

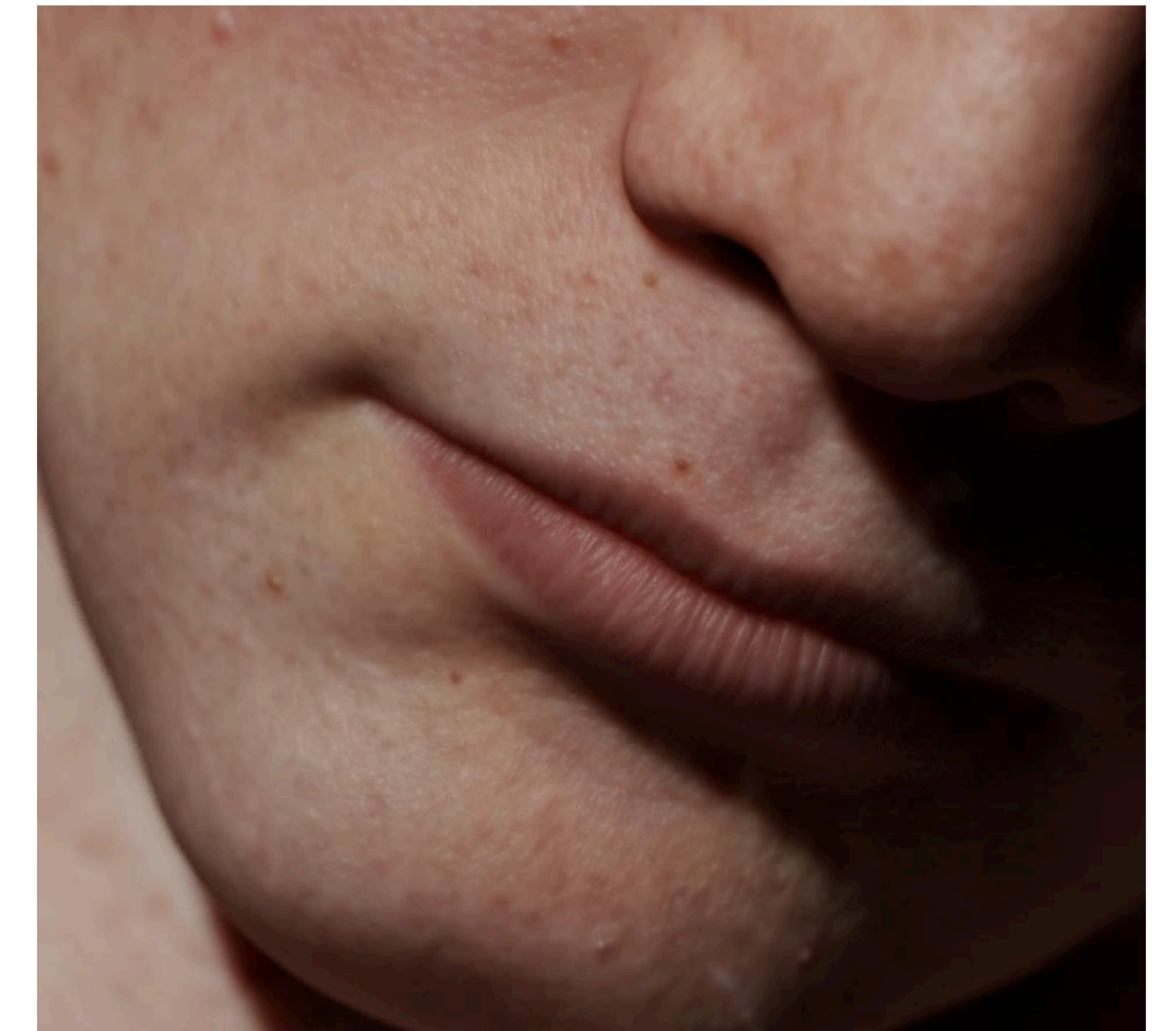
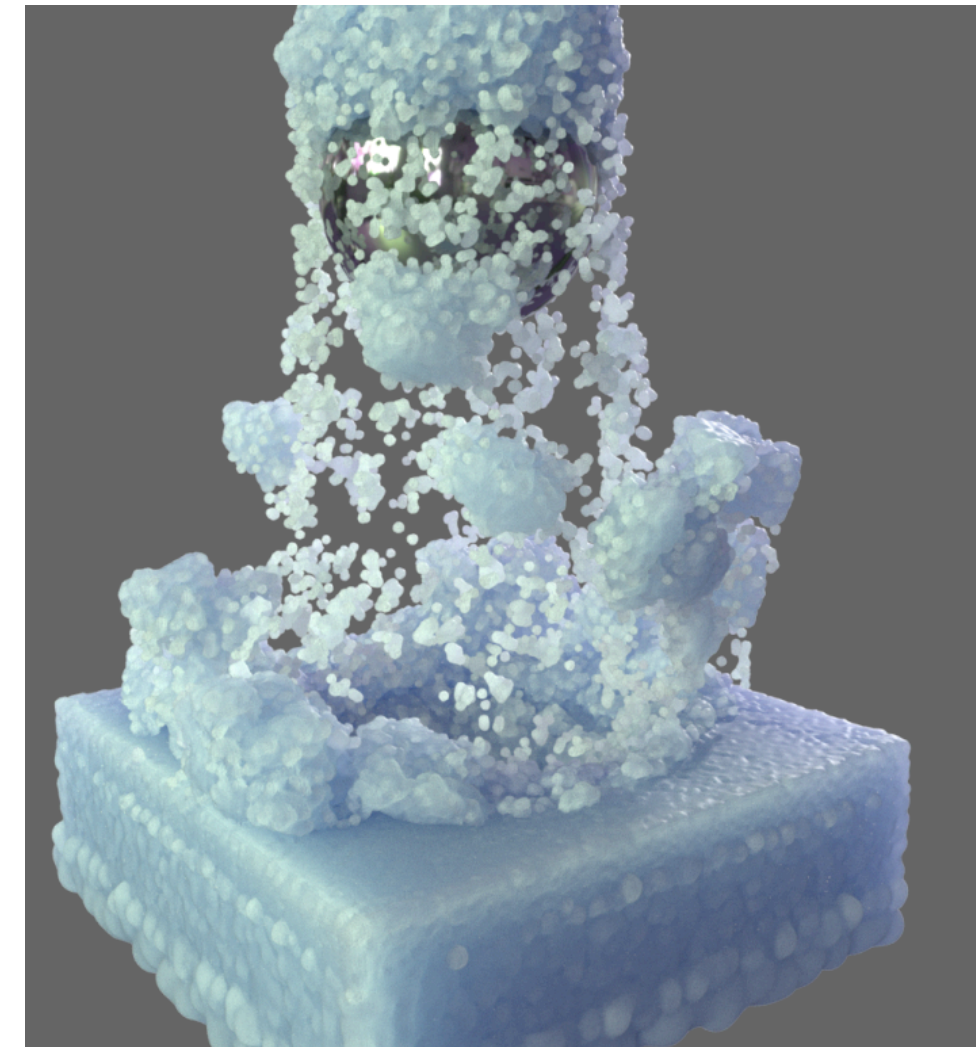
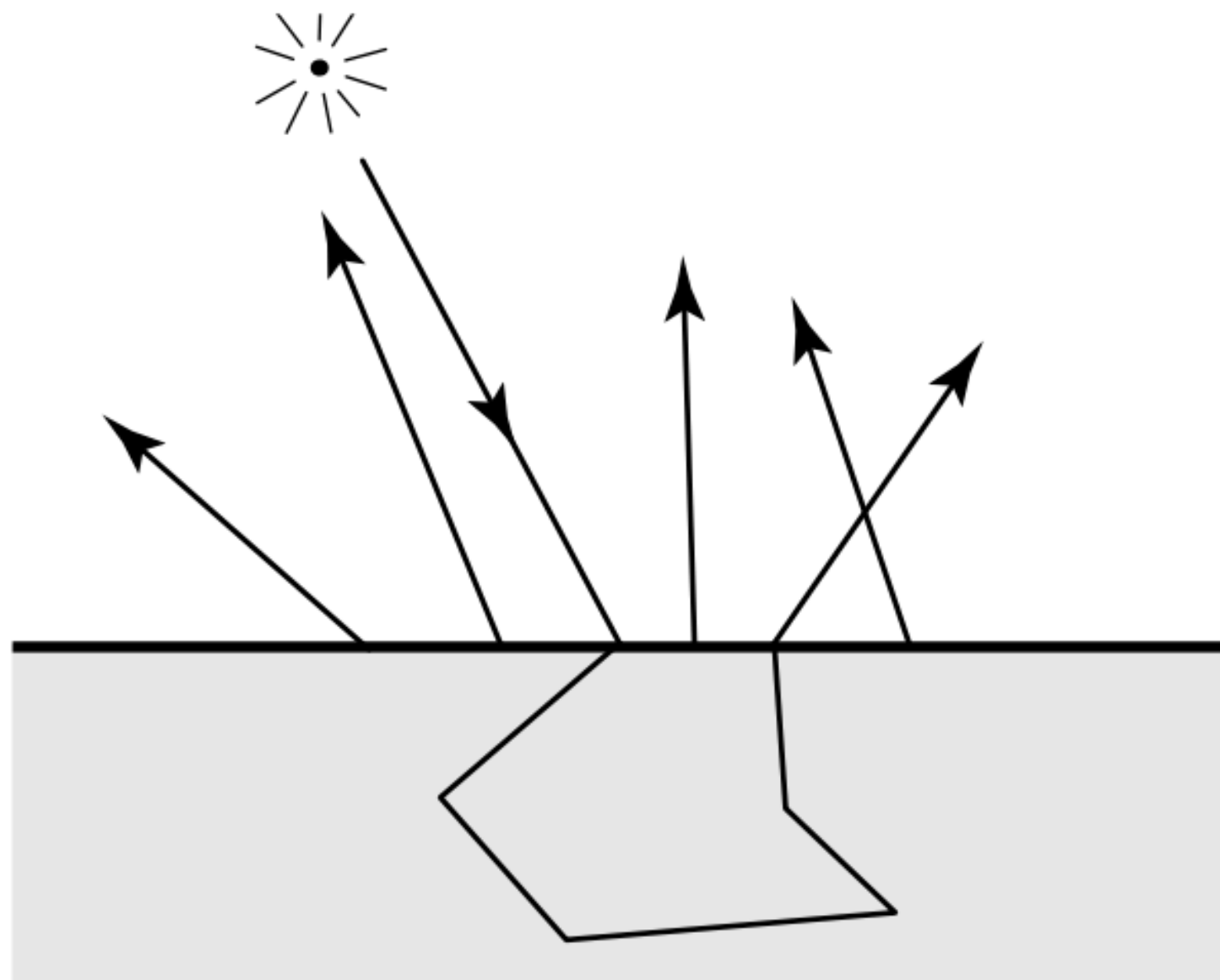
# Diffusion approximation

UCSD CSE 272

Advanced Image Synthesis

Tzu-Mao Li

# Today: multiple-scattering approximation



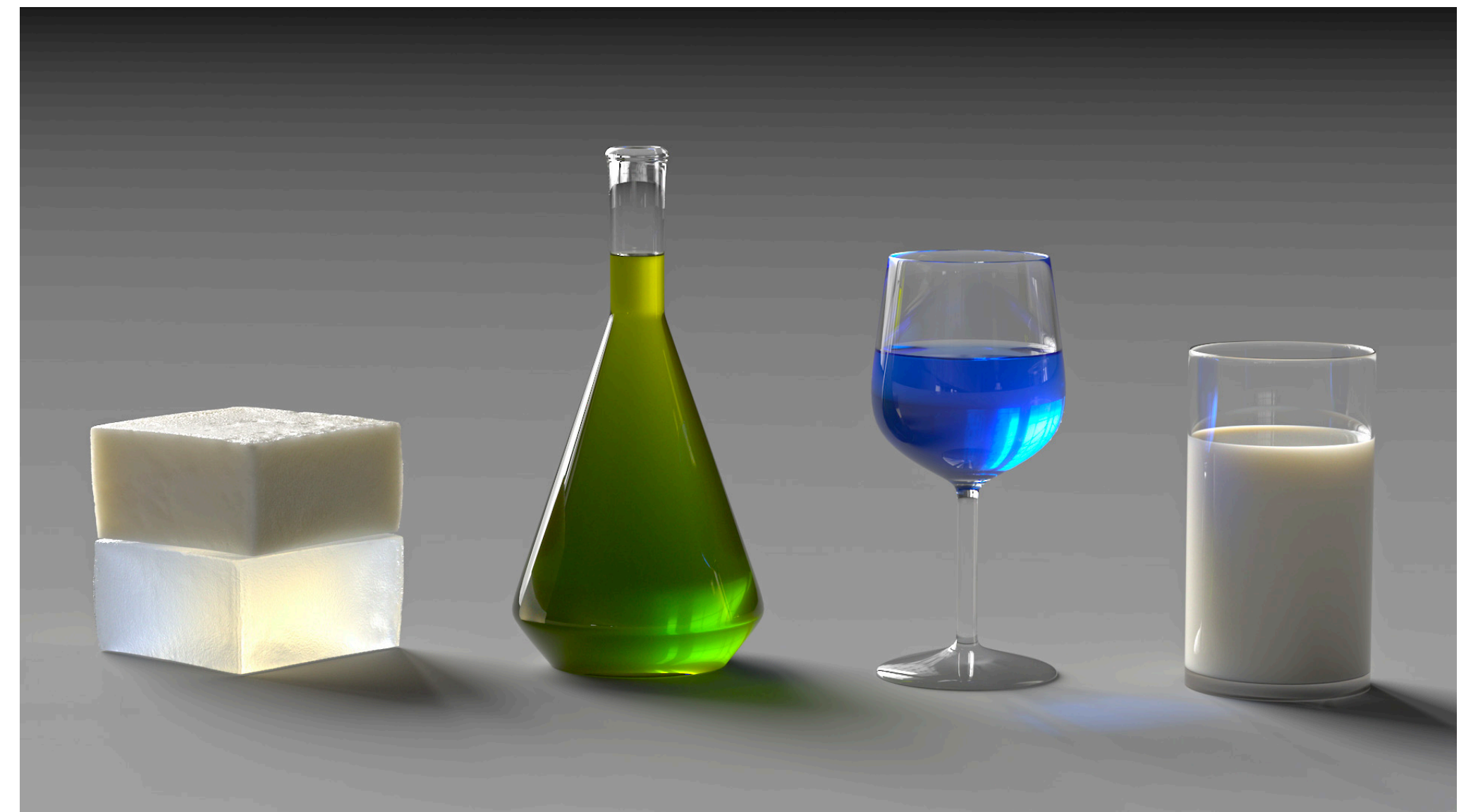
[https://blog.selfshadow.com/publications/s2015-shading-course/burley/s2015\\_pbs\\_disney\\_bsdf\\_notes.pdf](https://blog.selfshadow.com/publications/s2015-shading-course/burley/s2015_pbs_disney_bsdf_notes.pdf)

<http://graphics.ucsd.edu/~henrik/papers/bssrdf/>

[https://naml.us/paper/deon2011\\_subsurface.pdf](https://naml.us/paper/deon2011_subsurface.pdf)

# Challenge: multiple-scattering in dense media requires many bounces

these images usually require hundreds of bounces

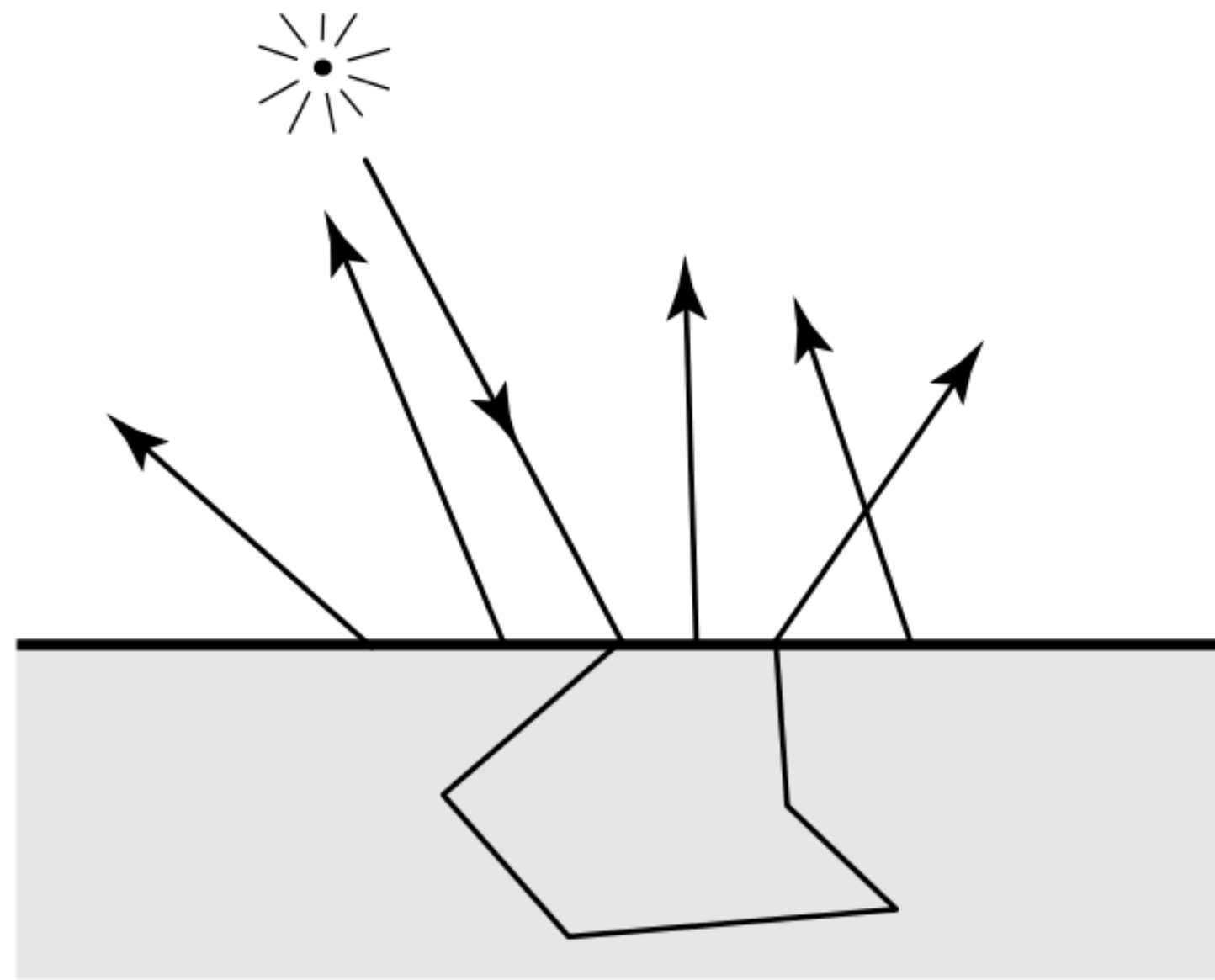


<https://rgl.epfl.ch/publications/Jakob2010Radiative>

<https://cs.dartmouth.edu/~wjarosz/publications/bitterli18framework.html>

<https://www.cs.cornell.edu/projects/translucency/#acquisition-sa13>

# Trick: aggregate multiple scattering events using a “BSSRDF”



Bidirectional Subsurface Scattering  
Reflectance Distribution Function

$$f(\omega, \omega')$$

BRDF / BSDF

$$f(p, \omega, p', \omega')$$

BSSRDF

# BRDF vs BSSRDF

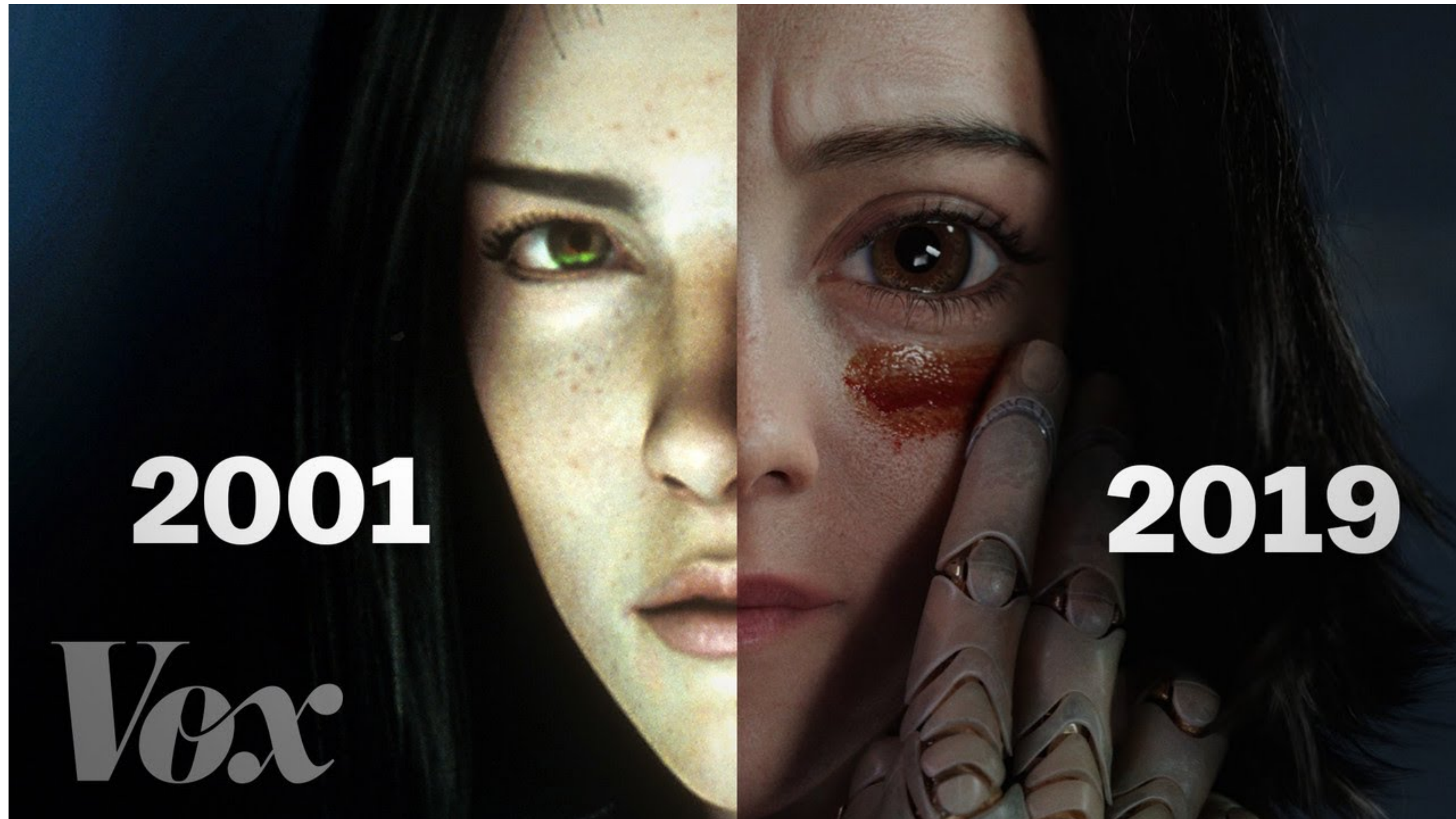


BRDF



BSSRDF

# Cool Vox video!



<https://www.youtube.com/watch?v=NvFoKkWyZ5Y>

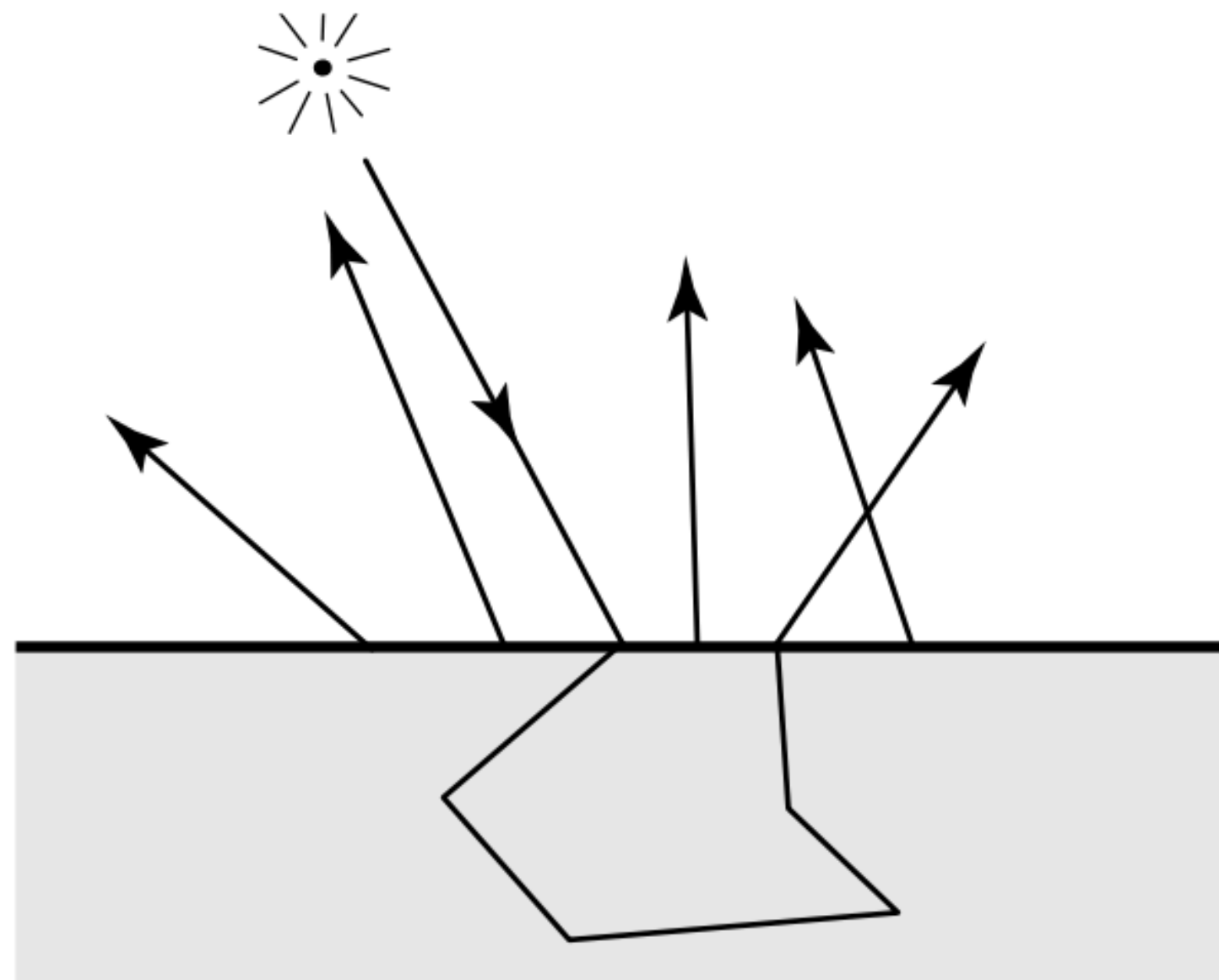
Goal: deriving BSSRDF from  
radiative transfer equation

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t), \omega') d\omega'$$



$$f(p, \omega, p', \omega')$$

# Simple BSSRDFs



$$f(p, \omega, p', \omega') = (1 - F(\omega)) R \left( \| p - p' \| \right) (1 - F(\omega'))$$

$$R(r) \propto e^{-\frac{r^2}{\sigma^2}}$$

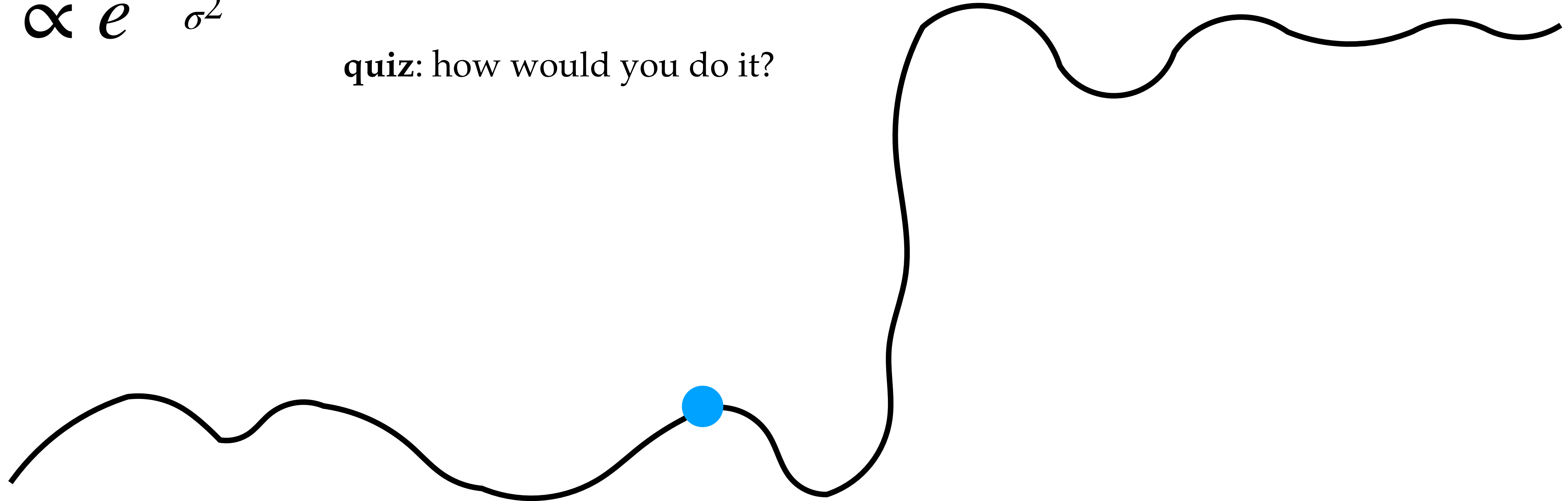
R: “diffuse reflectance profile”



# Sampling BSSRDFs

$$R(r) \propto e^{-\frac{r^2}{\sigma^2}}$$

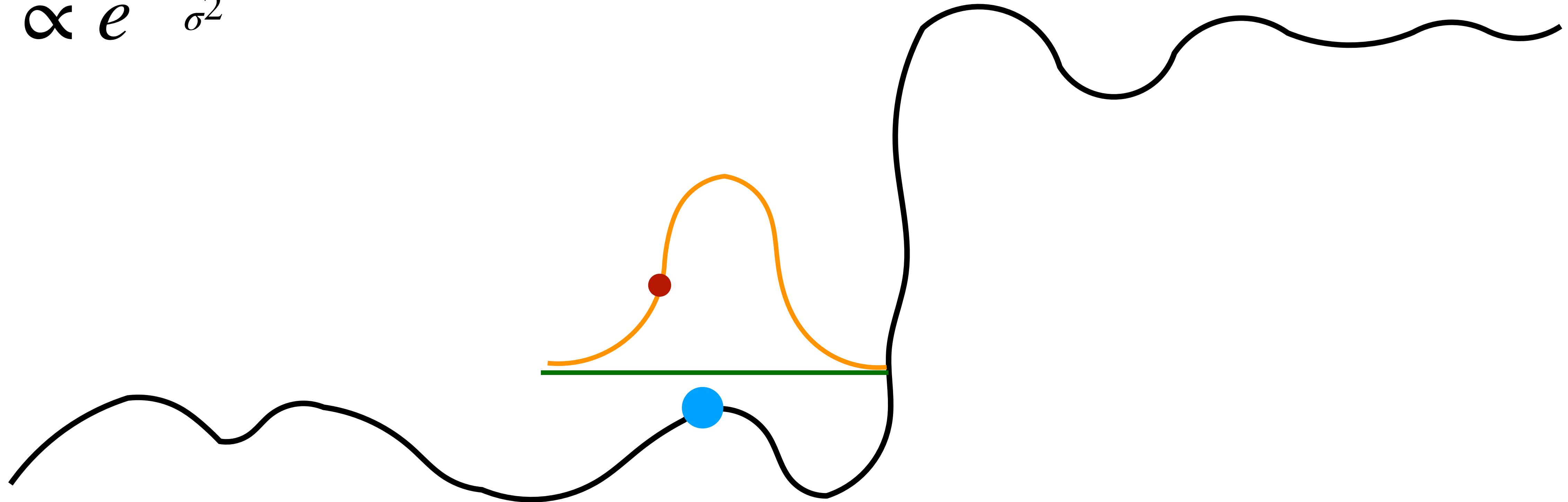
quiz: how would you do it?



# Sampling BSSRDFs

1. sample on a disk using  $R(r)$

$$R(r) \propto e^{-\frac{r^2}{\sigma^2}}$$



## BSSRDF Importance Sampling

Alan King  
Solid Angle

Christopher Kulla  
Sony Pictures Imageworks

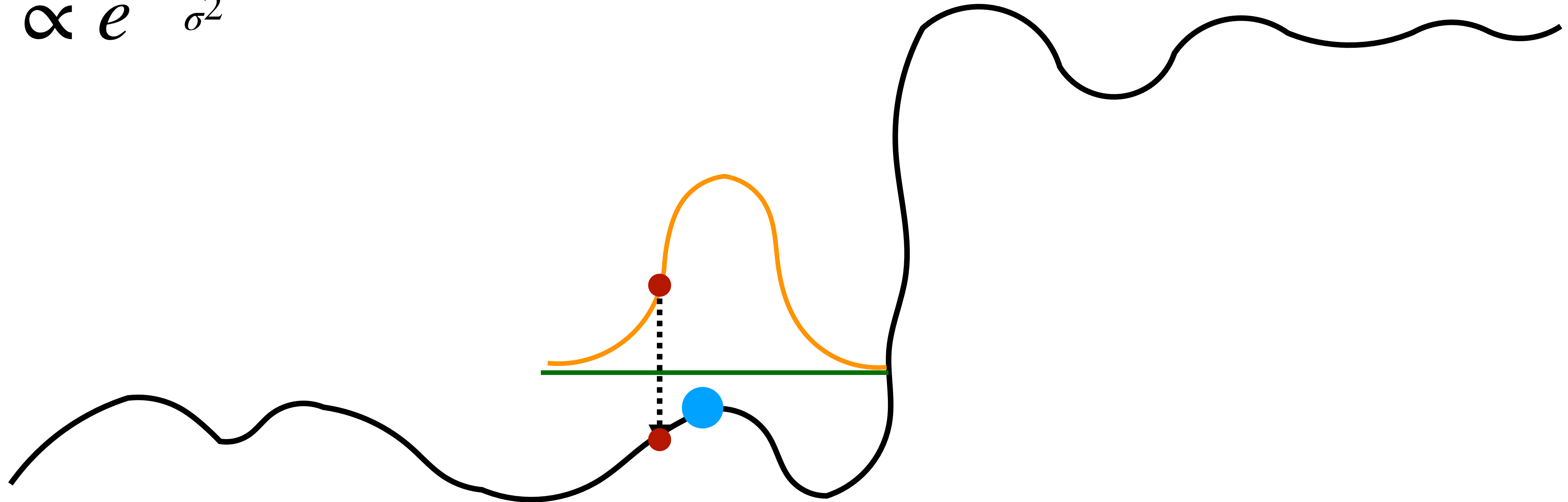
Alejandro Conty  
Sony Pictures Imageworks

Marcos Fajardo  
Solid Angle

# Sampling BSSRDFs

1. sample on a disk using  $R(r)$
2. project onto the surface

$$R(r) \propto e^{-\frac{r^2}{\sigma^2}}$$



## BSSRDF Importance Sampling

Alan King  
Solid Angle

Christopher Kulla  
Sony Pictures Imageworks

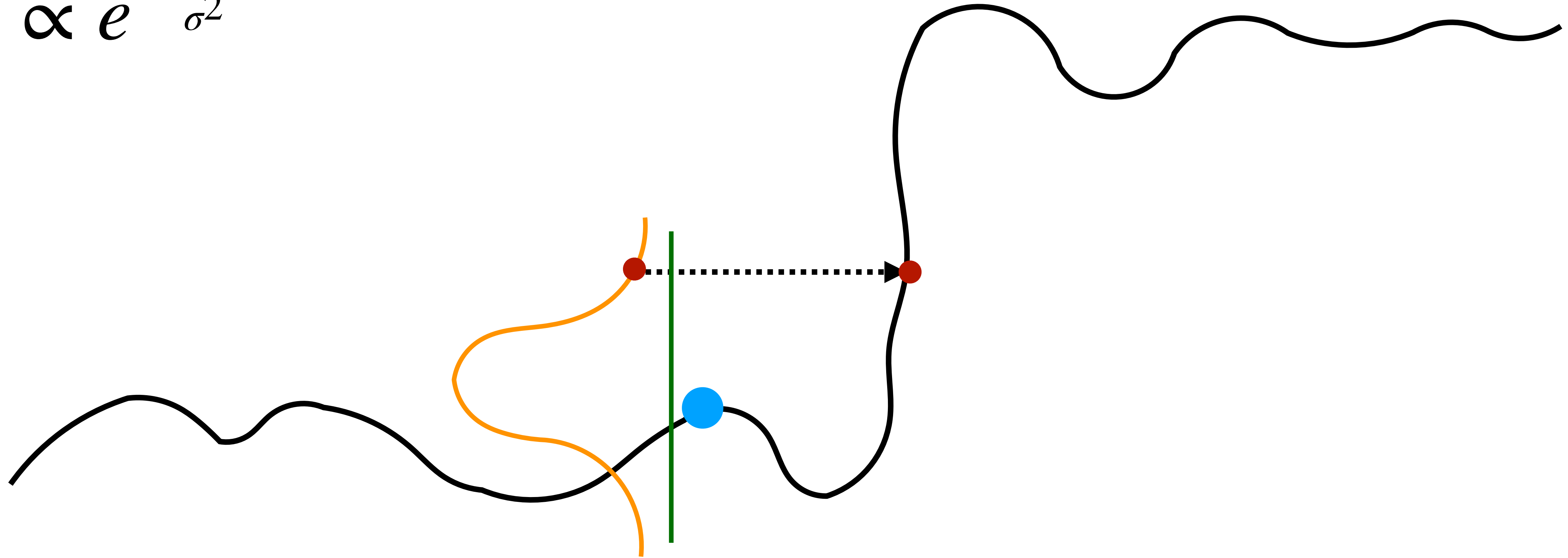
Alejandro Conty  
Sony Pictures Imageworks

Marcos Fajardo  
Solid Angle

# Sampling BSSRDFs

1. sample on a disk using  $R(r)$
2. project onto the surface
3. repeat this for different axes, combine with MIS

$$R(r) \propto e^{-\frac{r^2}{\sigma^2}}$$



## BSSRDF Importance Sampling

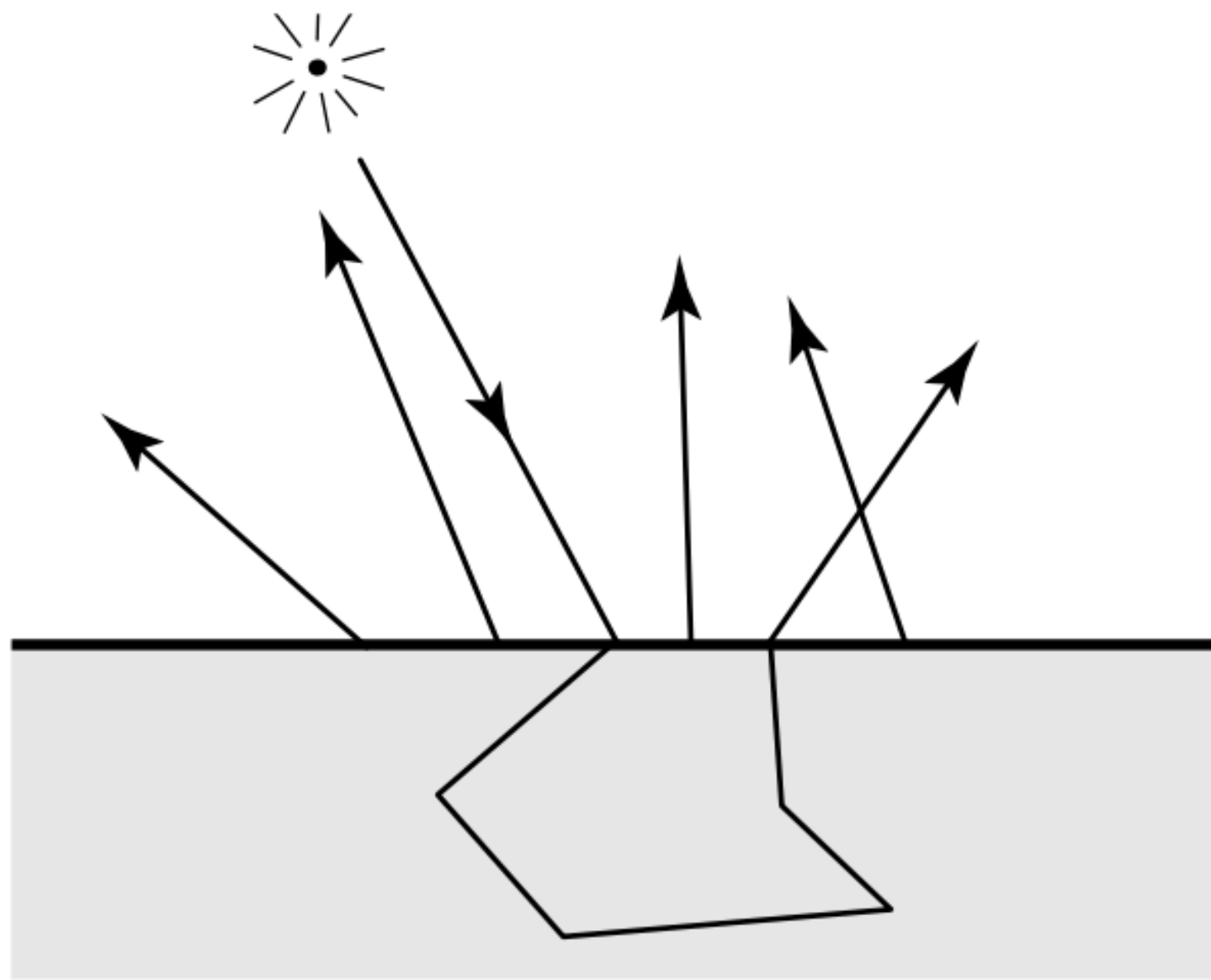
Alan King  
Solid Angle

Christopher Kulla  
Sony Pictures Imageworks

Alejandro Conty  
Sony Pictures Imageworks

Marcos Fajardo  
Solid Angle

# How do we know if simple BSSRDFs are sufficient?



$$f(p, \omega, p', \omega') = (1 - F(\omega)) R \left( \| p - p' \| \right) (1 - F(\omega'))$$

$$R(r) \propto e^{-\frac{r^2}{\sigma^2}}$$

R: “diffuse reflectance profile”

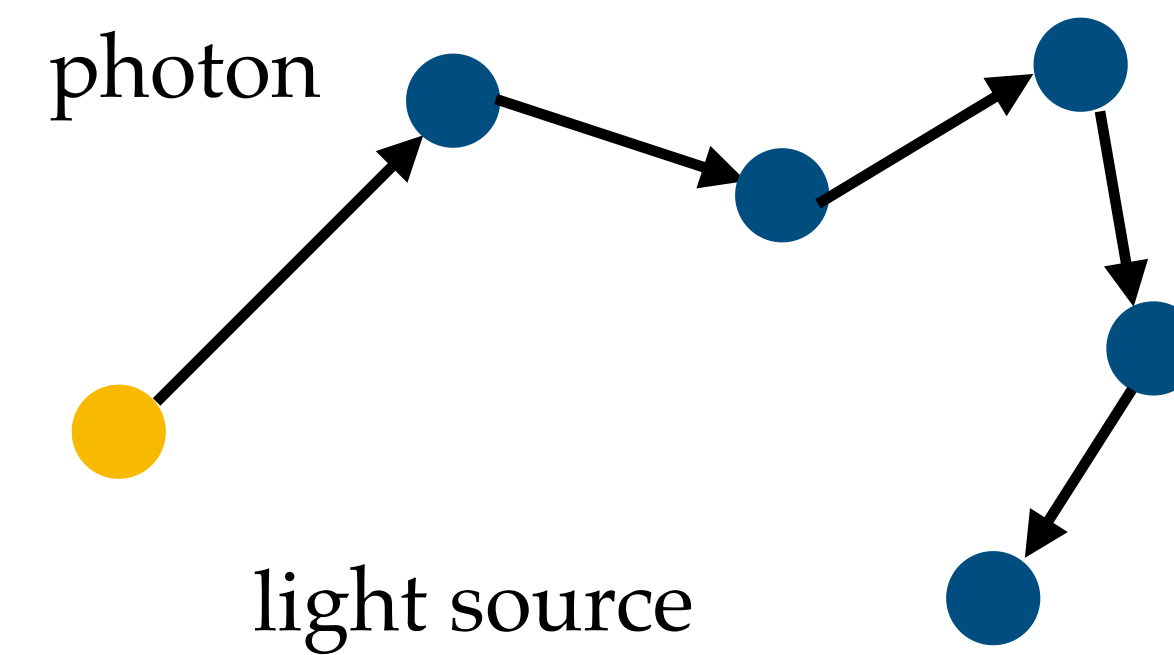
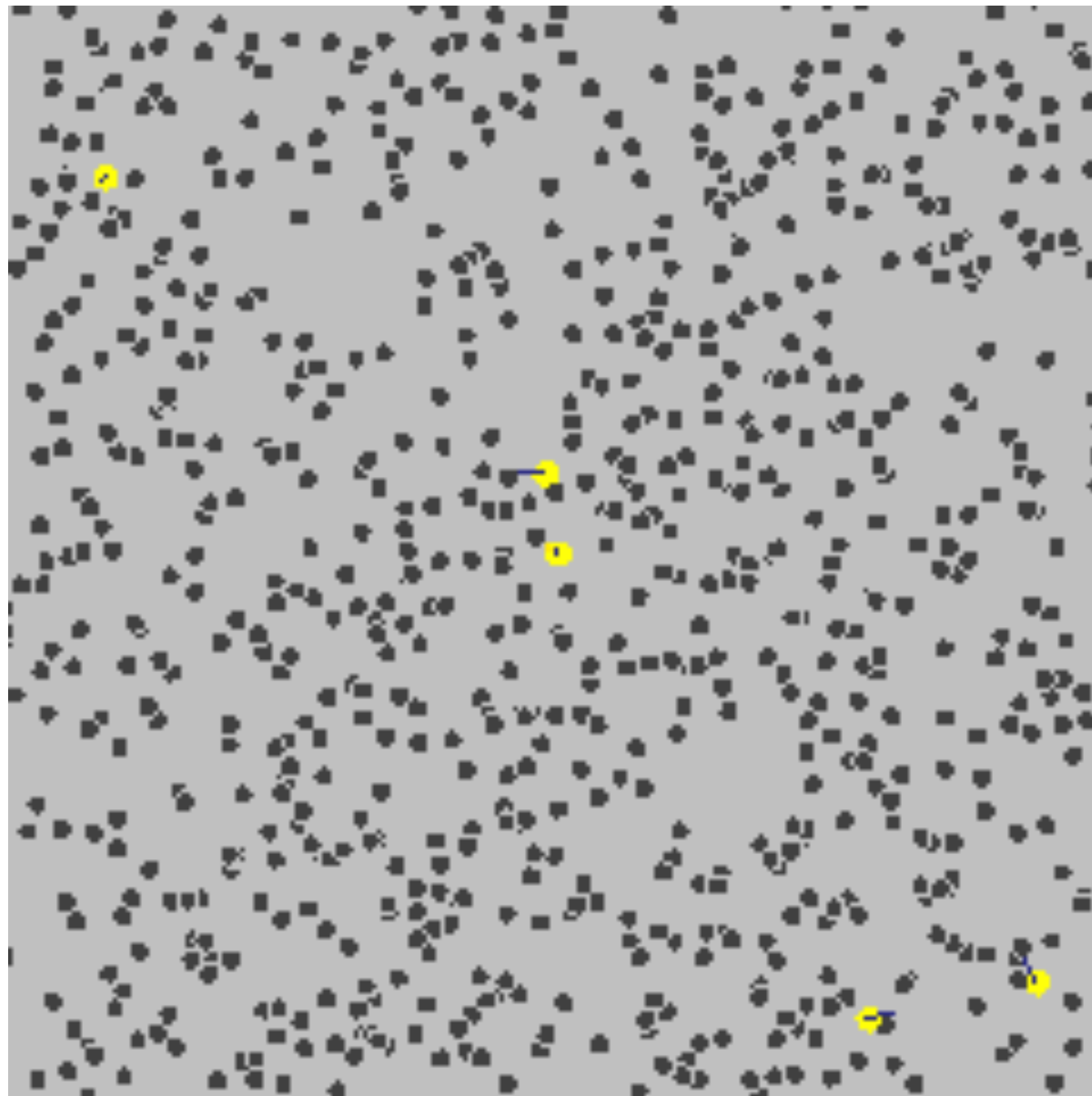
Goal: deriving BSSRDF from  
radiative transfer equation

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t), \omega') d\omega'$$



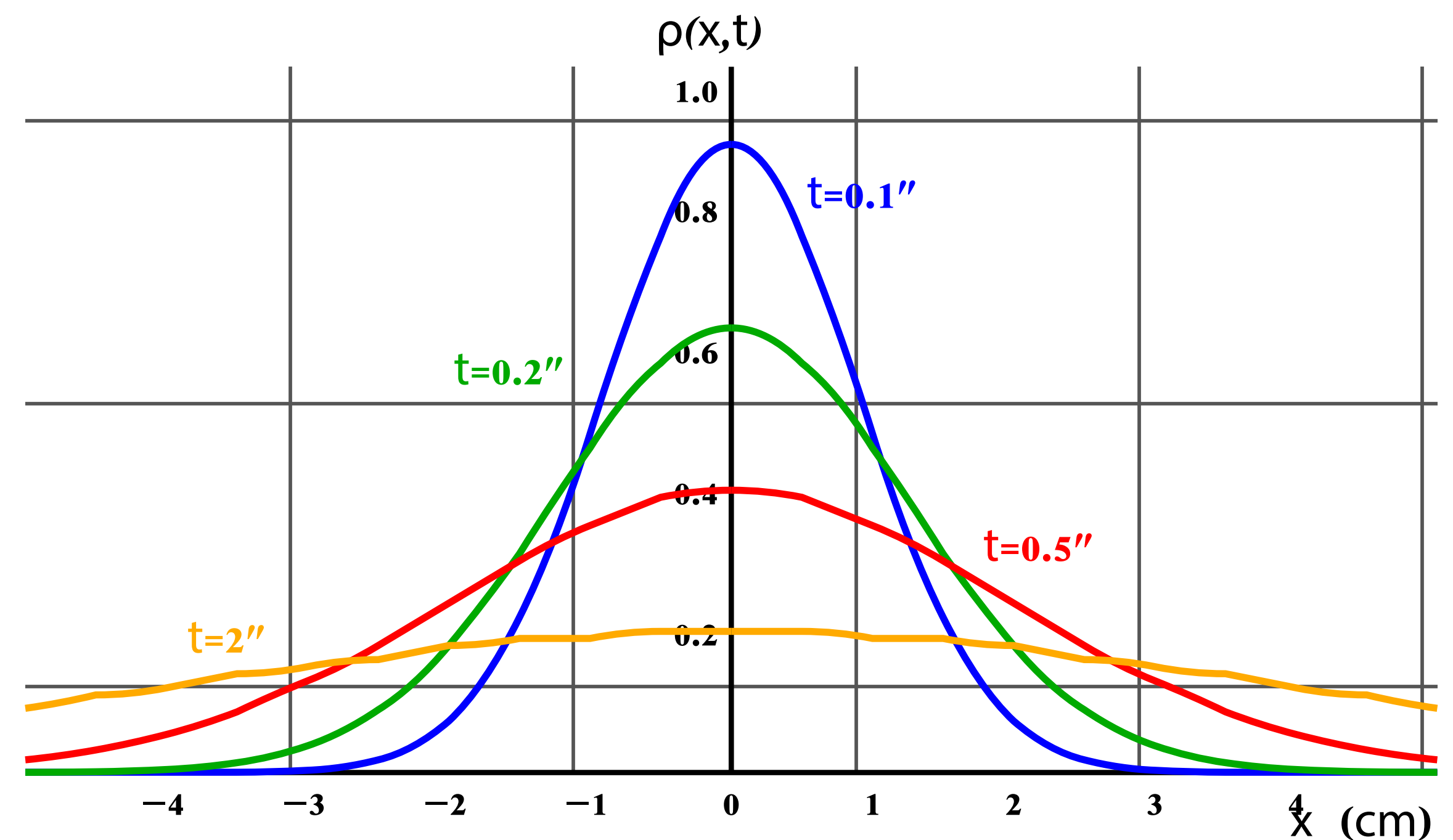
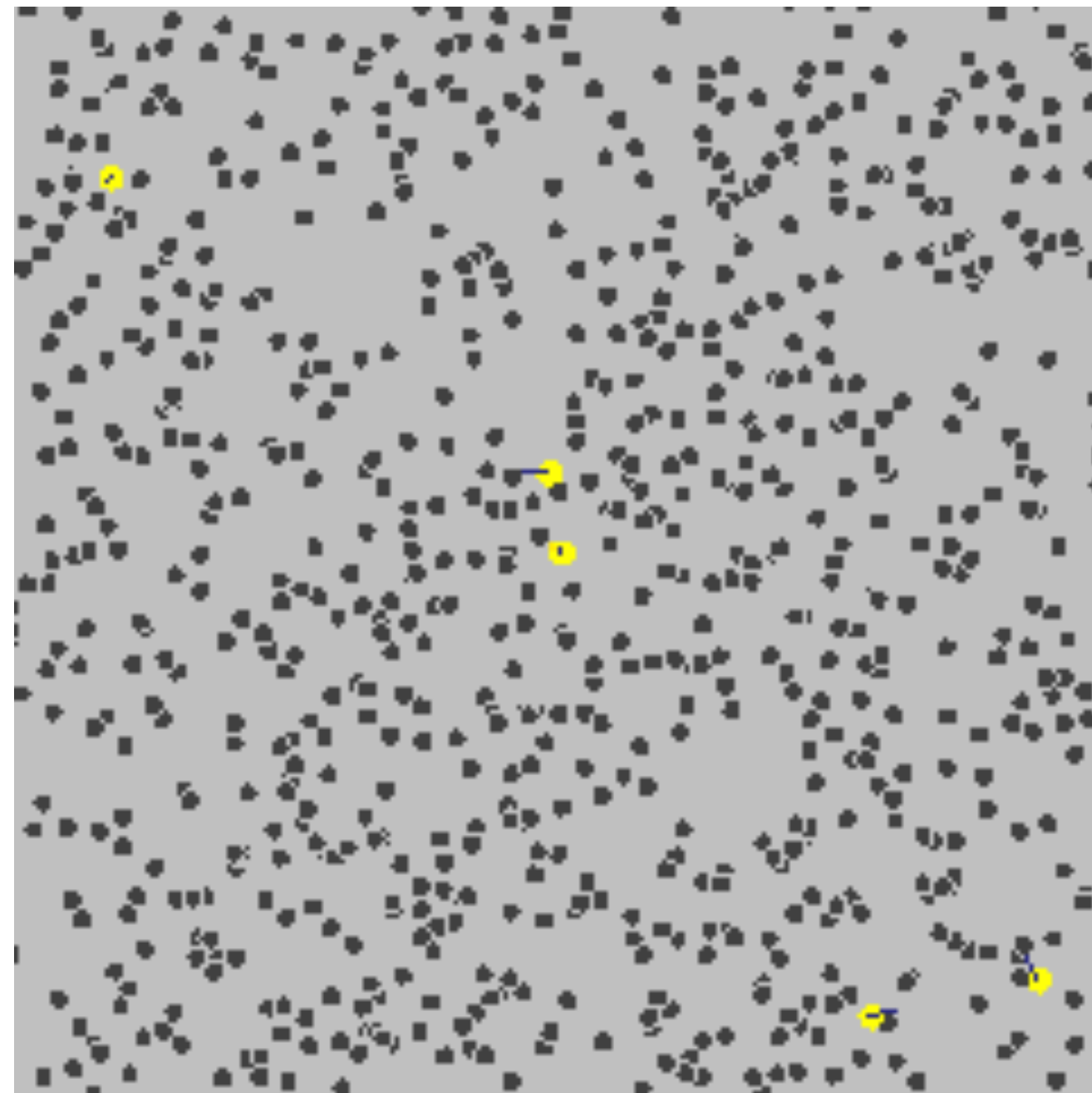
$$f(p, \omega, p', \omega')$$

# Intuition: volumetric path tracing looks like Brownian motion



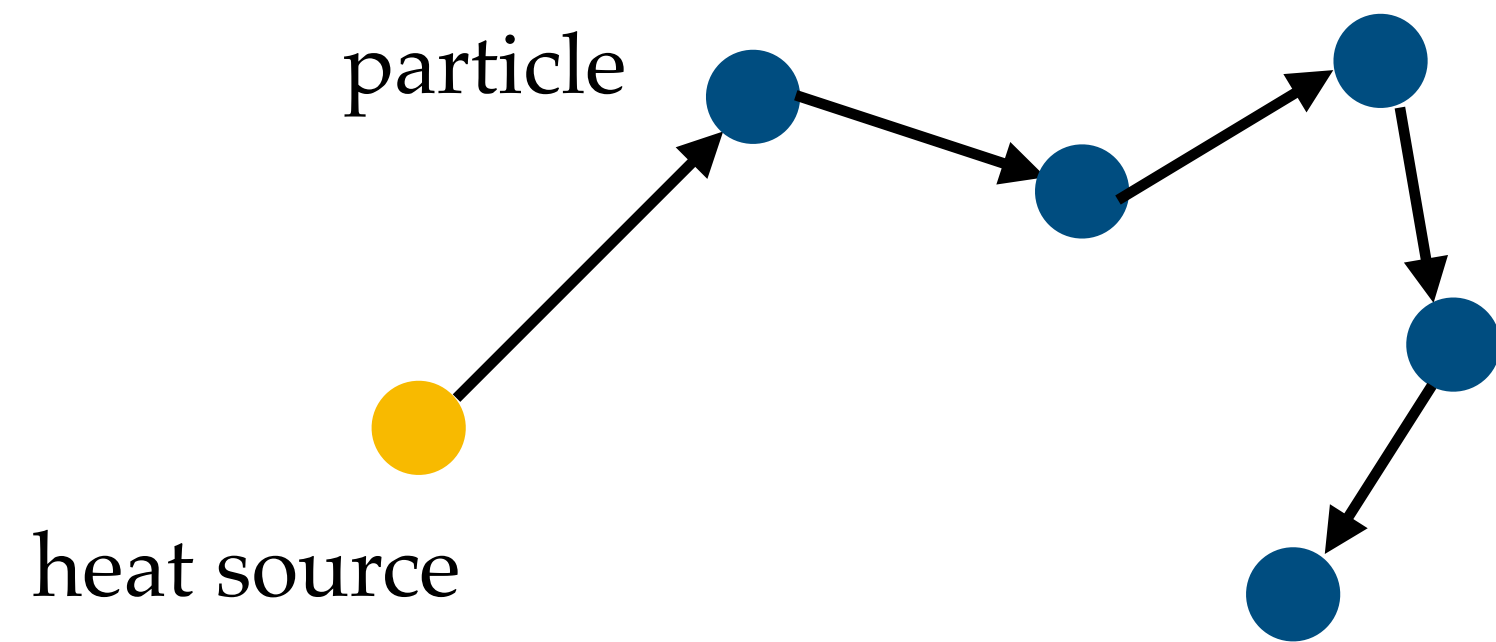
# Physics: expectation of Brownian motions is a solution to a PDE

- c.f. Fick, Einstein, Feynman-Kac formula





# Heat equation

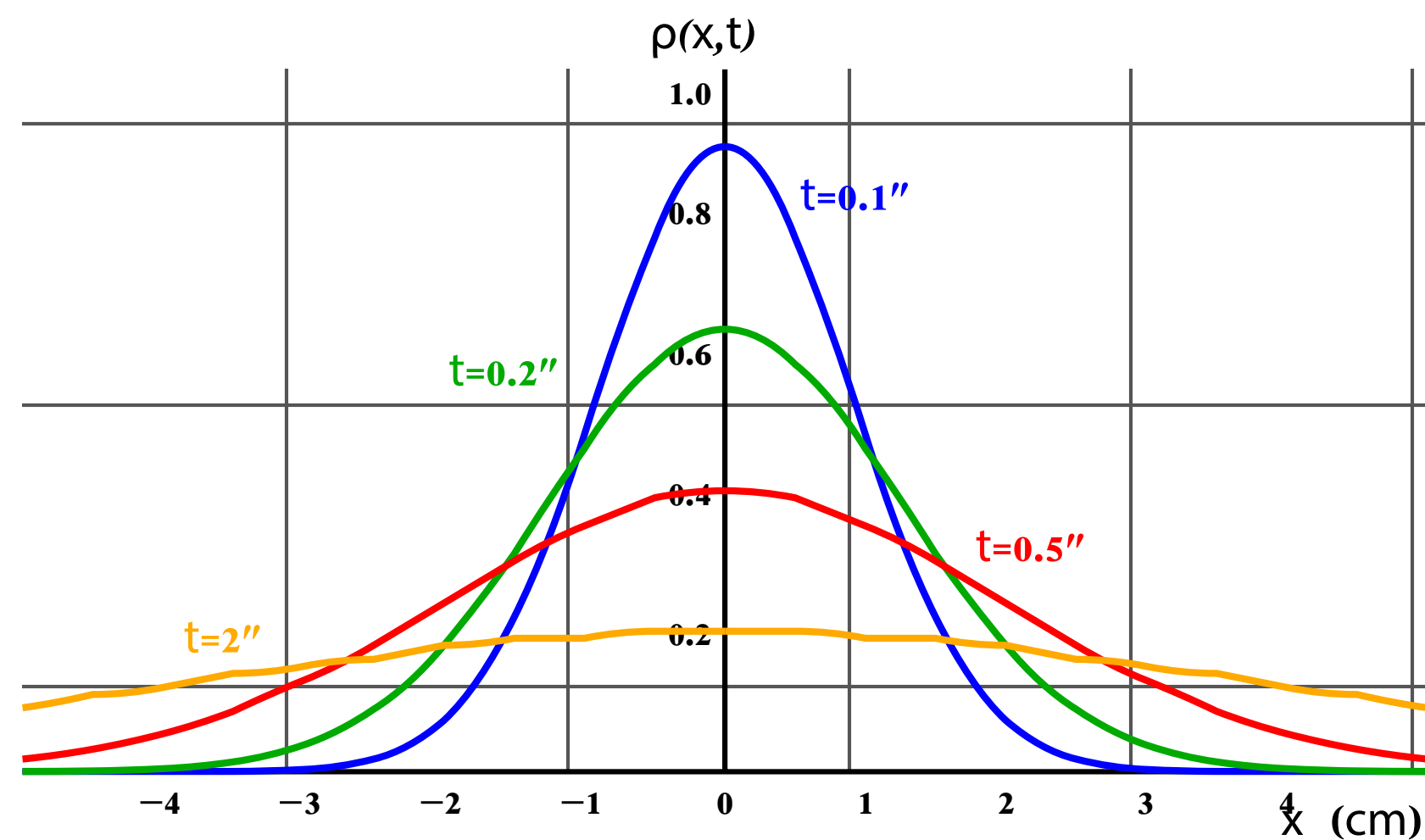


$$\frac{\partial u}{\partial \tau} = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + Q(x, y, z)$$

time  
derivative

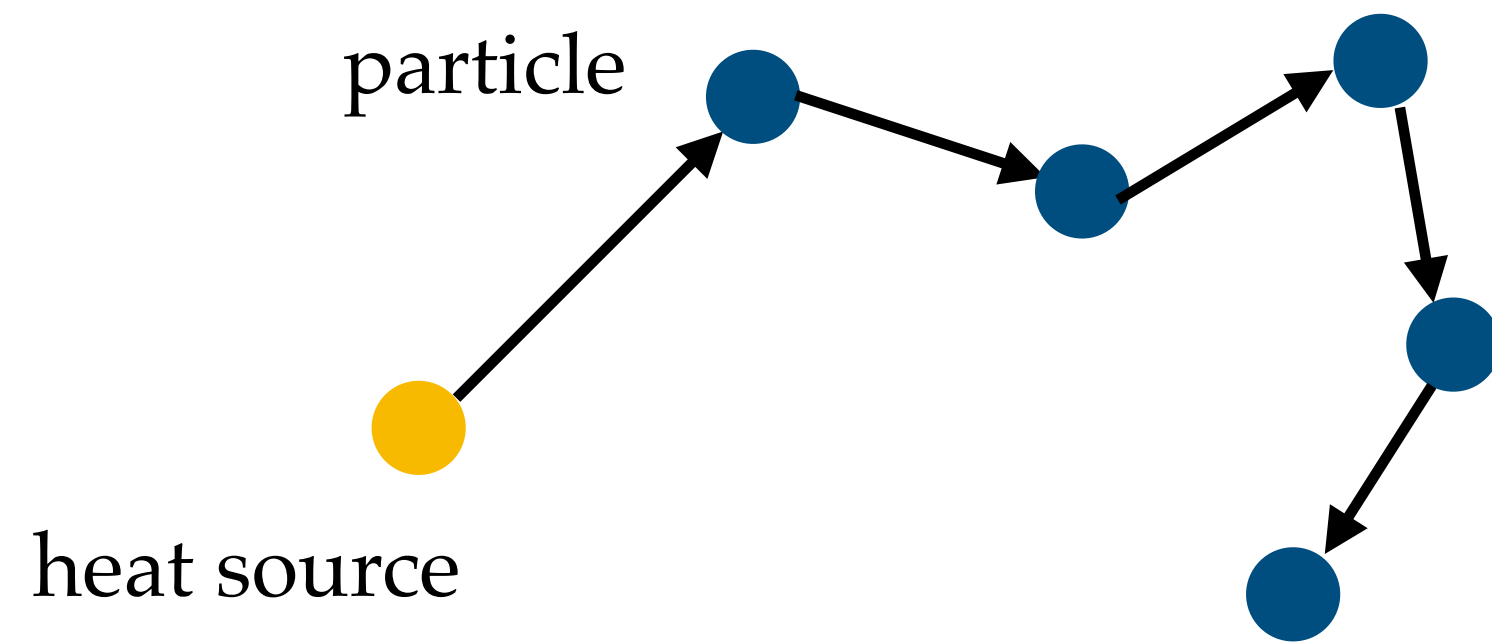
spatial  
diffusion

heat source



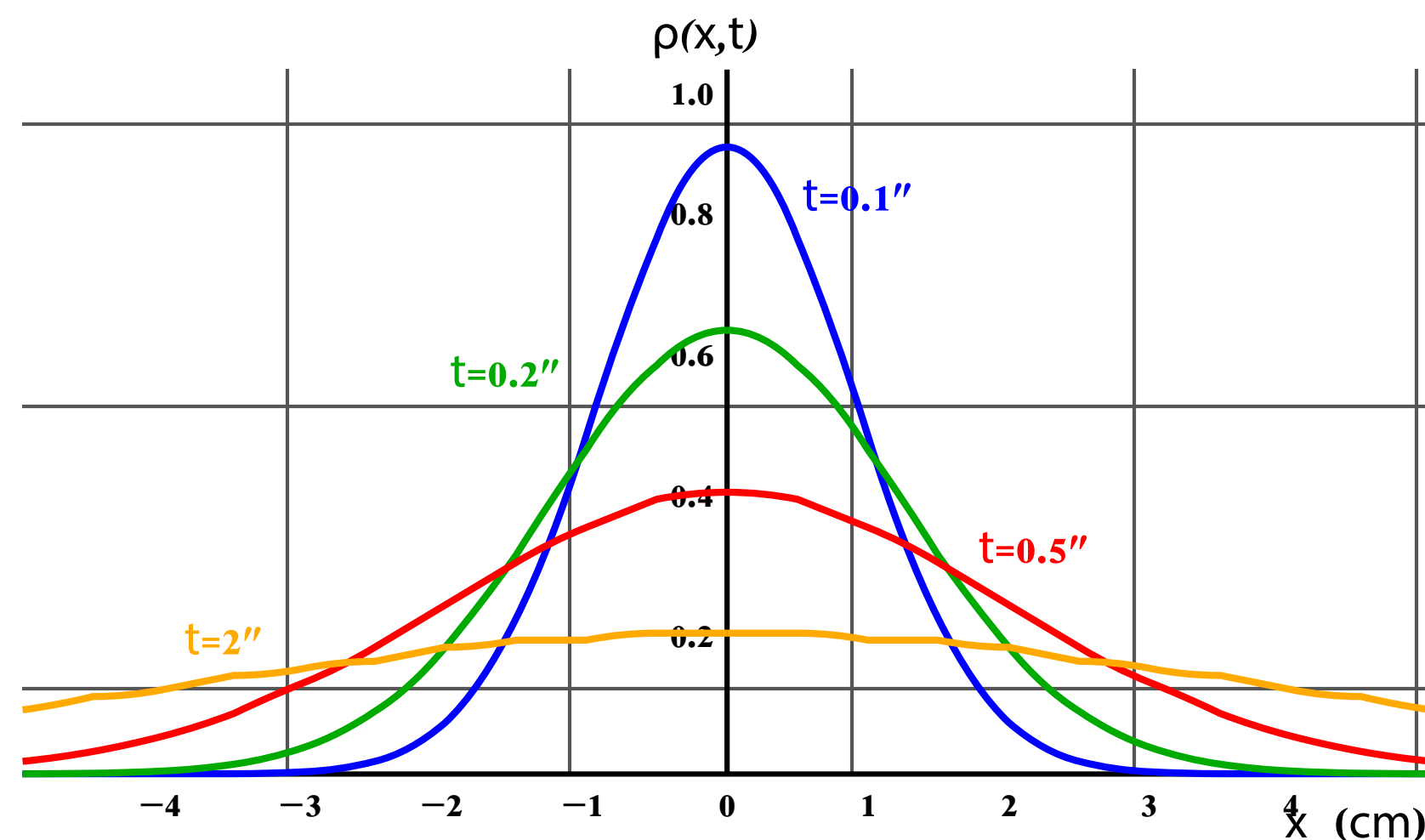
# Equilibrium of heat equation: Poisson equation

Poisson equation is also the equilibrium of a electric field  
assuming no magnetic field



$$\frac{\partial u}{\partial \tau} = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + Q(x, y, z)$$

time derivative
spatial diffusion
heat source



$$0 = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + Q(x, y, z)$$

$$= \Delta u + Q$$

Poisson equation

What is the connection between radiative transfer equation & Poisson equation?

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t), \omega') d\omega'$$

v.s.

$$\Delta u + Q = 0$$

# Assumption 1: isotropic phase function

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t), \omega') d\omega'$$



$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \frac{\sigma_s}{4\pi} \int_{S^2} L(\mathbf{p}(t), \omega') d\omega'$$

# Assumption 2:

first-order spherical moment expansion on  $L$

$$L(\mathbf{p}, \omega) \approx \frac{1}{4\pi} \int_{S^2} L(\mathbf{p}, \omega') d\omega' + \frac{3}{4\pi} \omega \cdot \int_{S^2} \omega' L(\mathbf{p}, \omega') d\omega'$$

zero-th order  
moment  
(total mass)

first order  
moment  
(center of mass)

$$= \frac{1}{4\pi} \phi(\mathbf{p}) + \frac{3}{4\pi} \omega \cdot \mathbf{E}(\mathbf{p})$$

$$\frac{d}{dt} L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \frac{\sigma_s}{4\pi} \int_{S^2} L(\mathbf{p}(t), \omega') d\omega'$$

# Assumption 3: matching spherical moments of RTE

plug in  $L(\mathbf{p}, \omega) \approx \frac{1}{4\pi}\phi(\mathbf{p}) + \frac{3}{4\pi}\omega \cdot \mathbf{E}(\mathbf{p})$

take 0-th order moment

take 0-th order moment

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \frac{\sigma_s}{4\pi} \int_{S^2} L(\mathbf{p}(t), \omega') d\omega'$$

plug in  $L(\mathbf{p}, \omega) \approx \frac{1}{4\pi}\phi(\mathbf{p}) + \frac{3}{4\pi}\omega \cdot \mathbf{E}(\mathbf{p})$

take 1st order moment

take 1st order moment

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \frac{\sigma_s}{4\pi} \int_{S^2} L(\mathbf{p}(t), \omega') d\omega'$$

# Diffusion approximation through moment matching

plug in  $L(\mathbf{p}, \omega) \approx \frac{1}{4\pi}\phi(\mathbf{p}) + \frac{3}{4\pi}\omega \cdot \mathbf{E}(\mathbf{p})$

take 0-th order moment

take 0-th order moment

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \frac{\sigma_s}{4\pi} \int_{S^2} L(\mathbf{p}(t), \omega') d\omega'$$



$$\nabla \cdot \mathbf{E}(\mathbf{p}) = -\sigma_a \phi(\mathbf{p}) + Q_0(\mathbf{p})$$

$$Q_0(\mathbf{p}) = \int L_e(\mathbf{p}, \omega') d\omega'$$

# Diffusion approximation through moment matching

plug in  $L(\mathbf{p}, \omega) \approx \frac{1}{4\pi}\phi(\mathbf{p}) + \frac{3}{4\pi}\omega \cdot \mathbf{E}(\mathbf{p})$

take 1st order moment

take 1st order moment

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \frac{\sigma_s}{4\pi} \int_{S^2} L(\mathbf{p}(t), \omega') d\omega'$$



$$\frac{1}{3} \nabla \phi(\mathbf{p}) = -\sigma_t \mathbf{E}(\mathbf{p}) + Q_1(\mathbf{p})$$

$$Q_1(\mathbf{p}) = \int \omega' \cdot L_e(\mathbf{p}, \omega') d\omega'$$



# Diffusion approximation through moment matching

$$\nabla \cdot \mathbf{E}(\mathbf{p}) = -\sigma_a \phi(\mathbf{p}) + Q_0(\mathbf{p})$$

$$\frac{1}{3} \nabla \phi(\mathbf{p}) = -\sigma_t \mathbf{E}(\mathbf{p}) + Q_1(\mathbf{p})$$

$$Q_0(\mathbf{p}) = \int L_e(\mathbf{p}, \omega') d\omega' \quad Q_1(\mathbf{p}) = \int \omega' \cdot L_e(\mathbf{p}, \omega') d\omega' \quad L(\mathbf{p}, \omega) \approx \frac{1}{4\pi} \int_{S^2} L(\mathbf{p}, \omega') d\omega' + \frac{3}{4\pi} \omega \cdot \int_{S^2} \omega' L(\mathbf{p}, \omega') d\omega' = \frac{1}{4\pi} \phi(\mathbf{p}) + \frac{3}{4\pi} \omega \cdot \mathbf{E}(\mathbf{p})$$

# Diffusion approximation through moment matching

$$\nabla \cdot \mathbf{E}(\mathbf{p}) = -\sigma_a \phi(\mathbf{p}) + Q_0(\mathbf{p})$$

$$\frac{1}{3} \nabla \phi(\mathbf{p}) = -\sigma_t \mathbf{E}(\mathbf{p}) + Q_1(\mathbf{p})$$

solve for  $\phi$

$$\frac{1}{3\sigma_t} \Delta \phi(\mathbf{p}) = \sigma_a \phi(\mathbf{p}) - Q_0(\mathbf{p}) + \frac{1}{\sigma_t} \nabla \cdot Q_1(\mathbf{p})$$

$$Q_0(\mathbf{p}) = \int L_e(\mathbf{p}, \omega') d\omega' \quad Q_1(\mathbf{p}) = \int \omega' \cdot L_e(\mathbf{p}, \omega') d\omega' \quad L(\mathbf{p}, \omega) \approx \frac{1}{4\pi} \int_{S^2} L(\mathbf{p}, \omega') d\omega' + \frac{3}{4\pi} \omega \cdot \int_{S^2} \omega' L(\mathbf{p}, \omega') d\omega' = \frac{1}{4\pi} \phi(\mathbf{p}) + \frac{3}{4\pi} \omega \cdot \mathbf{E}(\mathbf{p})$$

# Diffusion approximation through moment matching

$$\nabla \cdot \mathbf{E}(\mathbf{p}) = -\sigma_a \phi(\mathbf{p}) + Q_0(\mathbf{p})$$

$$\frac{1}{3} \nabla \phi(\mathbf{p}) = -\sigma_t \mathbf{E}(\mathbf{p}) + Q_1(\mathbf{p})$$

solve for  $\phi$

$$\frac{1}{3\sigma_t} \Delta \phi(\mathbf{p}) = \sigma_a \phi(\mathbf{p}) - Q_0(\mathbf{p}) + \frac{1}{\sigma_t} \nabla \cdot Q_1(\mathbf{p})$$

$\mathbf{E}$  can be computed from  $\phi$  and  $Q$

$$L(\mathbf{p}, \omega) \approx \frac{1}{4\pi} \int_{S^2} L(\mathbf{p}, \omega') d\omega' + \frac{3}{4\pi} \omega \cdot \int_{S^2} \omega' L(\mathbf{p}, \omega') d\omega' = \frac{1}{4\pi} \phi(\mathbf{p}) + \frac{3}{4\pi} \omega \cdot \mathbf{E}(\mathbf{p})$$

What is the connection between radiative transfer equation & Poisson equation?

$$\frac{d}{dt}L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega) + L_e(\mathbf{p}(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t), \omega') d\omega'$$

v.s.

$$\Delta u + Q = 0$$

What is the connection between radiative transfer equation & Poisson equation?

$$\frac{1}{3\sigma_t}\Delta\phi(\mathbf{p}) = \sigma_a\phi(\mathbf{p}) - Q_0(\mathbf{p}) + \frac{1}{\sigma_t}\nabla \cdot Q_1(\mathbf{p})$$

v.s.

$$\Delta u + Q = 0$$

# What is the connection between radiative transfer equation & Poisson equation?

$$\frac{1}{3\sigma_t} \Delta \phi(\mathbf{p}) = \boxed{\sigma_a \phi(\mathbf{p})} - Q_0(\mathbf{p}) + \frac{1}{\sigma_t} \nabla \cdot \mathbf{Q}_1(\mathbf{p})$$

energy loss due to absorption

v.s.

$$\Delta u + Q = 0$$

aka “screened Poisson equation” or  
Yukawa equation

[https://en.wikipedia.org/wiki/Screened\\_Poisson\\_equation](https://en.wikipedia.org/wiki/Screened_Poisson_equation)

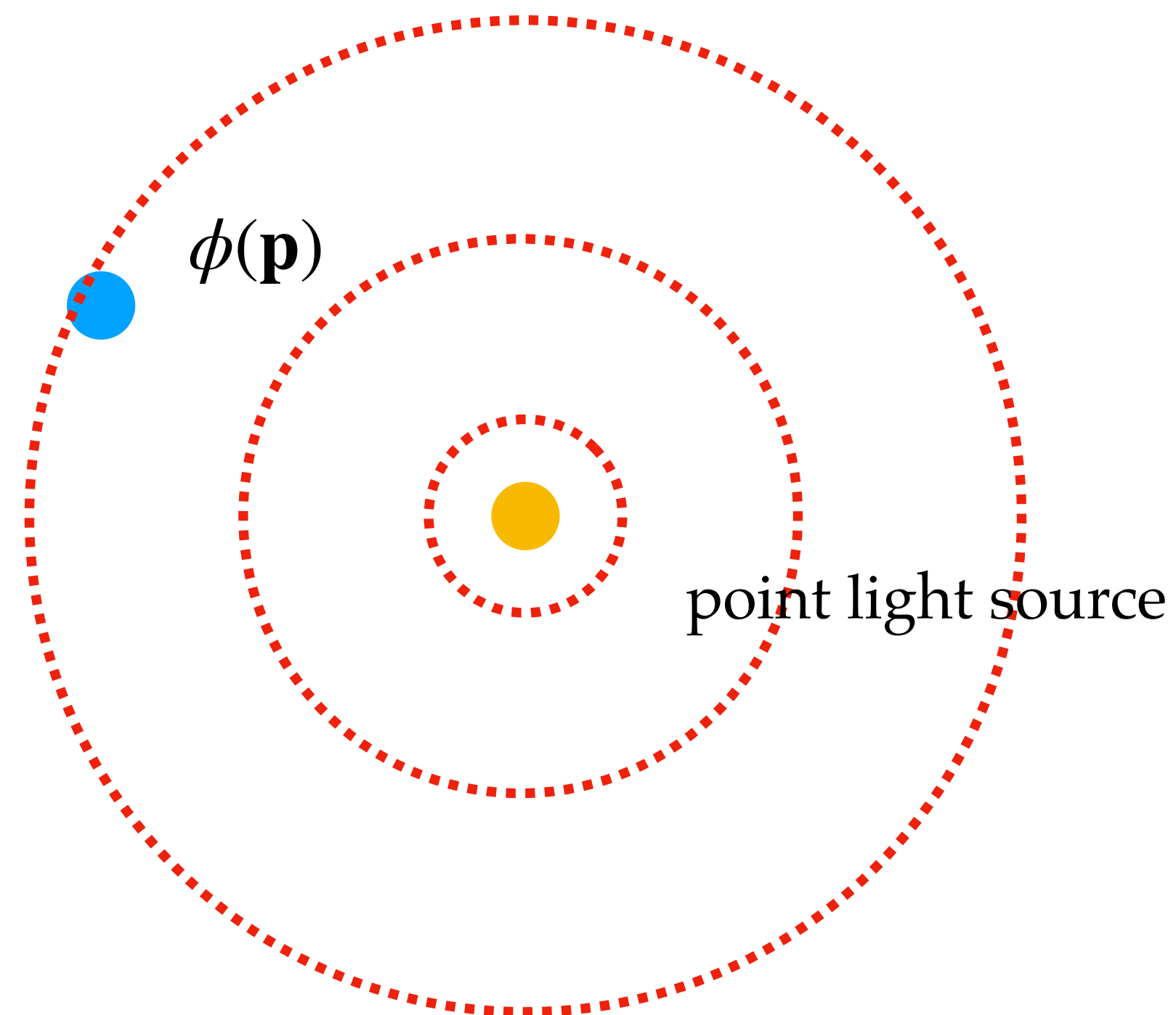
# Solving for $\phi$ in diffusion approximation

- $\phi$  depends on the choice of  $Q$  & boundary condition
- goal: setup  $Q$  & boundary conditions so that we have efficient solutions

$$\frac{1}{3\sigma_t}\Delta\phi(\mathbf{p}) = \sigma_a\phi(\mathbf{p}) - Q_0(\mathbf{p}) + \frac{1}{\sigma_t}\nabla \cdot Q_1(\mathbf{p})$$

# Monopole solution: a single point light source without boundary

$$\frac{1}{3\sigma_t}\Delta\phi(\mathbf{p}) = \sigma_a\phi(\mathbf{p}) - Q_0(\mathbf{p}) + \frac{1}{\sigma_t}\nabla \cdot Q_1(\mathbf{p})$$



$$Q_0 = \delta(\mathbf{p})$$

$$Q_1 = 0$$

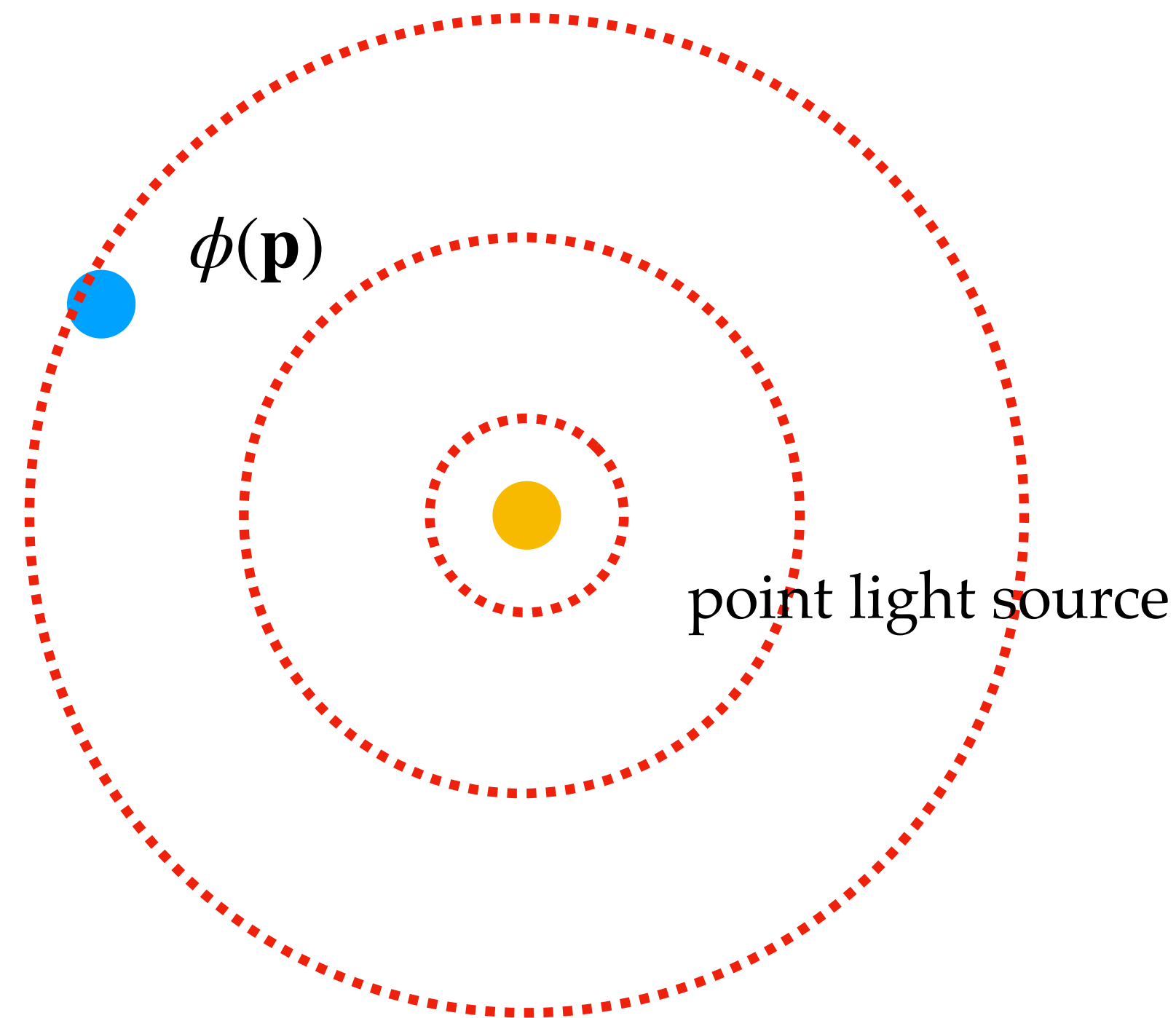


# Monopole solution: a single point light source without boundary

$$\frac{1}{3\sigma_t} \Delta \phi(\mathbf{p}) = \sigma_a \phi(\mathbf{p}) - Q_0(\mathbf{p}) + \frac{1}{\sigma_t} \nabla \cdot Q_1(\mathbf{p})$$

$$\phi_m(\mathbf{p}) = \frac{3\sigma_t}{4\pi} \frac{e^{-\sqrt{3\sigma_a\sigma_t}\|\mathbf{p}\|}}{\|\mathbf{p}\|}$$

“Green’s function”



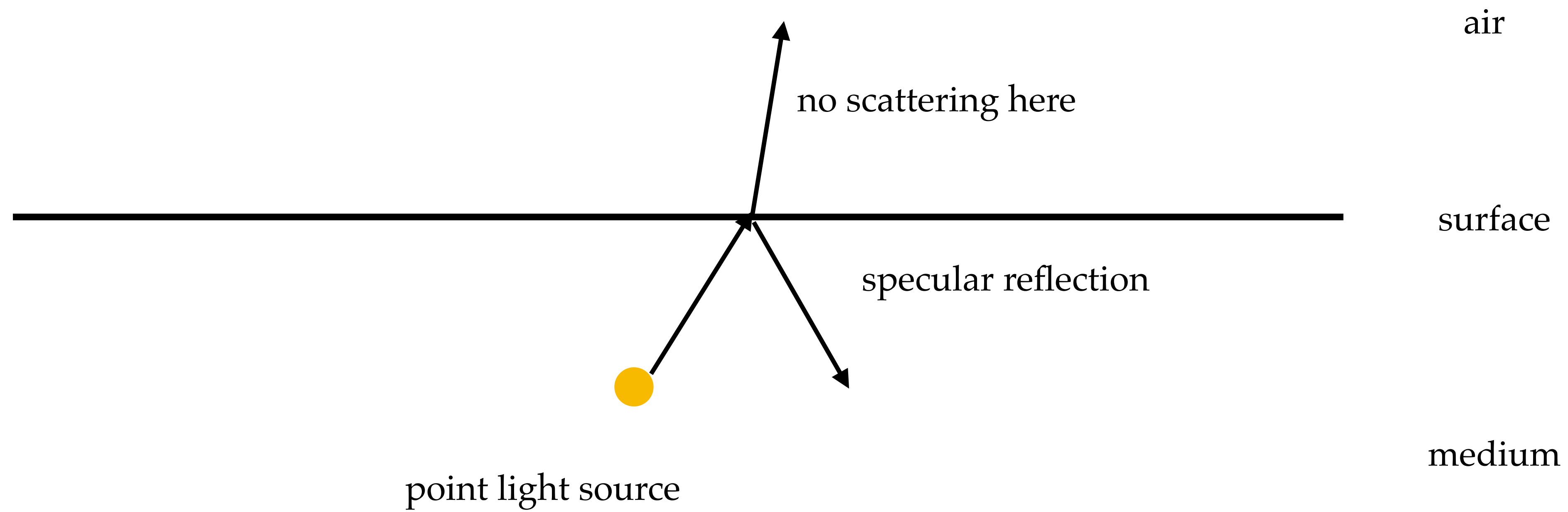
$$Q_0 = \delta(\mathbf{p})$$

$$Q_1 = 0$$

[https://en.wikipedia.org/wiki/Green%27s\\_function](https://en.wikipedia.org/wiki/Green%27s_function)

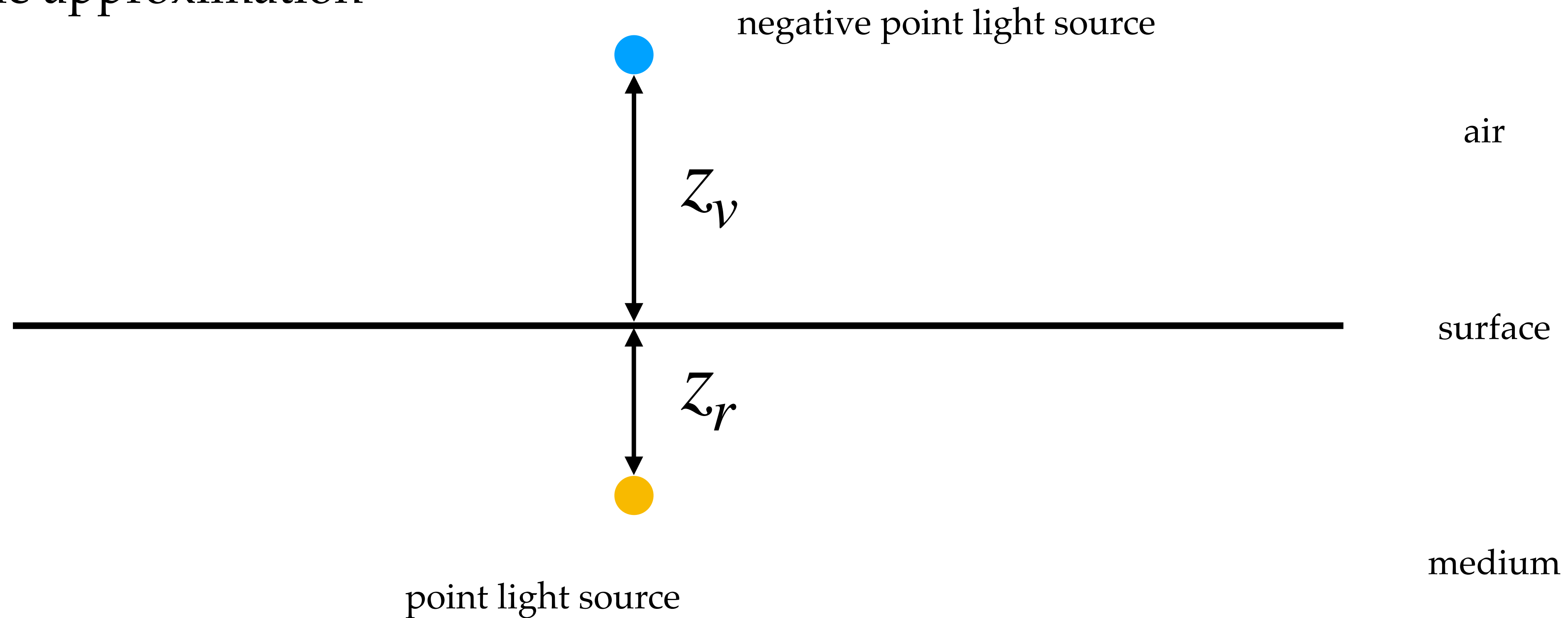
<https://www.youtube.com/watch?v=ism2SfZgFJg> (super cool video about Green’s function)

# Monopole fails to account for the boundary



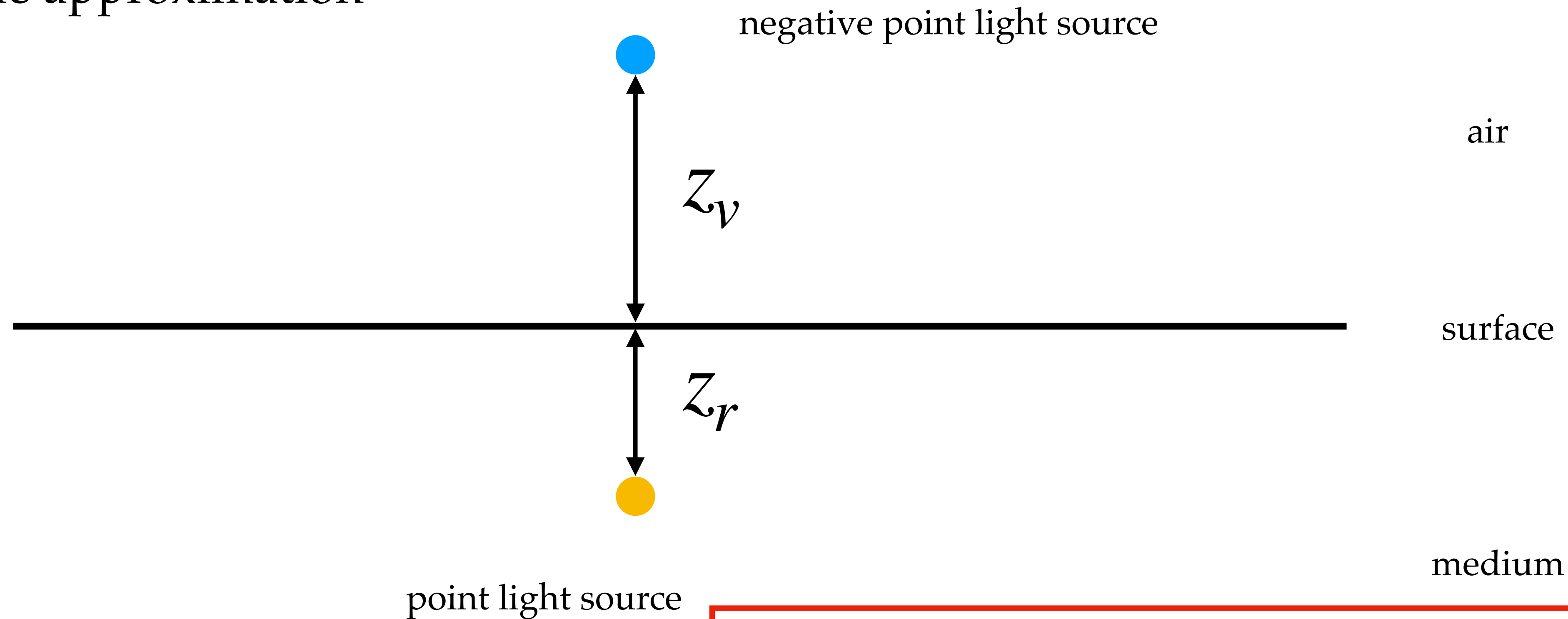
# Idea: put a **negative** light source to cancel out contribution

- “dipole approximation”



# Idea: put a **negative** light source to cancel out contribution

- “dipole approximation”

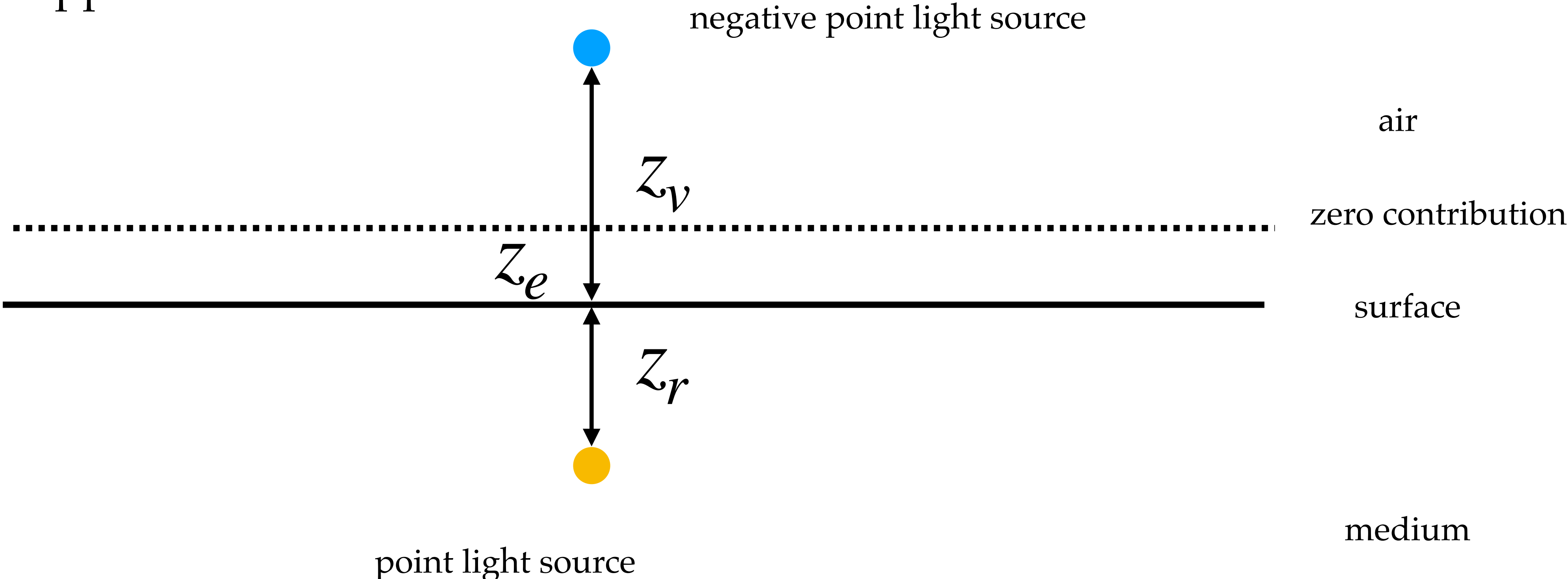


$$\phi_d(\mathbf{p}) = \frac{3\sigma_t}{4\pi} \frac{e^{-\sqrt{3\sigma_a\sigma_t}\|\mathbf{p}-\mathbf{p}_r\|}}{\|\mathbf{p}-\mathbf{p}_r\|} - \frac{3\sigma_t}{4\pi} \frac{e^{-\sqrt{3\sigma_a\sigma_t}\|\mathbf{p}-\mathbf{p}_v\|}}{\|\mathbf{p}-\mathbf{p}_v\|}$$

# Choose $z_v$ to cancel out contribution at $z_e$

read pbrt for how  $z_e$  is chosen

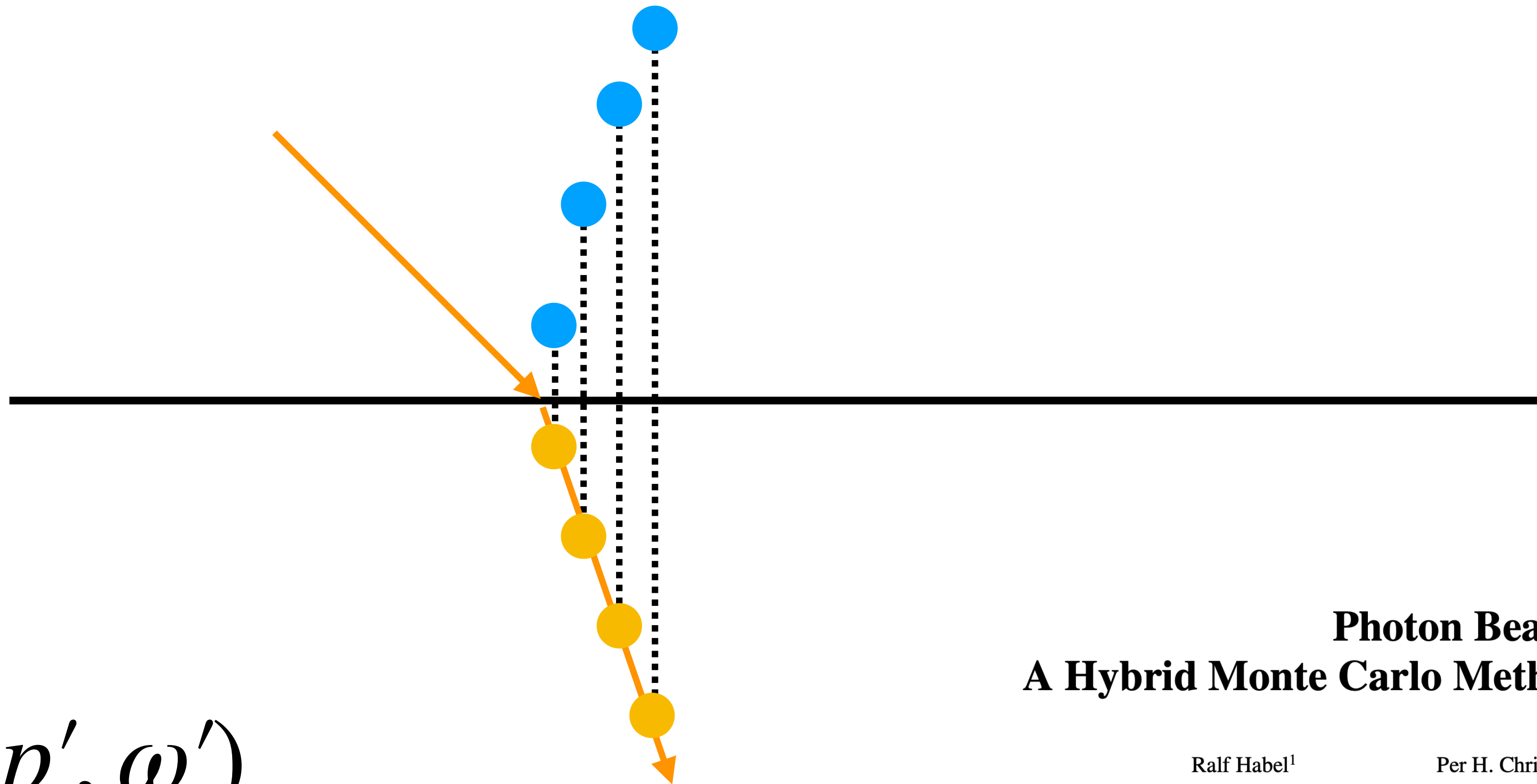
- “dipole approximation”



$$\phi_d(\mathbf{p}) = \frac{3\sigma_t}{4\pi} \frac{e^{-\sqrt{3\sigma_a\sigma_t}\|\mathbf{p}-\mathbf{p}_r\|}}{\|\mathbf{p}-\mathbf{p}_r\|} - \frac{3\sigma_t}{4\pi} \frac{e^{-\sqrt{3\sigma_a\sigma_t}\|\mathbf{p}-\mathbf{p}_v\|}}{\|\mathbf{p}-\mathbf{p}_v\|}$$

# Using dipole solutions for BSSRDF

- place point “light sources” along the incoming ray (using reciprocity of light transport)



**Photon Beam Diffusion:  
A Hybrid Monte Carlo Method for Subsurface Scattering**

Ralf Habel<sup>1</sup>

Per H. Christensen<sup>2</sup>

Wojciech Jarosz<sup>1</sup>

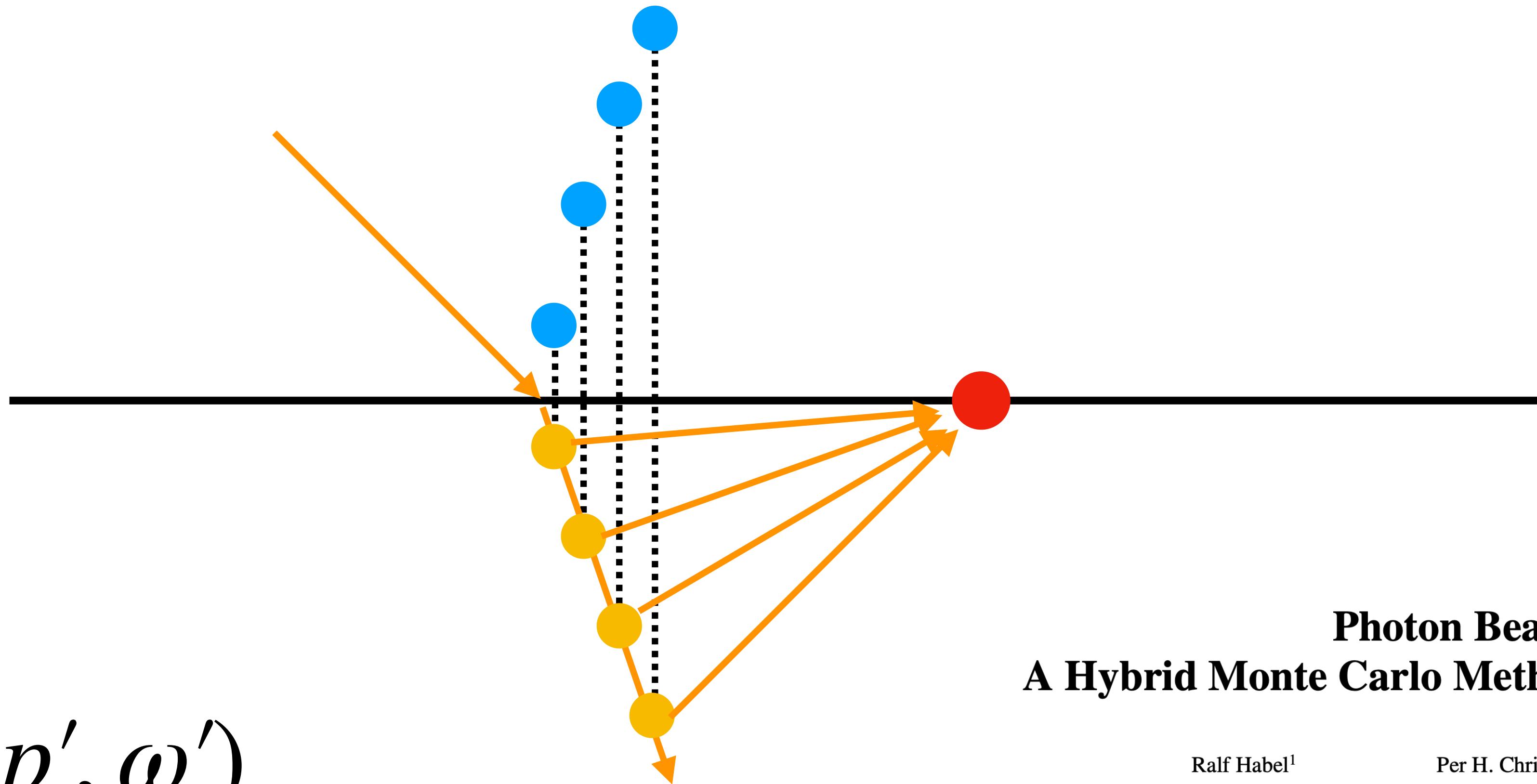
<sup>1</sup>Disney Research Zürich

<sup>2</sup>Pixar Animation Studios

$$f(p, \omega, p', \omega')$$

# Using dipole solutions for BSSRDF

- place point “light sources” along the incoming ray (using reciprocity of light transport)
- BSSRDF is defined as the sum of dipoles to all of them (multiply with Fresnel)



**Photon Beam Diffusion:  
A Hybrid Monte Carlo Method for Subsurface Scattering**

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<sup>2</sup>Pixar Animation Studios

$$f(p, \omega, p', \omega')$$

# Dipole vs volumetric path tracing



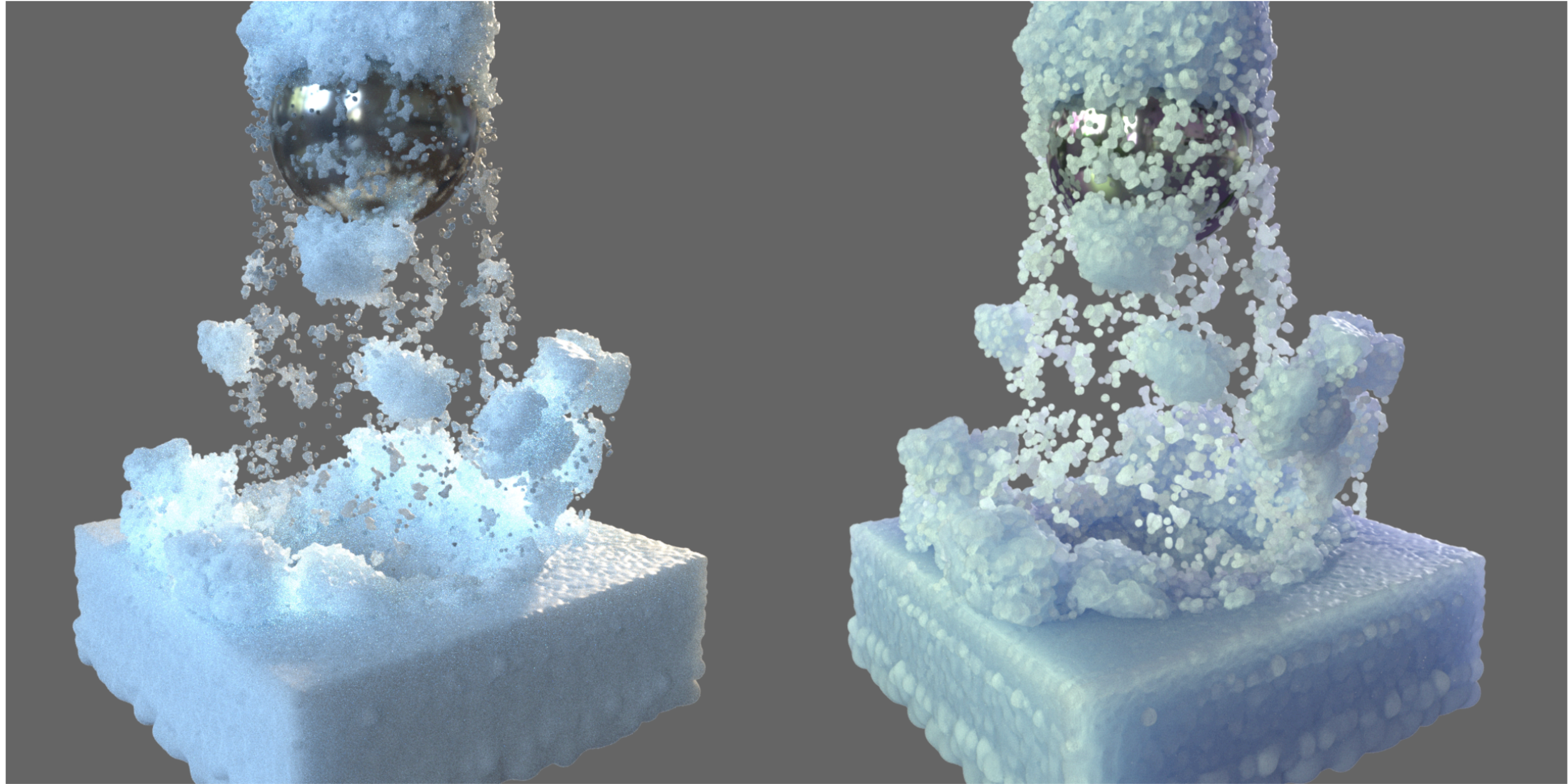
dipole (photon beam diffusion)



path tracing



# Dipole vs volumetric path tracing



dipole

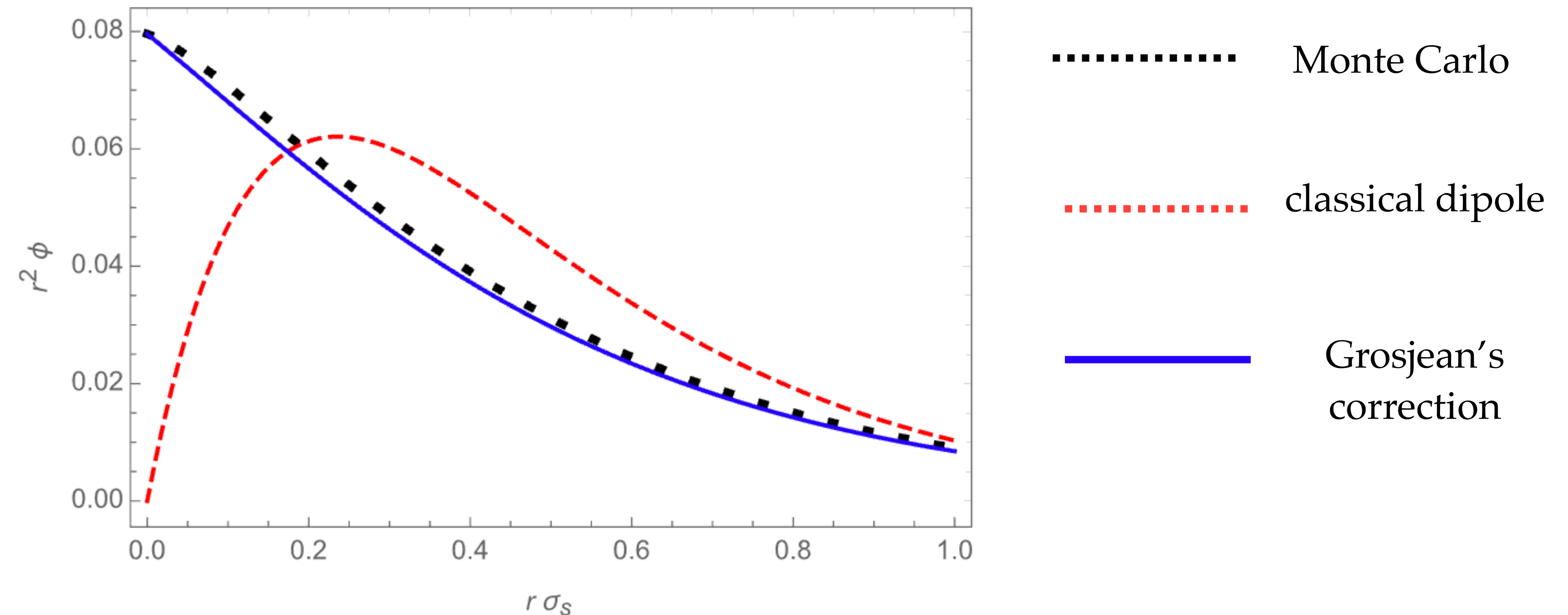
path tracing

# “Hacks” to improve dipoles

$$\phi_m(\mathbf{p}) = \frac{3\sigma_t e^{-\sqrt{3\sigma_a\sigma_t}\|\mathbf{p}\|}}{4\pi\|\mathbf{p}\|}$$

$$\phi_g(\mathbf{p}) = \frac{e^{-\sigma_t\|\mathbf{p}\|}}{4\pi\|\mathbf{p}\|^2} - \phi_m(\mathbf{p})$$

Grosjean's correction [1956]



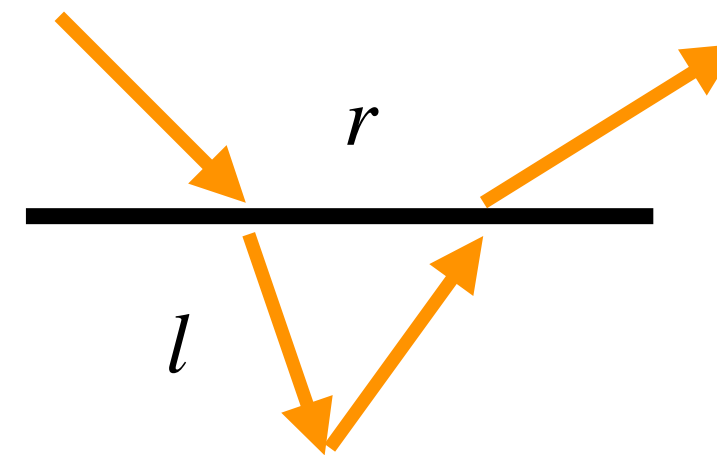
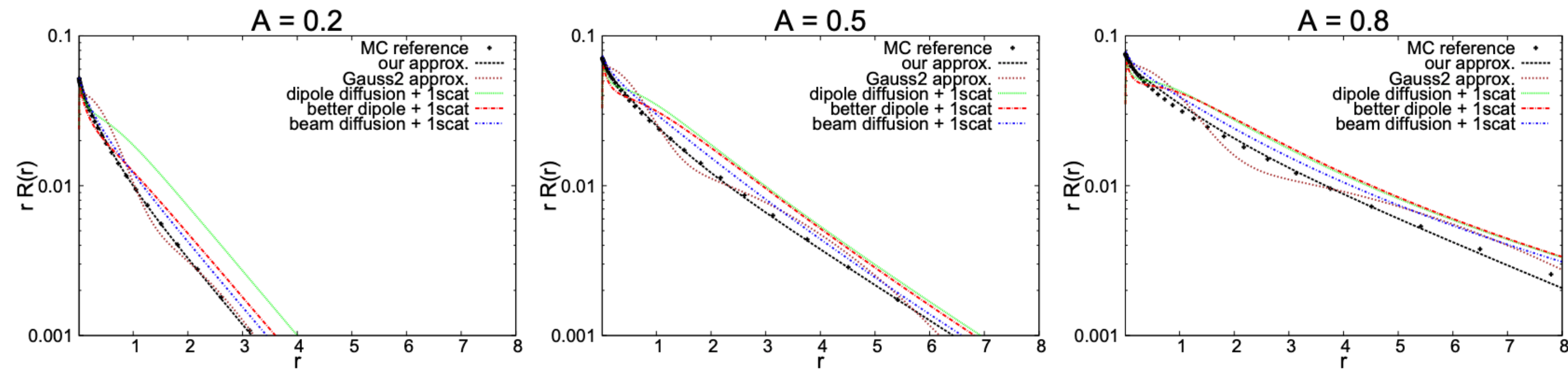
# “Hacks” to improve dipoles

Christensen & Burley's  
empirical model

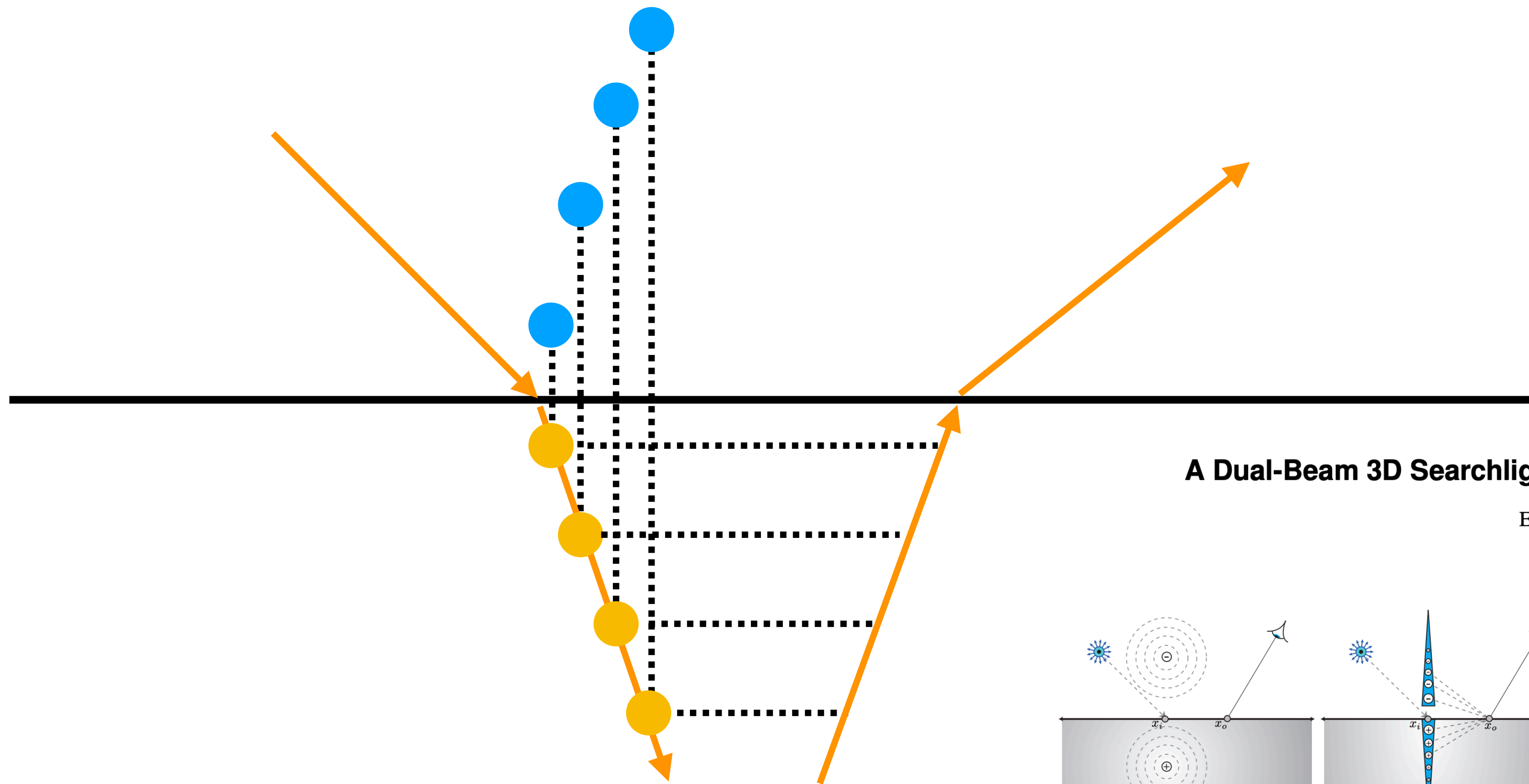
$$\phi_d(\mathbf{p}) = A \frac{e^{-\frac{sr}{l}} - e^{-\frac{sr}{3l}}}{8\pi lr}$$

A: albedo  $\frac{\sigma_s}{\sigma_t}$

$$s = 1.85 - A + 7|A - 0.8|^3$$

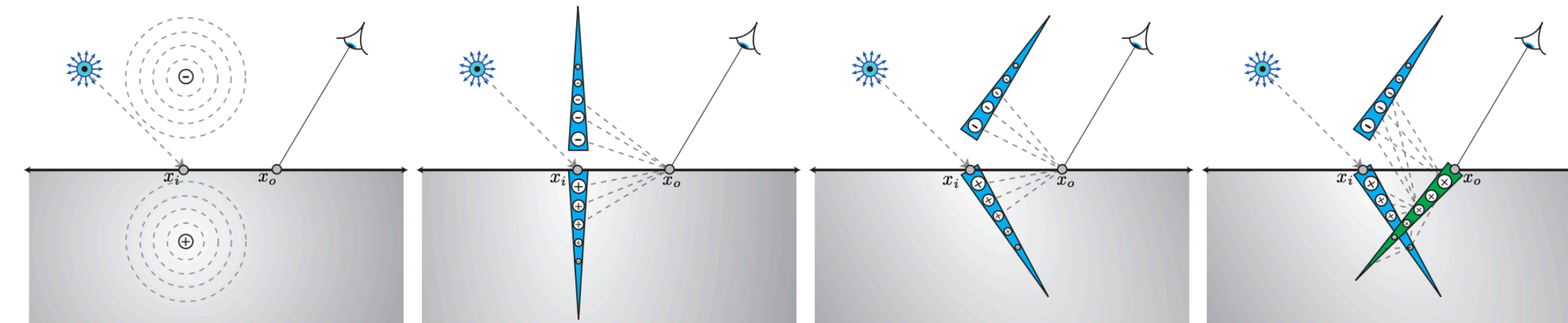


# Dual-beam diffusion



A Dual-Beam 3D Searchlight BSSRDF (Supplementary Doc)

Eugene d'Eon  
Jig Lab



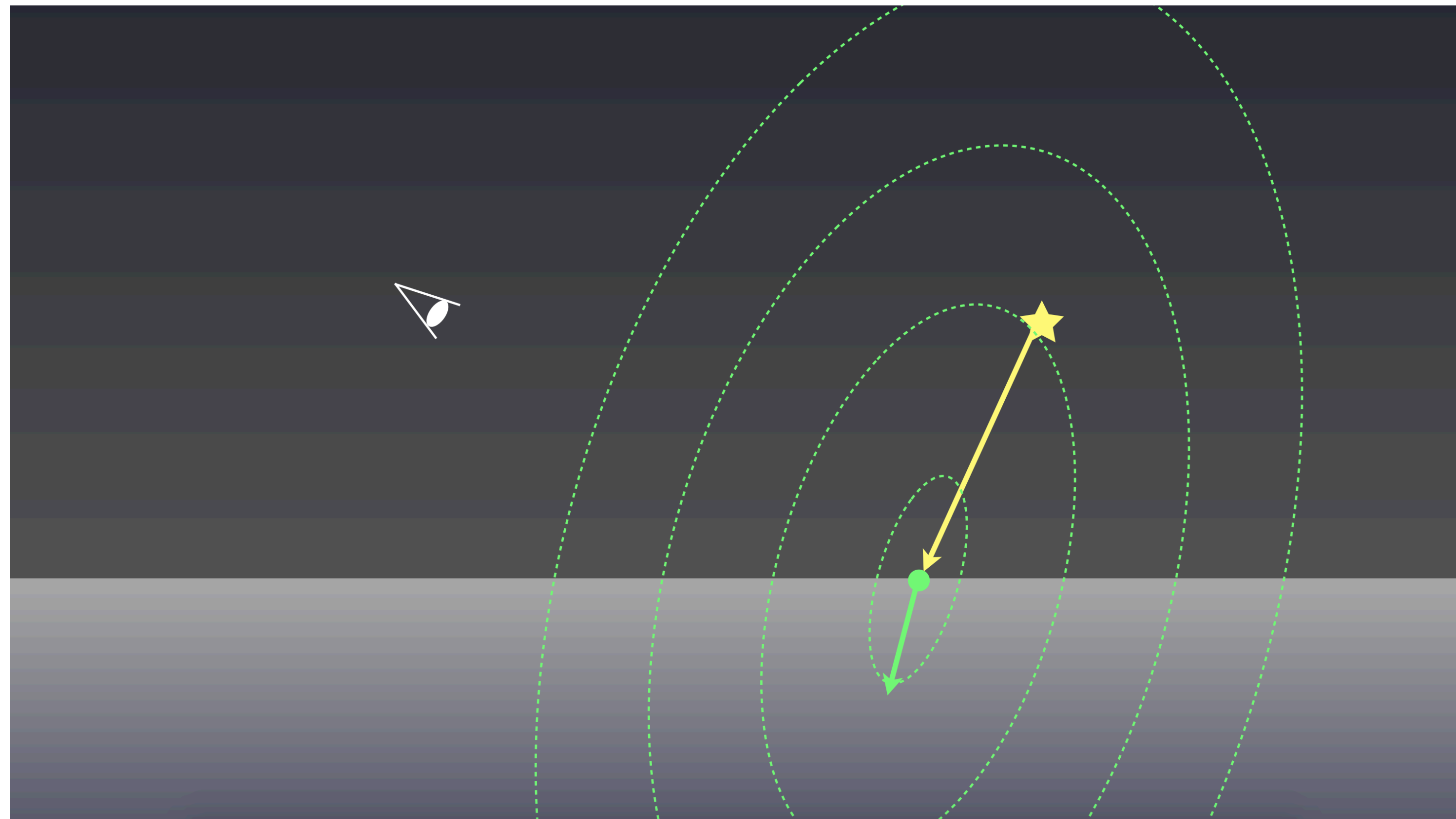
(a) Dipole

(b) Quantized Diffusion

(c) Photon Beam Diffusion

(d) Dual-Beam BSSRDF

# Directional dipole [Frisvad 2014]



(highly recommend Toshiya (UCSD phd!)’s slides!!)  
[https://cs.uwaterloo.ca/~thachisu/dirpole\\_slides.pdf](https://cs.uwaterloo.ca/~thachisu/dirpole_slides.pdf)

# Data-driven BSSRDFs

## An Empirical BSSRDF Model

Craig Donner\*    Jason Lawrence†    Ravi Ramamoorthi ‡  
Toshiya Hachisuka§    Henrik Wann Jensen§    Shree Nayar\*

\*Columbia University    †University of Virginia    ‡UC Berkeley    §UC San Diego



Diffusion Dipole + Single Scattering (10 min)



Our Model + Single Scattering (30 min)



Monte Carlo Path Tracing (30 hours)

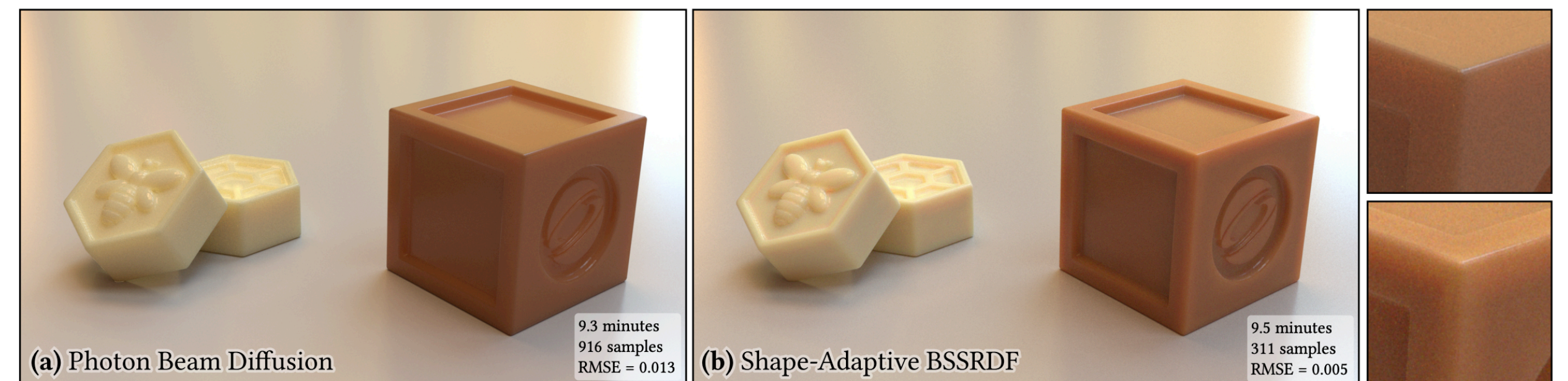


Single Scattering Only

tabular solution

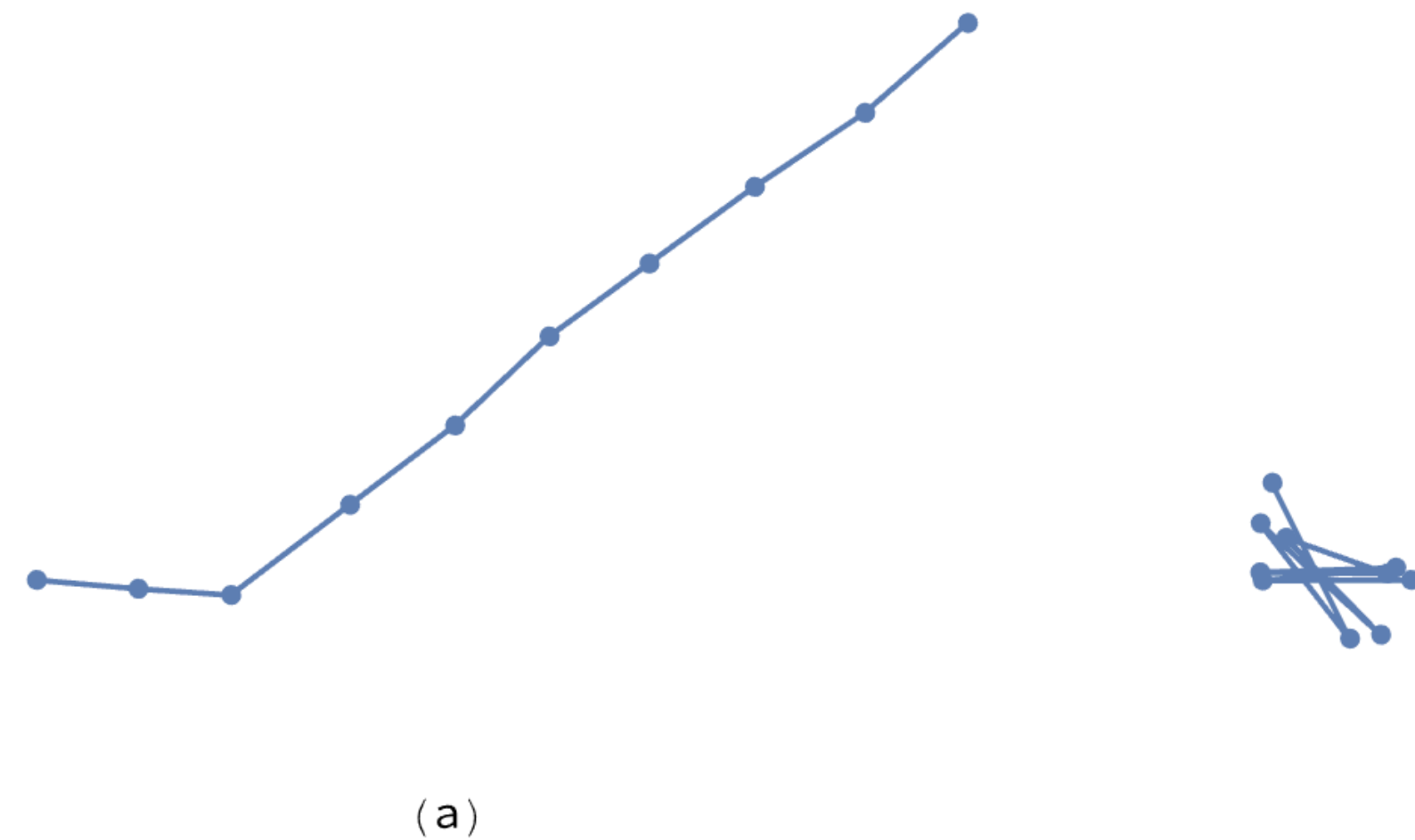
## A Learned Shape-Adaptive Subsurface Scattering Model

DELIO VICINI, Ecole Polytechnique Fédérale de Lausanne (EPFL)  
VLADLEN KOLTUN, Intel Labs  
WENZEL JAKOB, Ecole Polytechnique Fédérale de Lausanne (EPFL)



neural net solution

# Similarity relation for converting non-isotropic phase functions to isotropic ones



## High-Order Similarity Relations in Radiative Transfer

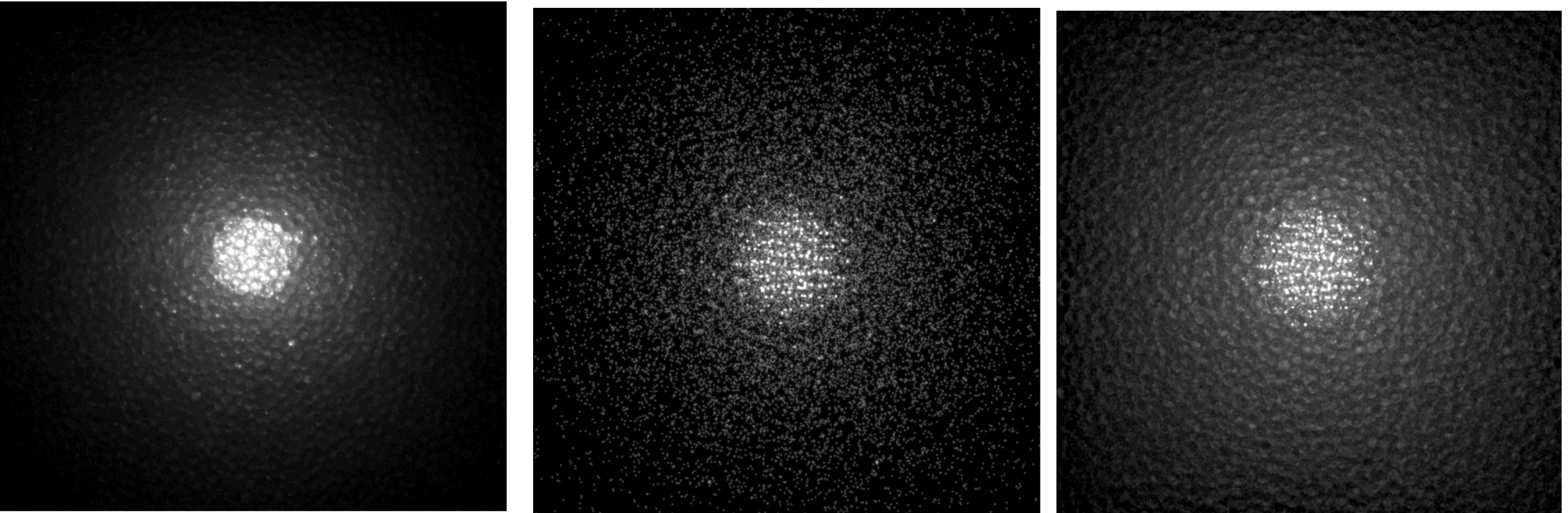
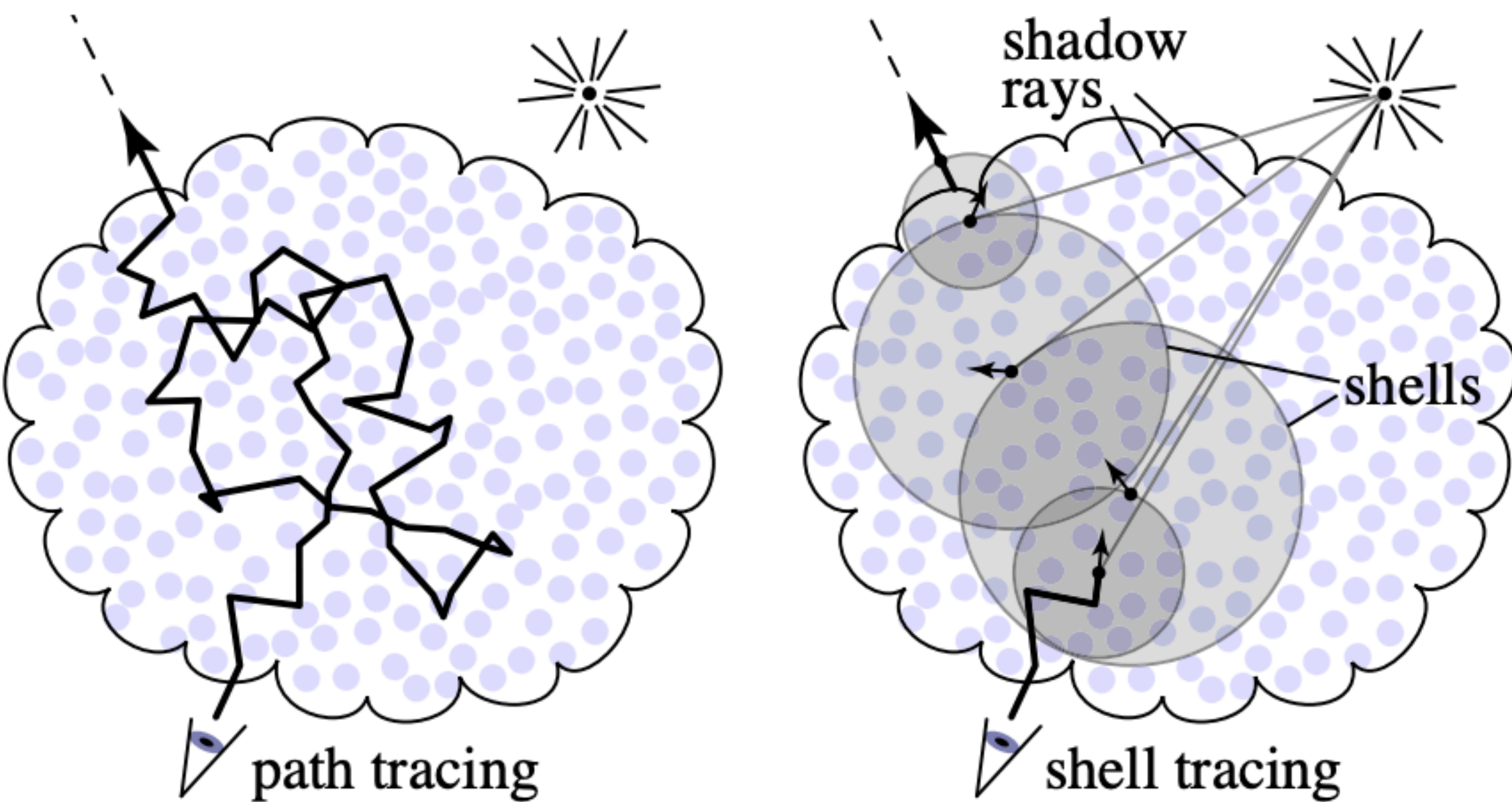
Shuang Zhao  
Cornell University

Ravi Ramamoorthi  
University of California, Berkeley

Kavita Bala  
Cornell University

Figure 15.15: Representative Light Paths for Highly Anisotropic Scattering Media. (a) Forward-scattering medium, with  $g = 0.9$ . Light generally scatters in the same direction it was originally traveling. (b) Backward-scattering medium, with  $g = -0.9$ . Light frequently bounces back and forth, making relatively little forward progress with respect to its original direction.

# Shell tracing: BSSRDF for discrete media



photograph

path tracing  
(28 hours)

shell tracing  
(1 hour)

## Rendering Discrete Random Media Using Precomputed Scattering Solutions

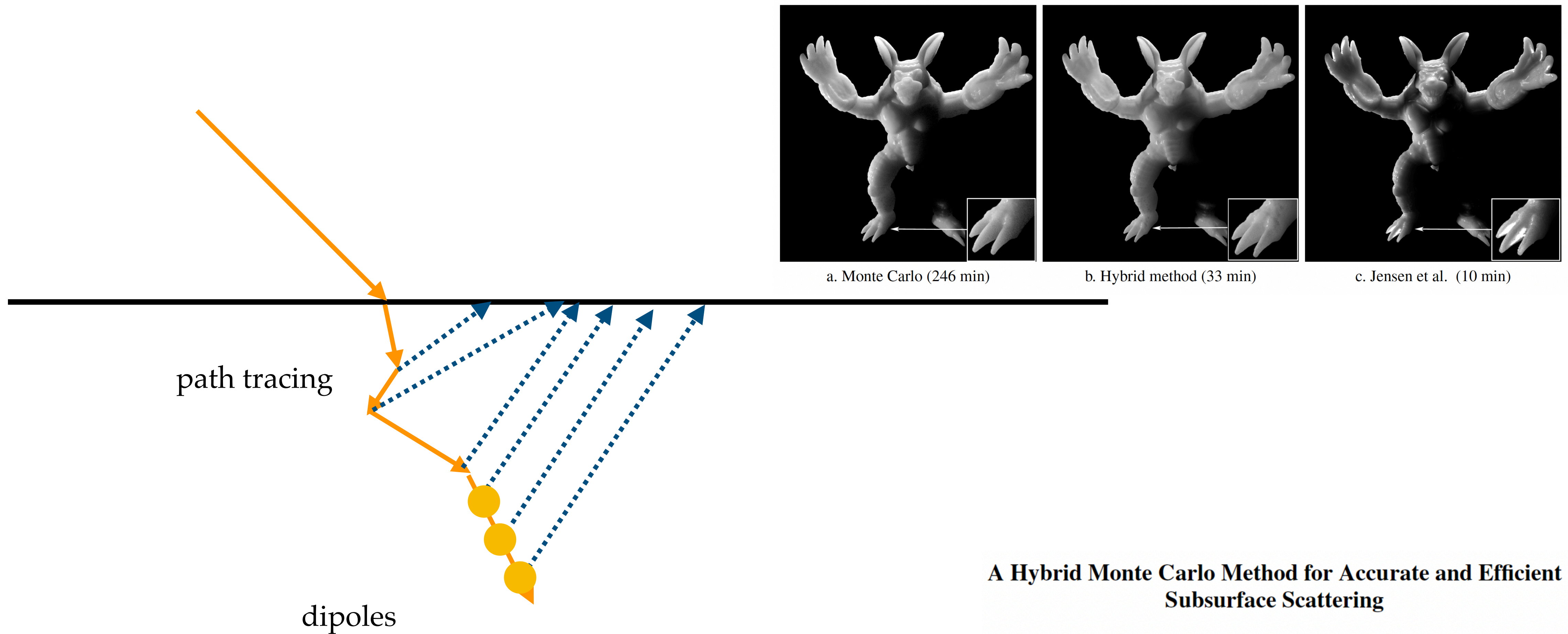
$$f(x, \omega, x', \omega')$$

Jonathan T. Moon, Bruce Walter, and Stephen R. Marschner

Department of Computer Science and Program of Computer Graphics, Cornell University



# Hybrid method: combining volumetric path tracing & BSSRDF

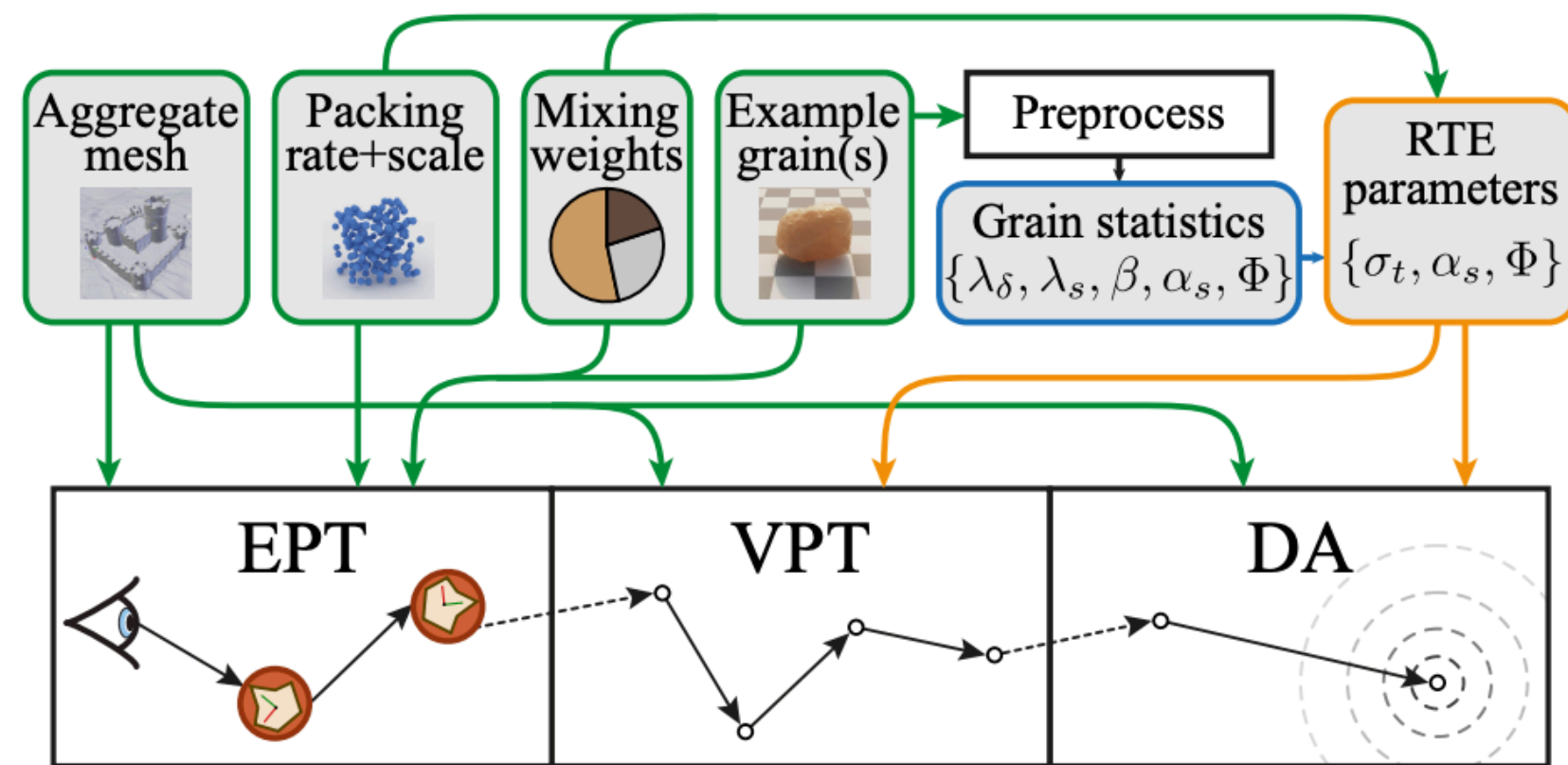


**A Hybrid Monte Carlo Method for Accurate and Efficient Subsurface Scattering**

Hongsong Li<sup>†</sup> Fabio Pellacini<sup>†</sup> Kenneth Torrance<sup>†</sup>

Program of Computer Graphics, Rhodes Hall, Cornell University, Ithaca, NY 14853, U.S.A.

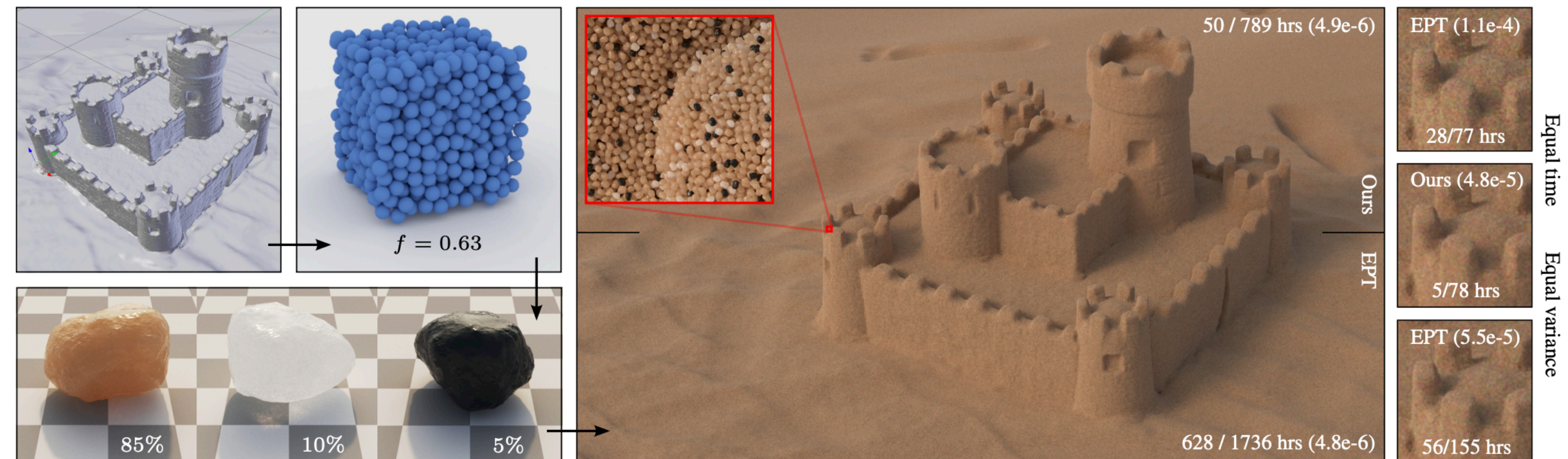
# Multi-scale methods: granular media rendering



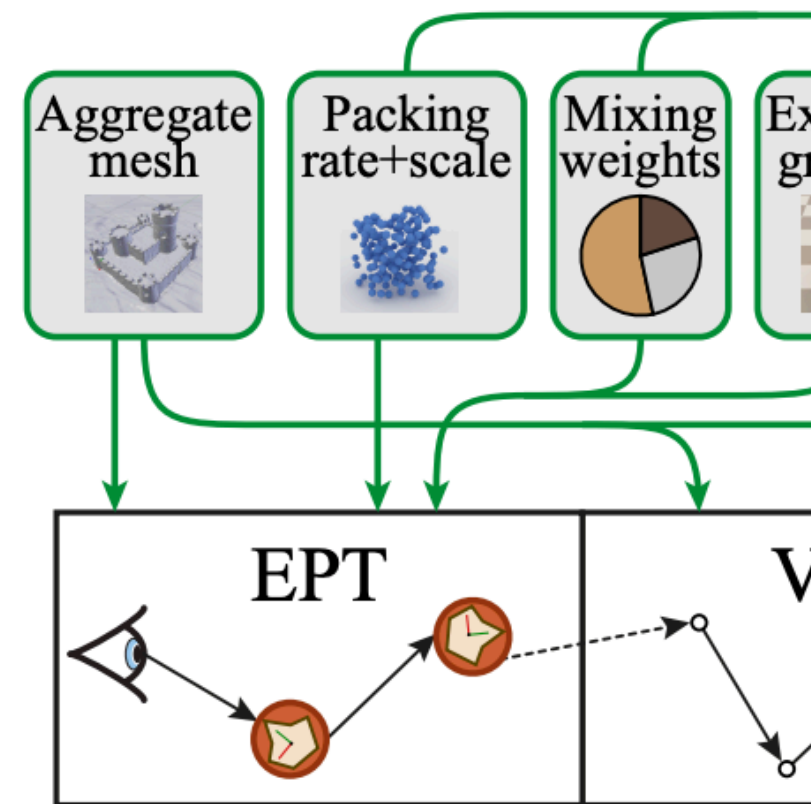
## Multi-Scale Modeling and Rendering of Granular Materials

Johannes Meng<sup>2,1</sup> Marios Papas<sup>1,3</sup> Ralf Habel<sup>1</sup>  
 Carsten Dachsbacher<sup>2</sup> Steve Marschner<sup>4</sup> Markus Gross<sup>1,3</sup> Wojciech Jarosz<sup>1,5\*</sup>

<sup>1</sup>Disney Research Zürich <sup>2</sup>Karlsruhe Institute of Technology <sup>3</sup>ETH Zürich <sup>4</sup>Cornell University <sup>5</sup>Dartmouth College



# Multi-scale methods: granular media rendering



## Multi-Scale Modeling and Rendering of Granular Materials

Johannes Meng<sup>2,1</sup> Marios Papas<sup>1,3</sup> Ralf Habel<sup>1</sup>

Carsten Dachsbacher<sup>2</sup> Steve Marschner<sup>4</sup> Markus Gross<sup>1,3</sup> Wojciech Jarosz<sup>1,5</sup>

<sup>1</sup>Disney Research Zürich <sup>2</sup>Karlsruhe Institute of Technology

<sup>3</sup>ETH Zürich <sup>4</sup>Cornell University <sup>5</sup>Dartmouth College

Dartmouth College

EPT (1.1e-4)

28/77 hrs

Equal time

Hours (4.8e-5)

5/78 hrs

Equal variance

EPT (5.5e-5)

56/155 hrs

# BSSRDF for fur rendering

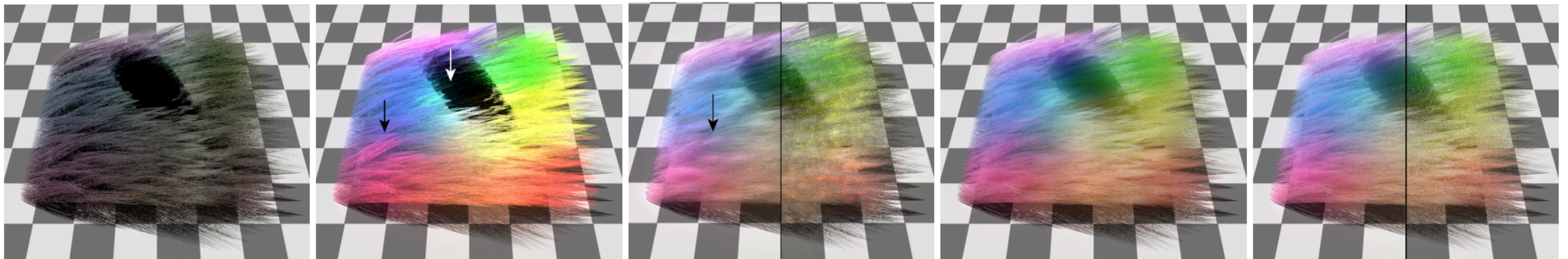
## A BSSRDF Model for Efficient Rendering of Fur with Global Illumination

LING-QI YAN, University of California, Berkeley

WEILUN SUN, University of California, Berkeley

HENRIK WANN JENSEN, University of California, San Diego

RAVI RAMAMOORTHY, University of California, San Diego



(a) Local illumination +  
Classic dual scattering  
16spp, 54s

(b) Local illumination +  
Extended dual scattering  
87spp, 7.2min

(c) Photon mapped  
Left: equal quality, 174.1min  
Right: equal time, 6.8min

Lingqi's  
(d) ~~our~~ method  
42spp, 7.0min

(e) Path traced reference  
Left: 1200spp, 72.9min  
Right: 85spp, 7.6min

# Next: differentiable rendering

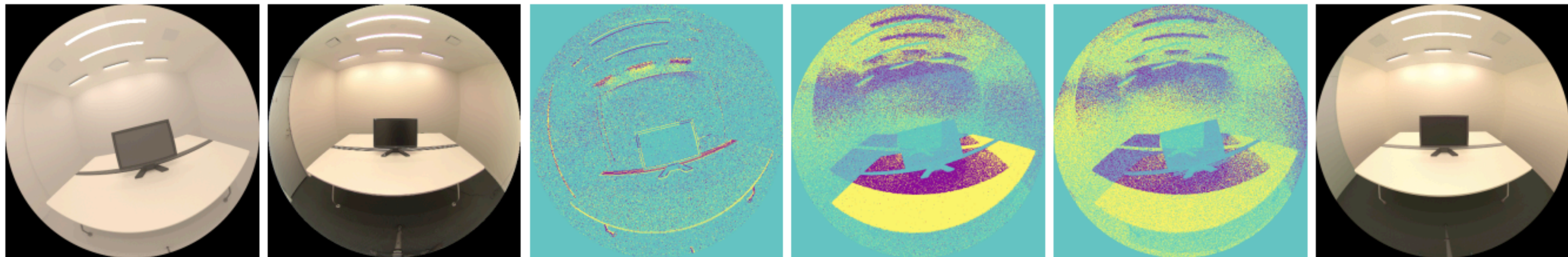
## Differentiable Monte Carlo Ray Tracing through Edge Sampling

TZU-MAO LI, MIT CSAIL

MIIKA AITTALA, MIT CSAIL

FRÉDO DURAND, MIT CSAIL

JAAKKO LEHTINEN, Aalto University & NVIDIA



(a) initial guess

(b) real photograph

(c) camera gradient  
(per-pixel contribution)

(d) table albedo gradient  
(per-pixel contribution)

(e) light gradient  
(per-pixel contribution)

(f) our fitted result