Transmittance sampling and evaluation

UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li

organization of the slides heavily borrowed from the SIGGRAPH course "Monte Carlo methods for physically-based volume rendering"
https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig.html

Last time: radiative transfer equation

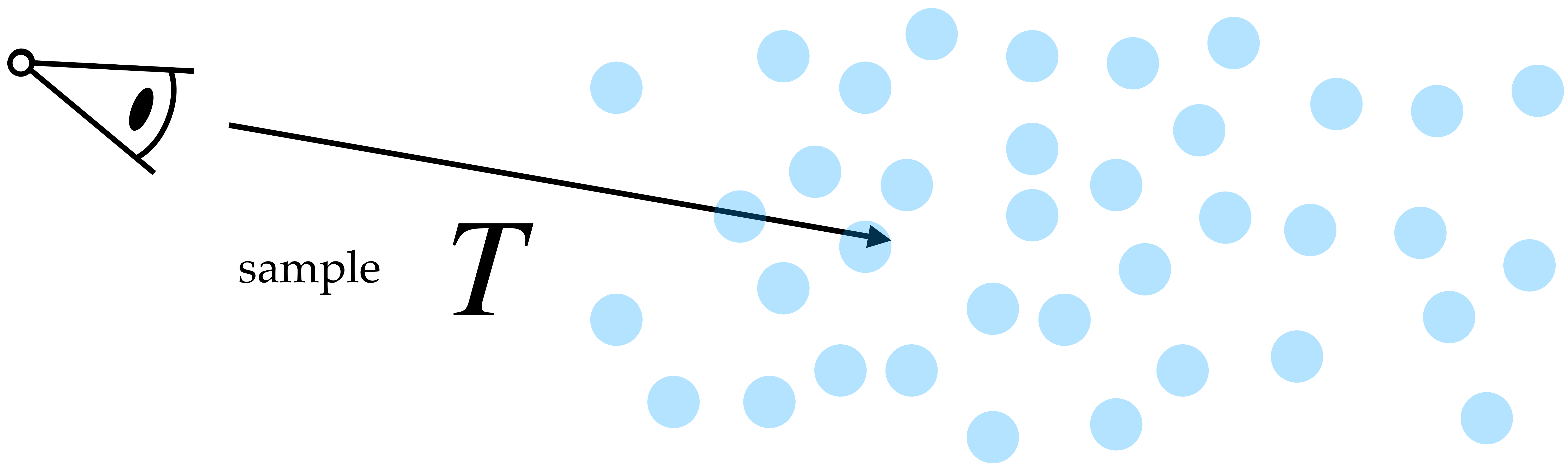
$$\frac{\mathrm{d}}{\mathrm{d}t} L(\mathbf{p}(t), \omega) = -\sigma_t L(\mathbf{p}(t), \omega)$$

$$+ L_e(\mathbf{p}(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t), \omega') \mathrm{d}\omega'$$

$$L(\mathbf{p}(0), \omega) = \int_0^t T\left(\mathbf{p}(0), \mathbf{p}(t')\right) \left[L_e(\mathbf{p}(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t'), \omega') \mathrm{d}\omega' \right] \mathrm{d}t'$$

$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'') \mathrm{d}t'' \right)$$

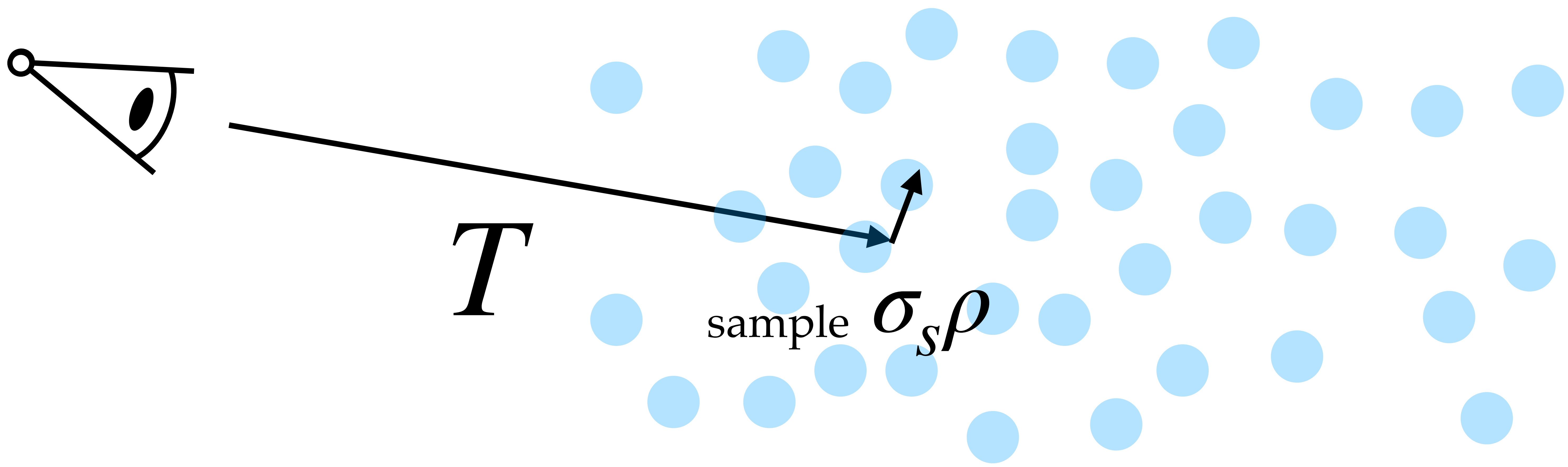
Recap: volumetric path tracing



$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right)$$

$$L(\mathbf{p}(0), \omega) = \int_0^t T\left(\mathbf{p}(0), \mathbf{p}(t')\right) \left[L_e(\mathbf{p}(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega')L(\mathbf{p}(t'), \omega')\mathrm{d}\omega'\right] \mathrm{d}t'$$

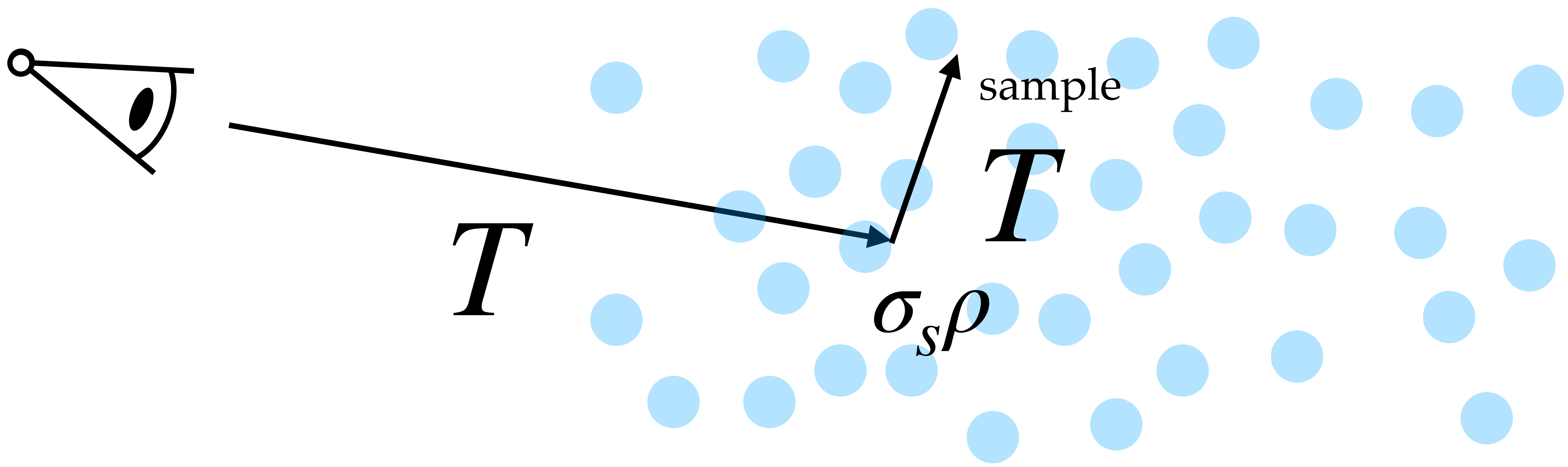
Recap: volumetric path tracing



$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'') \mathrm{d}t''\right)$$

$$L(\mathbf{p}(0), \omega) = \int_0^t T\left(\mathbf{p}(0), \mathbf{p}(t')\right) \left[L_e(\mathbf{p}(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t'), \omega') \mathrm{d}\omega'\right] \mathrm{d}t'$$

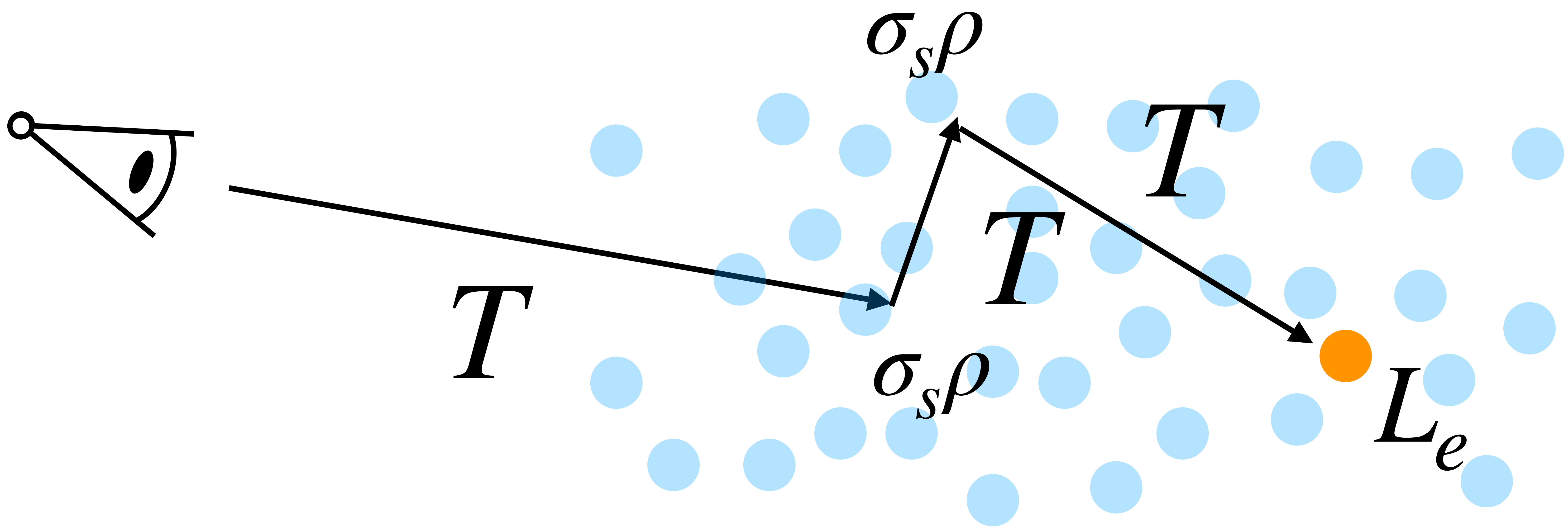
Recap: volumetric path tracing



$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right)$$

$$L(\mathbf{p}(0), \omega) = \int_0^t T\left(\mathbf{p}(0), \mathbf{p}(t')\right)\left[L_e(\mathbf{p}(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(\mathbf{p}(t'), \omega')\mathrm{d}\omega'\right]\mathrm{d}t'$$

Recap: volumetric path tracing



$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right)$$

$$L(\mathbf{p}(0), \omega) = \int_0^t T\left(\mathbf{p}(0), \mathbf{p}(t')\right)\left[L_e(\mathbf{p}(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega')L(\mathbf{p}(t'), \omega')\mathrm{d}\omega'\right]\mathrm{d}t'$$

How do we sample T?

often called "free-flight sampling" or "free-path sampling" in the literature

$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t'' \right)$$

goal: sample t' such that $p(t') \propto T(\mathbf{p}(0), \mathbf{p}(t'))$

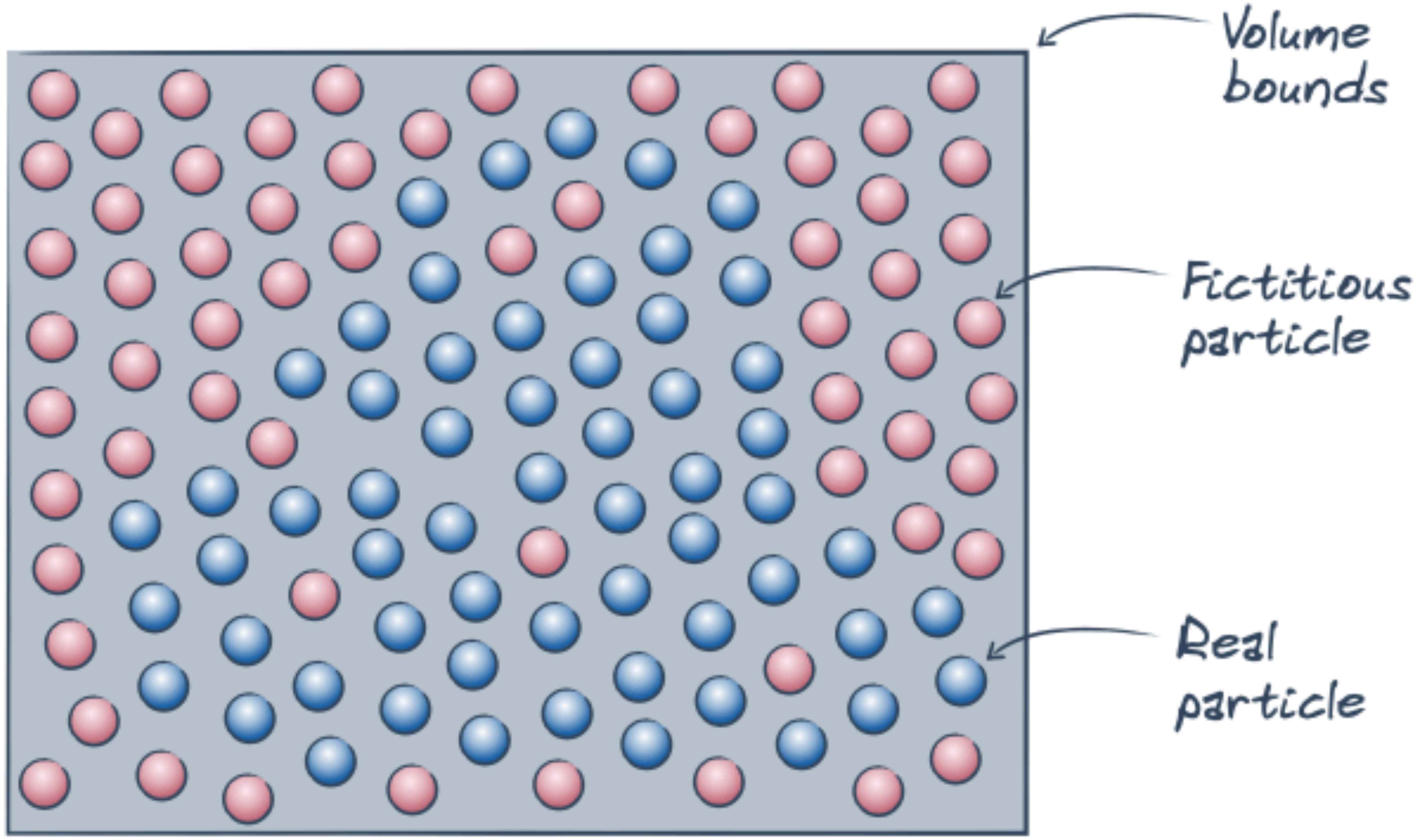
Observation: it's easy to sample T when σ_t is homogeneous

$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right) = \exp\left(-t'\sigma_t\right)$$

$$\text{let } t_i = \frac{\log(1-u_i)}{-\sigma_t}, p(t_i) \propto \exp\left(-t_i\sigma_t\right)$$

Idea: convert heterogeneous media to homogeneous media

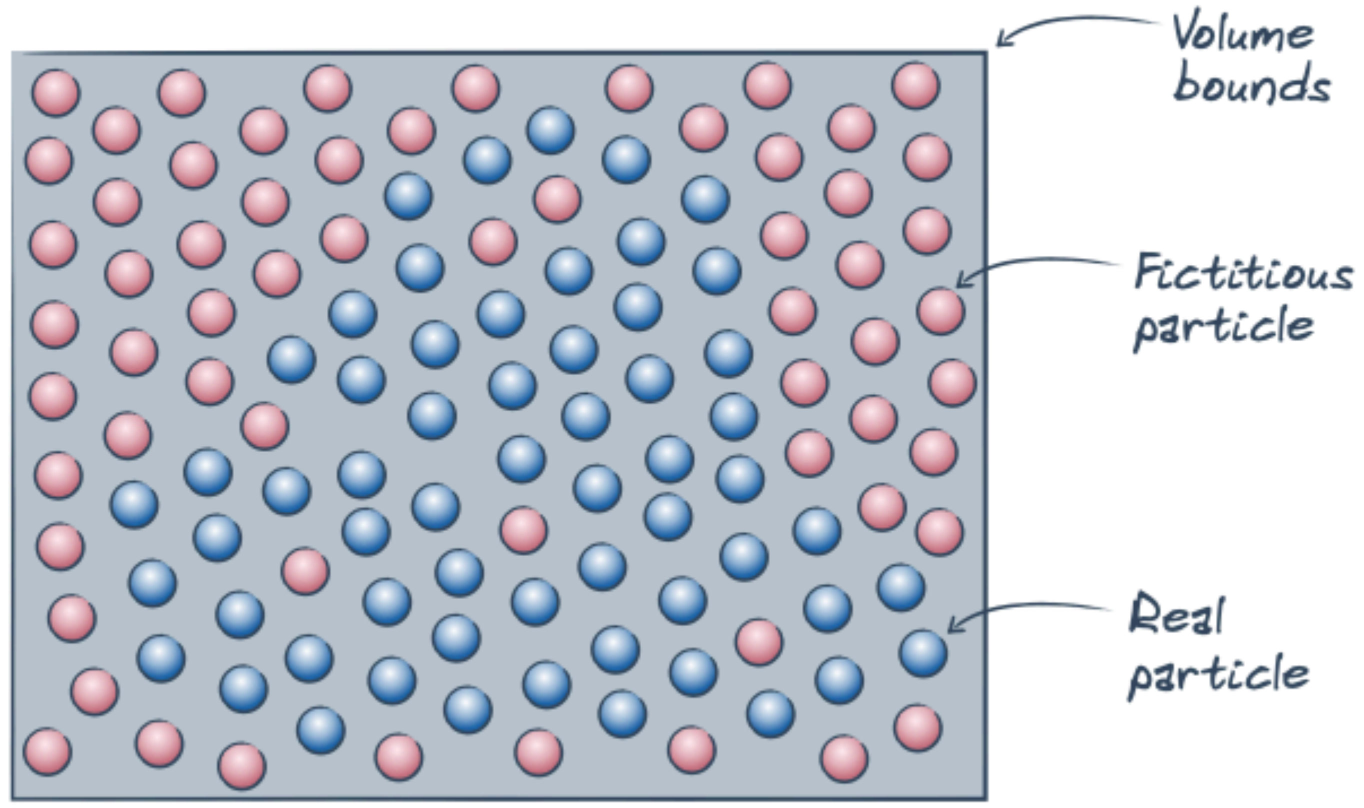
- often called "homogenization" or "null scattering"



https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Let's start from radiative transfer equation

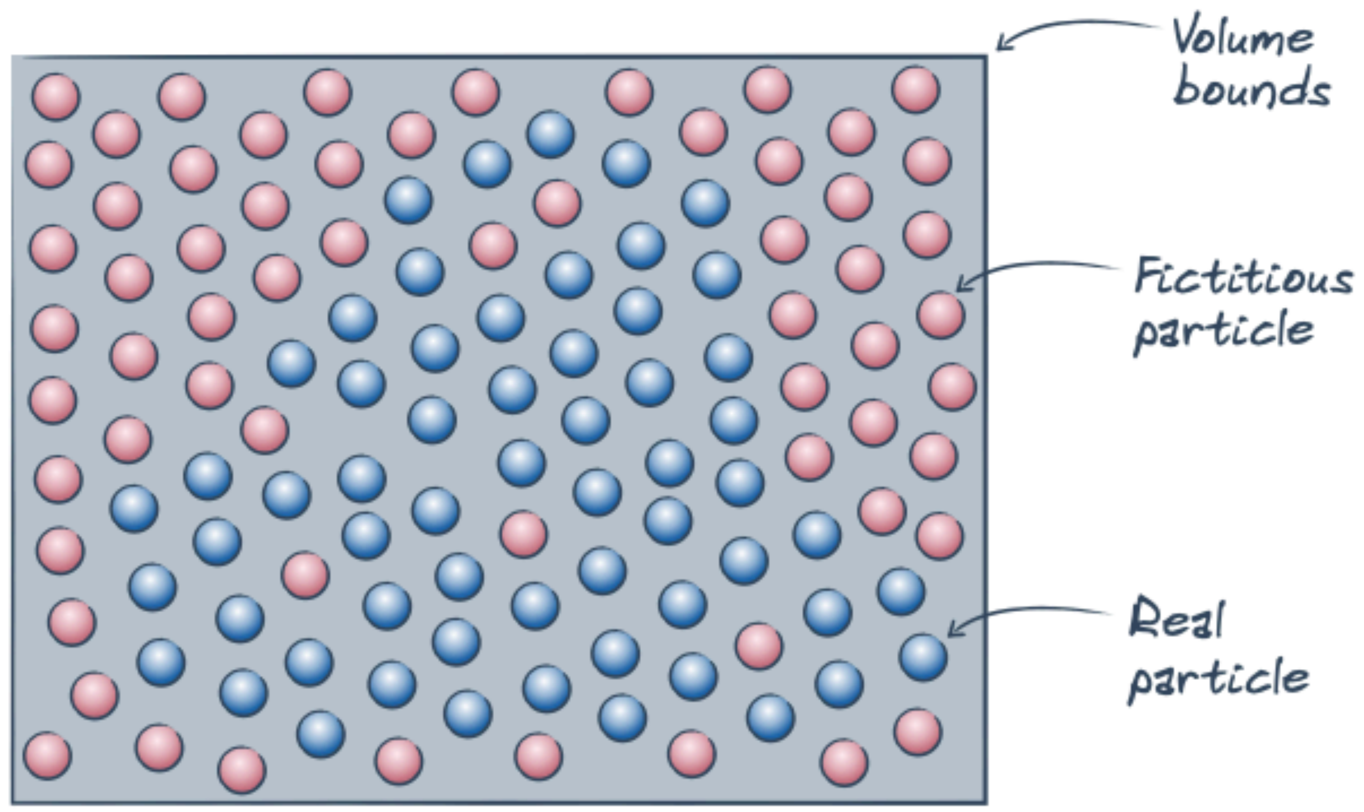
$$\frac{\mathrm{d}}{\mathrm{d}t}L(\mathbf{p}(t),\omega) = -\sigma_t(\mathbf{p})L(\mathbf{p}(t),\omega) + L_e(\mathbf{p}(t),\omega) + \sigma_s\int_{S^2}\rho(\omega,\omega')L(\mathbf{p}(t),\omega')\mathrm{d}\omega'$$



https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Combine L_e & scattering into L_{gain}

$$\frac{\mathrm{d}}{\mathrm{d}t}L(\mathbf{p}(t),\omega) = -\sigma_t(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$

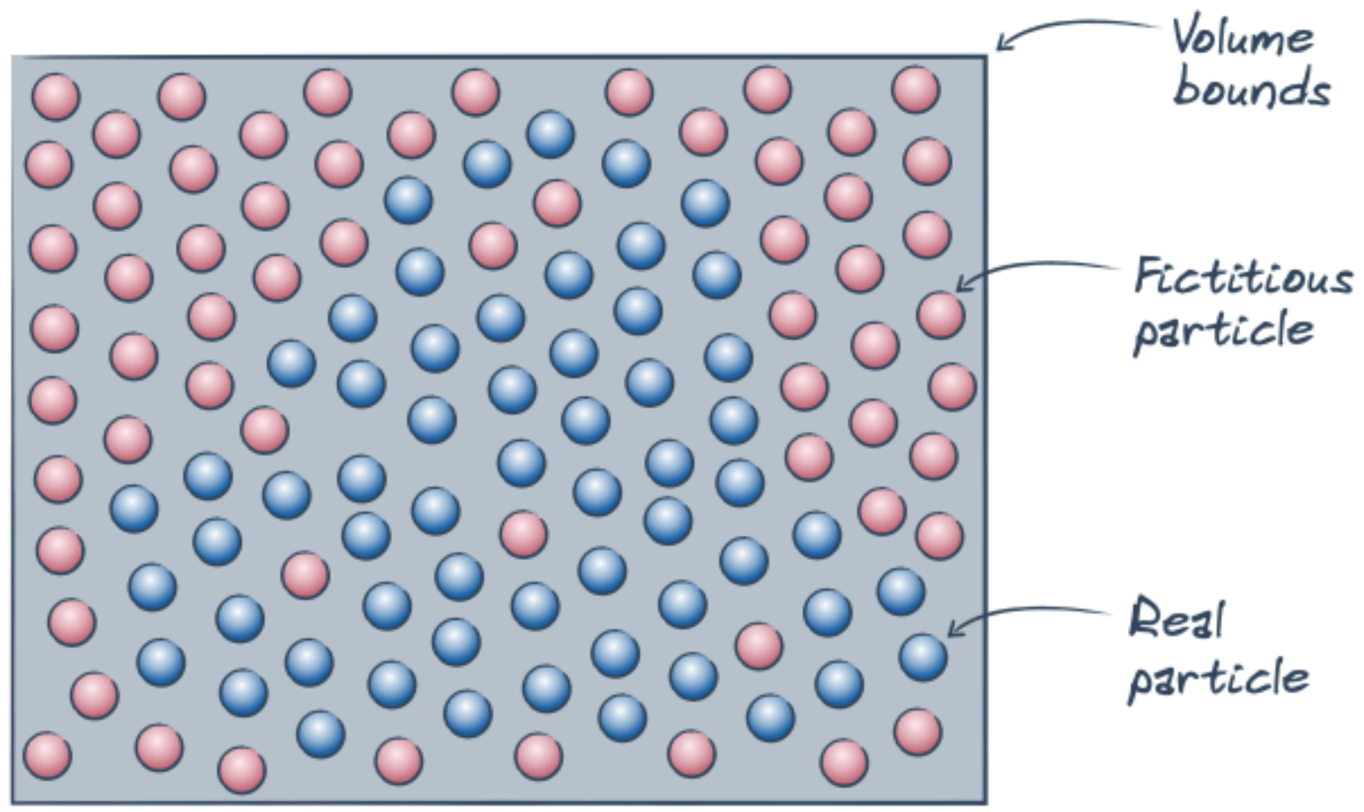


https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Introduce "null particles" with density $\sigma_n(\mathbf{p})$

$$\frac{\mathrm{d}}{\mathrm{d}t}L(\mathbf{p}(t),\omega) = -\sigma_t(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$

$$= -\left(\sigma_t(\mathbf{p}) + \sigma_n(\mathbf{p})\right) L(\mathbf{p}(t),\omega) + \sigma_n(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$



https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

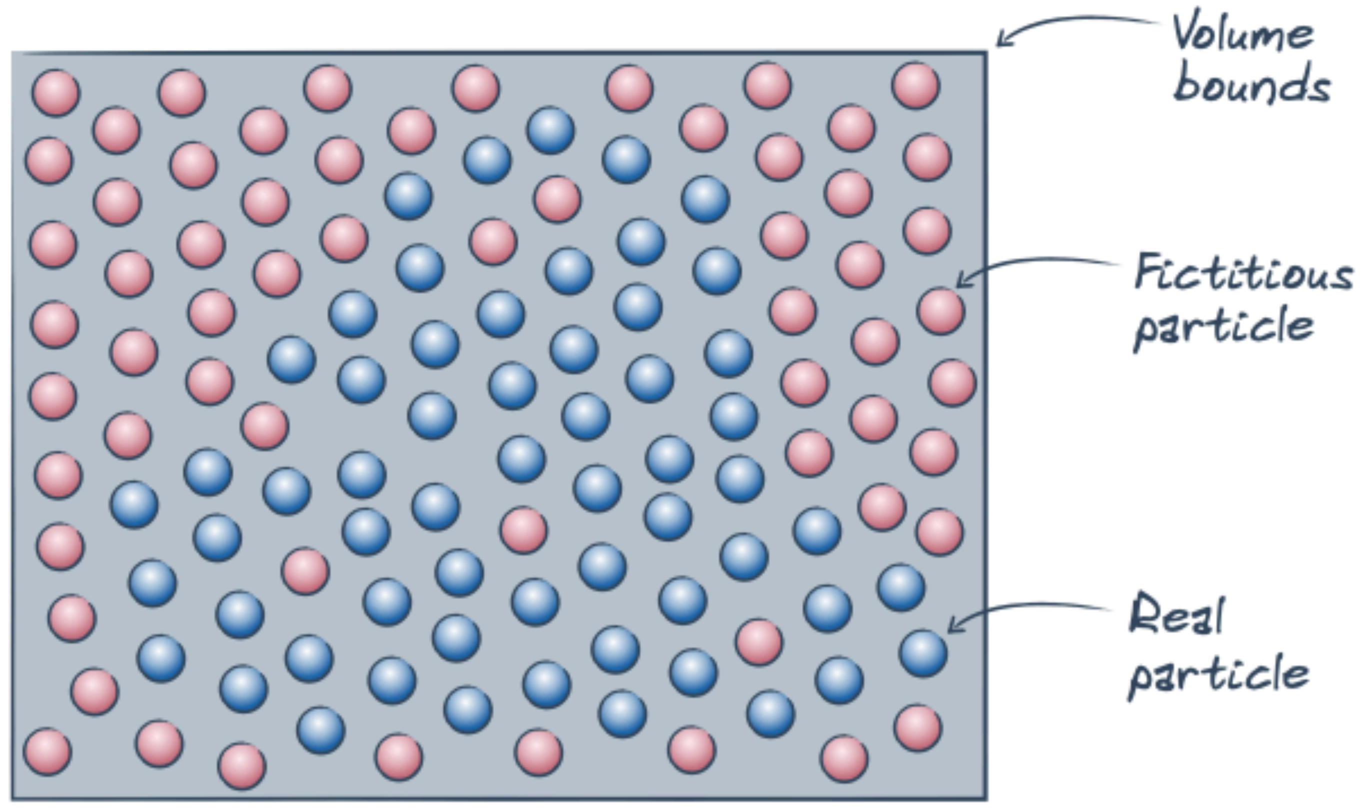
Introduce "null particles" with density $\boldsymbol{\sigma_n}(\mathbf{p})$

$$\frac{\mathrm{d}}{\mathrm{d}t}L(\mathbf{p}(t),\omega) = -\sigma_t(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$

$$= -\sigma_m L(\mathbf{p}(t),\omega) + \sigma_n(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$

choose σ_n such that $\sigma_t(\mathbf{p}) + \sigma_n(\mathbf{p}) = \text{const} = \sigma_m$

in practice, σ_m is the upper bound of $\sigma_t(\mathbf{p})$ (m = "majorant")



https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Integrate the ODE

can be derived using change of variable:

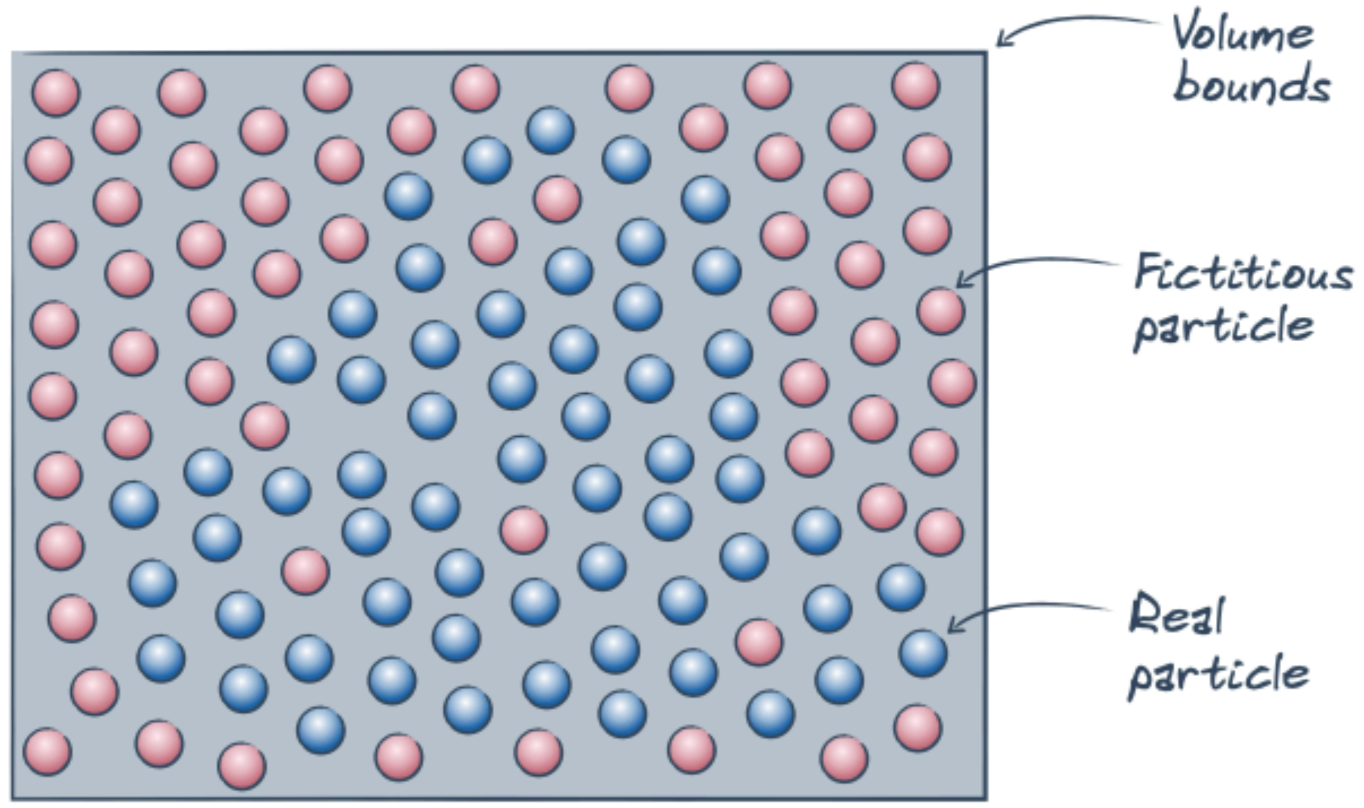
$\tilde{L} = \exp\left(\sigma_m t\right) L$

then integrate both sides

$$\frac{\mathrm{d}}{\mathrm{d}t} L(\mathbf{p}(t), \omega) = -\sigma_m L(\mathbf{p}(t), \omega) + \sigma_n(\mathbf{p}) L(\mathbf{p}(t), \omega) + L_{\text{gain}}$$

$$L(\mathbf{p}(0), \omega) = \int_0^t T_m(\mathbf{p}(0), \mathbf{p}(t')) \left(\sigma_n(\mathbf{p}) L(\mathbf{p}(t'), \omega) + L_{\text{gain}} \right) \mathrm{d}t'$$

$$T_m(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-t' \sigma_m\right)$$



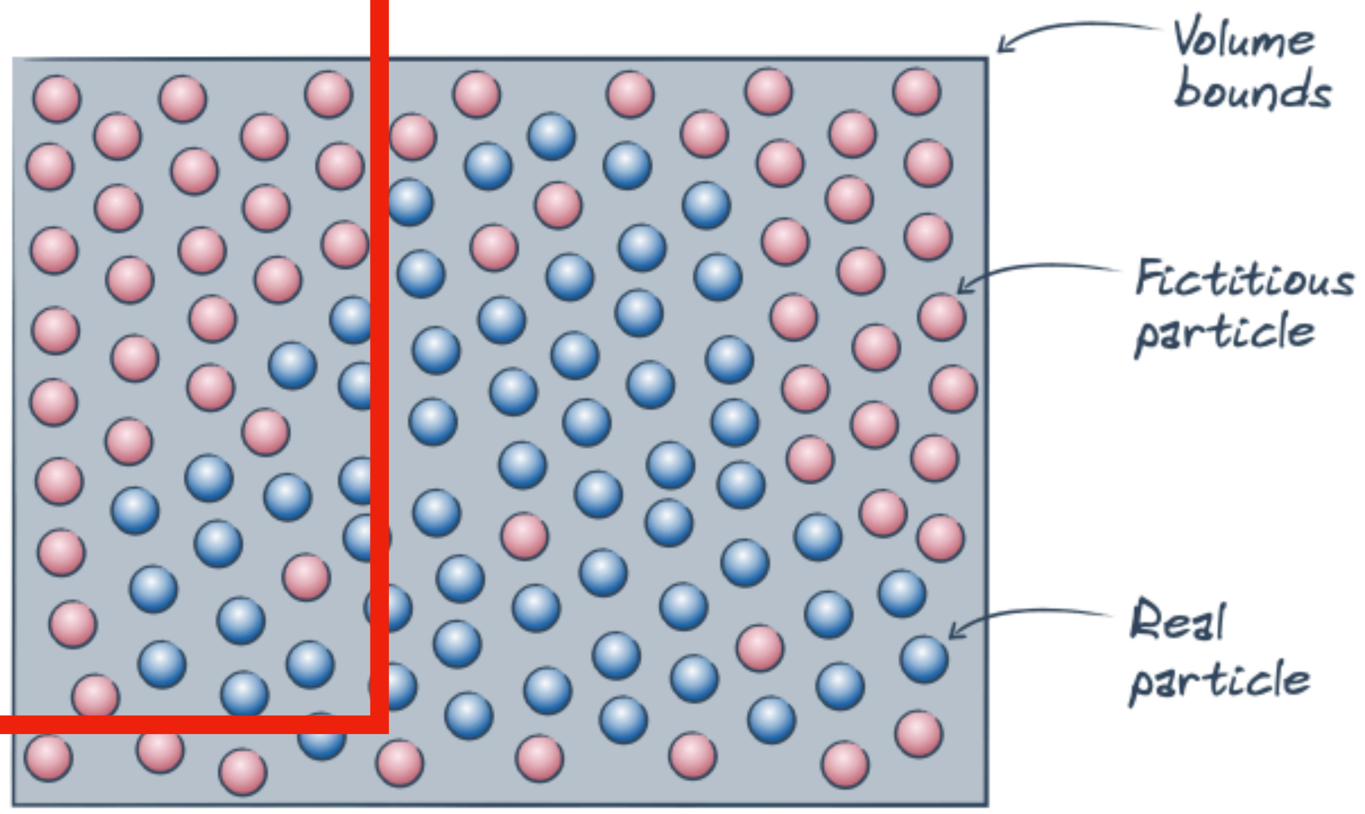
https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Homogenized!

$$\frac{\mathrm{d}}{\mathrm{d}t}L(\mathbf{p}(t),\omega) = -\sigma_m L(\mathbf{p}(t),\omega) + \sigma_n(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$

$$L(\mathbf{p}(0),\omega) = \int_0^t T_m(\mathbf{p}(0),\mathbf{p}(t'))\left(\sigma_n(\mathbf{p})L(\mathbf{p}(t'),\omega) + L_{\text{gain}}\right)\mathrm{d}t'$$

$$T_m(\mathbf{p}(0),\mathbf{p}(t')) = \exp\left(-t'\sigma_m\right)$$

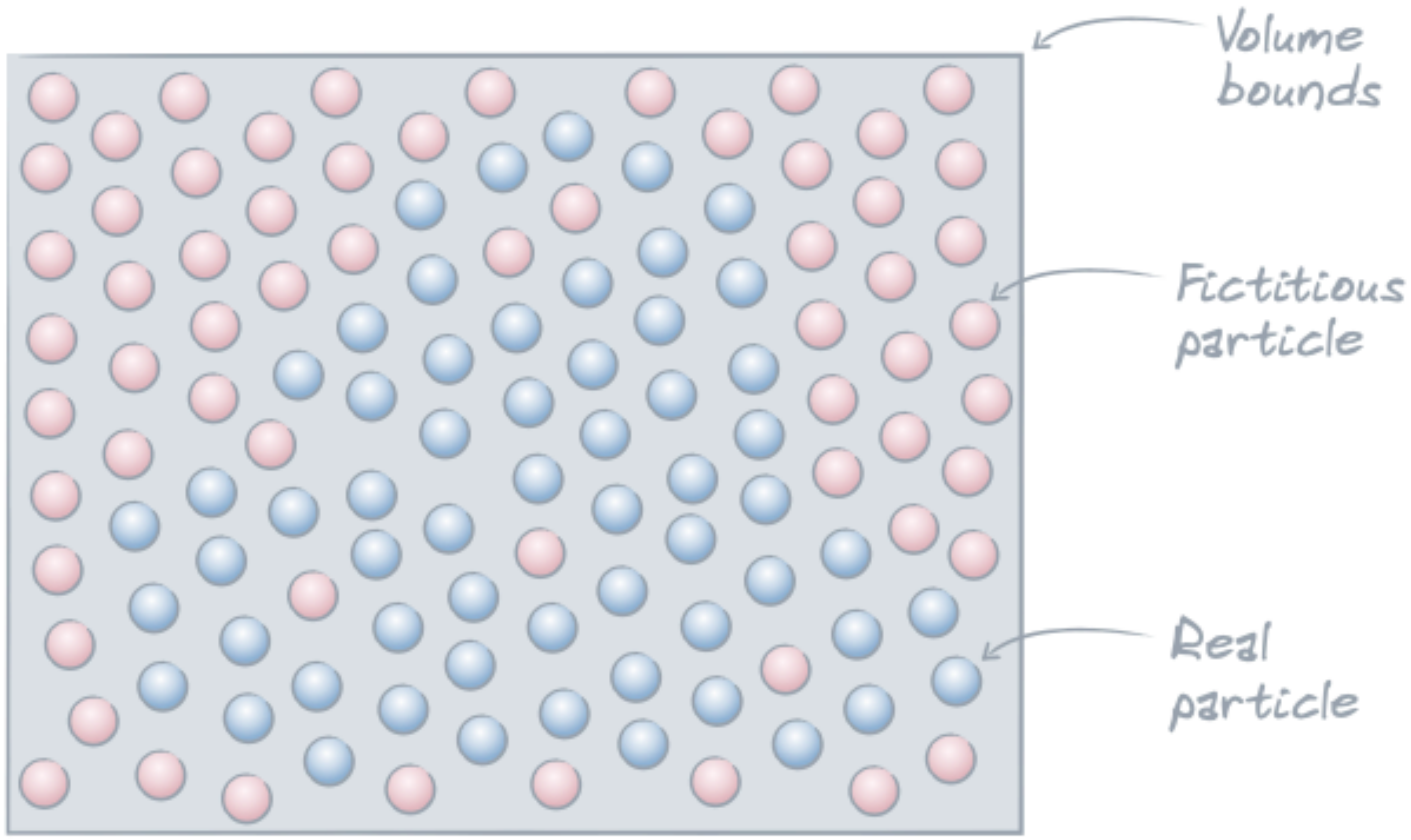


https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Tracing homogenized volumes ("delta tracking")

$$L(\mathbf{p}(0), \omega) = \int_0^t T_m(\mathbf{p}(0), \mathbf{p}(t')) \left(\sigma_n(\mathbf{p}) L(\mathbf{p}(t'), \omega) + L_{\text{gain}} \right) \mathrm{d}t'$$

$$T_m(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-t'\sigma_m\right)$$

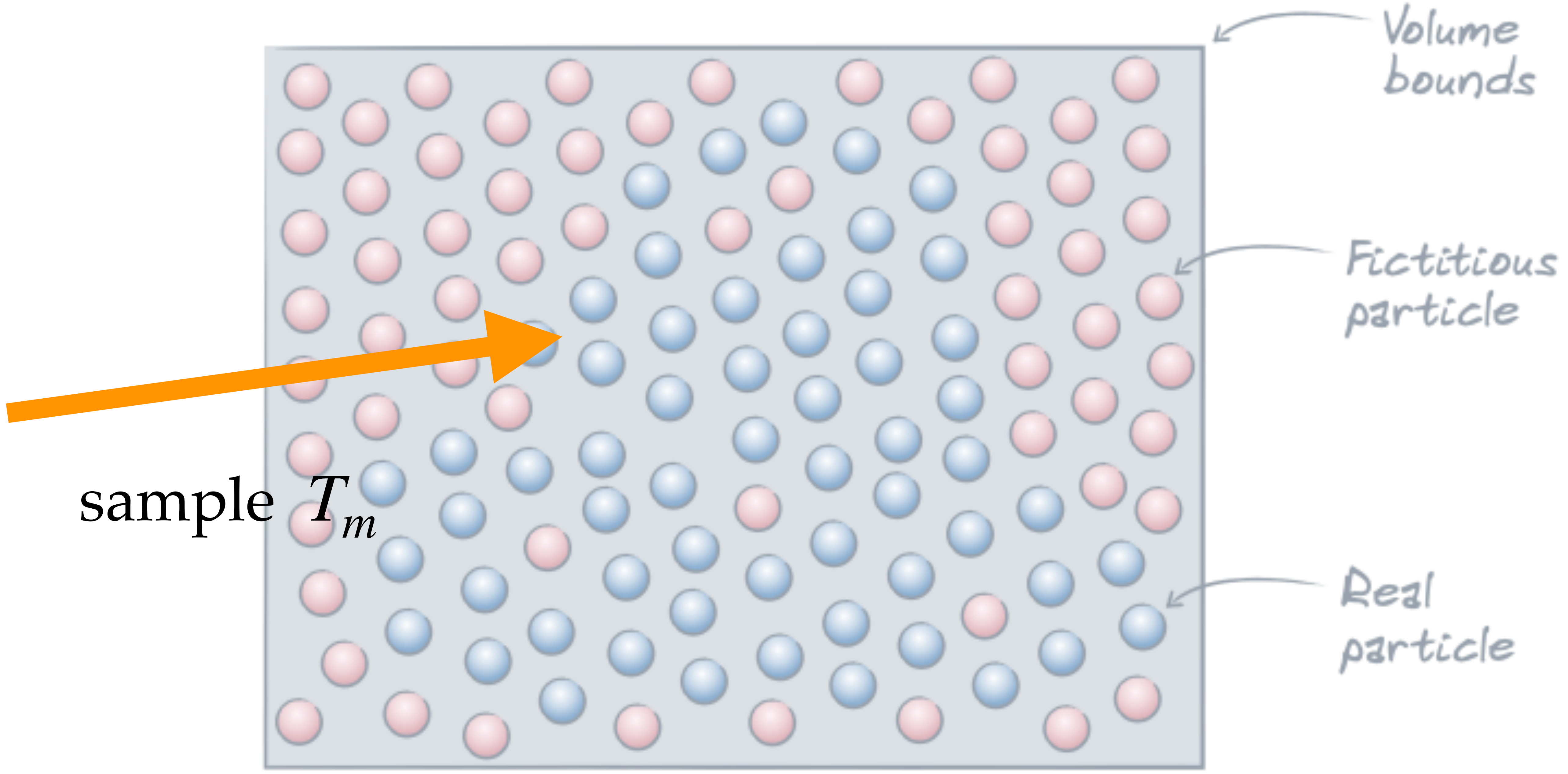


https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Tracing homogenized volumes ("delta tracking")

$$L(\mathbf{p}(0), \omega) = \int_0^t T_m(\mathbf{p}(0), \mathbf{p}(t')) \left(\sigma_n(\mathbf{p}) L(\mathbf{p}(t'), \omega) + L_{\text{gain}} \right) \mathrm{d}t'$$

$$T_m(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-t'\sigma_m\right)$$



https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Tracing homogenized volumes ("delta tracking")

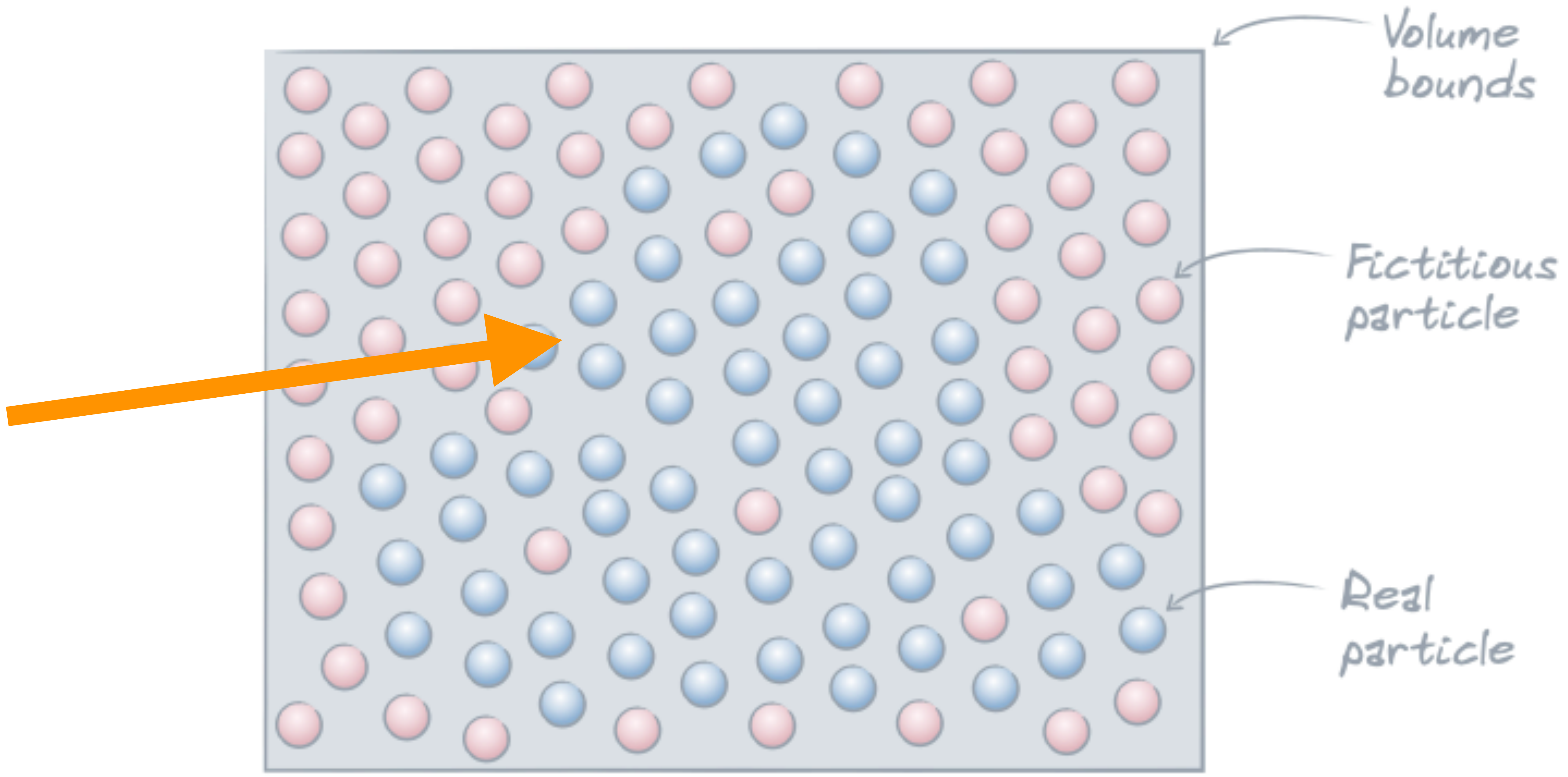
$$L(\mathbf{p}(0), \omega) = \int_0^t T_m(\mathbf{p}(0), \mathbf{p}(t'))\left(\boxed{\sigma_n(\mathbf{p})L(\mathbf{p}(t'), \omega)} + \boxed{L_{\text{gain}}}\right) \mathrm{d}t'$$

P_{null} $\quad P_{\text{gain}}$ $\quad$ sample "events"

$$T_m(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-t'\sigma_m\right)$$

in homework, we set

$$P_{\text{null}} = \frac{\sigma_n}{\sigma_m} \text{ and } P_{\text{gain}} = \frac{\sigma_t}{\sigma_m}$$



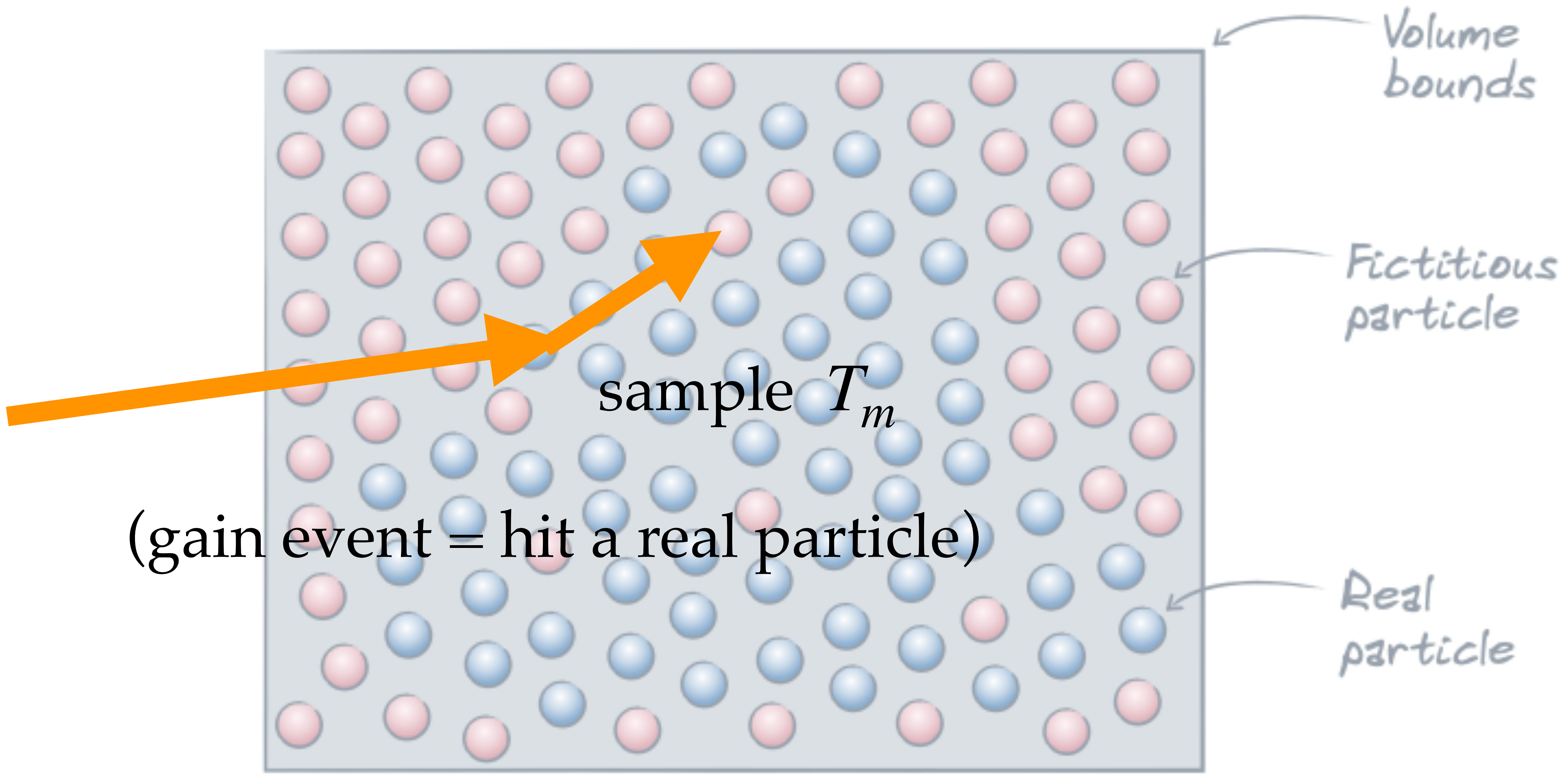
https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Tracing homogenized volumes (“delta tracking”)

$$L(\mathbf{p}(0), \omega) = \int_0^t T_m(\mathbf{p}(0), \mathbf{p}(t'))\left(\boxed{\sigma_n(\mathbf{p})L(\mathbf{p}(t'), \omega)} + \boxed{L_{\text{gain}}}\right) \mathrm{d}t'$$

P_{null} P_{gain}

$T_m(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-t'\sigma_m\right)$



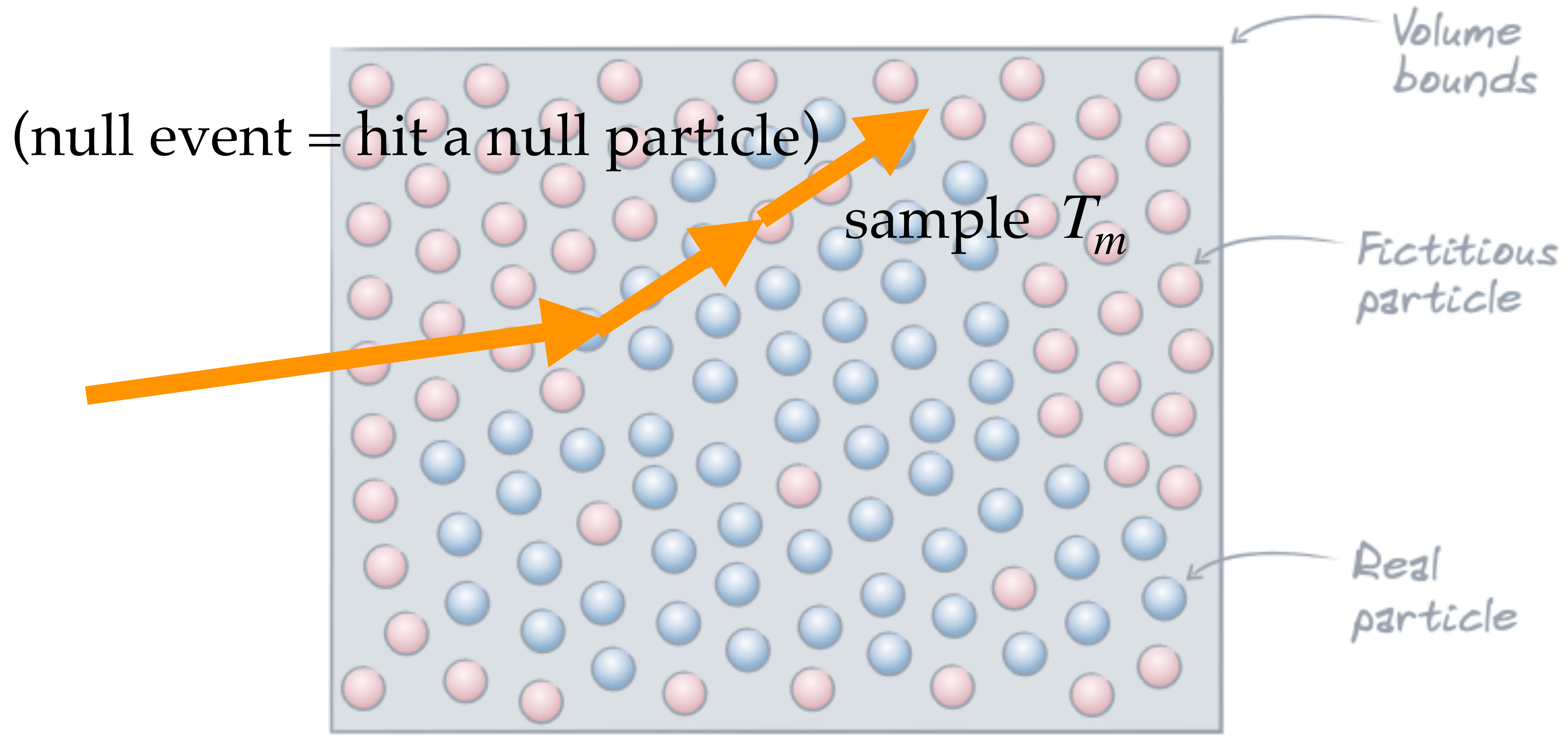
https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Tracing homogenized volumes ("delta tracking")

$$L(\mathbf{p}(0), \omega) = \int_0^t T_m(\mathbf{p}(0), \mathbf{p}(t')) \left(\boxed{\sigma_n(\mathbf{p}) L(\mathbf{p}(t'), \omega)} + \boxed{L_{\text{gain}}} \right) \mathrm{d}t'$$

P_{null} P_{gain}

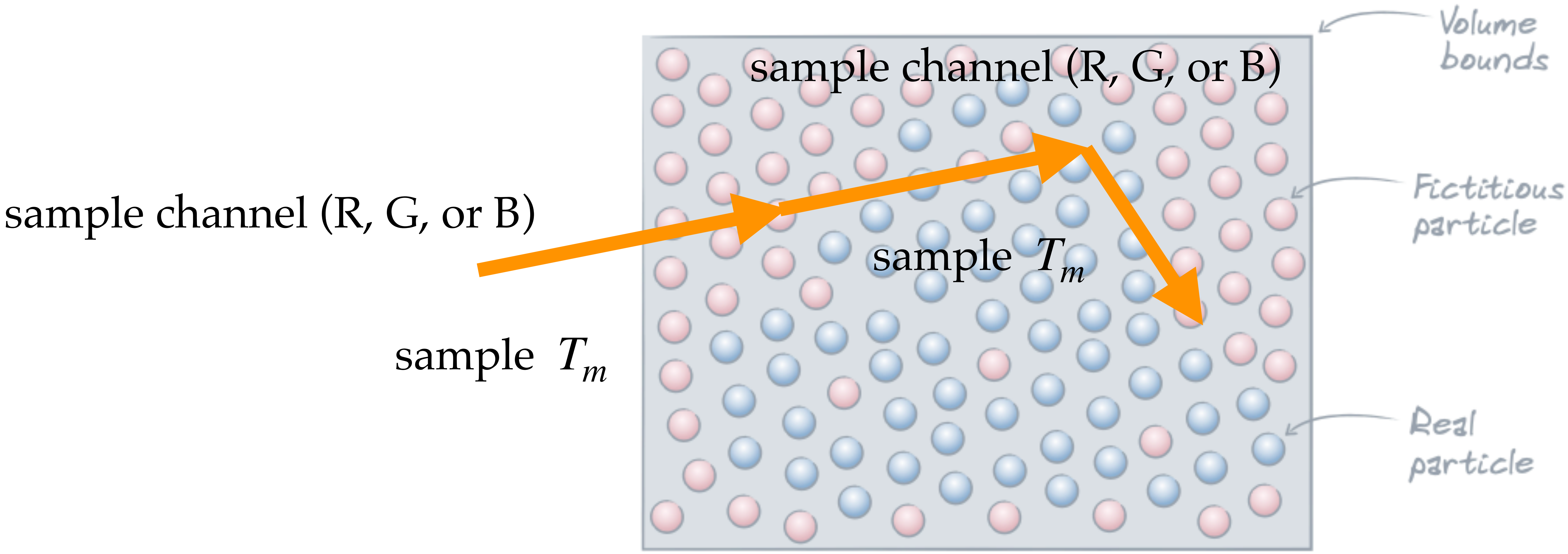
$T_m(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-t'\sigma_m\right)$



https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Handling colors

- randomly choose a channel for distance sampling at each scattering



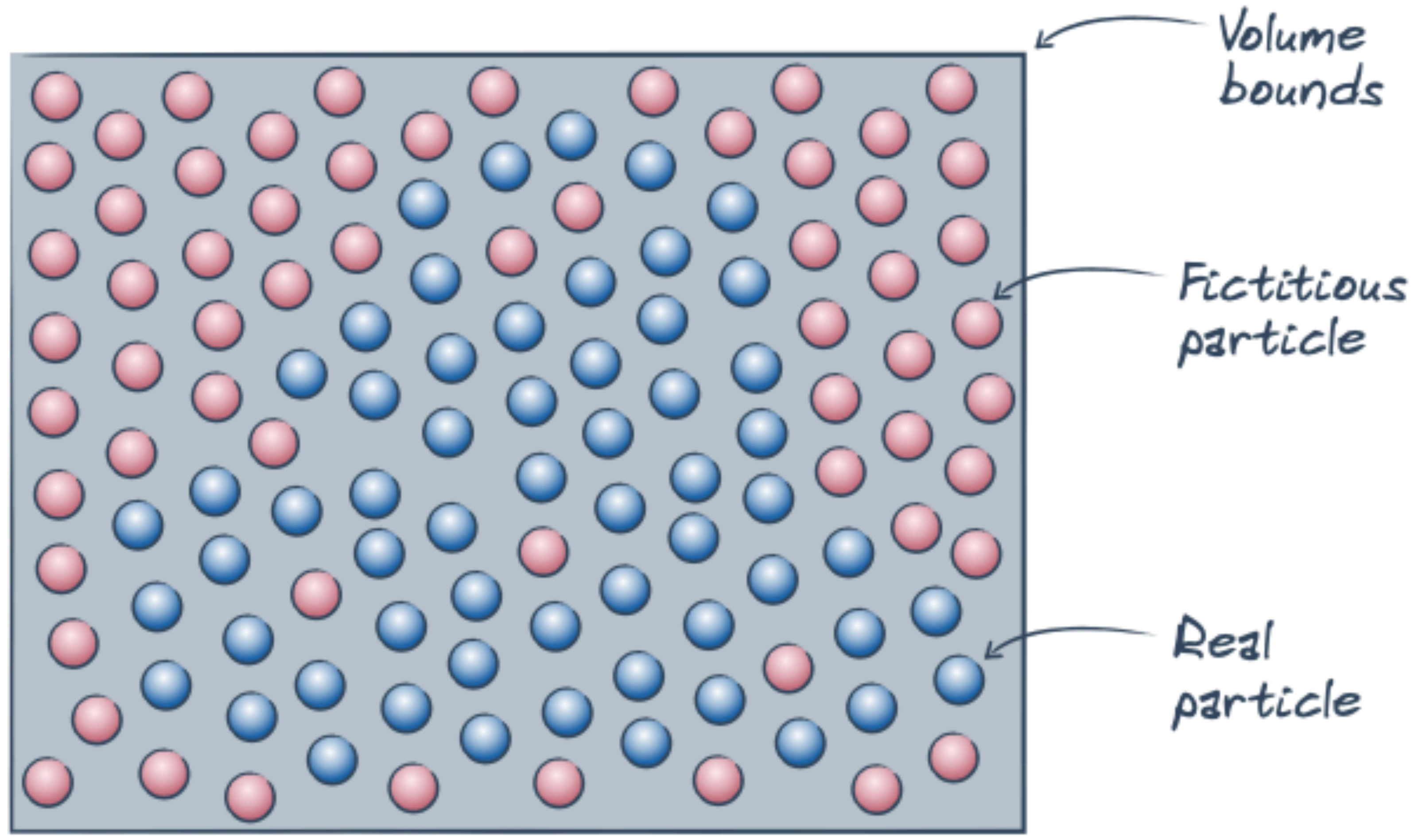
https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Recap: sampling T

- idea: homogenize the medium using the upper bound of $\sigma_t(\mathbf{p})$

$$\frac{\mathrm{d}}{\mathrm{d}t}L(\mathbf{p}(t),\omega) = -\,\sigma_t(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$

$$= -\,\sigma_m L(\mathbf{p}(t),\omega) + \sigma_n(\mathbf{p})L(\mathbf{p}(t),\omega) + L_{\text{gain}}$$

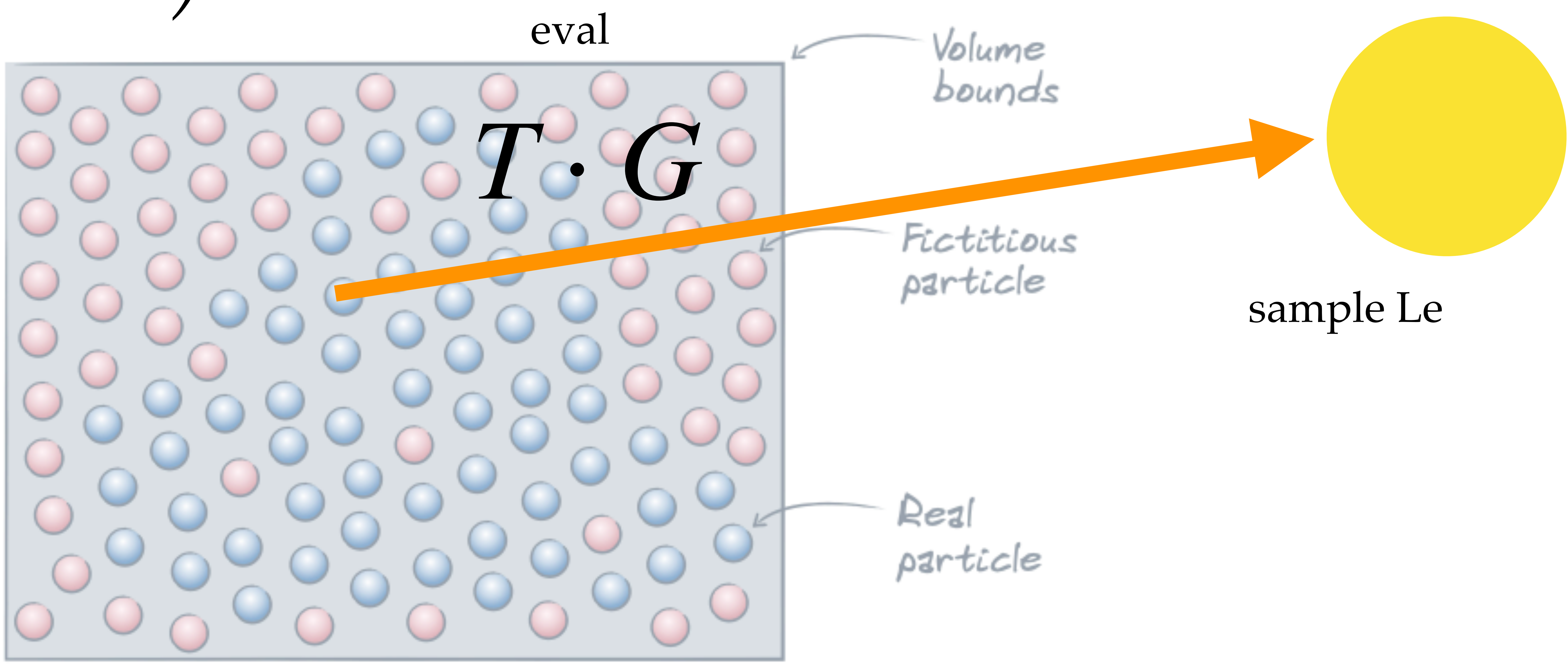


https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Next event estimation: need to evaluate T!

$$T(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right)$$

goal: unbiased estimation of T

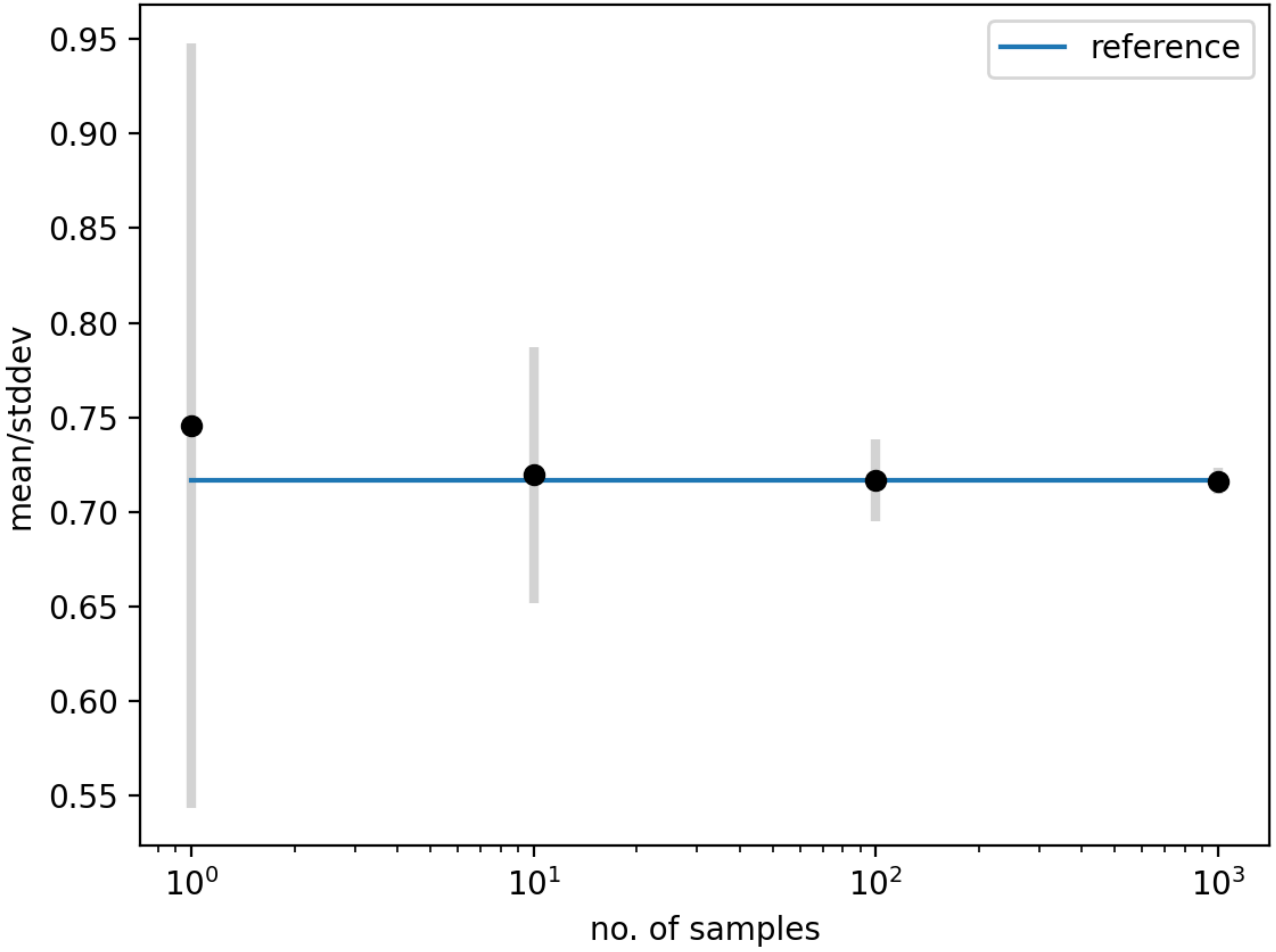


Challenge of computing an exponential integral

$$\int_0^{t'} \sigma_t(t'')\mathrm{d}t'' = E\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_t(t_i)}{p(t_i)}\right]$$

$$\exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right) \;??\; E\left[\exp\left(-\frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_t(t_i)}{p(t_i)}\right)\right]$$

Example: integrating $e^{\int_0^1 x^2 \mathrm{d}x}$



$$E[e^{\frac{1}{N}\sum_{i=1}^{N} x_i^2}] \neq e^{-\frac{1}{3}}$$

Idea: also apply the homogenization trick

- rewrite T as an ODE

$$T(\mathbf{p}(0), \mathbf{p}(t')) = T(t') = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right)$$

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_t(t')T(t')$$

$$T(0) = 1$$

Idea: also apply the homogenization trick

- rewrite T as an ODE

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_t(t')T(t')$$

$$T(0) = 1$$

Idea: also apply the homogenization trick

- add null particles

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_t(t')T(t') = -\sigma_m T(t') + \sigma_n(t')T(t')$$

$$T(0) = 1$$

choose σ_n such that $\sigma_t(\mathbf{p}) + \sigma_n(\mathbf{p}) = \text{const} = \sigma_m$

Idea: also apply the homogenization trick

- add null particles

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_m T(t') + \sigma_n(t')T(t')$$

$$T(0) = 1$$

Idea: also apply the homogenization trick

- integrate the ODE

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_m T(t') + \sigma_n(t')T(t')$$

$$T(0) = 1$$

$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'')T(t'')\mathrm{d}t''$$

Idea: also apply the homogenization trick

$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'')T(t'')\mathrm{d}t''$$

Observation: homogenized transmittance looks like a 1D rendering equation!

$$T(t') = \boxed{\exp\left(-\sigma_m t'\right)} + \int_0^{t'} \boxed{\exp\left(\sigma_m(t'' - t')\right)\sigma_n(t'')} T(t'')\mathrm{d}t''$$

"emission" "BRDF"

Integral formulations of volumetric transmittance

ILIYAN GEORGIEV*, Autodesk, United Kingdom
ZACKARY MISSO*, Dartmouth College, USA
TOSHIYA HACHISUKA, The University of Tokyo, Japan
DEREK NOWROUZEZAHRAI, McGill University, Canada
JAROSLAV KŘIVÁNEK, Charles University and Chaos Czech a. s., Czech Republic
WOJCIECH JAROSZ, Dartmouth College, USA

Ratio tracking

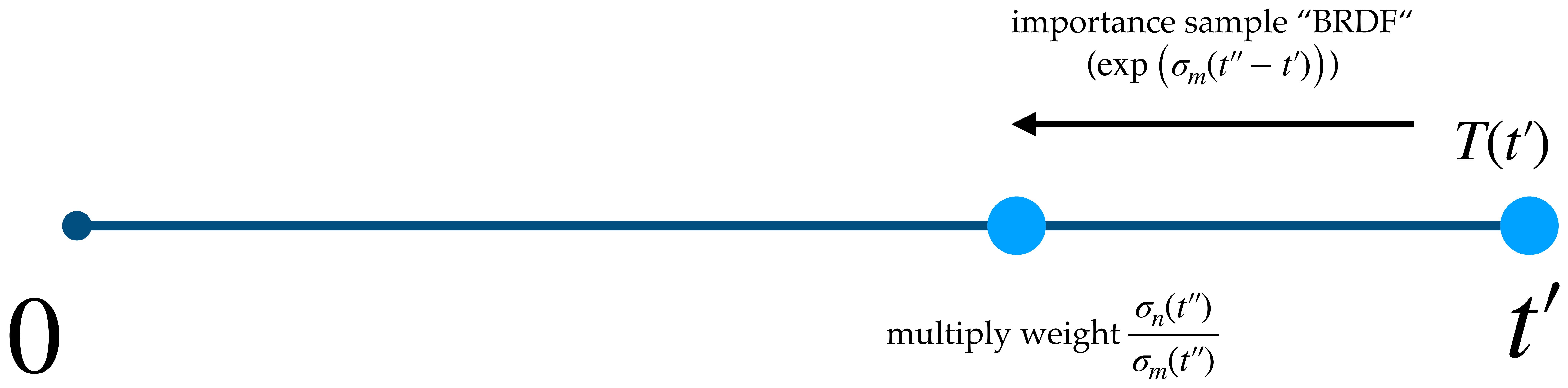
1. start from $z = t'$
2. sample t'', update current position $z = t' - t''$
3. if z reaches 0, evaluate the “emission”, otherwise evaluate the recursive term

$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m (t'' - t')\right) \sigma_n(t'') T(t'') \mathrm{d}t''$$

Ratio tracking

1. start from $z = t'$
2. sample t'', update current position $z = t' - t''$
3. if z reaches 0, evaluate the "emission", otherwise evaluate the recursive term

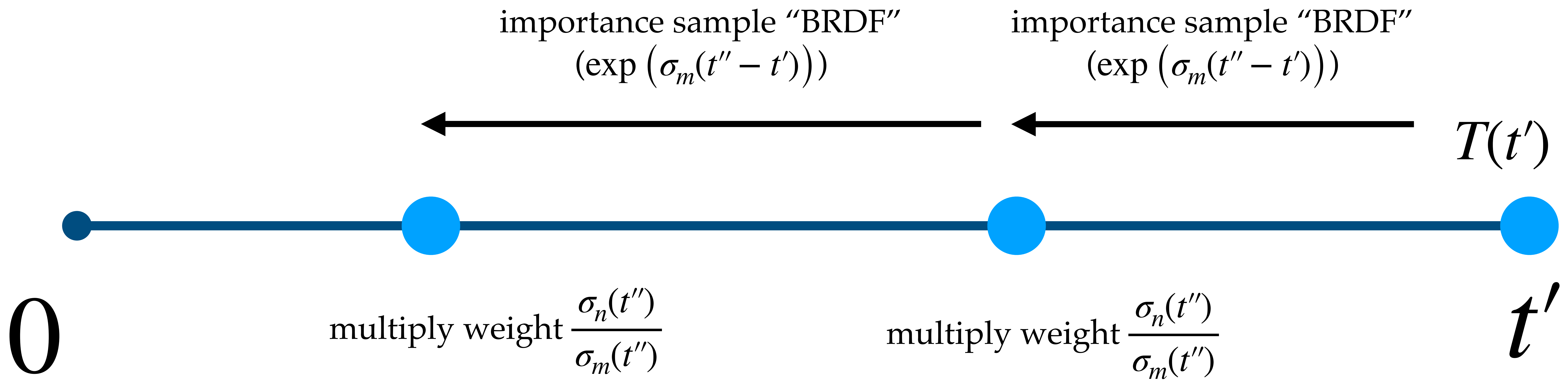
$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'') T(t'') \mathrm{d}t''$$



Ratio tracking

1. start from $z = t'$
2. sample t'', update current position $z = t' - t''$
3. if z reaches 0, evaluate the “emission”, otherwise evaluate the recursive term

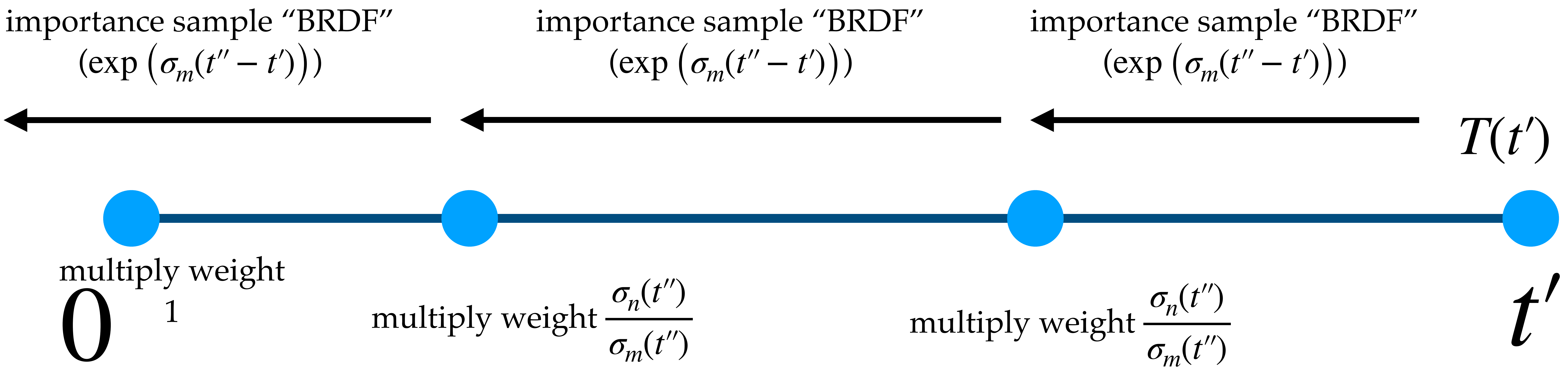
$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m (t'' - t')\right) \sigma_n(t'') T(t'') \mathrm{d}t''$$



Ratio tracking

1. start from $z = t'$
2. sample t'', update current position $z = t' - t''$
3. if z reaches 0, evaluate the "emission", otherwise evaluate the recursive term

$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'') T(t'') \mathrm{d}t''$$

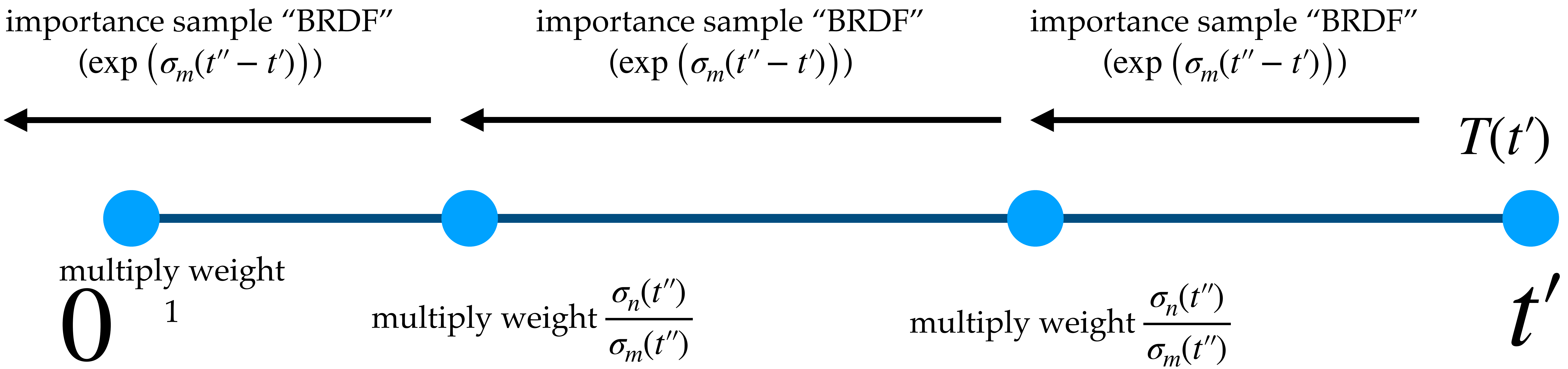


Ratio tracking

1. start from $z = t'$
2. sample t'', update current position $z = t' - t''$
3. if z reaches 0, evaluate the "emission", otherwise evaluate the recursive term

$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'')T(t'')\mathrm{d}t''$$

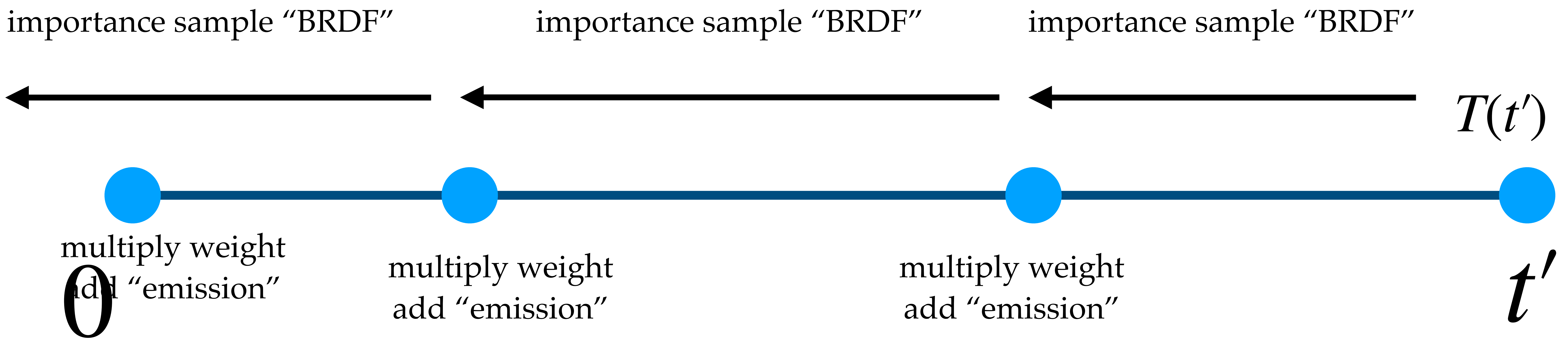
quiz: can you think of a different way to estimate T?



Next-flight estimator

1. start from $z = t'$
2. sample t'', update current position $z = t' - t''$
3. add the "emission" and evaluate the recursive term until z reaches 0

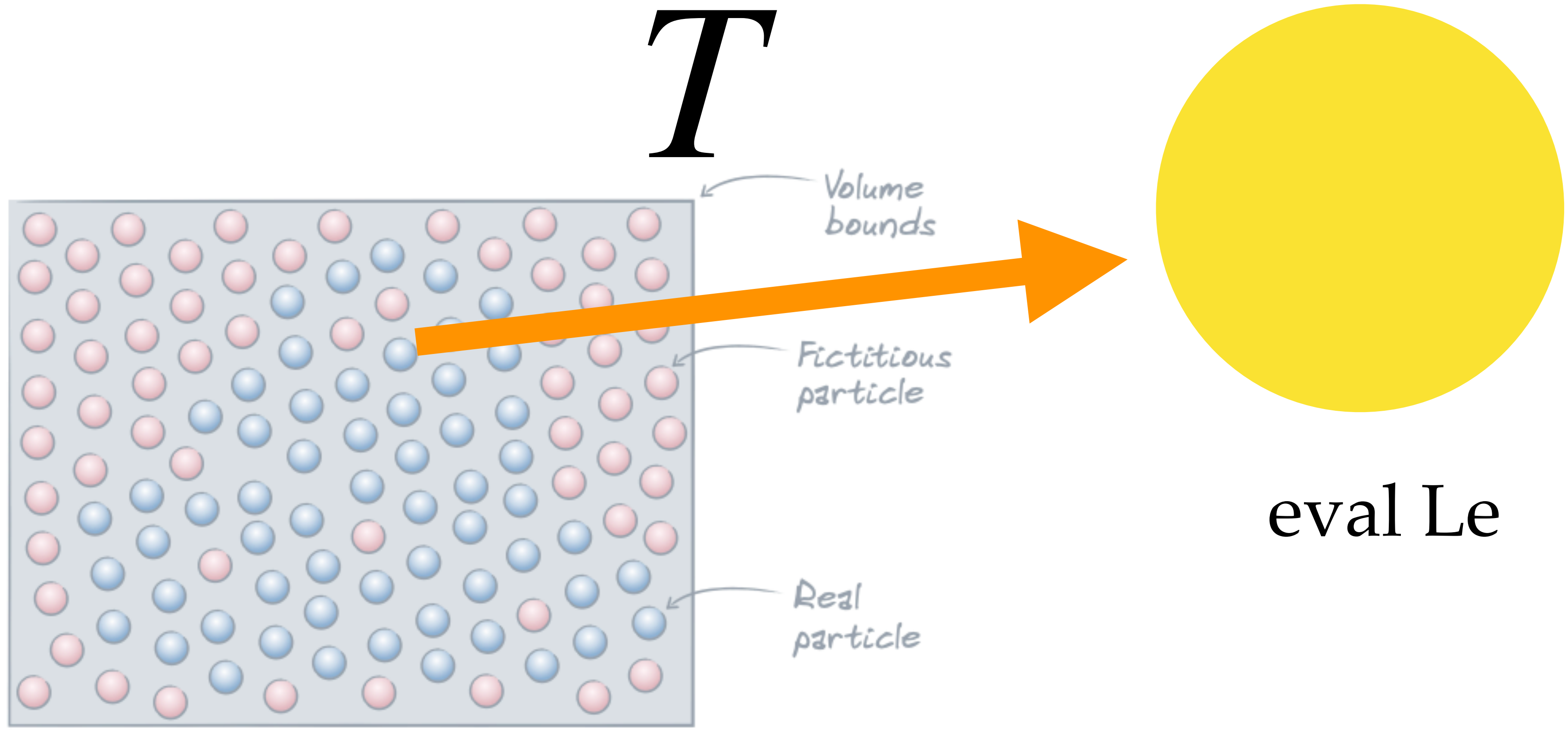
$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'')T(t'')\mathrm{d}t''$$



Multiple importance sampling of delta tracking & ratio tracking

sample

T

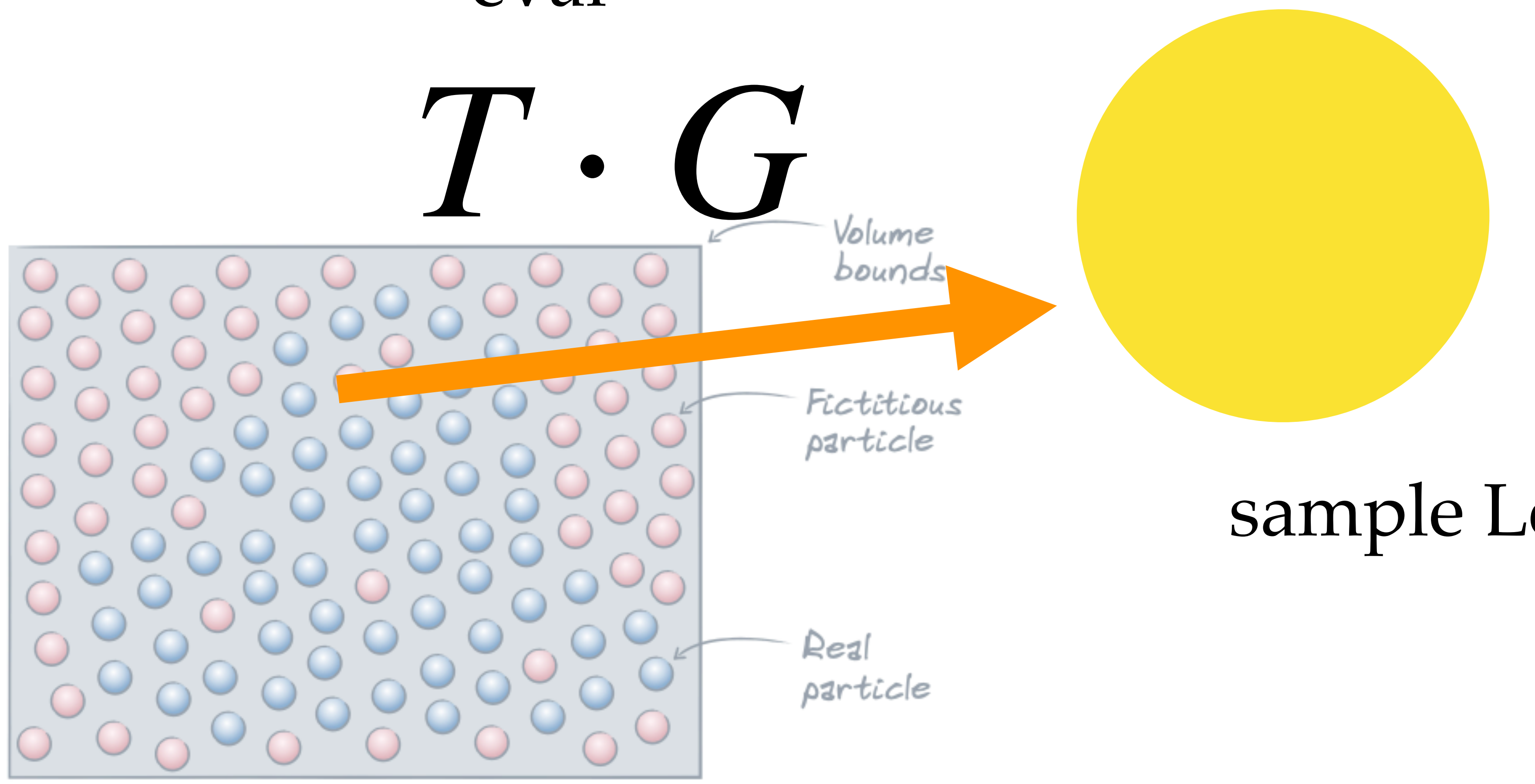


delta tracking

people didn't figure out how to do this until 2019!!

eval

$T \cdot G$



ratio tracking

A null-scattering path integral formulation of light transport

BAILEY MILLER*, Dartmouth College, USA
ILIYAN GEORGIEV*, Autodesk, United Kingdom
WOJCIECH JAROSZ, Dartmouth College, USA

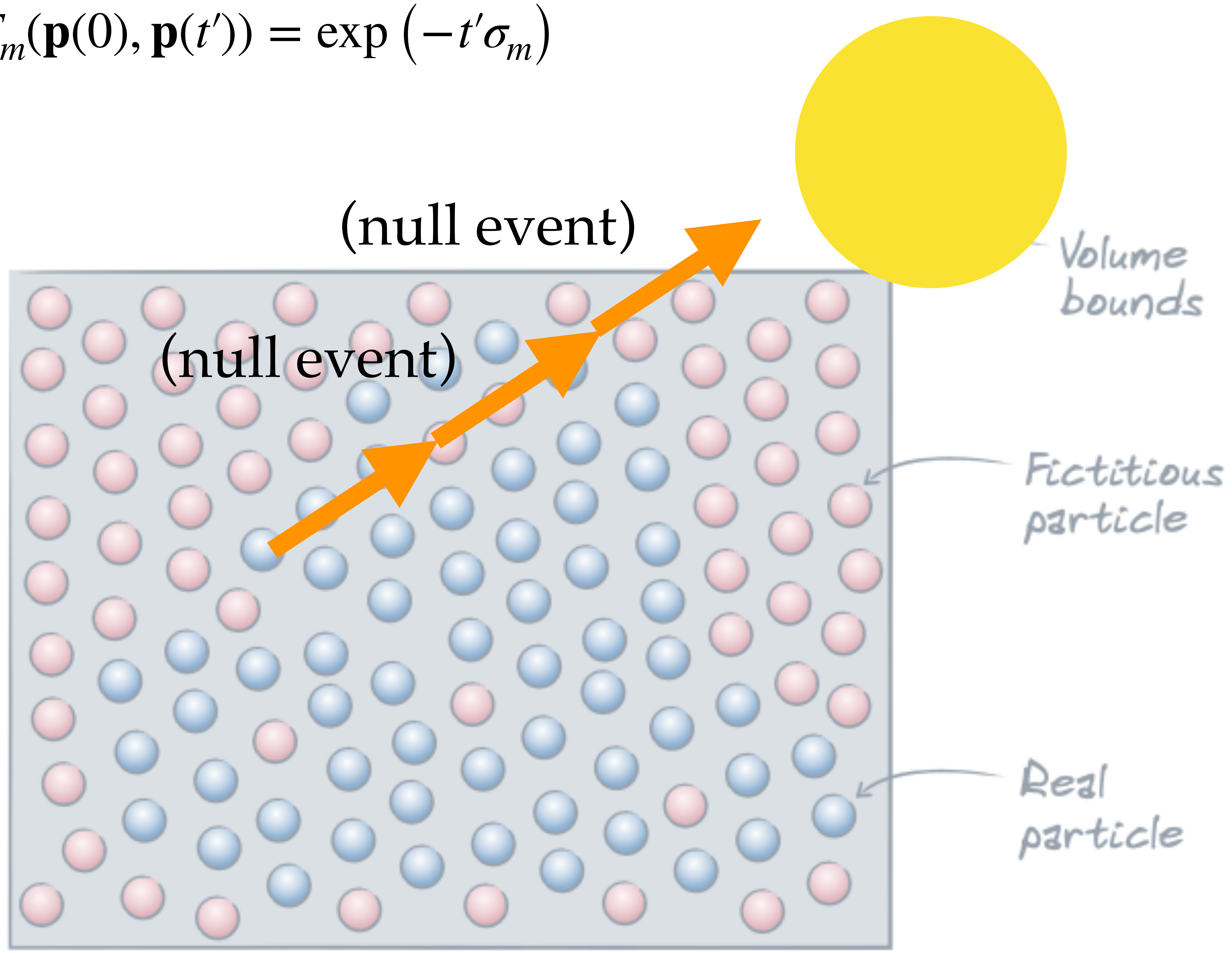
https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

Idea: keep track of event probabilities

$$L(\mathbf{p}(0), \omega) = \int_0^t T_m(\mathbf{p}(0), \mathbf{p}(t')) \left(\boxed{\sigma_n(\mathbf{p}) L(\mathbf{p}(t'), \omega)} + \boxed{L_{\text{gain}}} \right) \mathrm{d}t'$$

P_{null} P_{gain}

$$T_m(\mathbf{p}(0), \mathbf{p}(t')) = \exp\left(-t'\sigma_m\right)$$



$$p_{\text{delta}} = \exp(-t_1\sigma_m) P_{\text{null}} \exp(-t_2\sigma_m) P_{\text{null}} \exp(-t_3\sigma_m) \cdots$$

$$p_{\text{ratio}} = \exp(-t_1\sigma_m) \exp(-t_2\sigma_m) \exp(-t_3\sigma_m) \cdots$$

https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig-slides-3-distance-sampling-notes.pdf

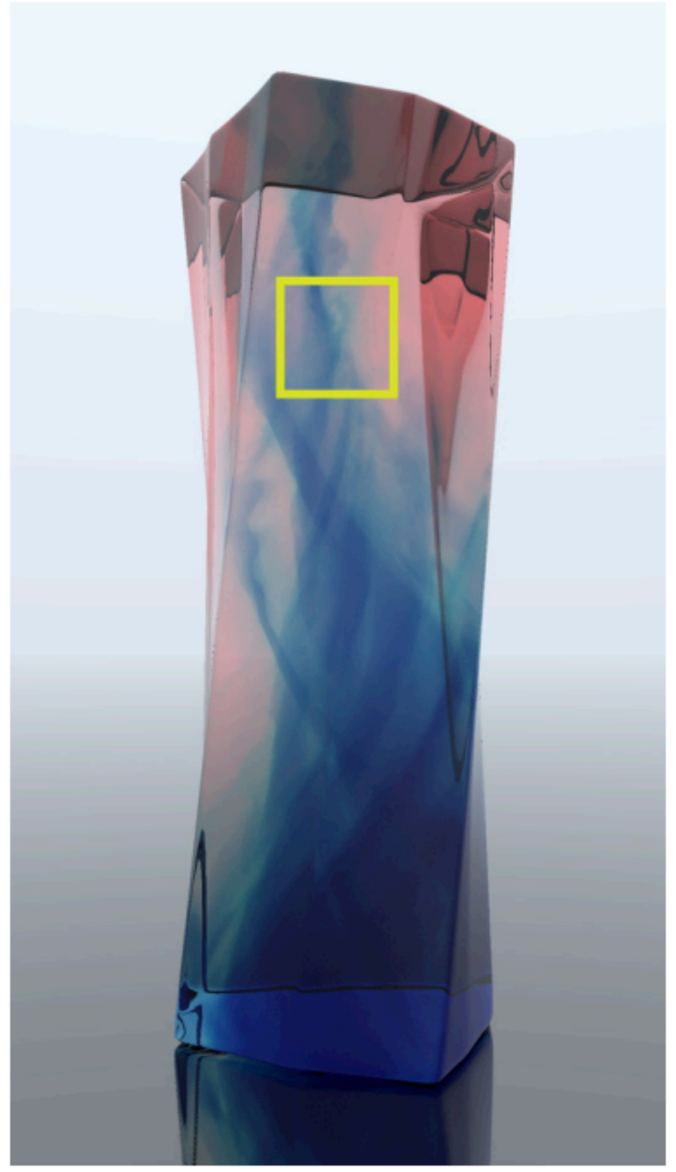
Advanced topic: other ways to look at transmittance

Integral formulations of volumetric transmittance

ILIYAN GEORGIEV*, Autodesk, United Kingdom
ZACKARY MISSO*, Dartmouth College, USA
TOSHIYA HACHISUKA, The University of Tokyo, Japan
DEREK NOWROUZEZAHRAI, McGill University, Canada
JAROSLAV KŘIVÁNEK, Charles University and Chaos Czech a. s., Czech Republic
WOJCIECH JAROSZ, Dartmouth College, USA

An unbiased ray-marching transmittance estimator

MARKUS KETTUNEN, NVIDIA
EUGENE D'EON, NVIDIA
JACOPO PANTALEONI, NVIDIA
JAN NOVÁK, NVIDIA



Back to the ODE view

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_t(t')T(t')$$

$$T(0) = 1$$

$$T(\mathbf{p}(0), \mathbf{p}(t')) = T(t') = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right)$$

Integrate both sides (without exponentiating)

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_t(t')T(t')$$

$$T(0) = 1$$

$$T(t') = 1 - \int_0^{t'} \sigma_t(t'')T(t'')\mathrm{d}t''$$

Integrate both sides (without exponentiating)

$$\frac{\mathrm{d}T(t')}{\mathrm{d}t'} = -\sigma_t(t')T(t')$$

$$T(0) = 1$$

$$T(t') = 1 - \int_0^{t'} \sigma_t(t'')T(t'')\mathrm{d}t''$$

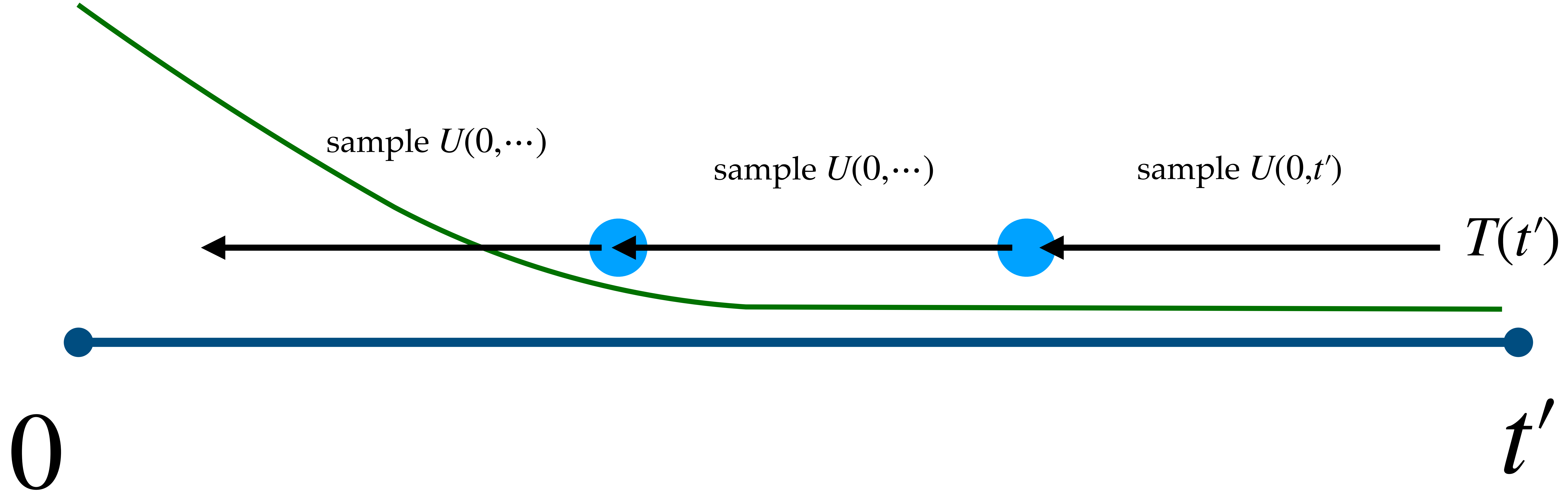
"Volterra integral equation"

why didn't we do this???

Volterra integral equation is inefficient

- doesn’t consider the exponential falloff

$$T(t') = 1 - \int_0^{t'} \sigma_t(t'')T(t'')\mathrm{d}t''$$



Homogenization is a **control variate**

- analytically solve for σ_m

$$T(t') = 1 - \int_0^{t'} \sigma_t(t'')T(t'')\mathrm{d}t''$$

$$\sigma_t(t') = \left[\sigma_t(t') + \sigma_m\right] - \sigma_m = \sigma_n(t) - \sigma_m$$

$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'')T(t'')\mathrm{d}t''$$

https://en.wikipedia.org/wiki/Control_variates

The recursive transmittance integral can be evaluated from both sides

- similar to “bidirectional path tracing” (more in the future), can combine using MIS

$$T(t') = \exp\left(-\sigma_m t'\right) + \int_0^{t'} \exp\left(\sigma_m(t'' - t')\right) \sigma_n(t'')T(t'')\mathrm{d}t''$$

Let's look at the exponential integral instead

$$T(t') = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t'' \right) = \exp\left(-\tau\right)$$

Take Taylor expansion

$$T(t') = \exp\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right) = \exp(-\tau)$$

$$= 1 - \frac{\tau}{1!} + \frac{\tau^2}{2!} \cdots$$

Can estimate this Taylor expansion using Monte Carlo!

$$T(t') = 1 - \frac{\tau}{1!} + \frac{\tau^2}{2!} \cdots$$

$$\tau = \int_0^{t'} \sigma_t(t'')\mathrm{d}t''$$

Can estimate this Taylor expansion using Monte Carlo!

$$T(t') = 1 - \frac{\tau}{1!} + \frac{\tau^2}{2!} \cdots$$

Let $\tau_1 = \sigma_t(t_1)$, $\tau_2 = \sigma_t(t_2)$, …

$t_i \sim U(0, t')$

$$T(t') \approx \frac{1}{P(k)} \left(1 - \frac{\tau_1}{1!} + \frac{\tau_1 \tau_2}{2!} - \cdots + \frac{\tau_1 \ldots \tau_k}{k!} \right)$$

$$\tau = \int_0^{t'} \sigma_t(t'') \mathrm{d}t''$$

Key idea from Kettunen et al. [2021]: use combined statistics to reduce variance

An unbiased ray-marching transmittance estimator

MARKUS KETTUNEN, NVIDIA
EUGENE D'EON, NVIDIA
JACOPO PANTALEONI, NVIDIA
JAN NOVÁK, NVIDIA

super efficient algorithm (Girad-Newton formula)
to compute the combined statistics

ALGORITHM 1: ElementaryMeans

Input : Samples $x_1, \cdots, x_N$; Evaluation order Z
Output: Elementary symmetric means $m_0, \ldots, m_Z$

$m_0 = 1$;
$m_k = 0$ (for $k = 1$ to Z) ;
for $n = 1$ *to* N **do**
  for $k = \min(n, Z)$ *to* 1 **do**
    $m_k = m_k + \frac{k}{n}\,(m_{k-1}x_n - m_k)$;
  end
end

$$T(t') = 1 - \frac{\tau}{1!} + \frac{\tau^2}{2!} \cdots$$

Let $\tau_1 = \sigma_t(t_1)$, $\tau_2 = \sigma_t(t_2)$, …

$t_i \sim U(0, t')$

$$T(t') \approx \frac{1}{P(k)} \left(1 - \frac{\tau_1}{1!} + \frac{\tau_1 \tau_2}{2!} - \cdots + \frac{\tau_1 \ldots \tau_k}{k!} \right)$$

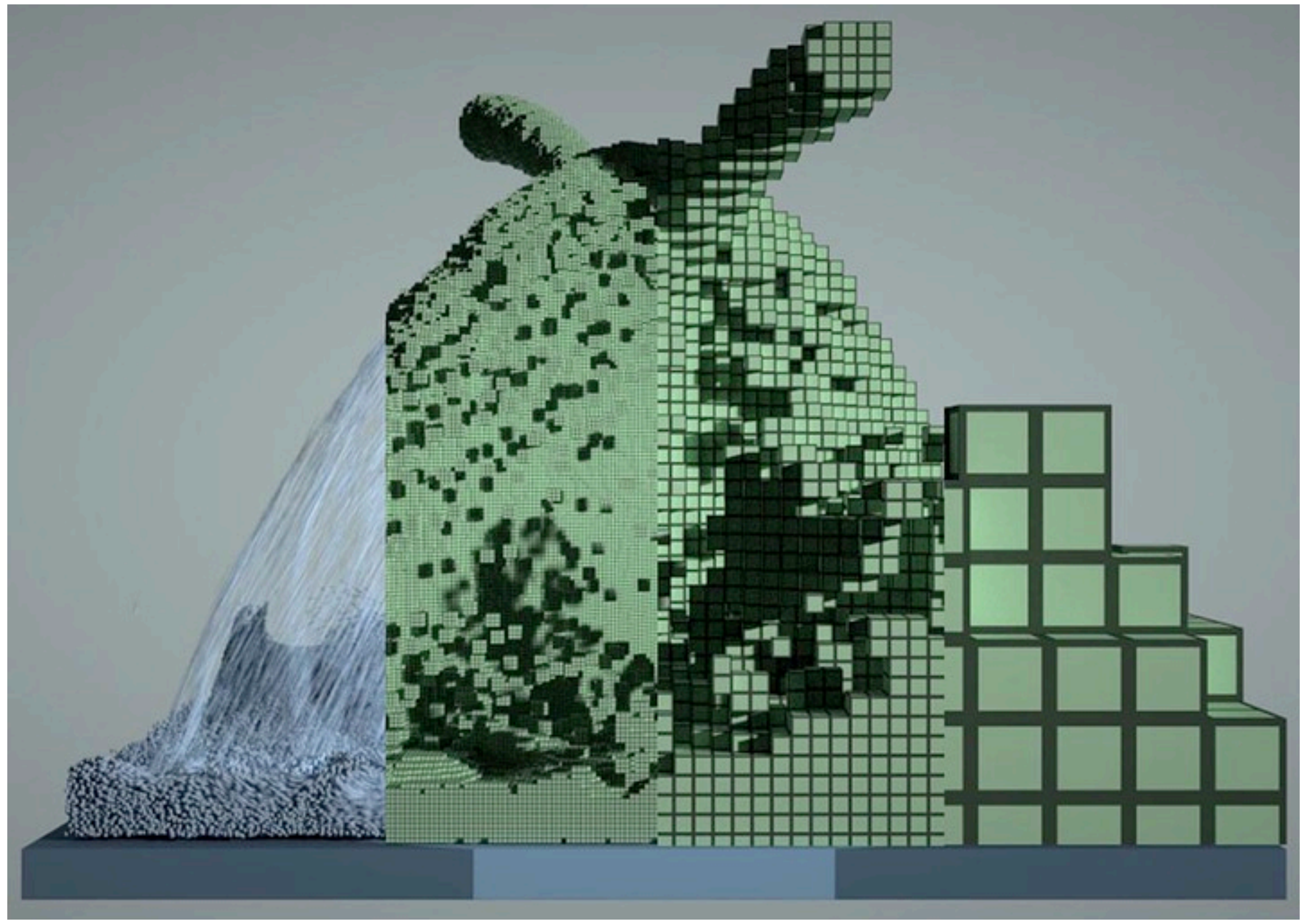
$$T(t') \approx \frac{1}{P(k)} \left(1 - \frac{\frac{1}{C_1^k}\left(\tau_1 + \tau_2 \ldots + \tau_k\right)}{1!} + \frac{\frac{1}{C_2^k}\left(\tau_1\tau_2 + \tau_1\tau_3 + \cdots \tau_{k-1}\tau_k\right)}{2!} - \cdots + \frac{\tau_1 \ldots \tau_k}{k!} \right)$$

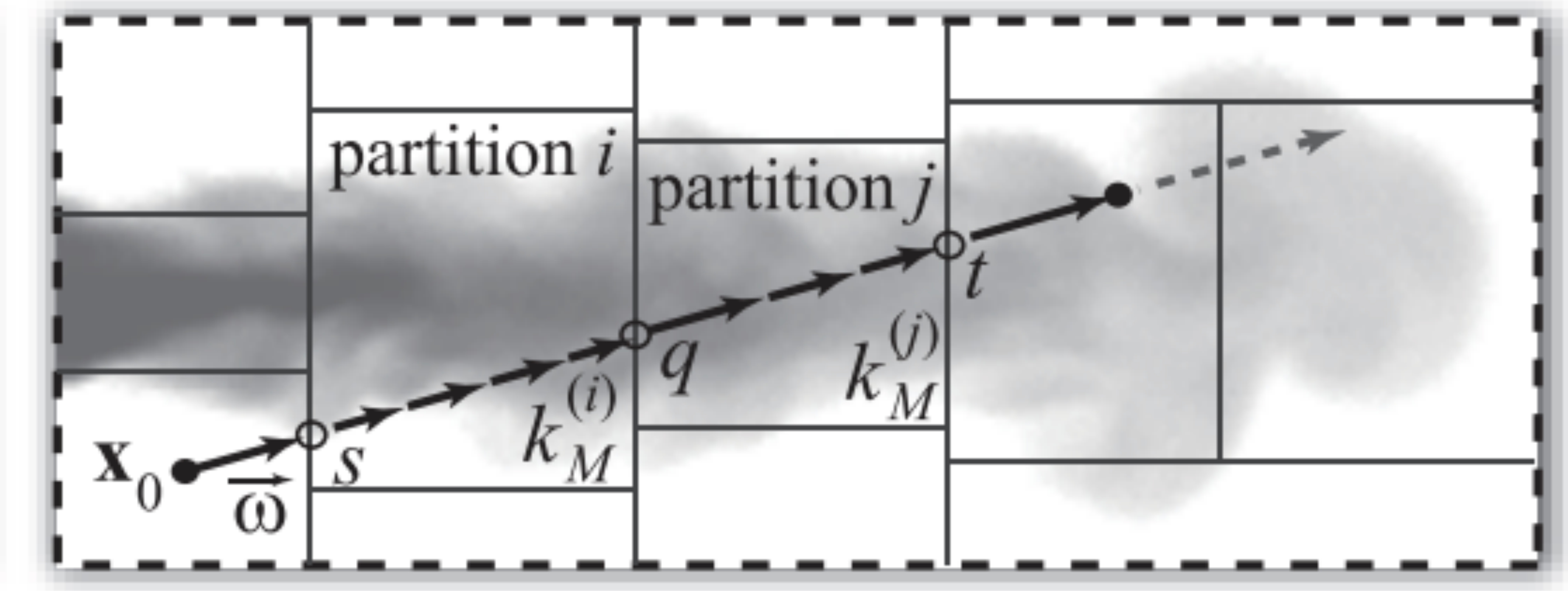
$$\tau = \int_0^{t'} \sigma_t(t'')\mathrm{d}t''$$

Bells and whistles: tighter upper bound σ_m using hierarchical data structures

Unbiased, Adaptive Stochastic Sampling for Rendering Inhomogeneous Participating Media

Yonghao Yue[1] Kei Iwasaki[2] Bing-Yu Chen[3] Yoshinori Dobashi[4] Tomoyuki Nishita[1]
[1]The University of Tokyo [2]Wakayama University [3]National Taiwan University [4]Hokkaido University

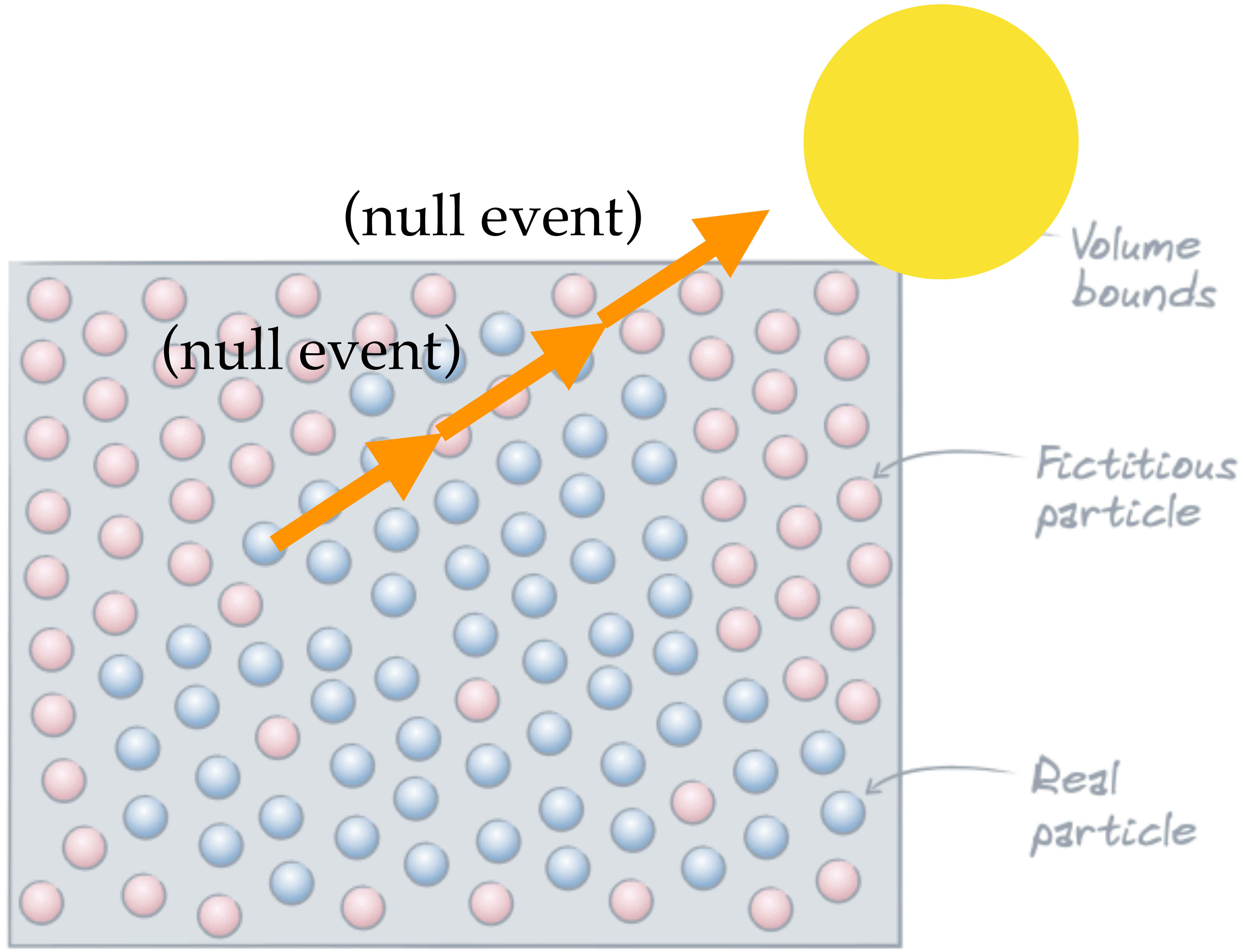




Open problem: connection between the Volterra integral & Taylor expansion w/ null-scattering formulation

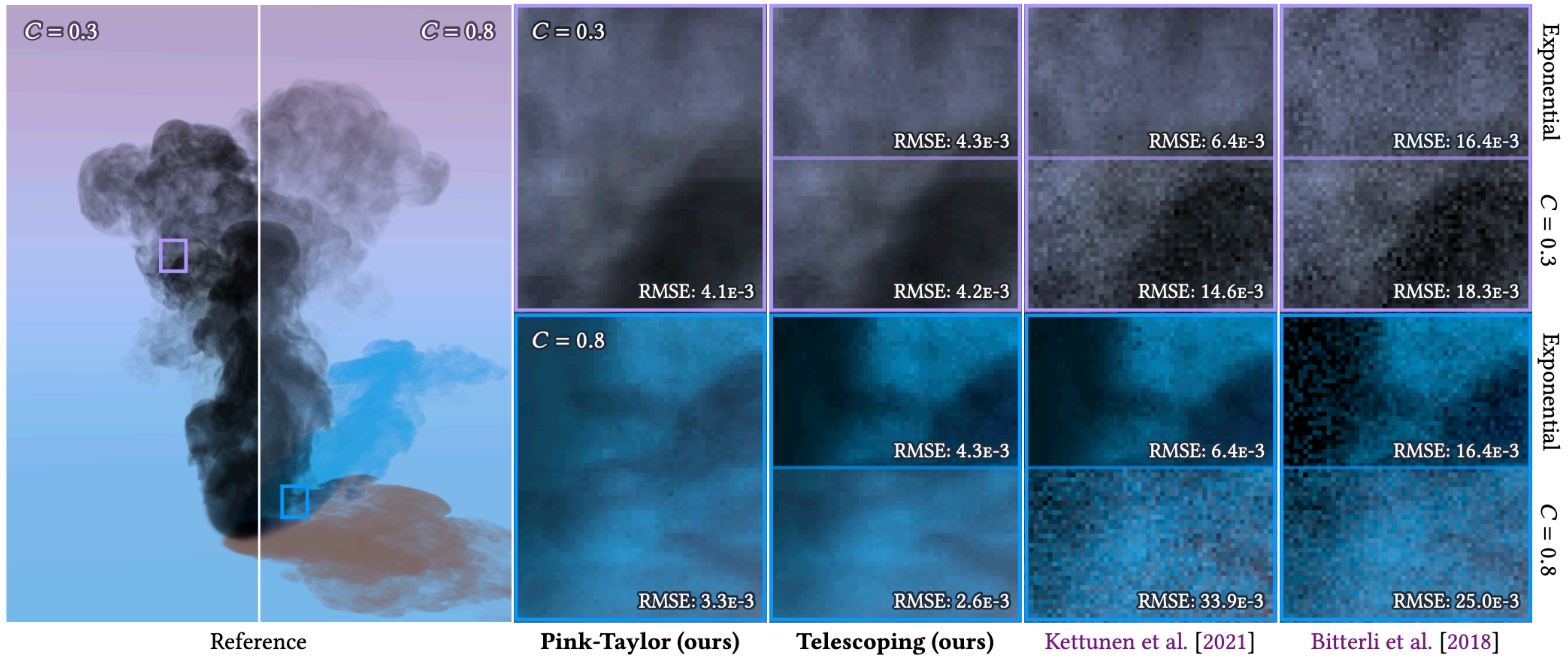
$$T(t') = 1 - \frac{\tau}{1!} + \frac{\tau^2}{2!} \cdots = \left(1 - \frac{\tau}{1}\left(1 - \frac{\tau}{2}(\ldots)\right)\right)$$

$$T(t') = 1 - \int_0^{t'} \sigma_t(t'')T(t'')\mathrm{d}t''$$



Fun research: unbiased estimators for non-exponential transmittance

- trick: use "Russian-roulette debiasing" to convert a consistent estimator to an unbiased one



Unbiased and consistent rendering using biased estimators

ZACKARY MISSO, Dartmouth College, USA
BENEDIKT BITTERLI, Dartmouth College, USA and NVIDIA, USA
ILIYAN GEORGIEV, Autodesk, United Kingdom
WOJCIECH JAROSZ, Dartmouth College, USA

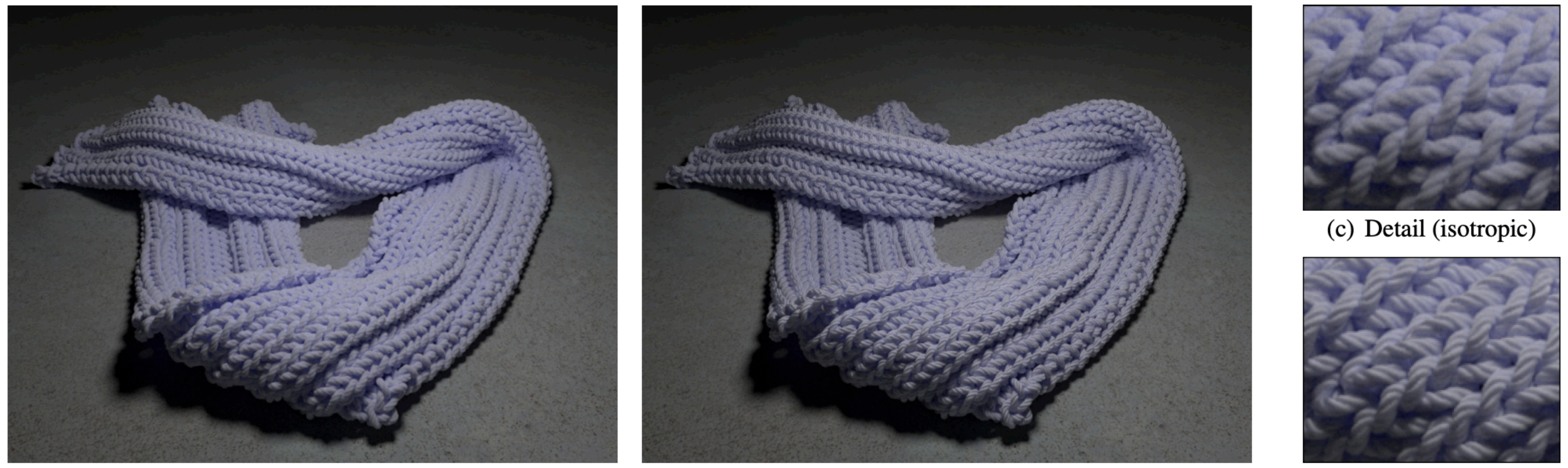
$$T(t') = f\left(-\int_0^{t'} \sigma_t(t'')\mathrm{d}t''\right)$$

Next: microflake theory

- how to design complex phase functions?

A radiative transfer framework for rendering materials with anisotropic structure

Wenzel Jakob Adam Arbree Jonathan T. Moon Kavita Bala Steve Marschner
Cornell University



(a) Isotropic scattering (b) Scattering by anisotropic micro-flakes (c) Detail (isotropic) (d) Detail (micro-flakes)

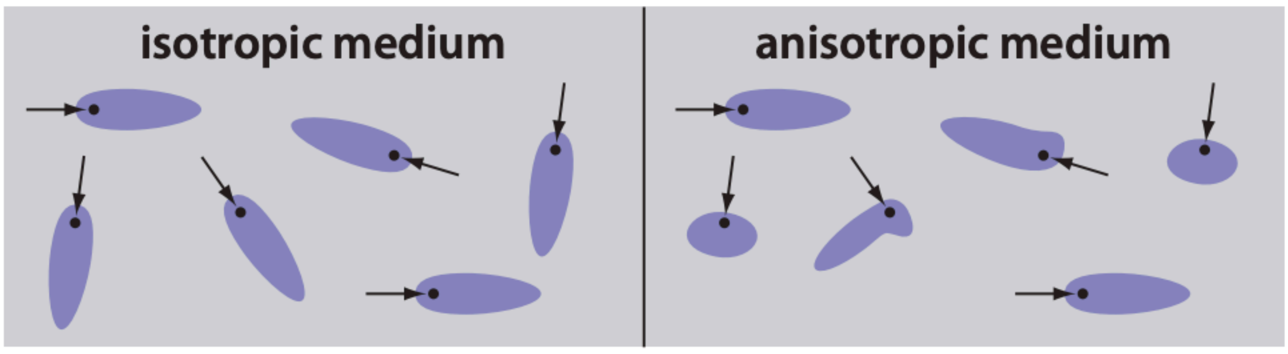
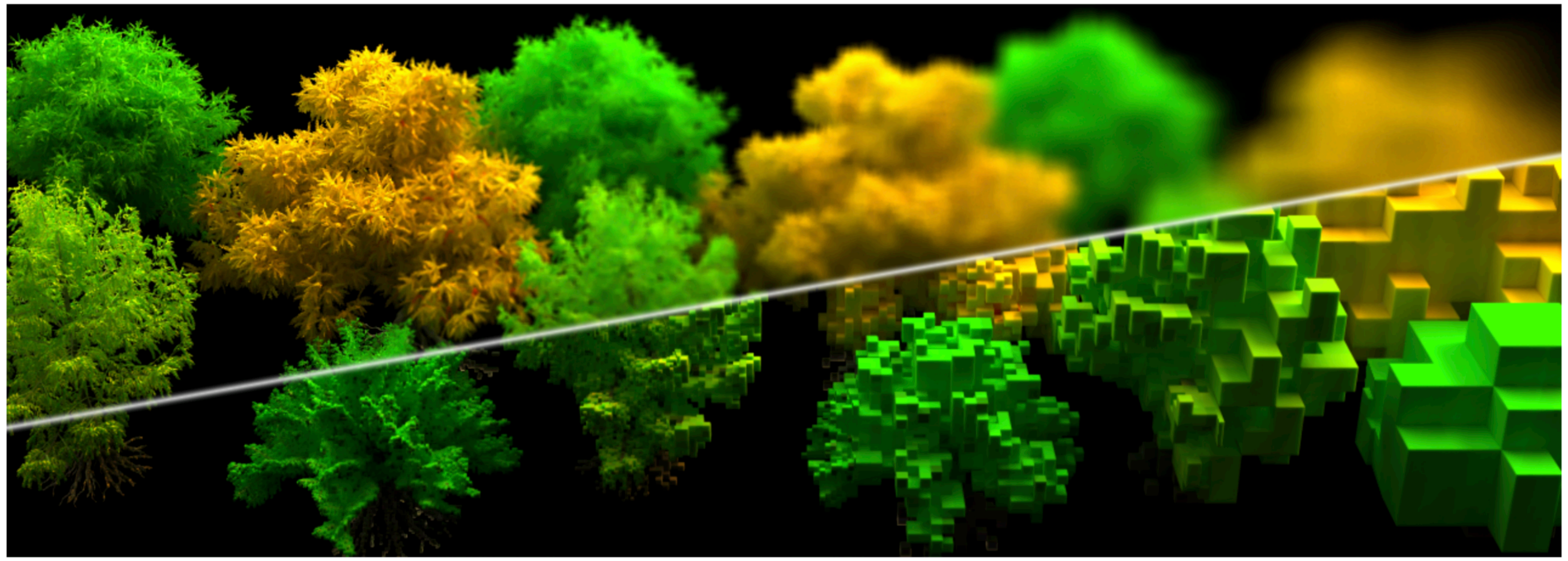


Figure 2: *The distinction between isotropic and anisotropic media.*

The SGGX Microflake Distribution

Eric Heitz[1,2] Jonathan Dupuy[3] Cyril Crassin[2] Carsten Dachsbacher[1]
[1]Karlsruhe Institute of Technology [2]NVIDIA [3]Univ. Montréal; LIRIS, Univ. Lyon 1



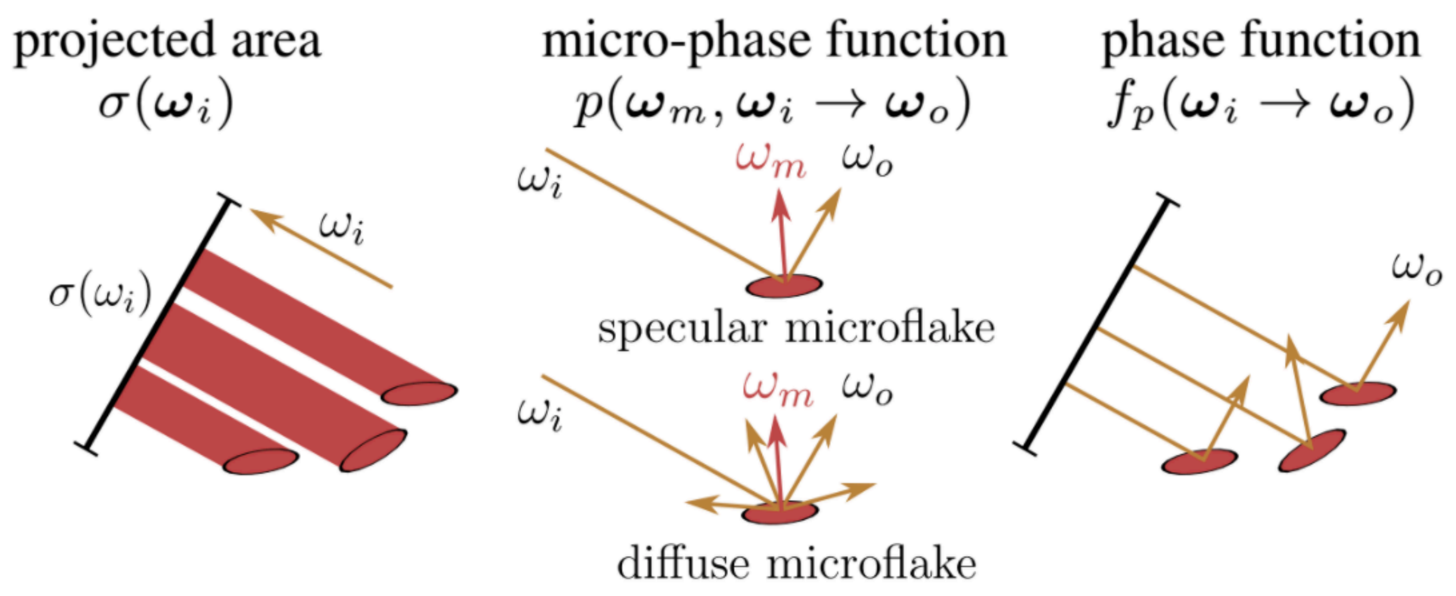


Figure 2: *Illustration of the notation used in microflake theory.*