}_\theta$

Our Approach $\rightarrow$ Filter Attempt 1 with harmonic weights

\[ k(\omega, \omega') = \frac{1}{D(\omega, \omega') + B(\omega')} \]

Distance function $\rightarrow$ Boundary test

+ Boundary consistent
+ Continuous
For each bounce:

1. Sample path using path tracer \(N\) paths

2. Sample auxiliary rays \(N'\) rays

3. Compute boundary term \(B()\) locally

4. Compute weight \(k(.,.)\) and \(\partial_\theta \omega\)

5. Find weighted mean
QUICK RECAP

- Used Reynolds transport theorem to find the boundary integral
  \[ \int_{\partial D} f \mathbf{v} \cdot \mathbf{n} \]

- Rewrote \[ \int_{\partial D} f \mathbf{v} \cdot \mathbf{n} \] to \[ \int_{D} \nabla \cdot (\mathbf{v}_{\theta} f) \] using the divergence theorem.

- Estimate consistent and continuous \( \mathbf{v}_{\theta} \) over domain D using auxiliary rays
MORE INTUITION: WARP-AREA SAMPLING CAN BE SEEN AS A CHANGE OF VARIABLE

\[ u = T(u'; \theta) \]

integration variable \( u \)
differentiating parameter \( \theta \)
MORE INTUITION: WARP-AREA SAMPLING CAN BE SEEN AS A CHANGE OF VARIABLE

\[
\frac{\partial}{\partial \theta} \int_D f \, du = \int_D f_\theta + \nabla \cdot (\tilde{V}_\theta f) \, du
\]
MORE INTUITION: WARP-AREA SAMPLING CAN BE SEEN AS A CHANGE OF VARIABLE

\[
\frac{\partial}{\partial \theta} \int_D f \, du = \int_D f_\theta + \nabla \cdot (\vec{V}_\theta f) \, du
\]

\[
= \int_D \frac{\partial}{\partial \theta} (f(T(u'; \theta)) J_T) \, du'
\]
MORE INTUITION: WARP-AREA SAMPLING CAN BE SEEN AS A CHANGE OF VARIABLE

\[ \frac{\partial}{\partial \theta} \int_D f \, du = \int_D f_\theta + \nabla \cdot (\tilde{V}_\theta f) \, du \]

\[ = \int_D \frac{\partial}{\partial \theta} (f (T(u'; \theta)) J_T) \, du' \]

\[ T = u' + (\theta - \theta_0) V_\theta \]
RESULTS
VARIANCE COMPARISON WITH EDGE-SAMPLING

Pot

Image $I$
Reference Derivative
Li et al. 2018
Ours without Russian roulette
Ours with Russian roulette

HEdge

Image $I$
Reference Derivative
Li et al. 2018
Ours without Russian roulette
Ours with Russian roulette
Rotating cylindrical objects present a complicated scenario for area-sampling.
BIAS COMPARISON WITH REPARAMETERIZATION

Extremely complex geometry like foliage can cause heuristic to fail
POSE ESTIMATION CAN FAIL WITH BIASED GRADIENTS

Multiple Initializations
WARPED-AREA SAMPLING CAN BE USED FOR SIGNED DISTANCE FIELDS RENDERING

Differentiable Signed Distance Function Rendering
DELIO VICINI, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland
SÉBASTIEN SPEIERER, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland
WENZEL JAKOB, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland

Differentiable Rendering of Neural SDFs through Reparameterization
Computing A Consistent $\nu(u)$ For An Arbitrary SDF

Find $\nu$ for silhouette rays, then interpolate.
Computing $\mathcal{V}$ by differentiating the silhouette position $u^*$.

The equation is:

$$\mathcal{V} = \nabla_\theta u^*(\theta)$$
Computing $\mathcal{V}$: Implicit Fn. Theorem + Chain Rule

1. **Compute $\nabla_\theta x^*(\theta)$ using implicit fn. theorem:**
   Derivative of any point in SDF can be computed by differentiating SDF function $\mathbf{f}$

\[
\nabla_\theta x(\theta) = \nabla_\theta \mathbf{f}(\mathbf{x}; \theta) \cdot \mathbf{n} = \mathbf{G}(\mathbf{x})
\]
Computing $\mathcal{V}$: Implicit Fn. Theorem + Chain Rule

1. Compute $\nabla_{\theta} x^*(\theta)$ using Implicit Fn. Theorem:

2. Propagate $G(x^*)$ to sample space through chain rule ($u \rightarrow x$):

\[ \mathcal{V} = G(x^*)^T \nabla_x u \]
Computing $\mathcal{V}(u)$ For An Arbitrary Ray

Ray-SDF Intersection: Sphere Tracing

What about non-silhouette rays?
Can Compute $\mathcal{V}(u)$ using the Geometry Derivative $G(x)$ of any Sphere Tracer point.

Using a single point can make $\mathcal{V}(u)$ discontinuous in $u$!
Computing $V(u)$ as Weighted Mean of $G(x)$ over Sphere Tracer Points

**Solution:** Compute silhouette weights $w(x)$

And use a weighted mean:

$$V(u) = \sum_i w(x_i) \cdot G(x_i) \cdot \nabla_x u$$
Weighted-Mean $\mathcal{V}(u)$ Is Both Consistent And Continuous

See Paper: Harmonic & Quadrature Weighting

(a) harmonic weight $w(x)$  
(b) quadrature approximation $w^*(x)$  
(c) top-k subset weight $\tilde{w}_k(x)$
Scaling Up From Simple 2D To Neural 3D

<table>
<thead>
<tr>
<th>Simple 2D SDF</th>
<th>Neural 3D SDF</th>
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<tr>
<td>$f(x, \theta)$</td>
<td></td>
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<tr>
<td>Single Parameter $\theta$</td>
<td>100,000s of Parameters $\theta$</td>
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<tr>
<td>2D Points $x : R^2$</td>
<td>3D Points $x : R^3$</td>
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<td>1D integration coord $u$</td>
<td>2D integration coords $\vec{u}$</td>
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<td>1D Scalar warp $\mathcal{V}(u)$</td>
<td>2D Vector warp $\vec{V}(\vec{u})$</td>
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<td>Weight adjustment $1 + \frac{\partial \mathcal{V}}{\partial u}$</td>
<td>Jacobian determinant $</td>
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Putting It All Together: First, Render SDF As Usual

Parameters $\theta$

Samples $u$

SDF Renderer
Putting It All Together: Then, Reparameterize Samples

Parameters $\theta$

Samples $u$

SDF Renderer

Calculate $\mathcal{V}(u)$ and $J_v$ from sphere tracer points
Putting It All Together: Finally, Differentiate With AD

Parameters $\theta$

$\nabla \theta$

$\nabla$ 

Samples $\mathbf{u}$

$\mathbf{u}$

SDF Renderer

RGB Image

Gradient Image

Calculate $\nabla (\mathbf{u})$ and $J_\theta$ from sphere tracer points
Comparisons Against IDR (Yariv et al. 2020): A Sharp-Surface Model With Segmentation Mask Inputs

**IDR (RGB + Masks)**

**Ours (RGB Only)**

**INPUTS**
Reconstructions On-Par With IDR *Without* Using Masks

**IDR (RGB + Masks)**

**Ours (RGB Only)**

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**Santa**

**Kitty**

**Duck**

**Input Image** | **IDR Radiance** | **IDR Depth** | **IDR 3D Geometry** | **Ours Radiance** | **Ours Depth** | **Ours 3D Geometry**
Cleaner Reconstructions Than IDR On Real Data with Poor Segmentation Masks

Poor Masks can Lead IDR to produce a **noisy/incorrect silhouette**
CONCLUSIONS

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