Participating media

UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li

organization of the slides heavily borrowed from the SIGGRAPH course “Monte Carlo methods for physically-based volume rendering”
https://cs.dartmouth.edu/~wjarosz/publications/novak18monte-sig.html
HW2 is out

- START EARLY
- ASK QUESTIONS
Today: foggy and transparent stuff

https://en.wikipedia.org/wiki/Sunbeam
"... in 10 years, all rendering will be volume rendering."

Jim Kajiya at SIGGRAPH '91

A Survey of Algorithms for Volume Visualization

T. Todd Elvins
San Diego Supercomputer Center
Foundation of modern rendering physics: radiative transfer [Chandrasekhar 1960]

- what happens when light hits particles in the space?
Infinitely many particles:
use ordinary differential equation to describe light’s behavior

\[
\frac{d}{dt} L(p(t), \omega) = ?
\]

\[L(p(t), \omega)\]
Three volumetric phenomenon

- absorption
- emission
- scattering

(smoke data from Duc Nguyen & Ron Fedkiw)
Absorption

\[ \frac{d}{dt} L(p(t), \omega) = -\sigma_a L(p(t), \omega) \]

the particles absorb light’s energy

(assumption: particles are independent to each other)
Emission

\[ \frac{d}{dt} L(p(t), \omega) = L_e(p(t), \omega) \]

the particles add to light’s energy

sometimes this is formulated as \[ \frac{d}{dt} L(p(t), \omega) = \sigma_a L_e(p(t), \omega) \]
Scattering

out-scattering

\[
\frac{d}{dt} L(p(t), \omega) = -\sigma_s L(p(t), \omega)
\]

in-scattering

\[
\frac{d}{dt} L(p(t), \omega) = \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega'
\]

\(\rho\): “phase function” (volume BSDF)
Radiative Transfer Equation

\[ \frac{d}{dt} L(p(t), \omega) = -\sigma_a L(p(t), \omega) - \sigma_s L(p(t), \omega) \]

- **Absorption**
- **Out-scattering**

\[ +L_e(p(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega' \]

- **Emission**
- **In-scattering**
- **Gain**
Radiative Transfer Equation

\[
\frac{d}{dt} L(p(t), \omega) = -\sigma_t L(p(t), \omega) + L_e(p(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega'
\]

\[\sigma_t = \sigma_a + \sigma_s\]
A simpler case: volume without scattering

\[
\frac{d}{dt} L(p(t), \omega) = -\sigma_t L(p(t), \omega)
\]

\[
+ L_e(p(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega'
\]

let \( \sigma_s = 0 \)

How would you solve for \( L \)?
A simpler case: volume without scattering

\[ \frac{d}{dt} L(p(t), \omega) = - \sigma_t L(p(t), \omega) + L_e(p(t), \omega) \]

let \( \sigma_s = 0 \)

\[ \frac{d}{dt} L(t) = a(t)L(t) + b(t) \]

it’s a linear ODE that has an analytical solution (quiz: what is it?)
A simpler case: volume without scattering

\[
\frac{d}{dt}L(p(t), \omega) = -\sigma_t L(p(t), \omega) + L_e(p(t), \omega)
\]

let \(\sigma_s = 0\)

\[
\frac{d}{dt}L(t) = a(t)L(t) + b(t)
\]

\[
L(t) = \int_0^t T(t)L_e(t)dt \\
T(t) = \exp \left( -\int_0^t \sigma_t(t')dt' \right)
\]
A simpler case: volume without scattering

\[
\frac{d}{dt} L(p(t), \omega) = -\sigma_t L(p(t), \omega) + L_e(p(t), \omega)
\]

let \(\sigma_s = 0\)

\[
\frac{d}{dt} L(t) = a(t)L(t) + b(t)
\]

\[
L(t) = \int_0^t T(t)L_e(t)dt
\]

\[
T(t) = \exp \left( -\int_0^t \sigma_t(t')dt' \right)
\]

"transmittance"
A simpler case: volume without scattering

\[ L(t) = \int_{0}^{t} T(t)L_e(t)\,dt \]

\[ T(t) = \exp \left( -\int_{0}^{t} \sigma(t')\,dt' \right) \]
The full radiative transfer equation is still a linear ODE

\[ \frac{d}{dt} L(p(t), \omega) = -\sigma_t L(p(t), \omega) + L_e(p(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega' \]

\[ \frac{d}{dt} L(t) = a(t) L(t) + b(t) \]
Integral form of radiative transfer equation

\[
\frac{d}{dt} L(p(t), \omega) = -\sigma_t L(p(t), \omega) + L_e(p(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega'
\]

\[
L(p(0), \omega) = \int_0^t T(p(0), p(t')) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t'), \omega') d\omega' \right] dt'
\]

\[
T(p(0), p(t)) = \exp \left( -\int_0^t \sigma_t(t') dt' \right)
\]
Volumetric path tracing

- the inclusion of the transmittance is the main difference to surface rendering equation

\[ T(p(0), p(t)) = \exp \left( -\int_0^t \sigma_t(t')dt' \right) \]

\[ L(p(0), \omega) = \int_0^t T(p(0), p(t')) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t'), \omega')d\omega' \right] dt' \]

quiz: how would you do it?
Volumetric path tracing

- the inclusion of the transmittance is the main difference to surface rendering equation

\[
T(p(0), p(t)) = \exp \left( -\int_0^t \sigma_t(t') dt' \right)
\]

\[
L(p(0), \omega) = \int_0^t T(p(0), p(t')) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t'), \omega') d\omega' \right] dt'
\]

will talk about how to sample T next time
Volumetric path tracing

- the inclusion of the transmittance is the main difference to surface rendering equation

\[
T(p(0), p(t)) = \exp \left( -\int_0^t \sigma_s(t') dt' \right)
\]

\[
L(p(0), \omega) = \int_0^t T(p(0), p(t')) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t'), \omega') d\omega' \right] dt'
\]
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Volumetric path tracing

- the inclusion of the transmittance is the main difference to surface rendering equation

\[
T(p(0), p(t)) = \exp\left(-\int_0^t \sigma_s(t')dt'\right)
\]

\[
L(p(0), \omega) = \int_0^t T(p(0), p(t')) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega')L(p(t'), \omega')d\omega' \right] dt'
\]
Next event estimation in volumetric path tracing

$$L(p(0), \omega) = \int_0^t T(p(0), p(t')) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega')L(p(t'), \omega')d\omega' \right] dt'$$

$$T(p(0), p(t)) = \exp \left( -\int_0^t \sigma_s(t')dt' \right)$$

G: geometry term

will talk about how to eval T next time
Inclusion surface lighting in volume rendering

- treat surface lighting as part of the volume "emission"

\[
L(p(0), \omega) = \int_{0}^{t} T(p(0), p(t')) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega')L(p(t'), \omega')d\omega' \right] dt'
\]

\[
T(p(0), p(t)) = \exp \left( -\int_{0}^{t} \sigma_i(t')dt' \right)
\]
Typical volume data structures in a path tracer

- use geometry as boundaries, store a medium inside each geometry
Smallvpt: volume path tracing in 150 lines

https://github.com/seifeddinedridi/smallvpt
Types of media

- Homogeneous medium
- Heterogeneous medium
- Isotropic phase function
- Anisotropic phase function
Homogeneous medium: $\sigma_t$ is constant

- significantly simplifies transmittance sampling/evaluation

$$T(p(0), p(t)) = \exp \left( - \int_0^t \sigma_t(p(t')) dt' \right) = \exp(-t\sigma_t)$$
Heterogeneous medium:
\( \sigma_t(p) \) varies spatially

\[
T(p(0), p(t)) = \exp \left( - \int_0^t \sigma_t(p(t')) dt' \right)
\]
Data structures for storing heterogeneous media

- hierarchical sparse arrays: exploiting spatial coherent sparsity

https://developer.nvidia.com/nanovdb

https://yuanming.taichi.graphics/publication/2019-taichi/
NeRF: spatial-directionally varying emission-absorption only volumes (no scattering)

\[
\frac{d}{dt} L(p(t), \omega) = -\sigma_t L(p(t), \omega) + L_e(p(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega'
\]

https://alexyu.net/plenoxels/?s=09

https://www.matthewtancik.com/nerf
Me in 2019, after submitted the Taichi paper

- I still think this is a cool direction!!

Tzu-Mao Li <bachi722@gmail.com>
to Yuanming, Luke, Fredo, Jonathan, Bill  
Tue, 21 May 2019, 06:49

I start to feel that there is some opportunities for inverse rendering with volumetric path tracing, even in the surface lighting case.

Combining the CNN and path tracing code in this project we already have a pretty efficient forward model for large scale rendering. The rest is to add an anisotropic phase function (E.g. microflake) and level of detail (e.g. sggx). Then we can autodiff the compiler. Efficient scatter to gather conversion is again an issue.

I am also thinking about the deep image prior thing applied to volume. A simple thing to try is to take a few measurement images and reconstruct the volumes using DIP.
Phase function

• very little work on this!

• in physics:
  • very very small particles: Rayleigh scattering
  • very small particles: Mie scattering
  • large particles: just treat them as surfaces…
  • phenomenological model: Henyey-Greenstein

http://homework.uoregon.edu/pub/class/atm/scatter.html
Rayleigh scattering

- based on “dipole approximation” of electromagnetic fields

$$\rho(\omega, \omega') = \frac{8\pi^4 \alpha^2}{\lambda^4} \left( 1 + (\omega \cdot \omega')^2 \right)$$

$\alpha$: “polarizibility”
(https://en.wikipedia.org/wiki/Polarizability)
$\lambda$: wavelength

Rayleigh scattering explains the color of sky

$$\rho(\omega, \omega') = \frac{8\pi^4 \alpha^2}{\lambda^4} \left( 1 + (\omega \cdot \omega')^2 \right)$$

http://hyperphysics.phy-astr.gsu.edu/hbase/atmos/blusky.html
Phase function

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http://homework.uoregon.edu/pub/class/atm/scatter.html
Mie scattering

• derive the electric field by solving Maxwell’s equation directly on a spherical particle

• less wavelength dependent

https://en.wikipedia.org/wiki/Mie_scattering
Mie-scattering exhibits “forward scattering”

http://homework.uoregon.edu/pub/class/atm/scatter.html
Rayleigh and Mie scattering

When there is large particulate matter in the air, the forward lobe of Mie scattering is dominant. Since it is not very wavelength dependent, we see a white glare around the sun.

From overhead, the Rayleigh scattering is dominant, the Mie scattered intensity being projected forward. Since Rayleigh scattering strongly favors short wavelengths, we see a blue sky.

http://hyperphysics.phy-astr.gsu.edu/hbase/atmos/blusky.html
Phase function

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http://homework.uoregon.edu/pub/class/atm/scatter.html
Henyey-Greenstein phase function [1944]

\[ \rho(\omega, \omega') = \frac{1}{4\pi} \frac{1 - g^2}{\left(1 + g^2 + 2g \omega \cdot \omega'\right)^{\frac{3}{2}}} \]

g = -0.35, -0.7, 0.67, 0.7

Microflake: microfacets for phase functions

- more about it in the future
Phase function

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- in physics:
  - very very small particles: Rayleigh scattering
  - very small particles: Mie scattering
  - large particles: just treat them as surfaces...
  - phenomenological model: Henyey-Greenstein

http://homework.uoregon.edu/pub/class/atm/scatter.html
Shell tracing: precomputed phase function

- also more about this in the future, probably

Figure 13: Renderings of 10,000 glass Buddha meshes, each with 10,000 triangles, using our new method. Left: the full bowl of Buddhas, 133 minutes on the cluster. Center: inset of the left edge of the bowl of Buddhas, 175 minutes on the cluster. Top right: path traced low order scattering for the inset, in 135 minutes. Bottom right: high order scattering using shells, in 40 minutes. Precomputation time was 17 minutes on a single machine.
Other fun research: non-exponential radiative transfer

traditional RTE assumes linear ODEs, can we use arbitrary ODEs?

\[
\frac{d}{dt} L(p(t), \omega) = -\sigma_t L(p(t), \omega) + L_e(p(t), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t), \omega') d\omega'
\]
Other fun research:
non-exponential radiative transfer

traditional RTE assumes linear ODEs, can we use arbitrary ODEs?

\[
L(p(0), \omega) = \int_0^t T\left(p(0), p(t')\right) \left[ L_e(p(t'), \omega) + \sigma_s \int_{S^2} \rho(\omega, \omega') L(p(t'), \omega') d\omega' \right] dt'
\]

T: arbitrary functions!
Other fun research: non-exponential radiative transfer

non exponential T corresponds to correlated particles

A radiative transfer framework for non-exponential media

BENEDIKT BITTERLI, Dartmouth College, USA
SRINATH RAVICHANDRAN, Dartmouth College, USA
THOMAS MÜLLER, Disney Research, ETH Zürich, Switzerland
MAGNUS WRENNINGE, Pixar Animation Studios, USA
JAN NOVÁK, Disney Research, Switzerland
STEVE MARSCHNER, Cornell University, USA
WOJCIECH JAROSZ, Dartmouth College, USA
Other fun research: beyond Mie scattering

Computing the Scattering Properties of Participating Media Using Lorenz-Mie Theory
Jeppe Revall Frisvad¹ Niels Jørgen Christensen¹ Henrik Wann Jensen²
¹Informatics and Mathematical Modelling, Technical University of Denmark
²University of California, San Diego

Beyond Mie Theory: Systematic Computation of Bulk Scattering Parameters based on Microphysical Wave Optics
Yu Guo¹, Adrian Jarabo², and Shuang Zhao³
¹University of California, Irvine
²Universidad de Zaragoza - I3A
ACM Transactions on Graphics (SIGGRAPH Asia 2021), 40(6), 2021
Other fun research: phase functions for rainbows

Physically-Based Simulation of Rainbows

IMAN SADEGHI
University of California, San Diego

ADOLFO MUNOZ
Universidad de Zaragoza

PHILIP LAVEN
Horley, UK

WOJCIech JAROSZ
Disney Research Zürich, University of California, San Diego

FRANCISCO SERON and DIEGO GUTIERREZ
Universidad de Zaragoza

and

HENRIK WANN JENSEN
University of California, San Diego

Fig. 8. Steps of the algorithm. (1) Casting the grid of rays toward the particle. (2) Rays are reflected and refracted toward the water drop, forming patches. (3) Outgoing patches are collected in an infinite collecting sphere. (4) The stored patches in the collecting sphere are queried at specific directions, sampling the phase function.
Other fun research:
perception study of phase functions

Understanding the Role of Phase Function in Translucent Appearance

Joannis Gkioulekas\textsuperscript{1}, Bei Xiao\textsuperscript{2}, Shuang Zhao\textsuperscript{3}, Edward Adelson\textsuperscript{2}, Todd Zickler\textsuperscript{1}, and Kavita Bala\textsuperscript{3}

\textsuperscript{1}Harvard School of Engineering and Applied Sciences, \textsuperscript{2}Massachusetts Institute of Technology, \textsuperscript{3}Cornell University

ACM Transactions on Graphics, 32(5), September 2013
Other fun research: acquiring phase functions
Next time:
Monte Carlo evaluation of transmittance

\[ \exp \left( -\int_{0}^{t} \sigma_{t}(t')dt \right) \]