

# Layered BSDFs

UCSD CSE 272

Advanced Image Synthesis

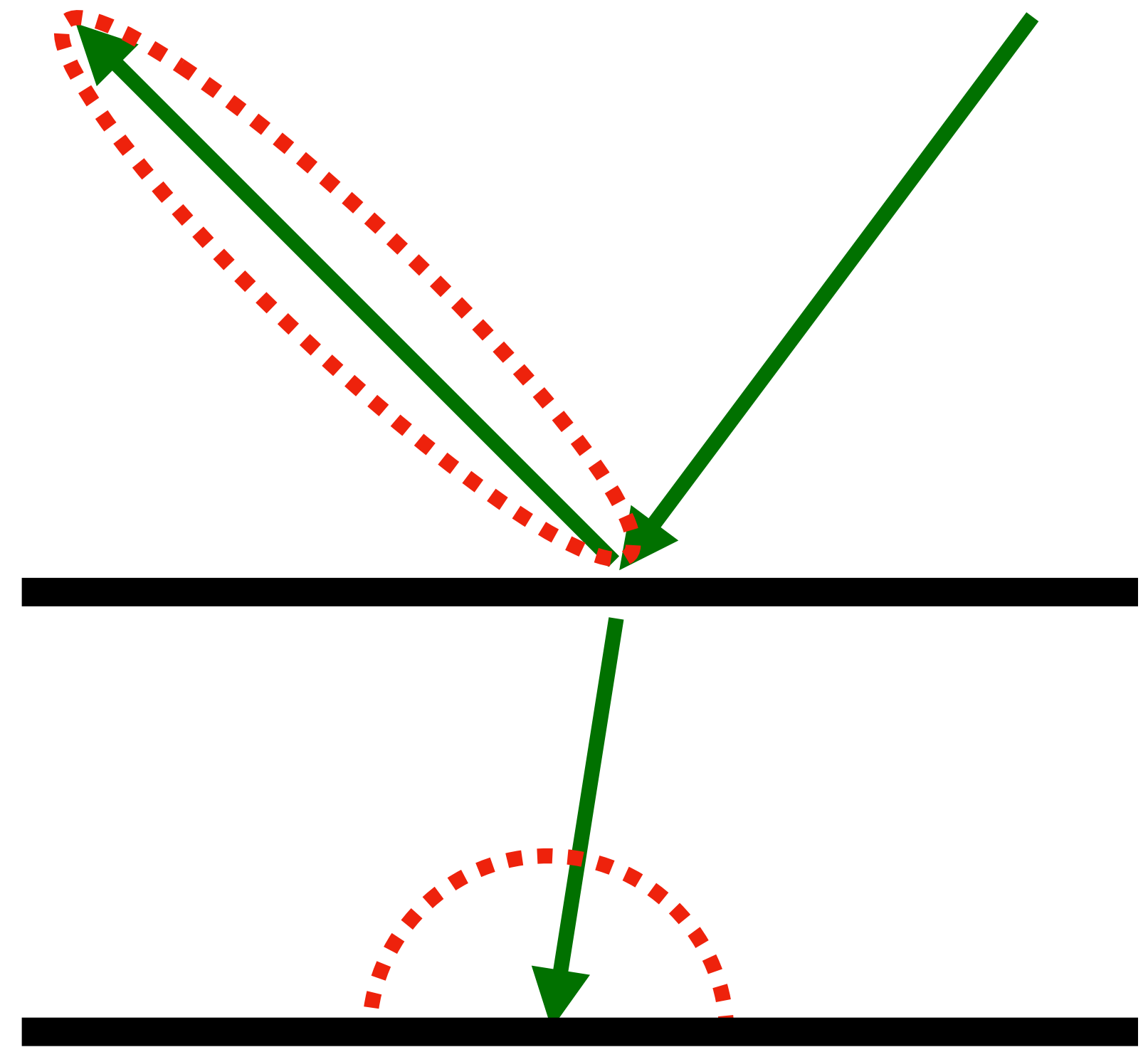
Tzu-Mao Li

# Real-world materials are multi-layered

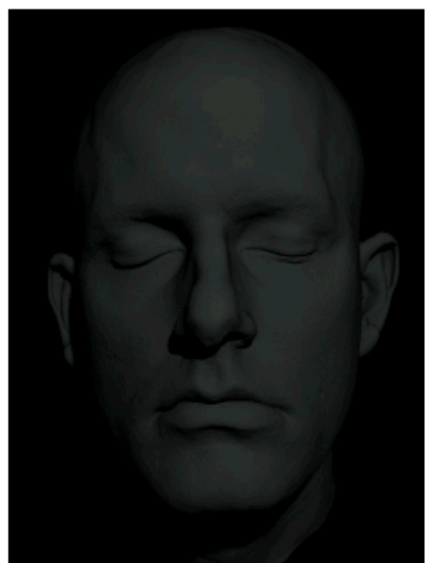


dielectric layer

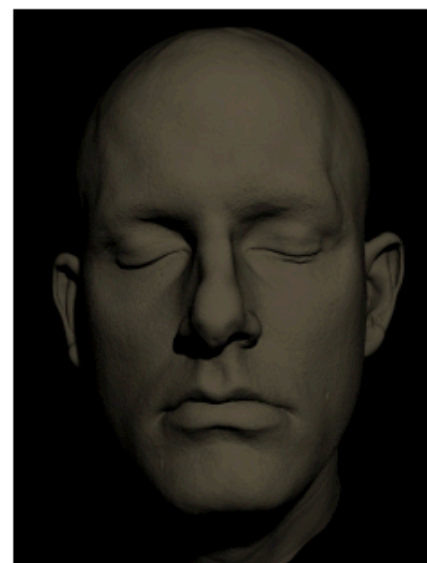
diffuse layer



# Real-world materials are multi-layered



Epidermis Reflectance



Epidermis Transmittance



Upper Dermis Reflectance



Upper Dermis Transmittance



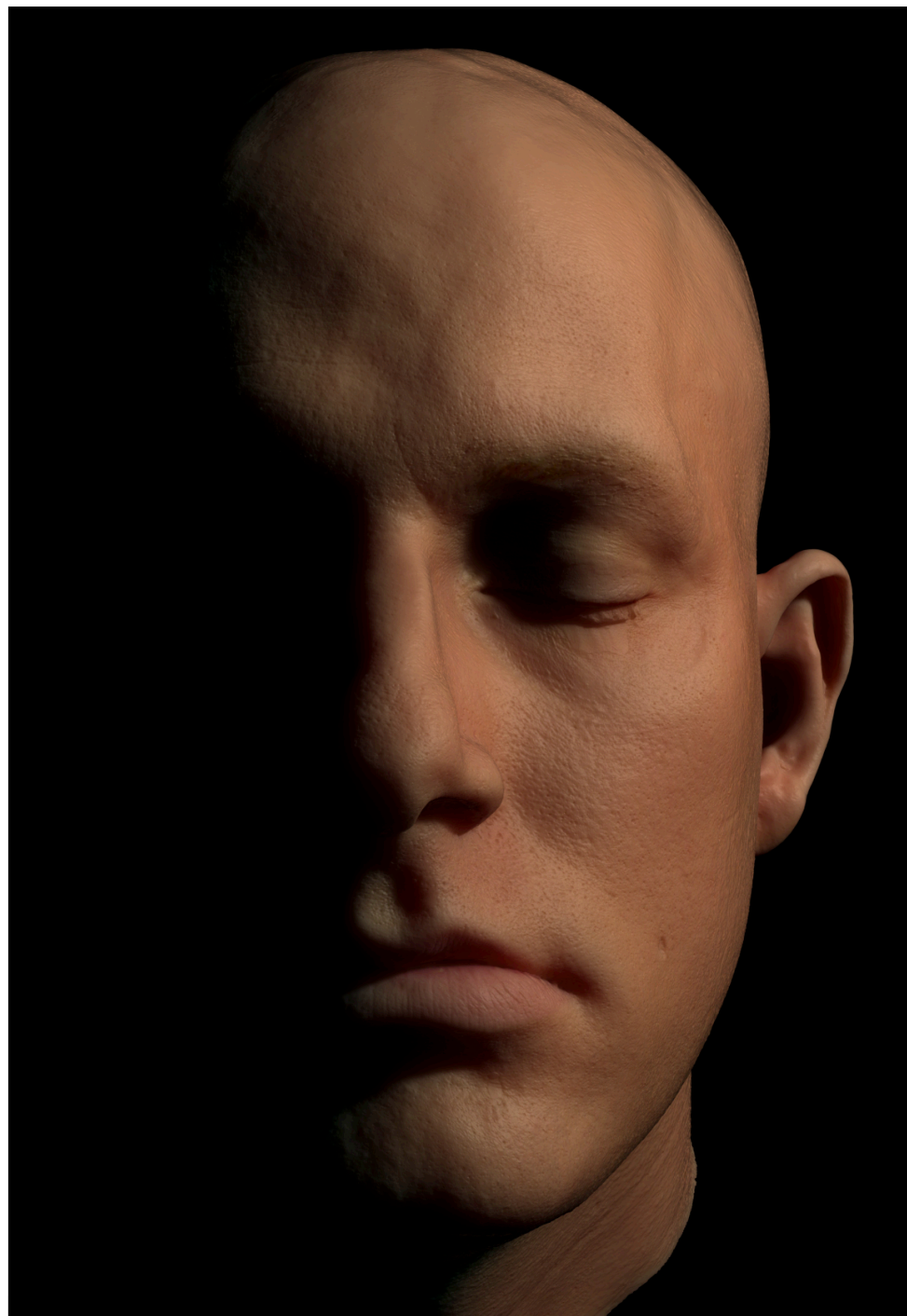
Bloody Dermis Reflectance



Surface Roughness



All Layers

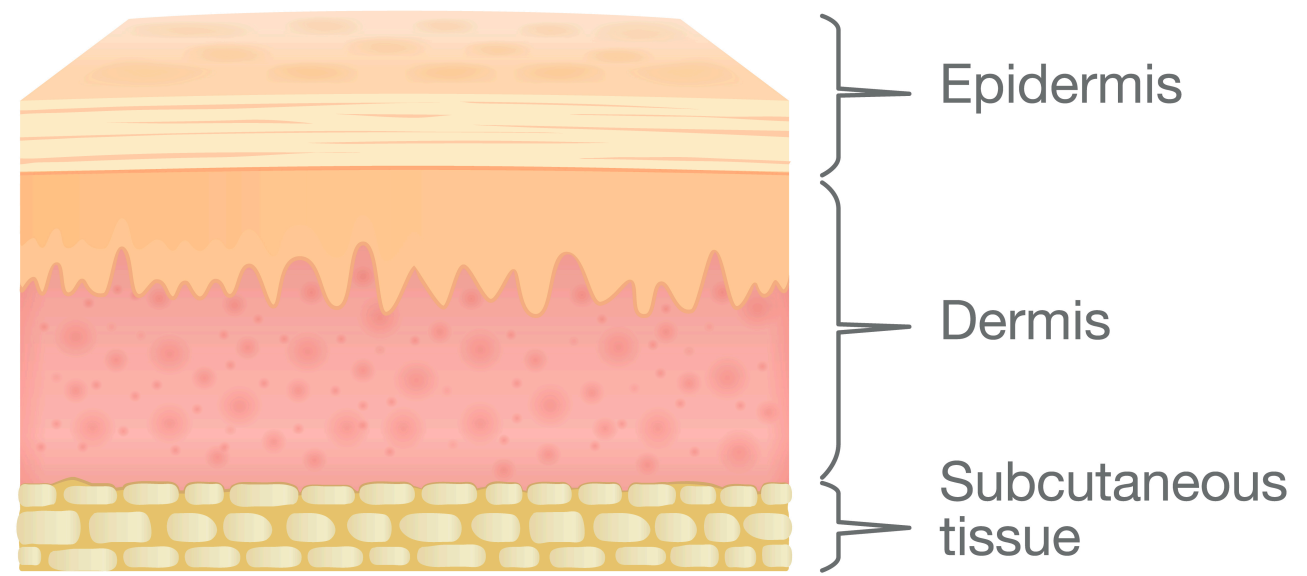


single-layer



multi-layer

## The Layers of Skin



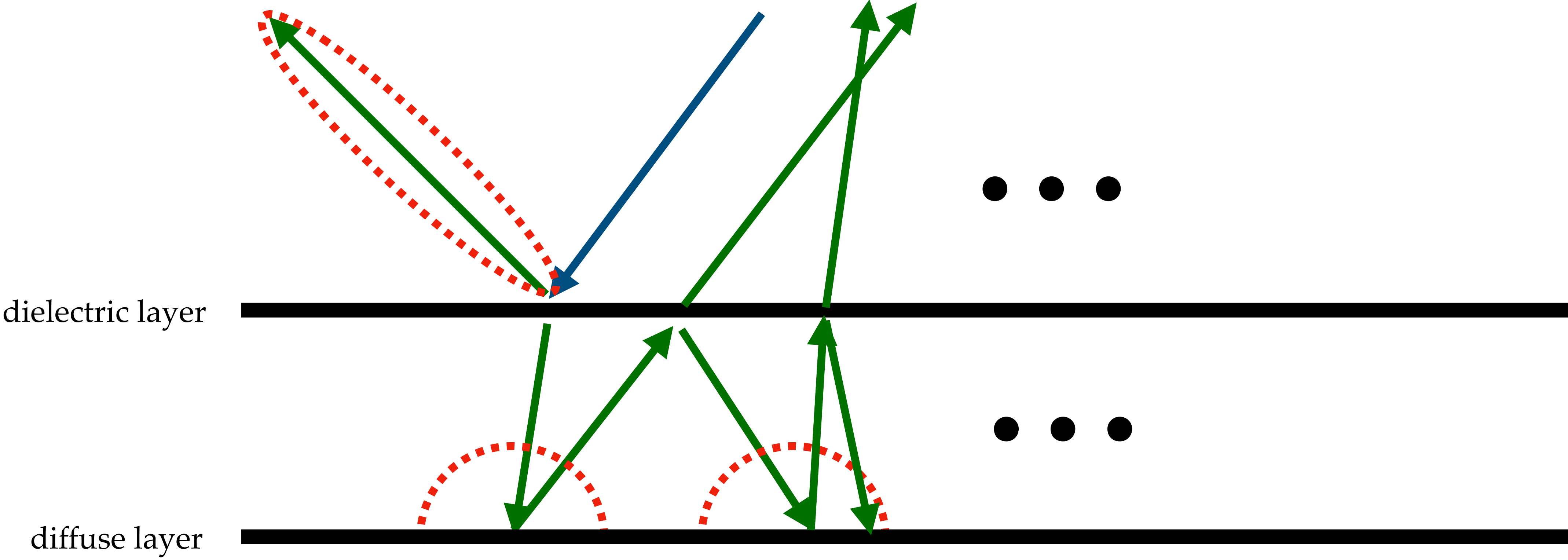
<https://fldscc.com/2020/01/30/three-layers-skin/>

### Light Diffusion in Multi-Layered Translucent Materials

Craig Donner      Henrik Wann Jensen  
University of California, San Diego

# Naive combination of BSDF lobes

does not accurately model scattering between interfaces



# Why is wet material darker?



(rendering)



(photo)

image credit

<https://rnrtyres.com/tips-guides/top-tips-for-driving-on-wet-roads/>

## Rendering of Wet Materials

Henrik Wann Jensen  
Justin Legakis  
Julie Dorsey

Massachusetts Institute of Technology

# Why is wet material darker?

A thin water film on a Lambertian surface can also cause the surface to become darker [10]. The main cause for this darkening is the possibility of total internal reflection at the water-air boundary. Some of the light reflected from the Lambertian surface will be reflected back to the surface by the water-air interface. This light is then subject to another round of absorption by the surface (see Figure 1) before it is reflected again. This can lead to a sequence of multiple absorptions, resulting in a darkening of the surface.

## Rendering of Wet Materials

Henrik Wann Jensen  
Justin Legakis  
Julie Dorsey

Massachusetts Institute of Technology

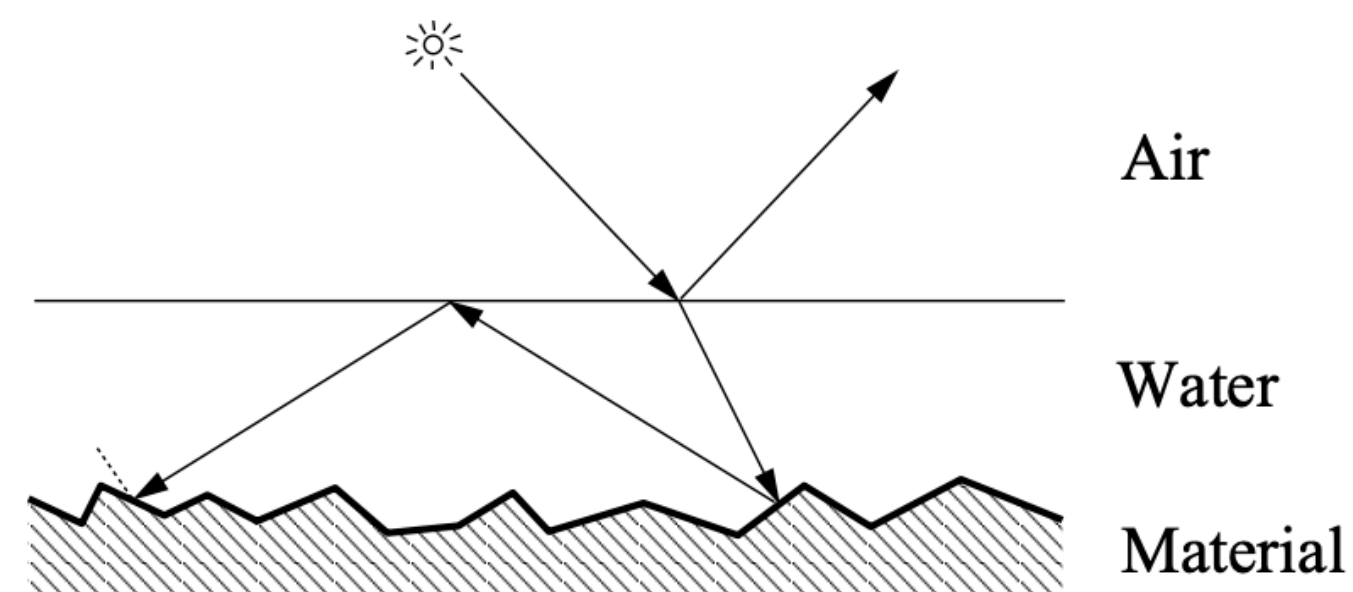
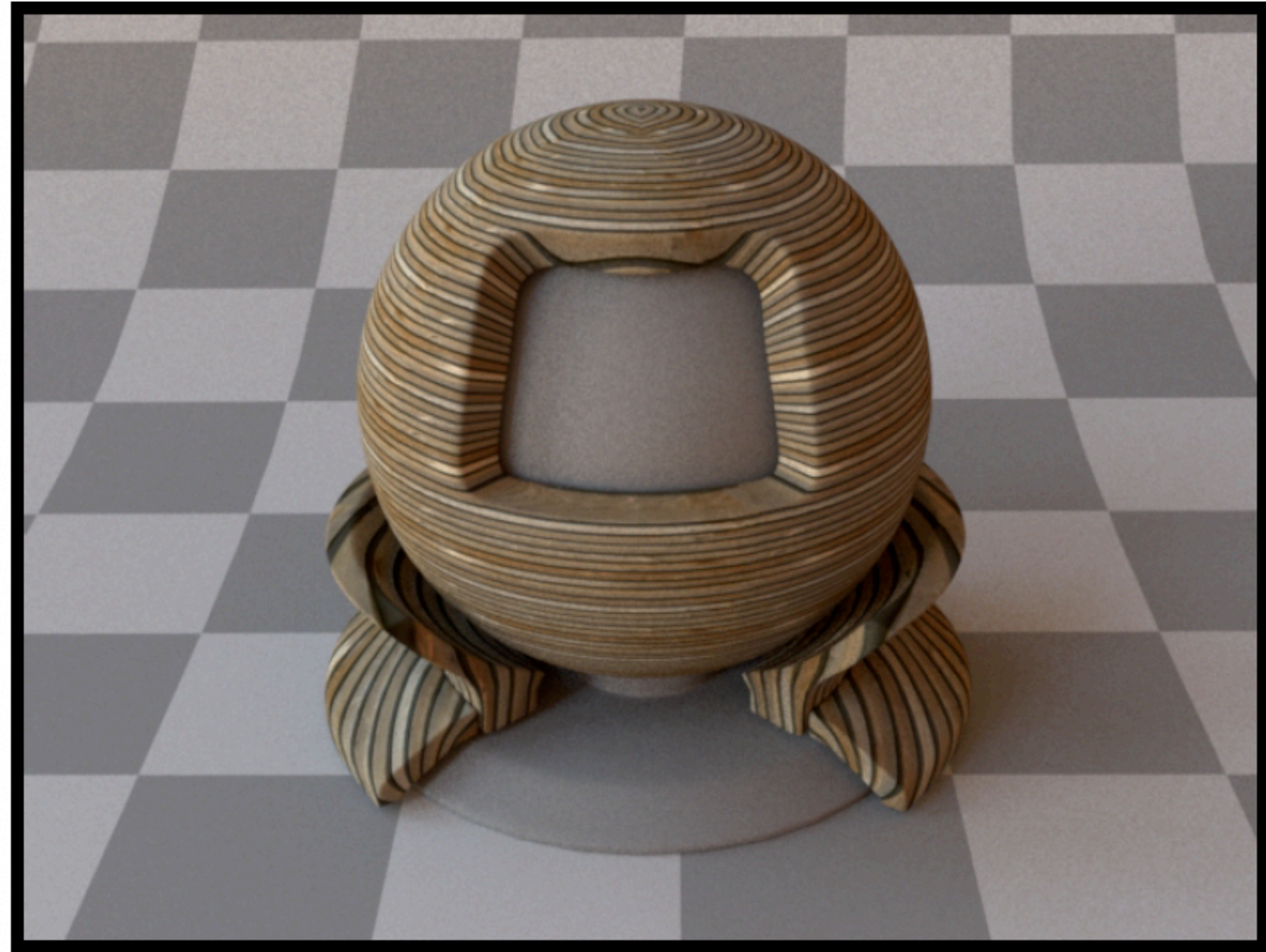
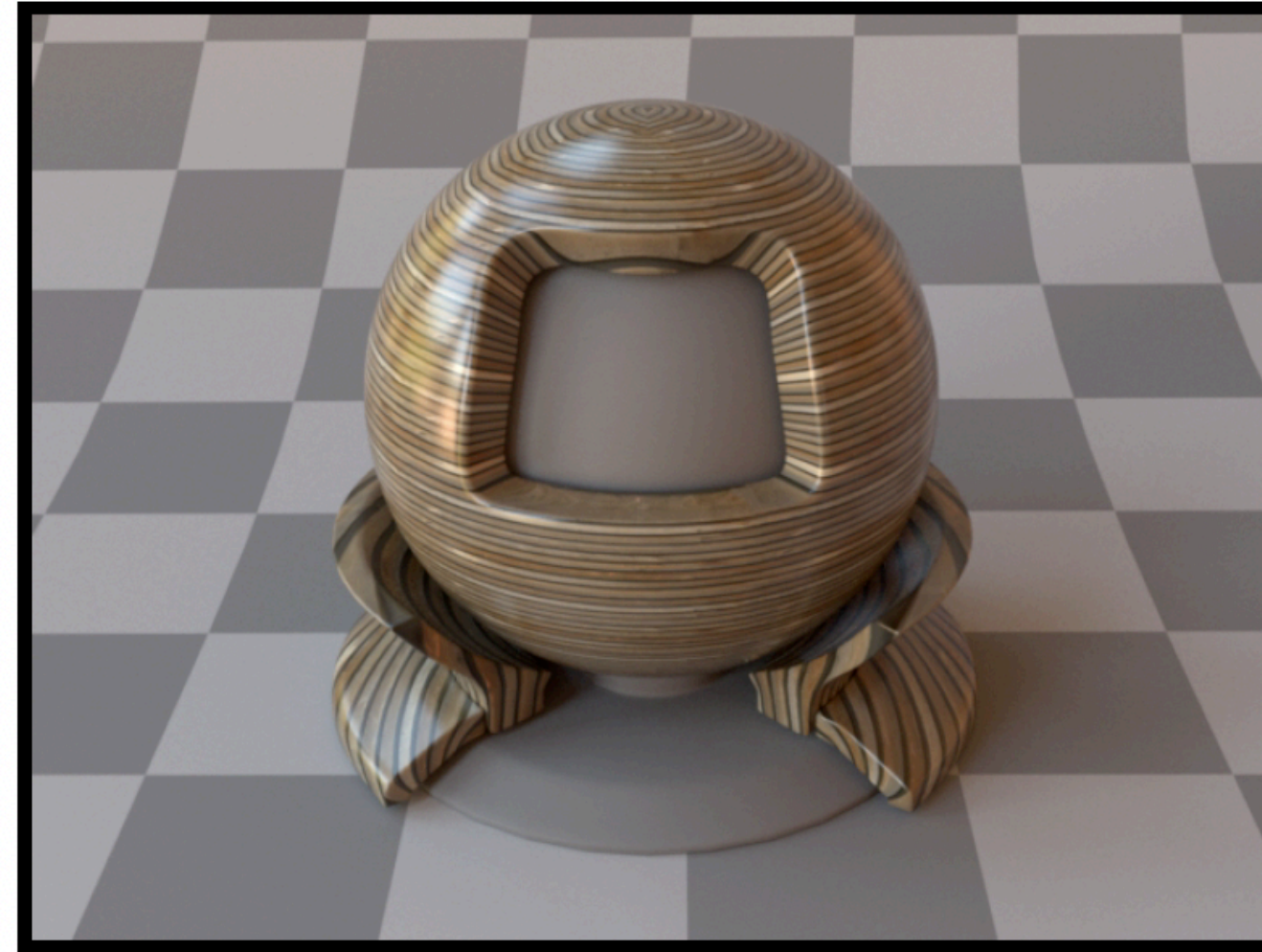


Fig. 1. A layer of water above the surface reflects less light due to the internal reflection at the water-air interface.

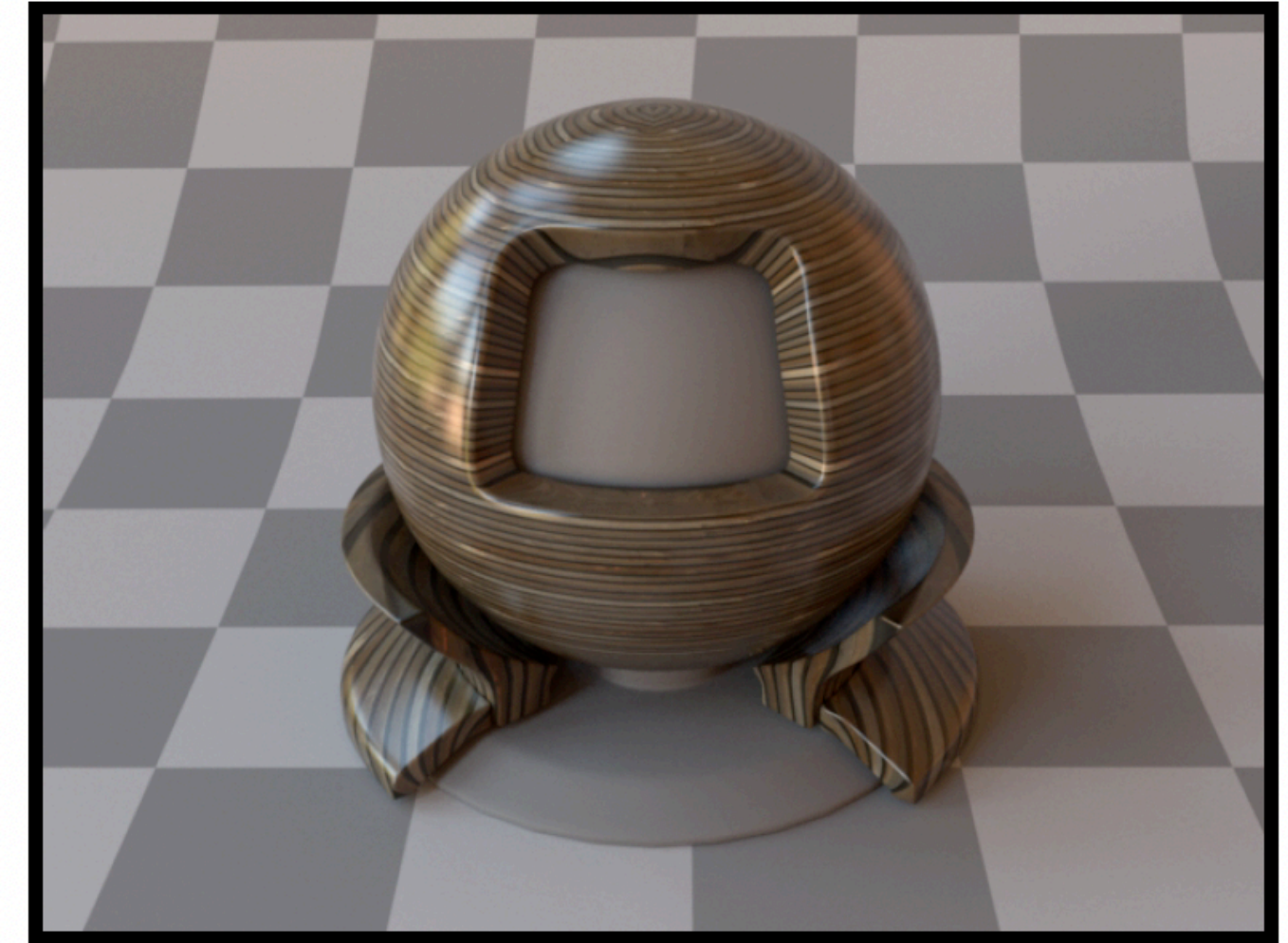
# Multiple-scattering causes non-linear shift on reflectance



diffuse only

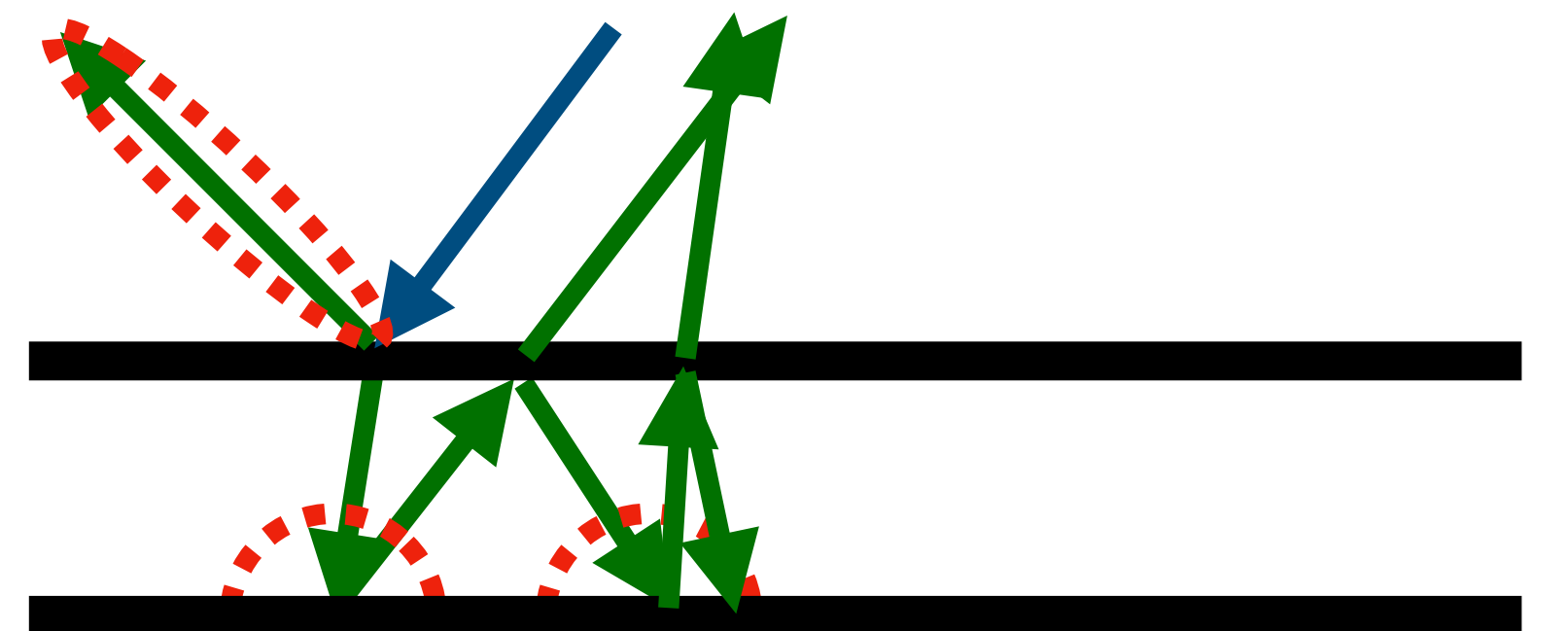


diffuse + specular lobes



multiple scattering  
between diffuse & specular

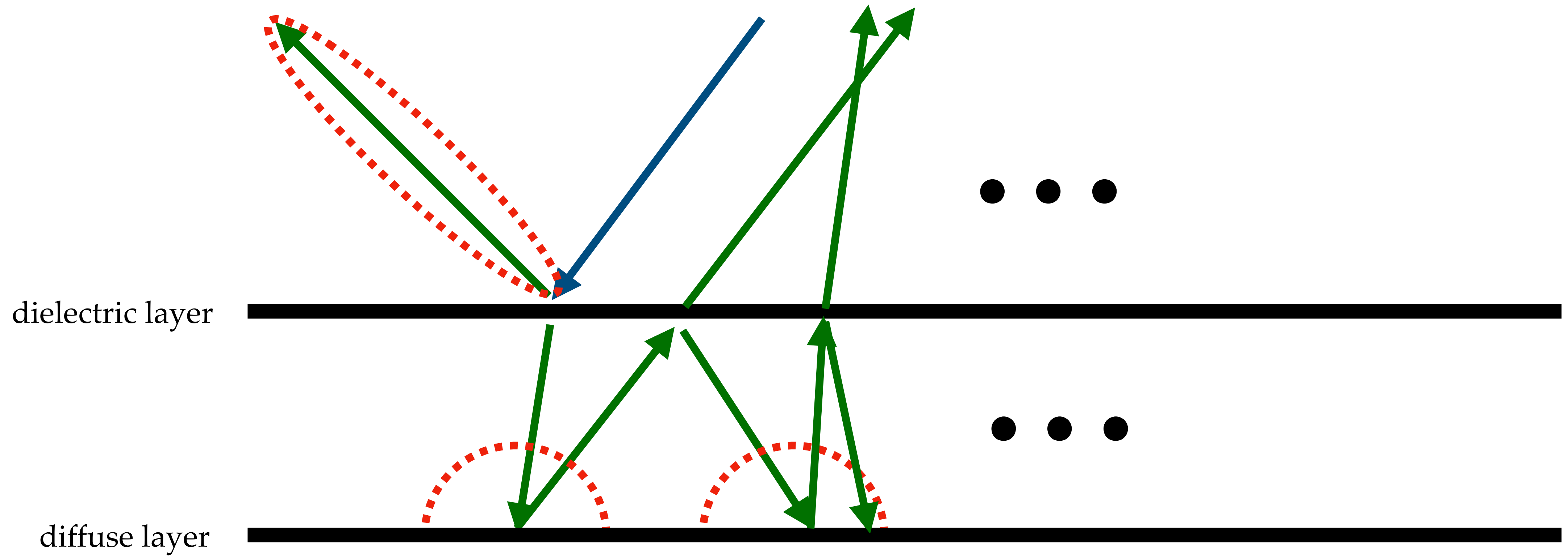
**quiz:** what is the consequence of this to the end users of the renderer?



from Mitsuba documentation

<https://www.mitsuba-renderer.org/releases/current/documentation.pdf>

# How would you model the internal scattering?





# Today: 3 approaches of modeling layered BSDFs

## A Comprehensive Framework for Rendering Layered Materials

Wenzel Jakob Eugene D'Eon Otto Jakob Steve Marschner

In ACM Transactions on Graphics (Proceedings of SIGGRAPH 2014)



## Efficient Rendering of Layered Materials using an Atomic Decomposition with Statistical Operators

Laurent Belcour (Unity Technologies)  
Published in ACM Transactions on Graphics (proc. of SIGGRAPH 2018)

paper TeX bib paper code video slides



# Today: 3 approaches of modeling layered BSDFs

## A Comprehensive Framework for Rendering Layered Materials

Wenzel Jakob Eugene D'Eon Otto Jakob Steve Marschner

In ACM Transactions on Graphics (Proceedings of SIGGRAPH 2014)



## Efficient Rendering of Layered Materials using an Atomic Decomposition with Statistical Operators

Laurent Belcour (Unity Technologies)  
Published in ACM Transactions on Graphics (proc. of SIGGRAPH 2018)

paper TeX bib paper code video slides



## Position-Free Monte Carlo Simulation for Arbitrary Layered BSDFs

Yu Guo<sup>1</sup>, Miloš Hašan<sup>2</sup>, Shuang Zhao<sup>1</sup>

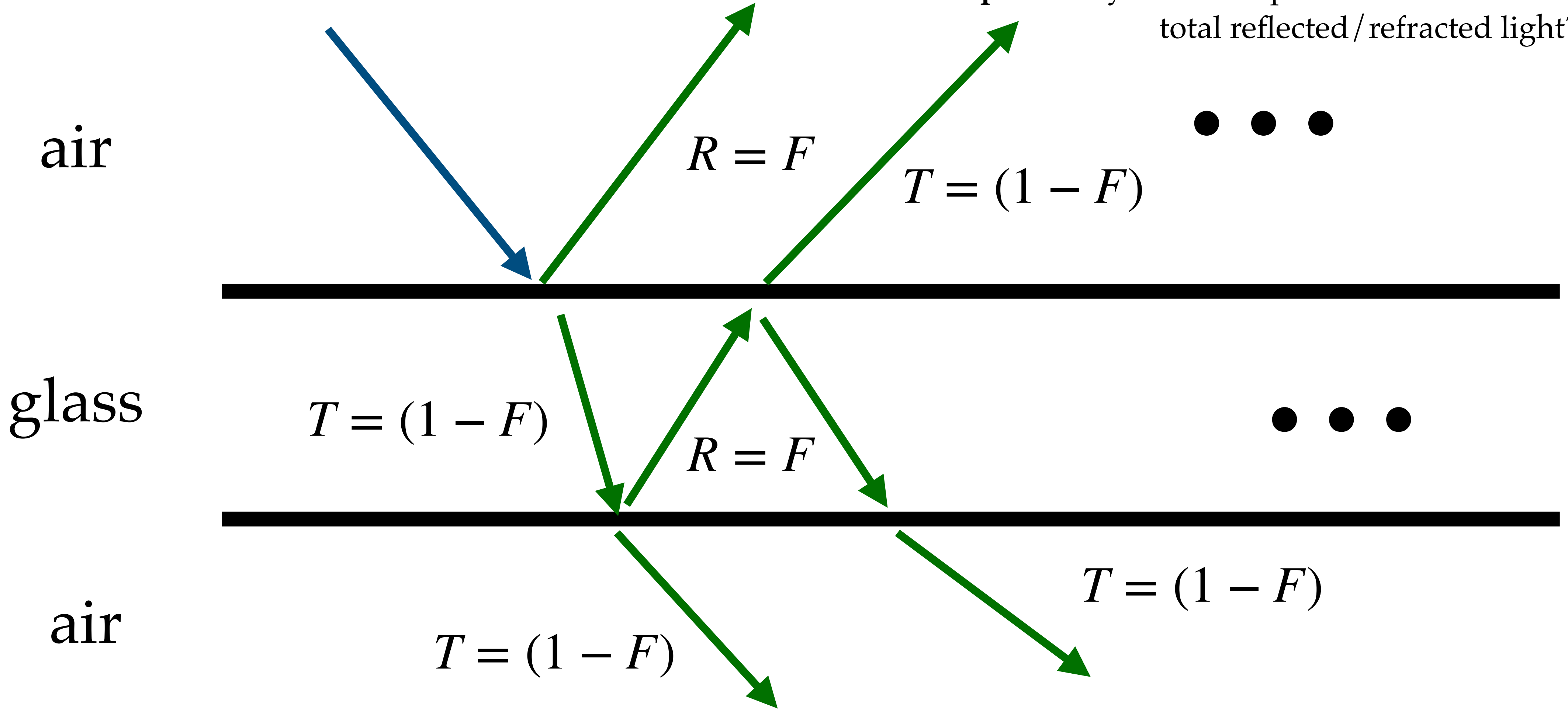
<sup>1</sup>University of California, Irvine <sup>2</sup>Autodesk

ACM Transactions on Graphics (SIGGRAPH Asia 2018), 37(6), 2018



# Special case: pile of glass plates [Stokes 1860]

quiz: can you come up with a closed-form solution of the total reflected/refracted light?



air

$$R = F$$

$$T = (1 - F)$$

$\dots$

glass

$$T = (1 - F)$$

$$R = F$$

$\dots$

air

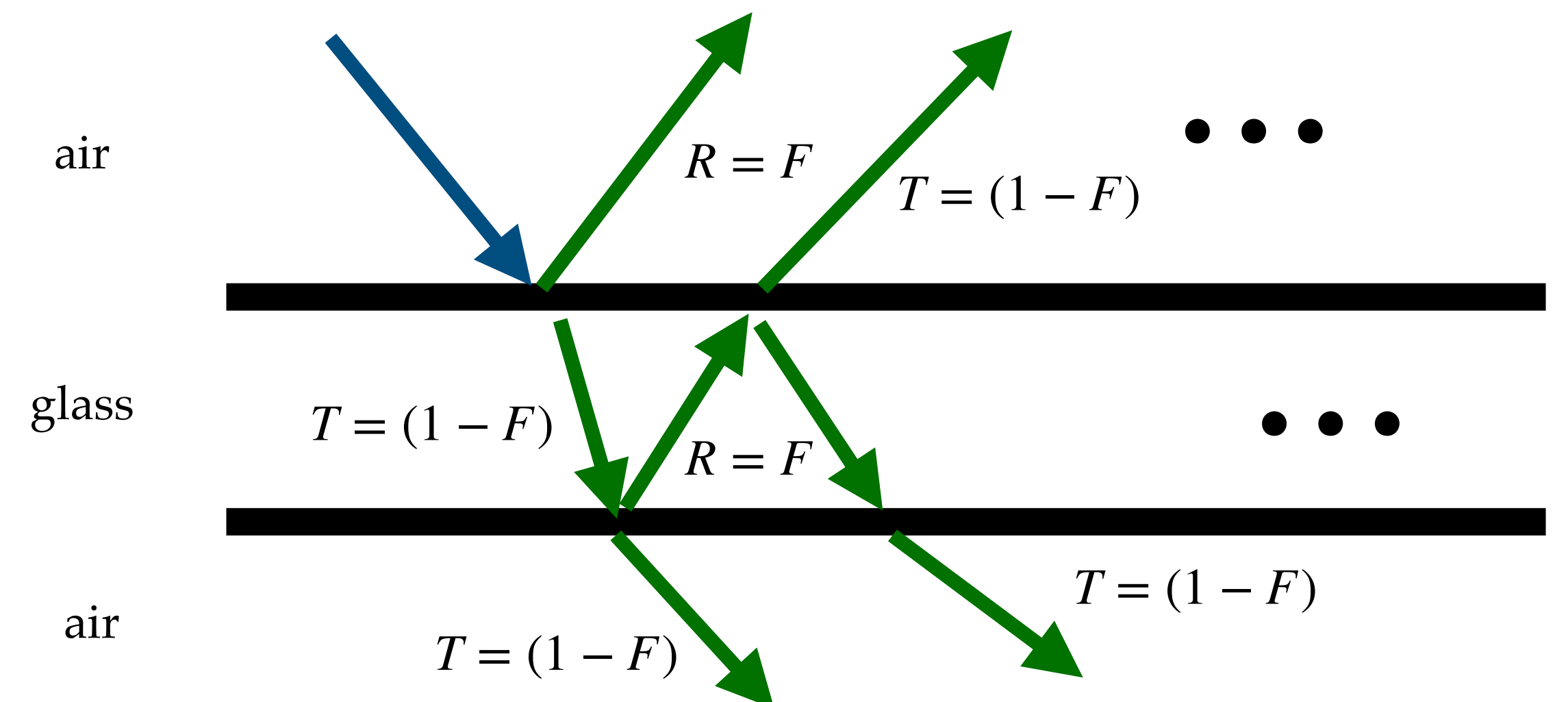
$$T = (1 - F)$$

$$T = (1 - F)$$

# Special case: pile of glass plates [Stokes 1860]

$$R_{\text{total}} = R + TRT + TR^3T + \dots = R + T^2 \sum_{i=0}^{\infty} R^{2i+1} = R + \frac{RT^2}{1 - R^2}$$

$$T_{\text{total}} = TT + TR^2T + TR^4T + \dots = T^2 \sum_{i=0}^{\infty} R^{2i} = \frac{T^2}{1 - R^2}$$

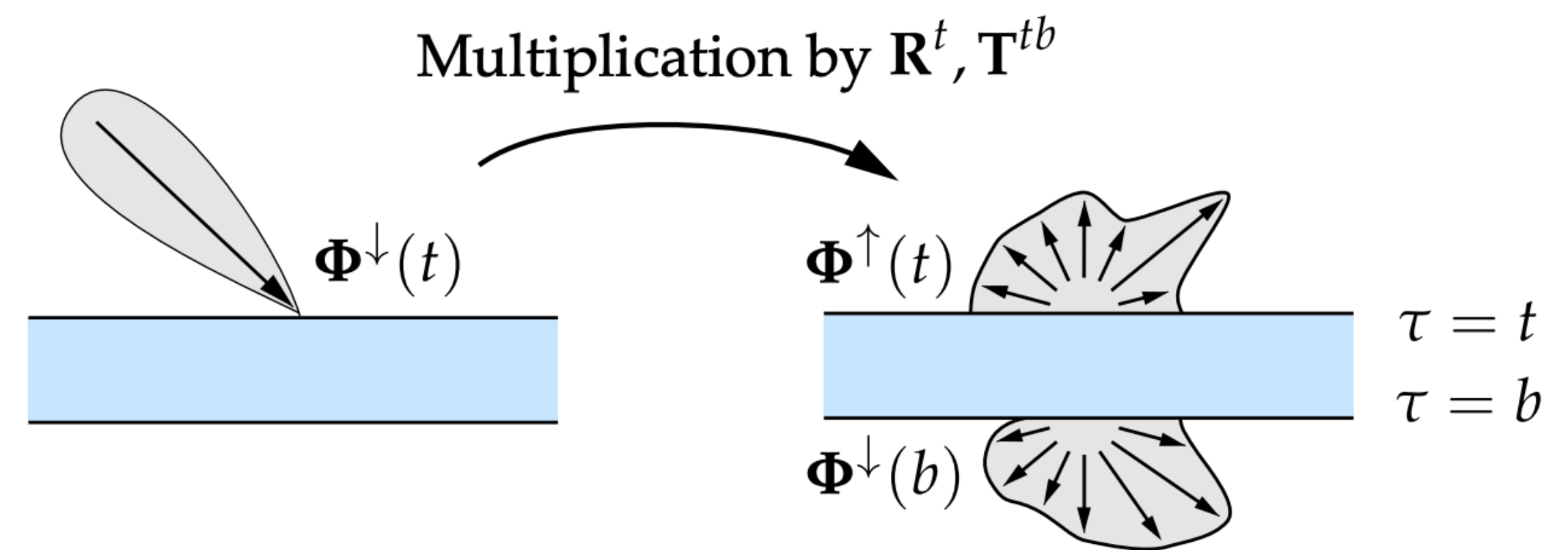


# General case: adding equation [Grant and Hunt 1969]

- replace Fresnel equations with matrix vector multiplications

$$\Phi^\uparrow(t) = \mathbf{R}^t \Phi^\downarrow(t) + \mathbf{T}^{bt} \Phi^\uparrow(b)$$

$$\Phi^\downarrow(b) = \mathbf{R}^b \Phi^\uparrow(b) + \mathbf{T}^{tb} \Phi^\downarrow(t)$$

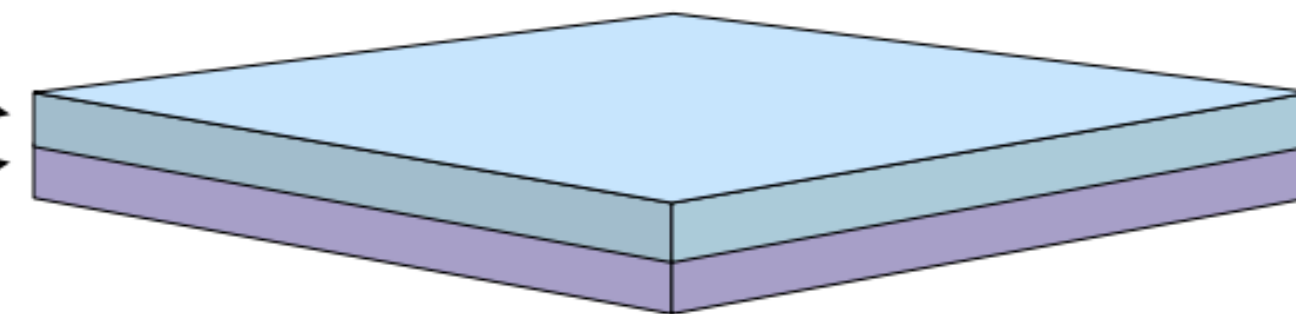
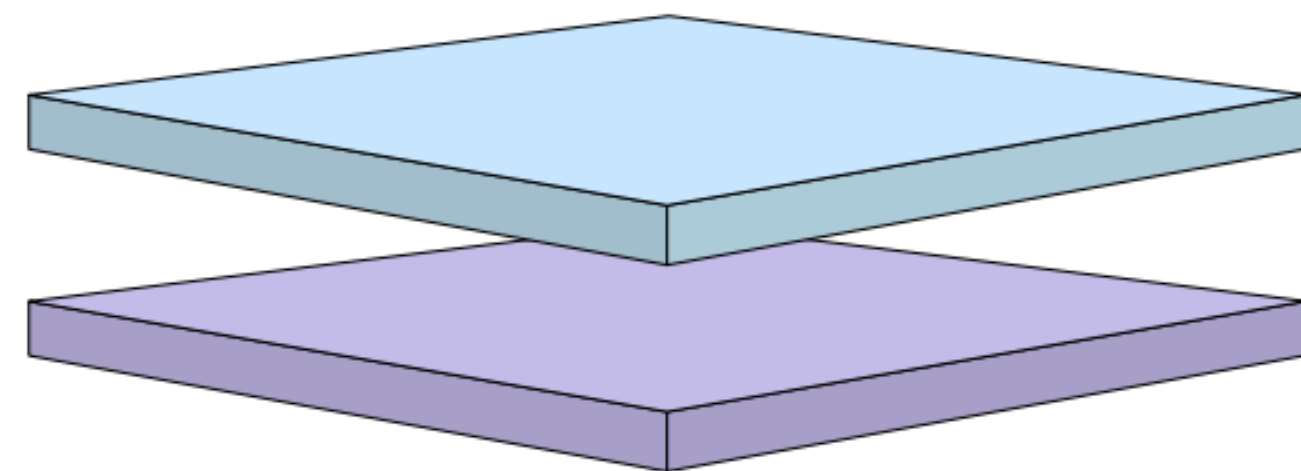


(aka discrete ordinates method [https://en.wikipedia.org/wiki/Discrete\\_ordinates\\_method](https://en.wikipedia.org/wiki/Discrete_ordinates_method))

# General case: adding equation [Grant and Hunt 1969]

goal: figure out the aggregate reflectance/transmittance when we stack layers

$$\mathbf{R}_1^t, \mathbf{T}_1^{bt}, \mathbf{R}_1^b, \mathbf{T}_1^{tb}$$



$$\tilde{\mathbf{R}}^t, \tilde{\mathbf{T}}^{bt}, \tilde{\mathbf{R}}^b, \tilde{\mathbf{T}}^{tb}$$

the air/glass layers are a special case!

$$\mathbf{R}_2^t, \mathbf{T}_2^{bt}, \mathbf{R}_2^b, \mathbf{T}_2^{tb}$$



Figure from Wenzel Jakob et al.

# General case: adding equation

## [Grant and Hunt 1969]

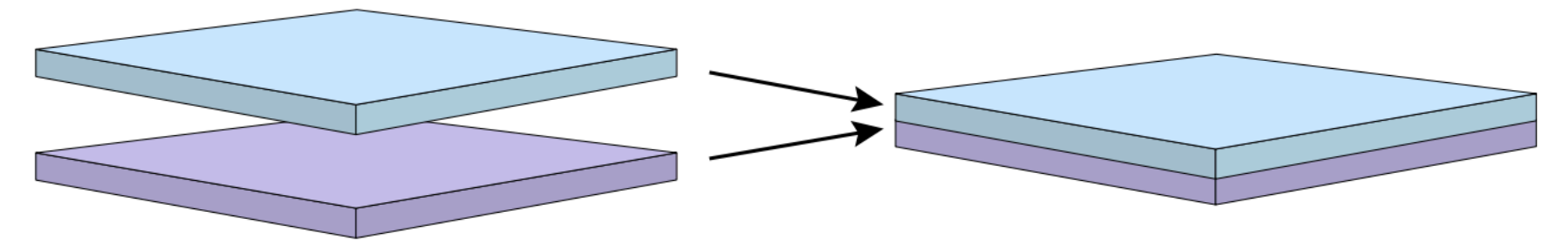
- stacking layer is analogous to summing geometric series for matrices (be careful with non-commutativity of matrix multiplication)

$$\tilde{\mathbf{R}}^t = \mathbf{R}_1^t + \mathbf{T}_1^{bt} (\mathbf{I} - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{R}_2^t \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{R}}^b = \mathbf{R}_1^b + \mathbf{T}_1^{tb} (\mathbf{I} - \mathbf{R}_2^b \mathbf{R}_1^t)^{-1} \mathbf{R}_2^b \mathbf{T}_1^{bt}$$

$$\tilde{\mathbf{T}}^{tb} = \mathbf{T}_2^{tb} (\mathbf{I} - \mathbf{R}_1^b \mathbf{R}_2^t)^{-1} \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{T}}^{bt} = \mathbf{T}_2^{bt} (\mathbf{I} - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{T}_1^{bt}$$



$$R_{\text{total}} = R + TRT + TR^3T + \dots = R + T^2 \sum_{i=0}^{\infty} R^{2i+1} = R + \frac{RT^2}{1 - R^2}$$

$$T_{\text{total}} = TT + TR^2T + TR^4T + \dots = T^2 \sum_{i=0}^{\infty} R^{2i} = \frac{T^2}{1 - R^2}$$

# Adding equation for thick interfaces: adding-doubling method [van de Hulst 1980]

- start from a thin layer, stack the same layer repeatedly

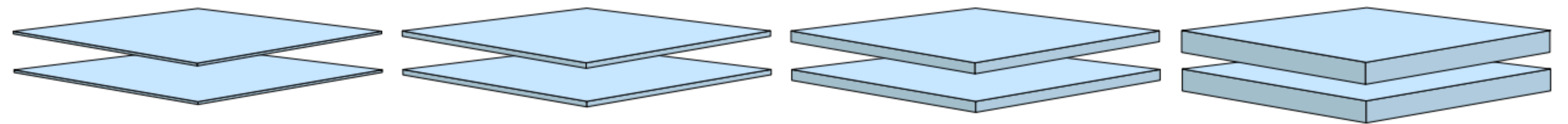
$$\tilde{\mathbf{R}}^t = \mathbf{R}_1^t + \mathbf{T}_1^{bt} (I - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{R}_2^t \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{R}}^b = \mathbf{R}_1^b + \mathbf{T}_1^{tb} (I - \mathbf{R}_2^b \mathbf{R}_1^t)^{-1} \mathbf{R}_2^b \mathbf{T}_1^{bt}$$

$$\tilde{\mathbf{T}}^{tb} = \mathbf{T}_2^{tb} (I - \mathbf{R}_1^b \mathbf{R}_2^t)^{-1} \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{T}}^{bt} = \mathbf{T}_2^{bt} (I - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{T}_1^{bt}$$

the thickness grows exponentially

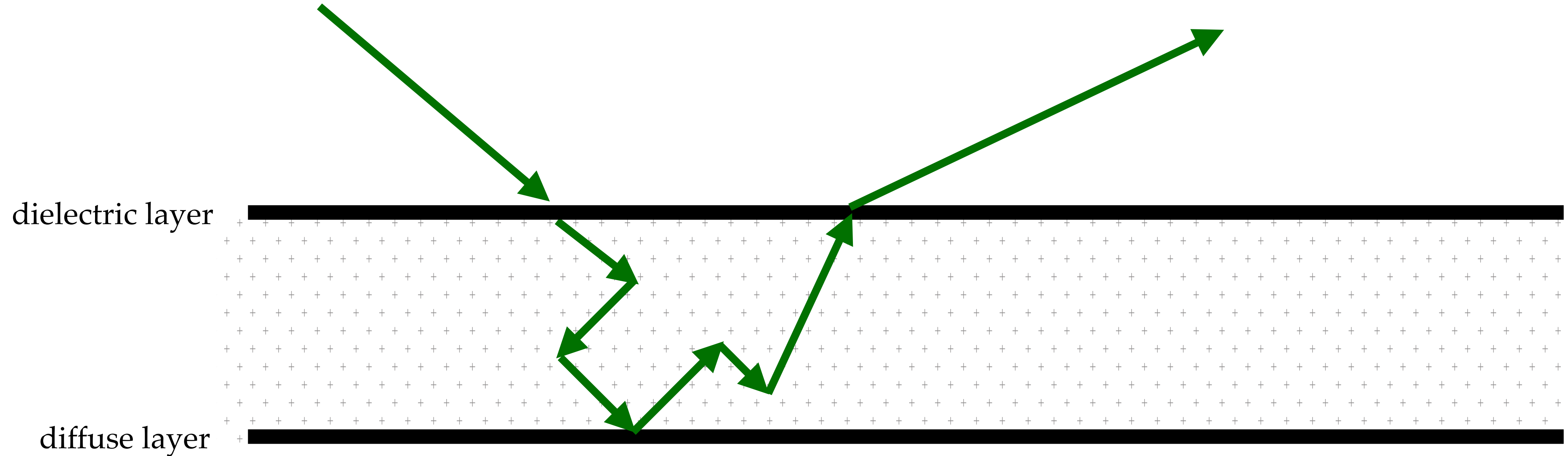


adding-doubling is commonly used for solving light transport in biomedical optics and atmospheric science!



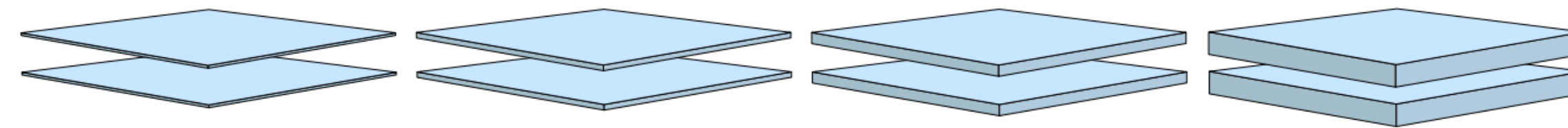
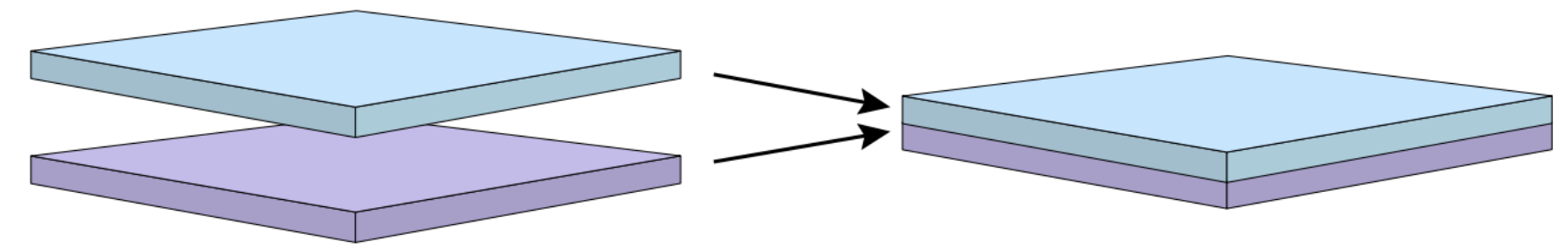
# Detail: need to also account for volumetric effects between layers

- all methods we talked about today can be easily extended to do this



# General layered BSDF construction [Jakob 2014]

- measure R & T of a single (thin) layer (either an analytical BSDF or measured one)
- store a big matrix for all different angles
- combine different layers using equation below



$$\tilde{\mathbf{R}}^t = \mathbf{R}_1^t + \mathbf{T}_1^{bt} (\mathbf{I} - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{R}_2^t \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{R}}^b = \mathbf{R}_1^b + \mathbf{T}_1^{tb} (\mathbf{I} - \mathbf{R}_2^b \mathbf{R}_1^t)^{-1} \mathbf{R}_2^b \mathbf{T}_1^{bt}$$

$$\tilde{\mathbf{T}}^{tb} = \mathbf{T}_2^{tb} (\mathbf{I} - \mathbf{R}_1^b \mathbf{R}_2^t)^{-1} \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{T}}^{bt} = \mathbf{T}_2^{bt} (\mathbf{I} - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{T}_1^{bt}$$

Figure from Wenzel Jakob et al.

# Detail: use a Fourier basis to compress the matrices

- observation: changes in azimuthal direction are usually low frequency

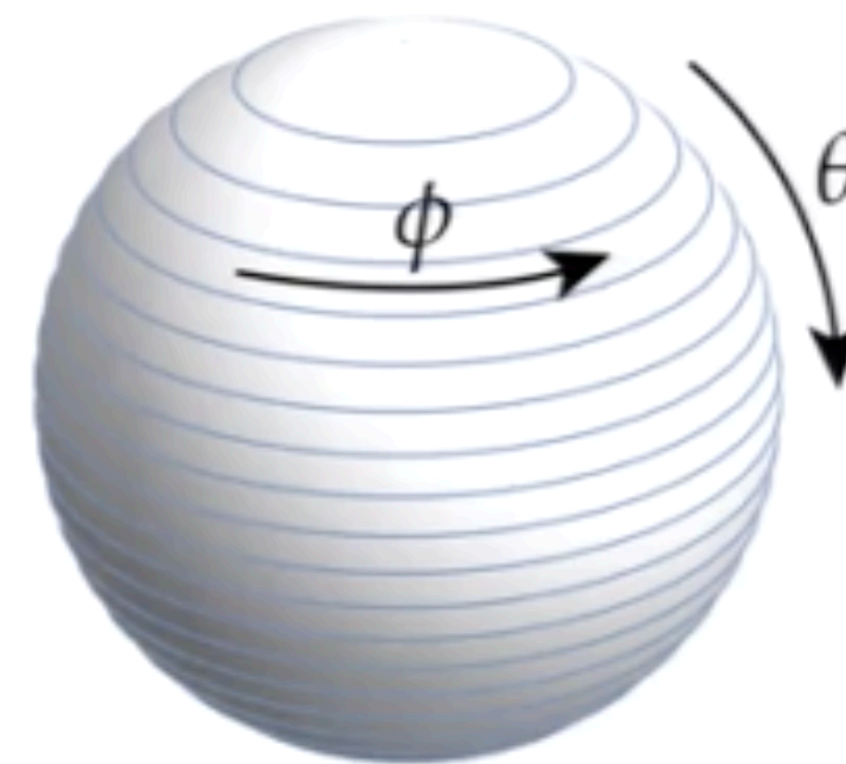
## Chandrasekhar basis

$$L_o(\omega_o) = \int_{S^2} f(\omega_i, \omega_o) L_i(\omega_i) |\cos \theta_i| d\omega_i$$



Discretize in  $\theta$

Fourier transform in  $\phi$



$$\mathbf{L}_o^{(k)} = \mathbf{F}^{(k)} \mathbf{L}_i^{(k)}$$

$n$ -vector    $n \times n$ -matrix    $n$ -vector

the Fourier basis BSDF is implemented in pbrt

[https://www.pbr-book.org/3ed-2018/Reflection\\_Models/Fourier\\_Basis\\_BSDFs](https://www.pbr-book.org/3ed-2018/Reflection_Models/Fourier_Basis_BSDFs)

# Layered-BSDFs match well with real-world materials!



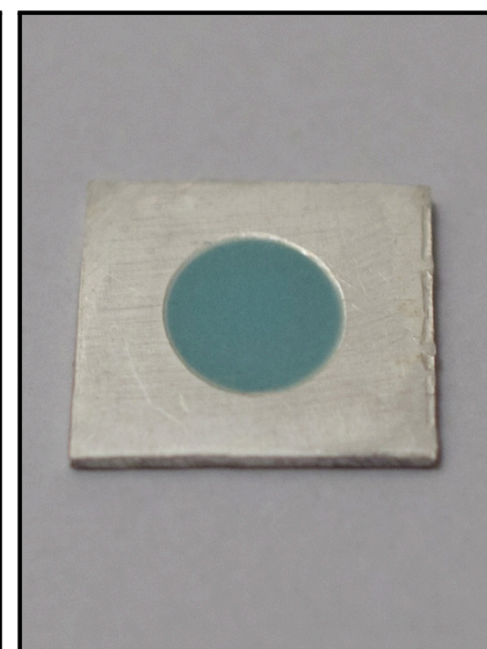
photo



rendering



(a) A sample of jewelry enamels in powdered form



(b) Enamel test swatch

Figure from Wenzel Jakob et al.

[https://rgl.s3.eu-central-1.amazonaws.com/media/papers/Jakob2014Comprehensive\\_1.pdf](https://rgl.s3.eu-central-1.amazonaws.com/media/papers/Jakob2014Comprehensive_1.pdf)

# Downside of this method?

$$\tilde{\mathbf{R}}^t = \mathbf{R}_1^t + \mathbf{T}_1^{bt} (I - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{R}_2^t \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{R}}^b = \mathbf{R}_1^b + \mathbf{T}_1^{tb} (I - \mathbf{R}_2^b \mathbf{R}_1^t)^{-1} \mathbf{R}_2^b \mathbf{T}_1^{bt}$$

$$\tilde{\mathbf{T}}^{tb} = \mathbf{T}_2^{tb} (I - \mathbf{R}_1^b \mathbf{R}_2^t)^{-1} \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{T}}^{bt} = \mathbf{T}_2^{bt} (I - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{T}_1^{bt}$$

# Downside of the direct implementation of adding equation

- need to store a huge sparse matrix (in Fourier domain)
- a few megabytes per BSDF
- texturing is hard

## Chandrasekhar basis

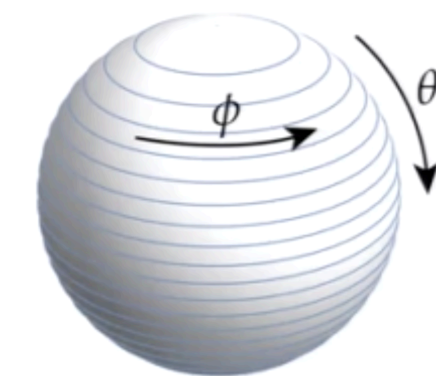
$$L_o(\omega_o) = \int_{S^2} f(\omega_i, \omega_o) L_i(\omega_i) |\cos \theta_i| d\omega_i$$



Discretize in  $\theta$   
Fourier transform in  $\phi$

$$\mathbf{L}_o^{(k)} = \mathbf{F}^{(k)} \mathbf{L}_i^{(k)}$$

$n$ -vector    $n \times n$ -matrix    $n$ -vector



$$\tilde{\mathbf{R}}^t = \mathbf{R}_1^t + \mathbf{T}_1^{bt} (I - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{R}_2^t \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{R}}^b = \mathbf{R}_1^b + \mathbf{T}_1^{tb} (I - \mathbf{R}_2^b \mathbf{R}_1^t)^{-1} \mathbf{R}_2^b \mathbf{T}_1^{bt}$$

$$\tilde{\mathbf{T}}^{tb} = \mathbf{T}_2^{tb} (I - \mathbf{R}_1^b \mathbf{R}_2^t)^{-1} \mathbf{T}_1^{tb}$$

$$\tilde{\mathbf{T}}^{bt} = \mathbf{T}_2^{bt} (I - \mathbf{R}_2^t \mathbf{R}_1^b)^{-1} \mathbf{T}_1^{bt}$$

# Next: a more compact representation

## A Comprehensive Framework for Rendering Layered Materials

Wenzel Jakob Eugene D'Eon Otto Jakob Steve Marschner

In ACM Transactions on Graphics (Proceedings of SIGGRAPH 2014)



## Efficient Rendering of Layered Materials using an Atomic Decomposition with Statistical Operators

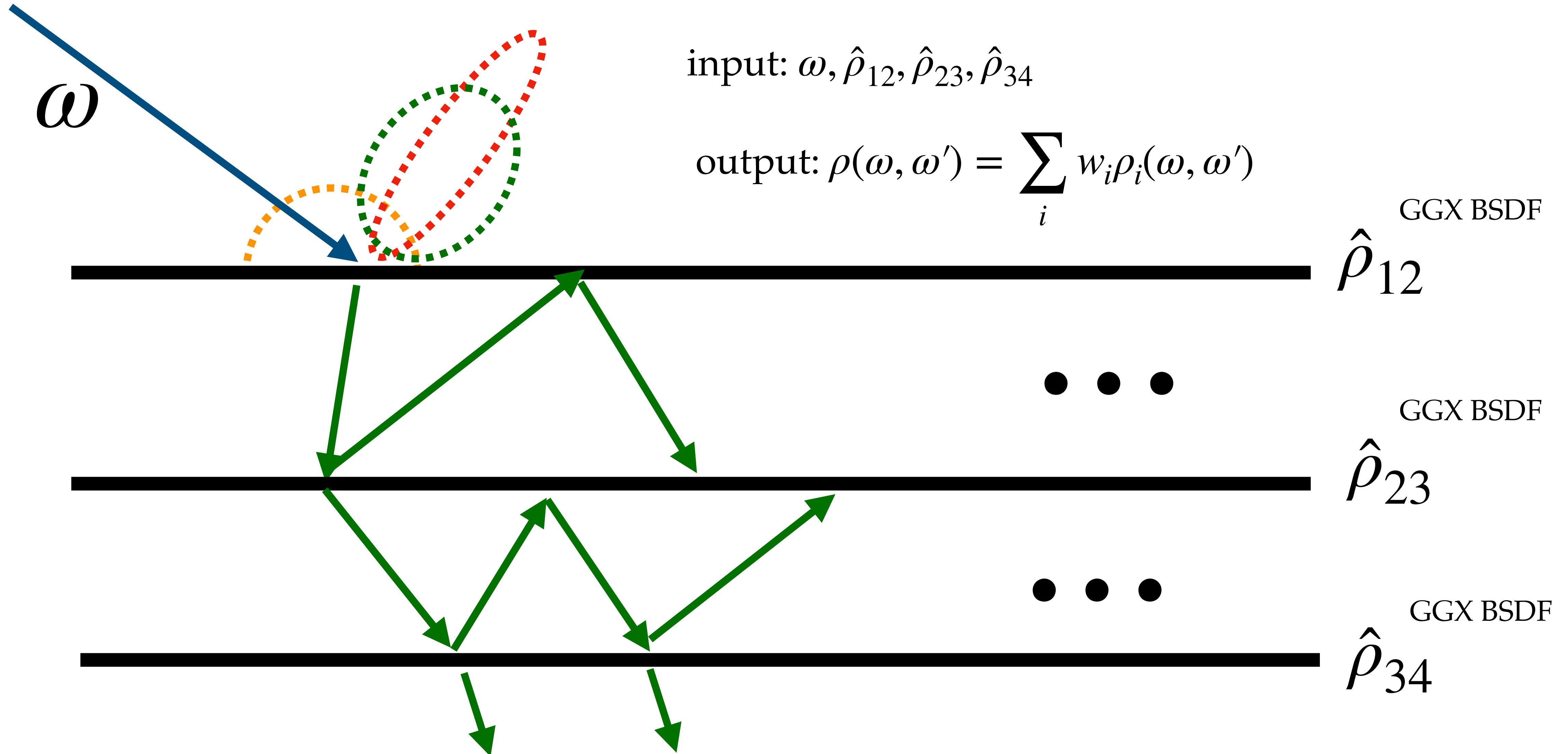
Laurent Belcour (Unity Technologies)

Published in ACM Transactions on Graphics (proc. of SIGGRAPH 2018)

[paper](#) [bib](#) [code](#) [video](#) [slides](#)



# Goal: use sum of GGX lobes to represent multiple scattering between layers



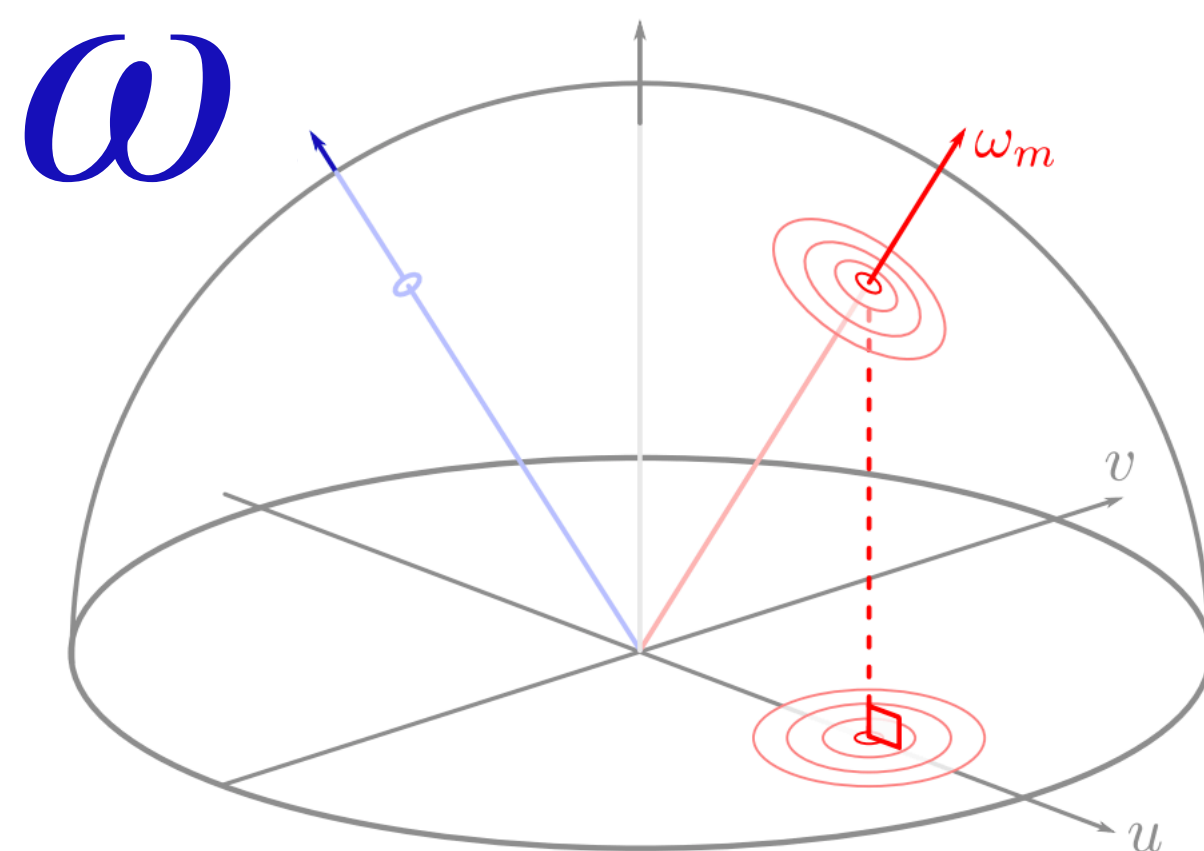
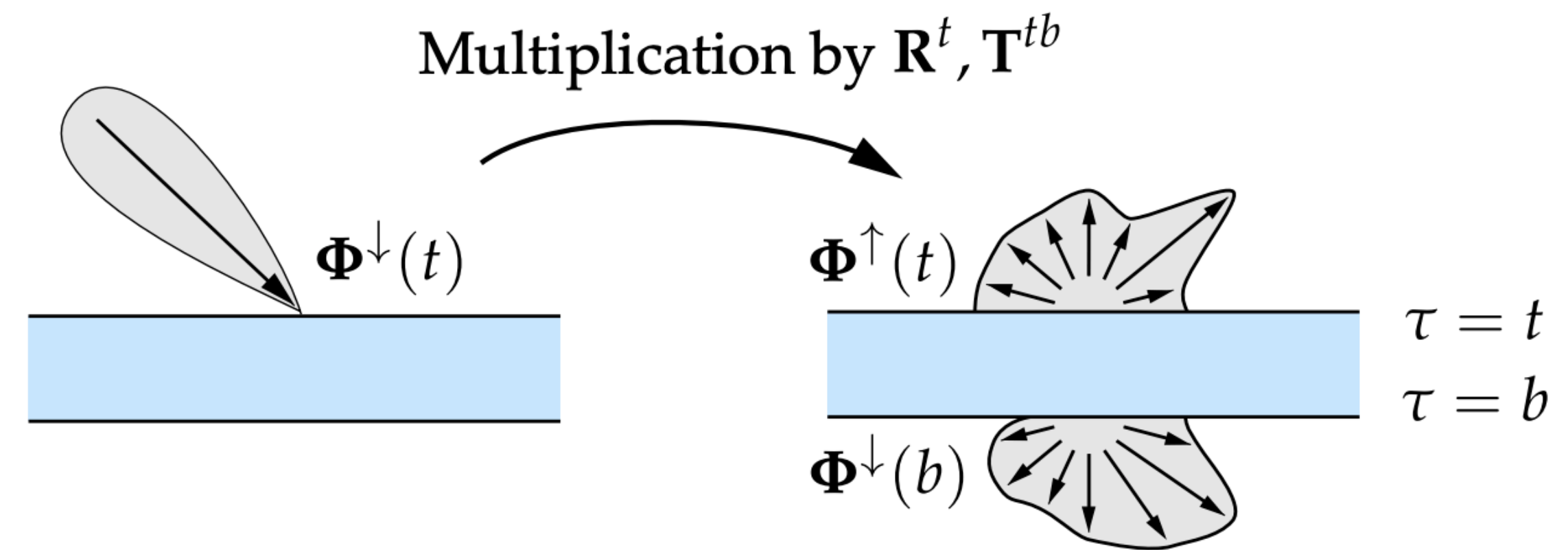


# Idea: approximate the R&T matrices using Gaussians for a given direction $\omega$

for a given direction, the matrix vector multiplication falls back to the scalar case

$$\Phi^\uparrow(t) = \mathbf{R}^t \Phi^\downarrow(t) + \mathbf{T}^{bt} \Phi^\uparrow(b)$$

$$\Phi^\downarrow(b) = \mathbf{R}^b \Phi^\uparrow(b) + \mathbf{T}^{tb} \Phi^\downarrow(t)$$

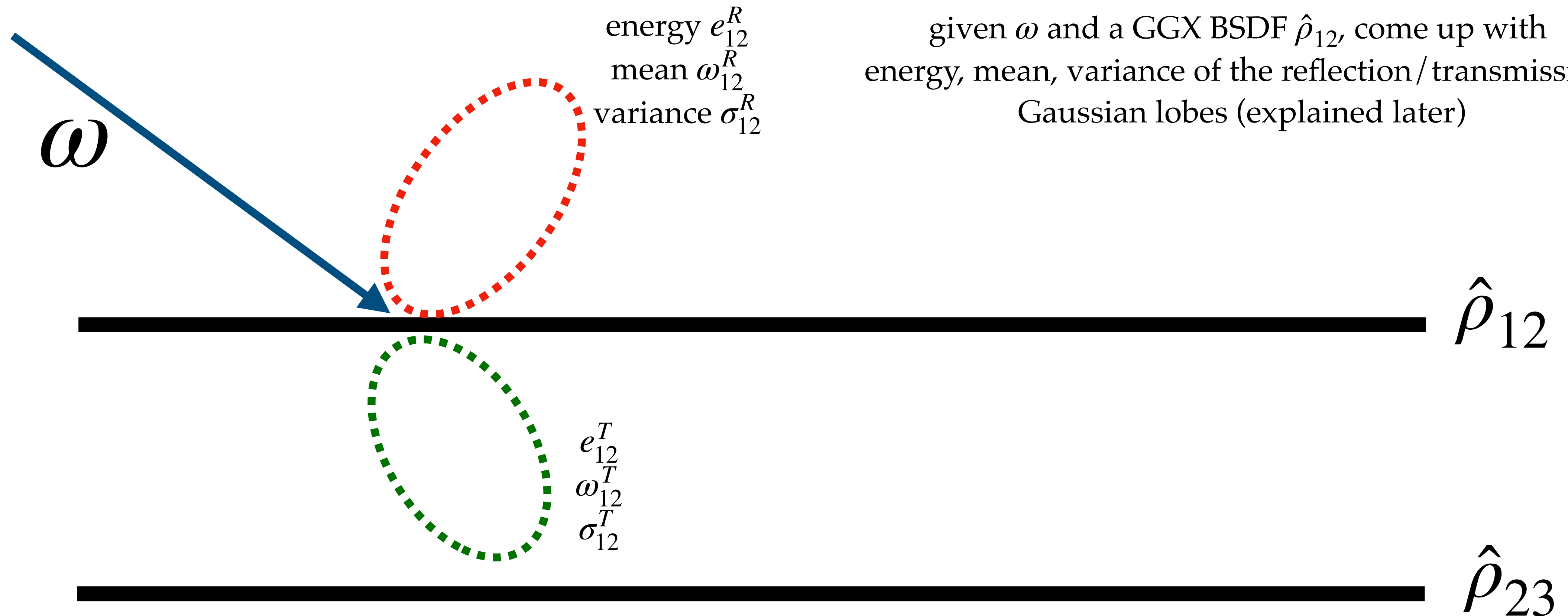


in practice, put the Gaussian in the projected disk (slope space)

input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

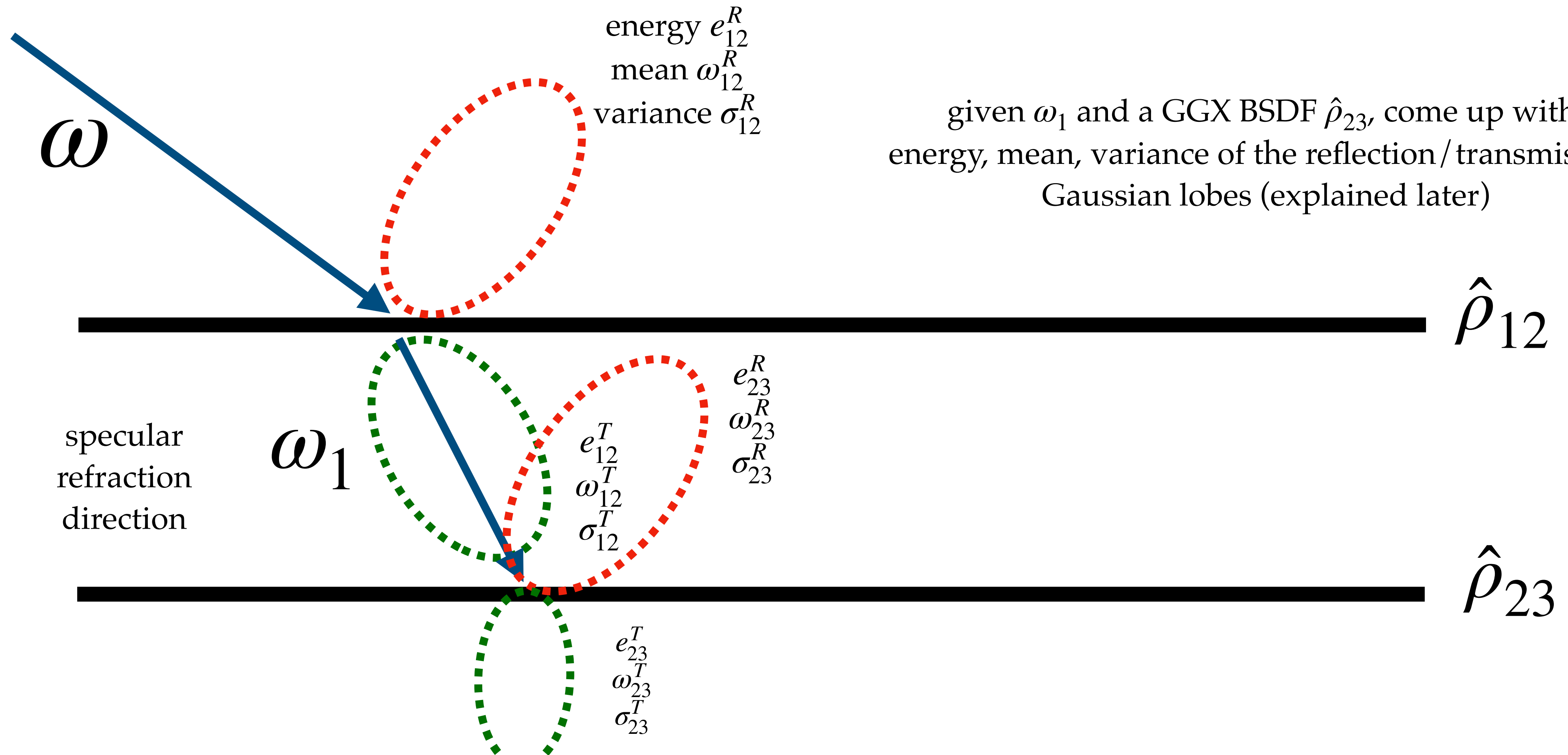


input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$



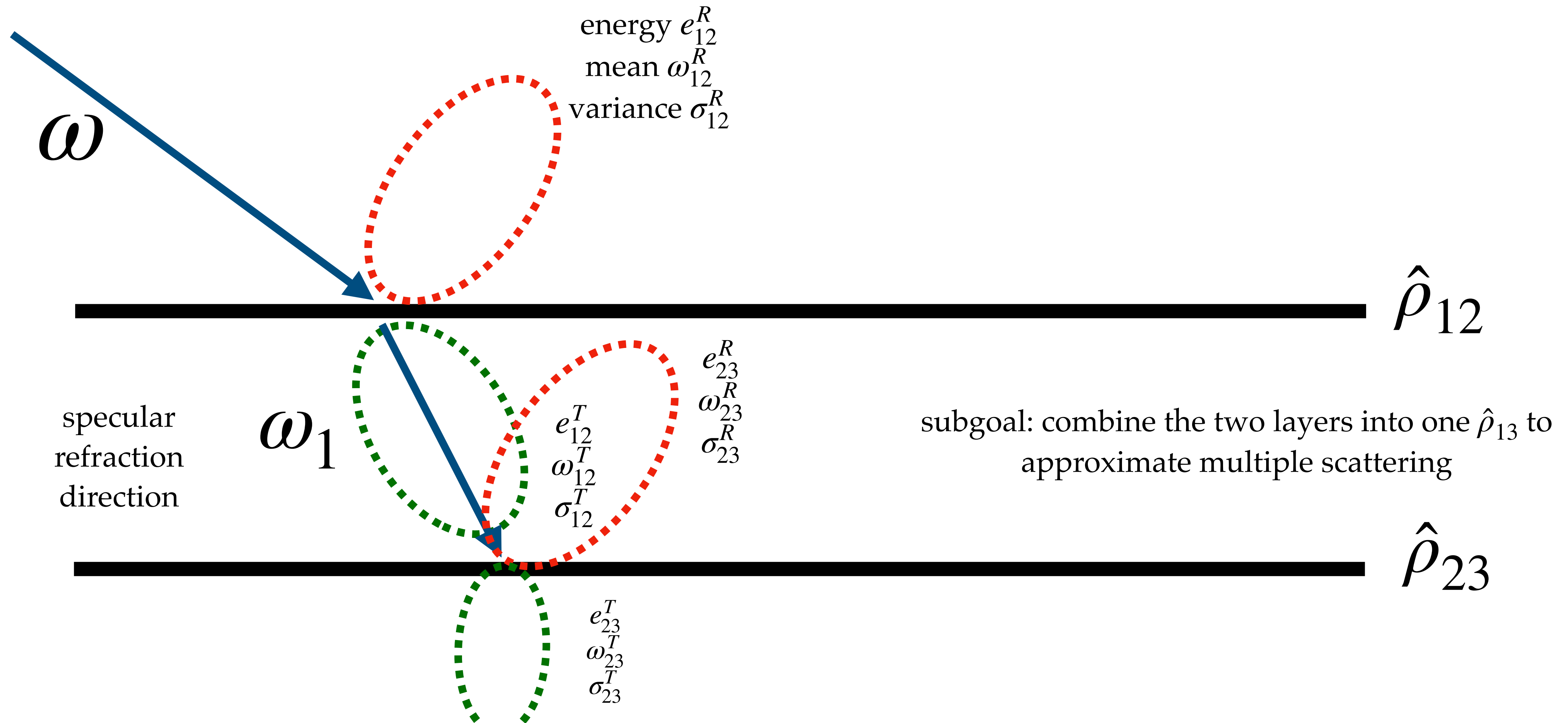
given  $\omega_1$  and a GGX BSDF  $\hat{\rho}_{23}$ , come up with energy, mean, variance of the reflection/transmission  
Gaussian lobes (explained later)

input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$



input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

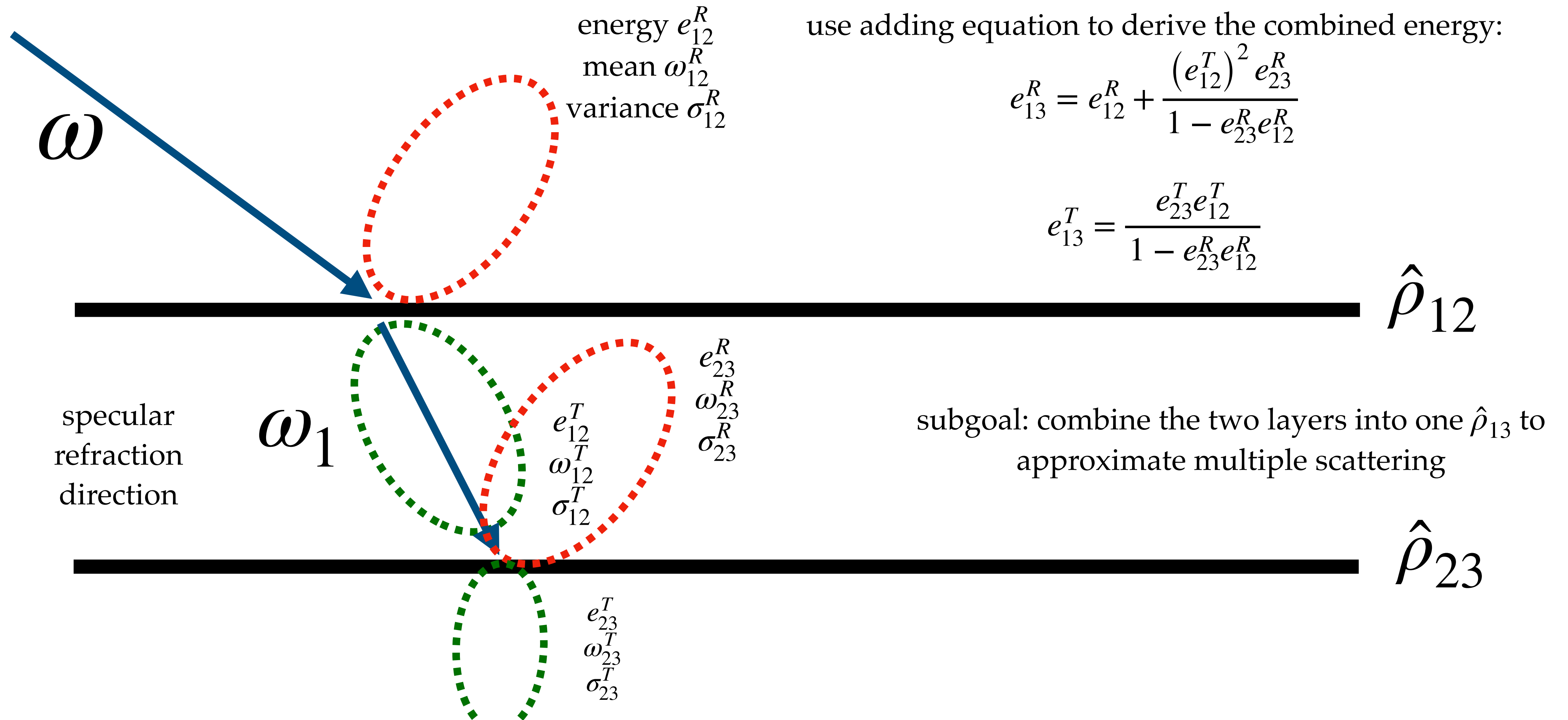
# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$

use adding equation to derive the combined energy:

$$e_{13}^R = e_{12}^R + \frac{(e_{12}^T)^2 e_{23}^R}{1 - e_{23}^R e_{12}^R}$$

$$e_{13}^T = \frac{e_{23}^T e_{12}^T}{1 - e_{23}^R e_{12}^R}$$

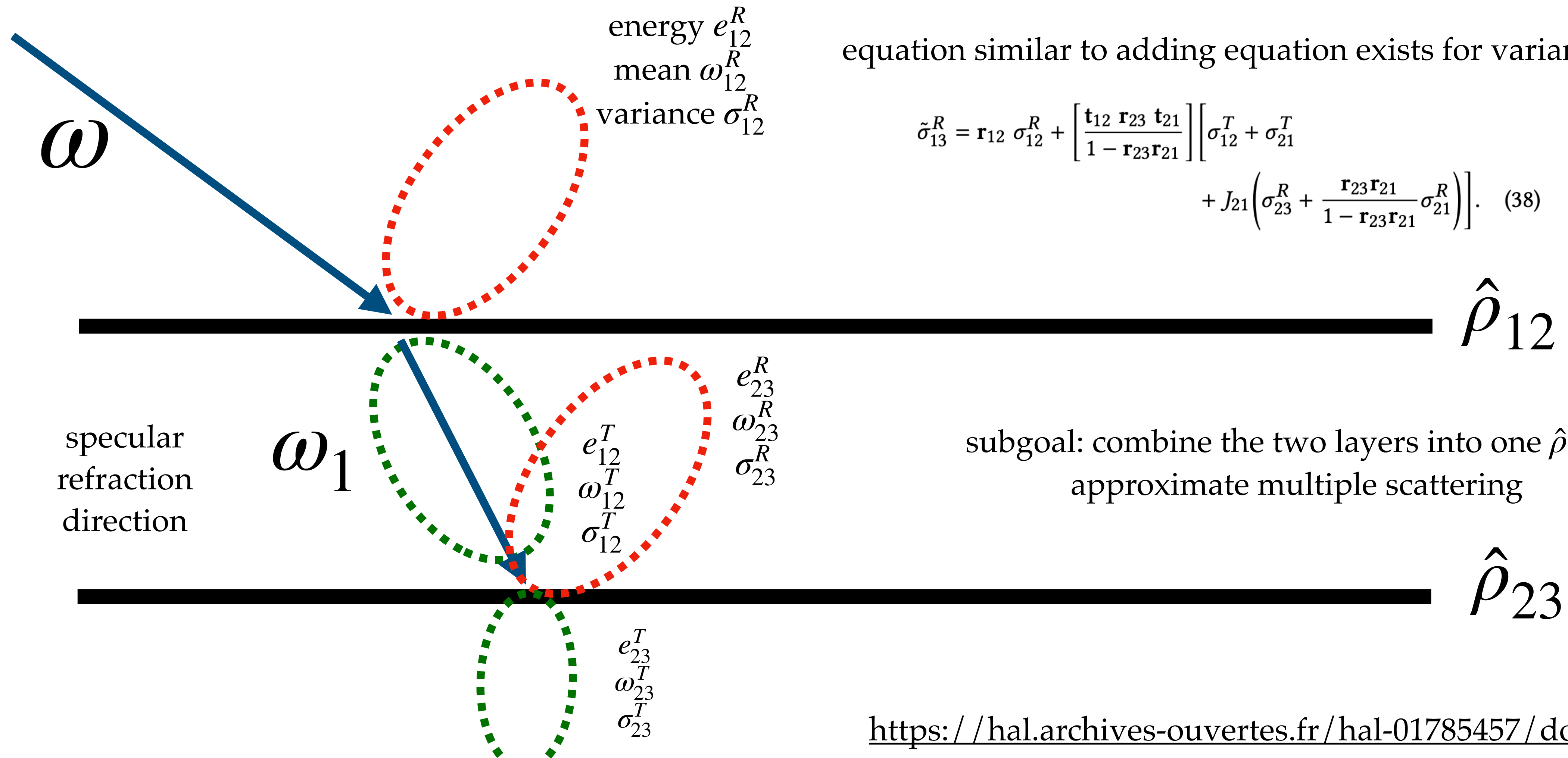


input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$



equation similar to adding equation exists for variance

$$\tilde{\sigma}_{13}^R = r_{12} \sigma_{12}^R + \left[ \frac{t_{12} \mathbf{r}_{23} t_{21}}{1 - \mathbf{r}_{23} \mathbf{r}_{21}} \right] \left[ \sigma_{12}^T + \sigma_{21}^T + J_{21} \left( \sigma_{23}^R + \frac{\mathbf{r}_{23} \mathbf{r}_{21}}{1 - \mathbf{r}_{23} \mathbf{r}_{21}} \sigma_{21}^R \right) \right]. \quad (38)$$

subgoal: combine the two layers into one  $\hat{\rho}_{13}$  to approximate multiple scattering

input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

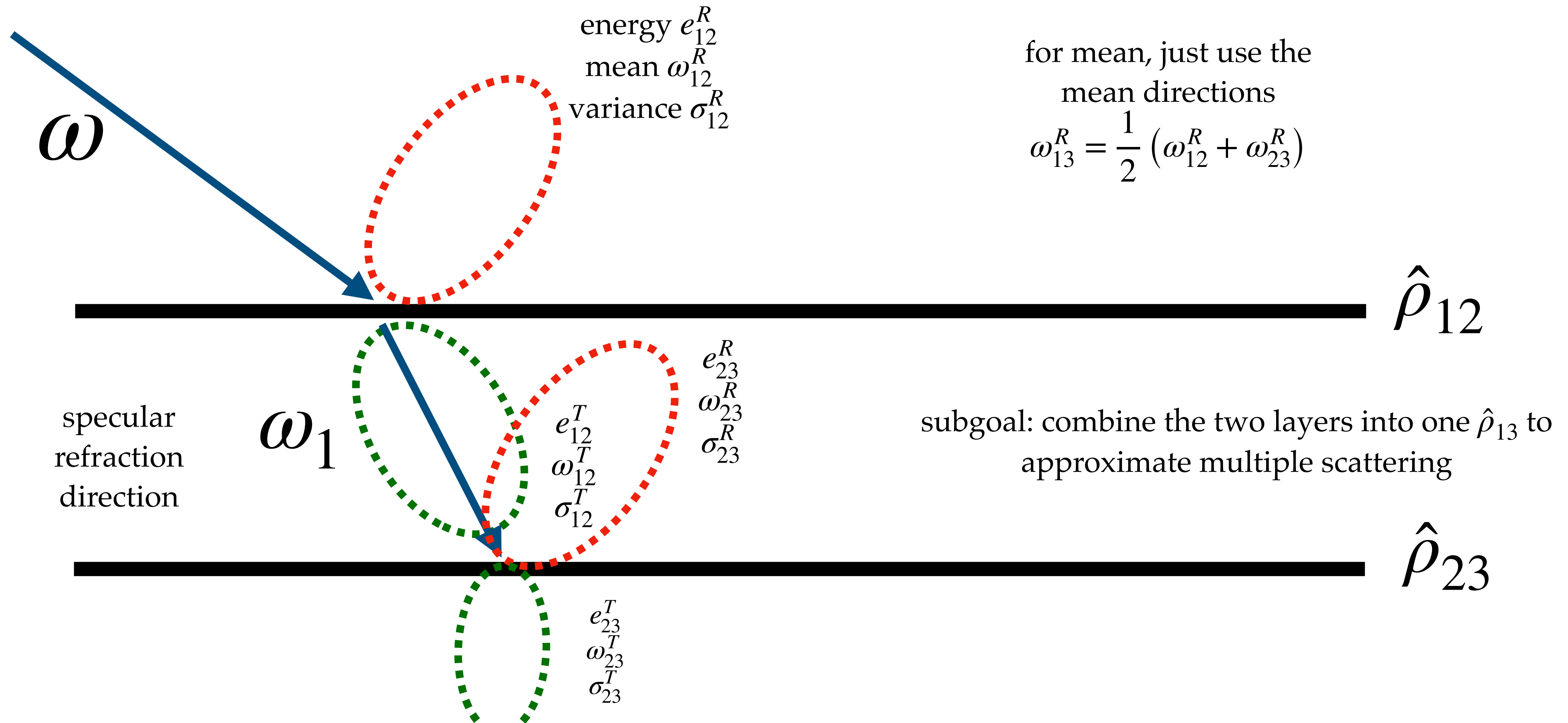
output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$

for mean, just use the mean directions

$$\omega_{13}^R = \frac{1}{2} (\omega_{12}^R + \omega_{23}^R)$$

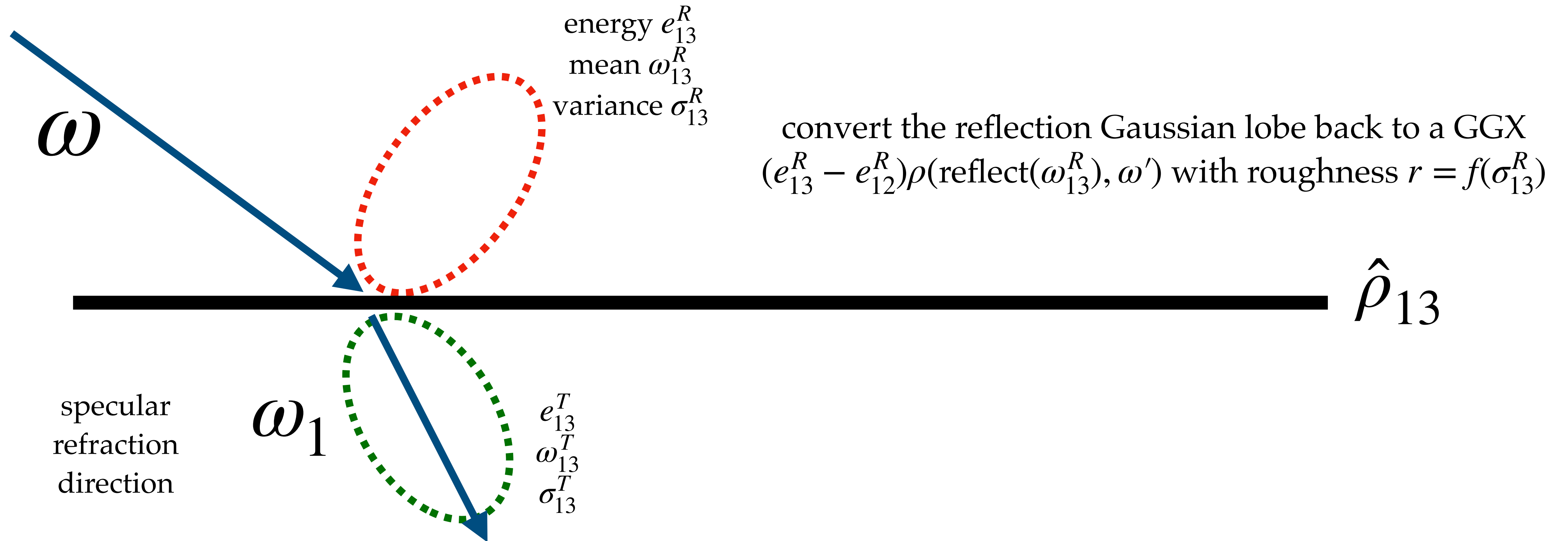


input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$





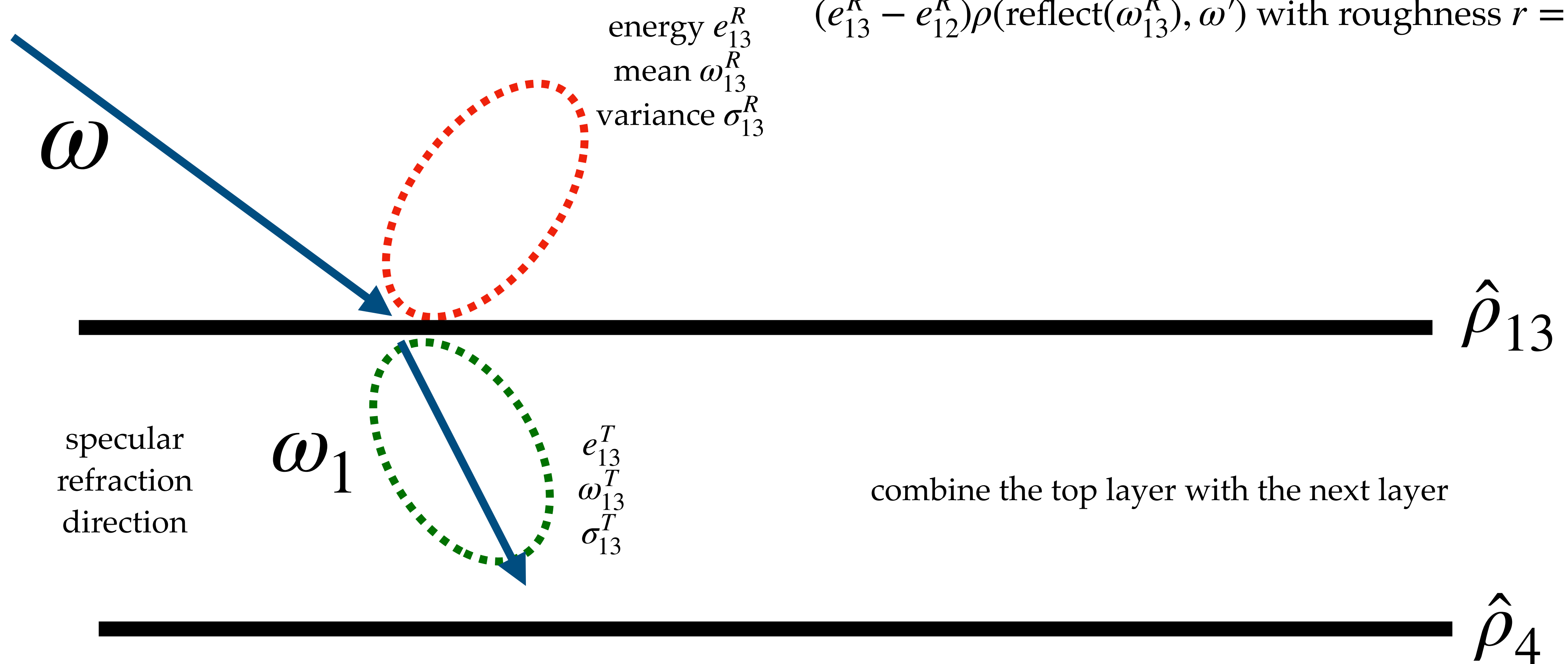
input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$

$(e_{13}^R - e_{12}^R) \rho(\text{reflect}(\omega_{13}^R), \omega')$  with roughness  $r = f_R(\sigma_{13}^R)$



input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

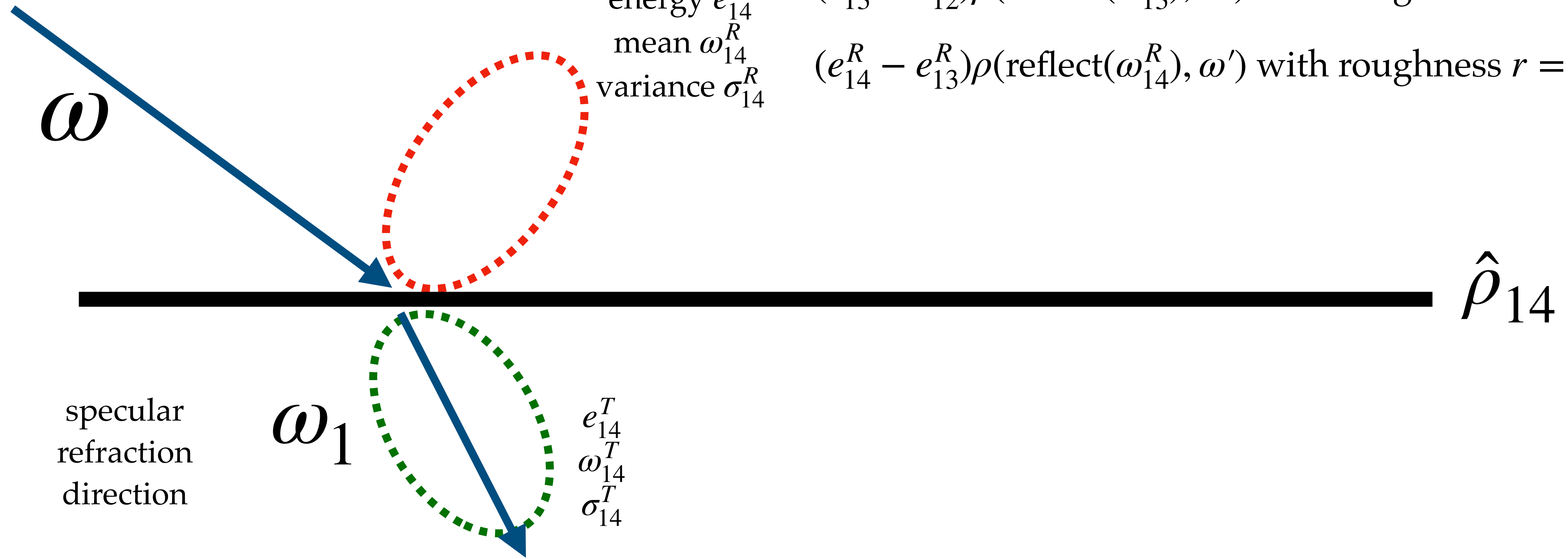
output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

# Belcour's algorithm

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$

$(e_{13}^R - e_{12}^R) \rho(\text{reflect}(\omega_{13}^R), \omega')$  with roughness  $r = f_R(\sigma_{13}^R)$

$(e_{14}^R - e_{13}^R) \rho(\text{reflect}(\omega_{14}^R), \omega')$  with roughness  $r = f_R(\sigma_{14}^R)$



# Belcour's algorithm

input:  $\omega, \hat{\rho}_{12}, \hat{\rho}_{23}, \hat{\rho}_{34}$

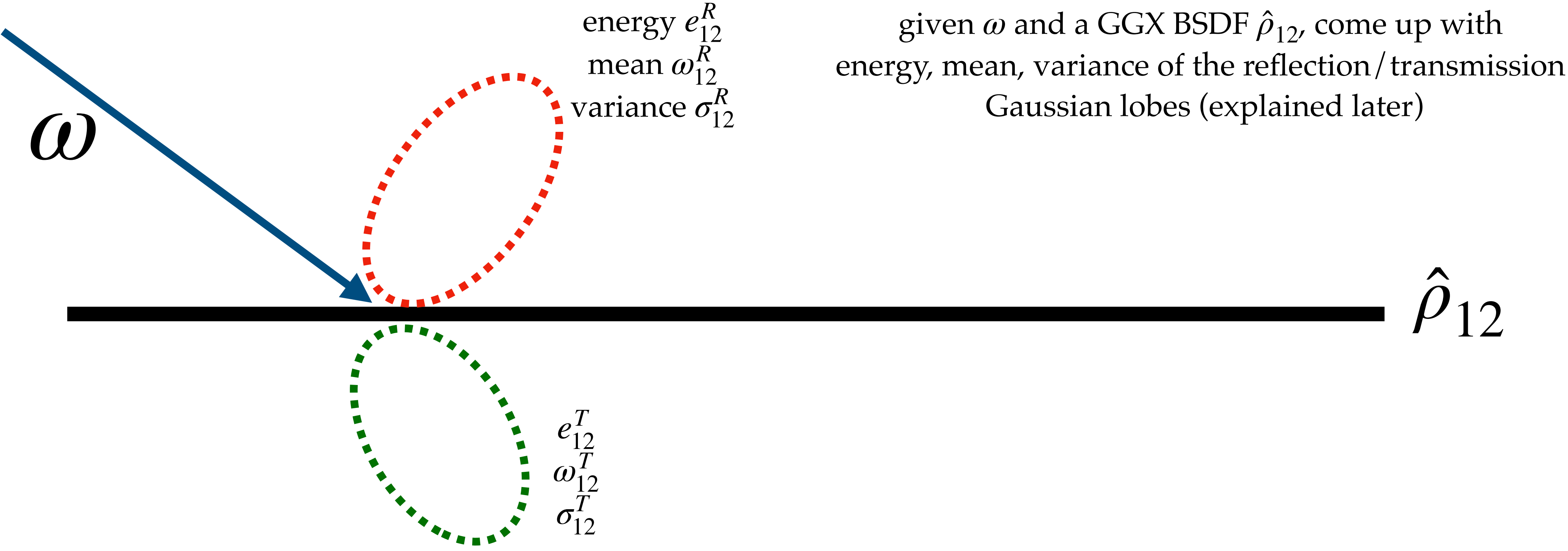
output:  $\rho(\omega, \omega') = \sum_i w_i \rho_i(\omega, \omega')$

$e_{12}^R \rho(\omega, \omega')$  with roughness  $r = f_R(\sigma_{12}^R)$

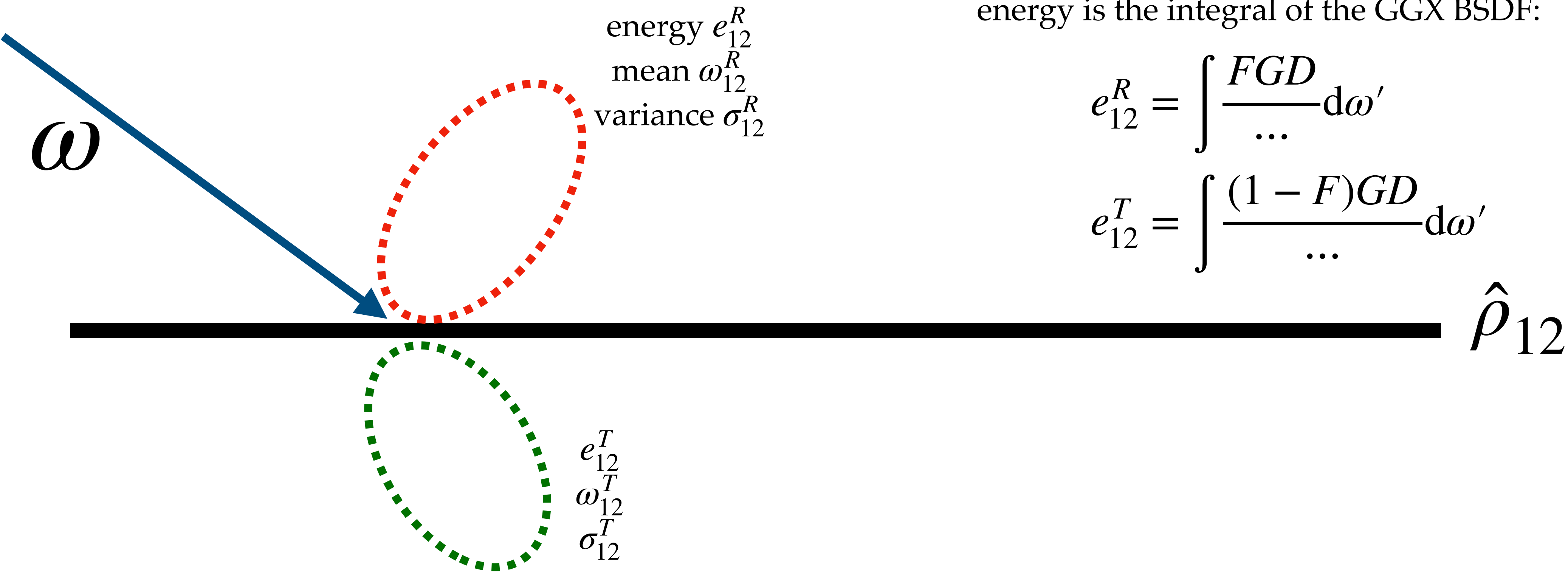
$(e_{13}^R - e_{12}^R) \rho(\text{reflect}(\omega_{13}^R), \omega')$  with roughness  $r = f_R(\sigma_{13}^R)$

$(e_{14}^R - e_{13}^R) \rho(\text{reflect}(\omega_{14}^R), \omega')$  with roughness  $r = f_R(\sigma_{14}^R)$

# How do obtain the Gaussian lobes given GGX?



# How do obtain the Gaussian lobes given GGX?

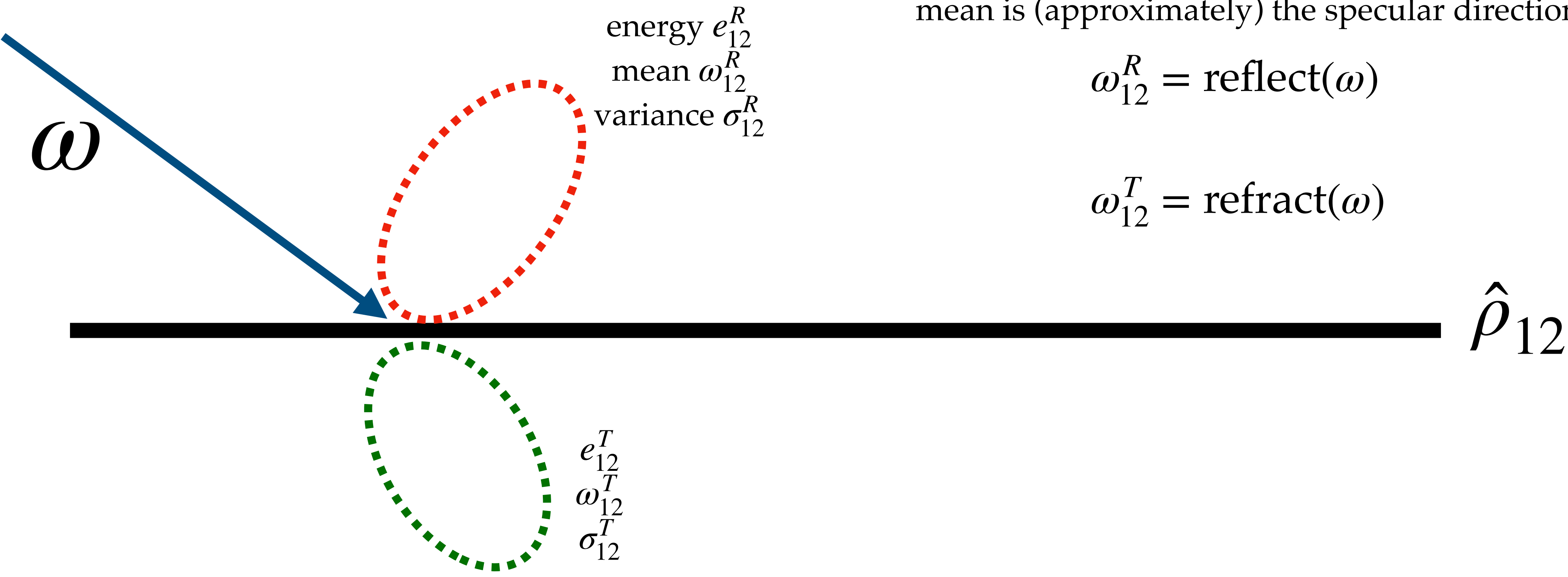


energy is the integral of the GGX BSDF:

$$e_{12}^R = \int \frac{FGD}{\dots} d\omega'$$

$$e_{12}^T = \int \frac{(1 - F)GD}{\dots} d\omega'$$

# How do obtain the Gaussian lobes given GGX?

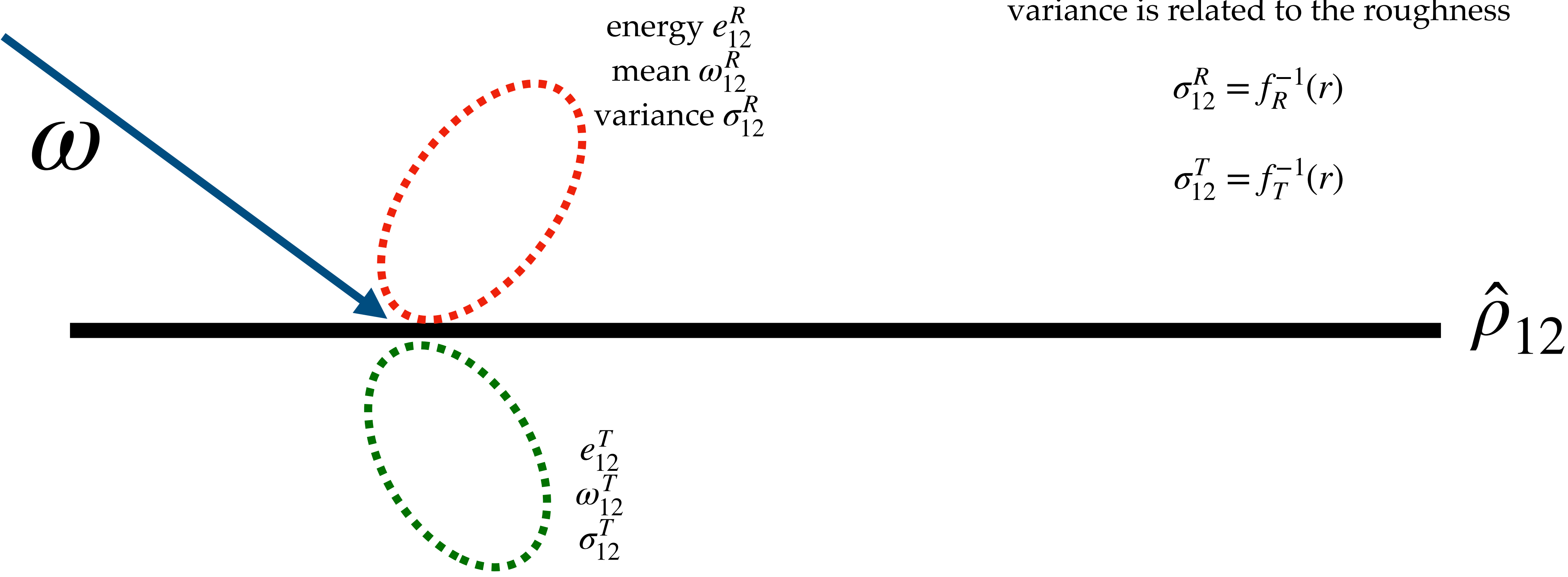


mean is (approximately) the specular directions:

$$\omega_{12}^R = \text{reflect}(\omega)$$

$$\omega_{12}^T = \text{refract}(\omega)$$

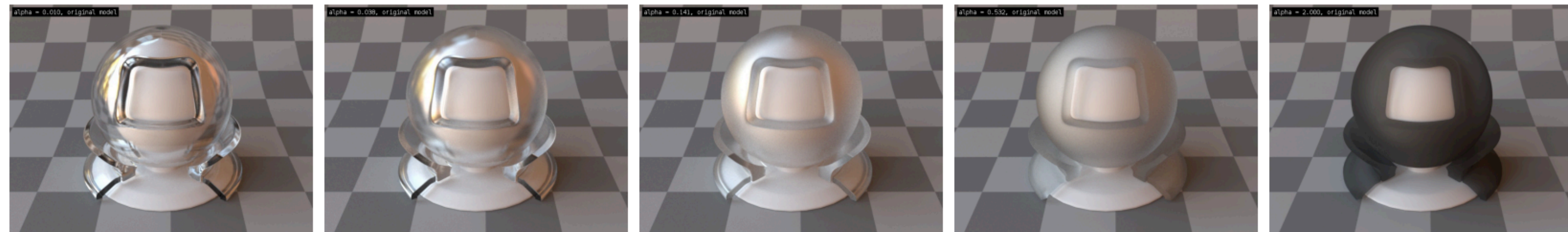
# How do obtain the Gaussian lobes given GGX?



# Detail: important to consider multiple-scattering inside microfacet models as well

increased roughness

single-scattering  
microfacet



(a)  $\alpha = 0.01$

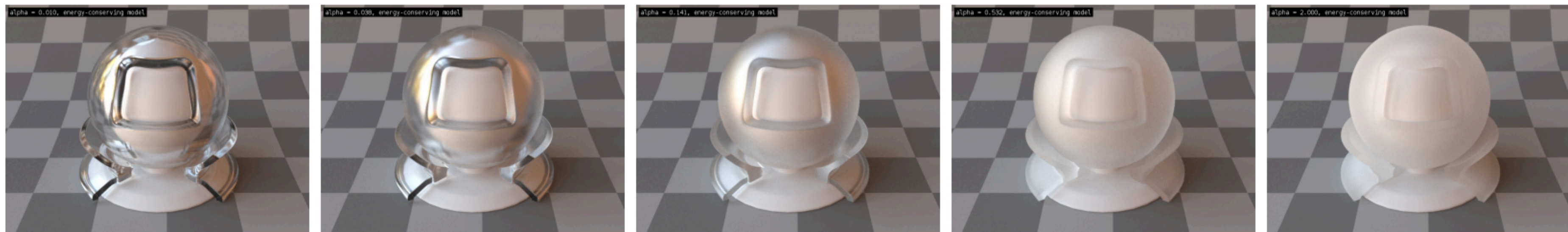
(b)  $\alpha = 0.04$

(c)  $\alpha = 0.14$

(d)  $\alpha = 0.53$

(e)  $\alpha = 2$

multiple-scattering  
microfacet



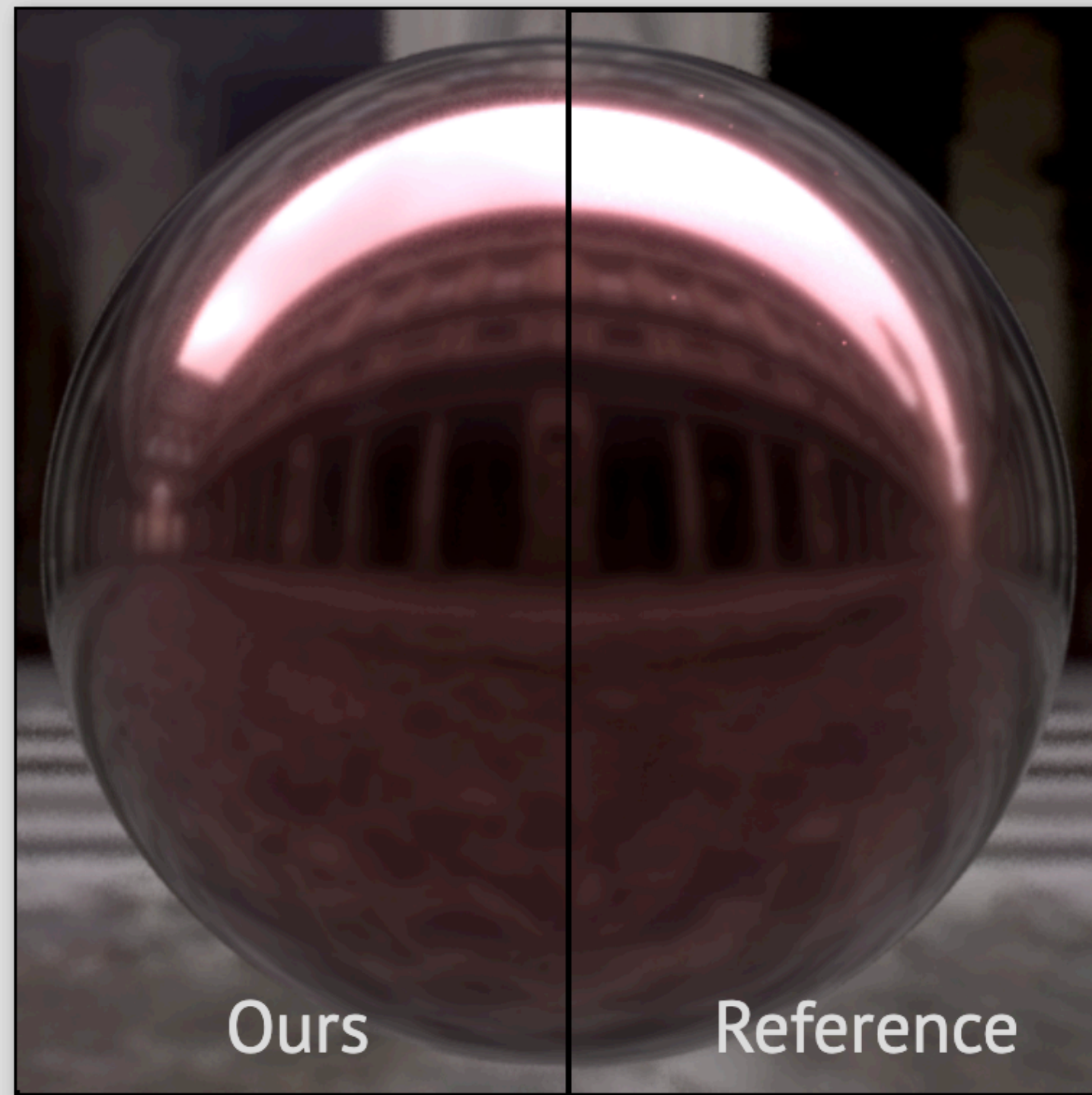
images from Wenzel Jakob et al.

[https://rgl.s3.eu-central-1.amazonaws.com/media/papers/Jakob2014Comprehensive\\_1.pdf](https://rgl.s3.eu-central-1.amazonaws.com/media/papers/Jakob2014Comprehensive_1.pdf)

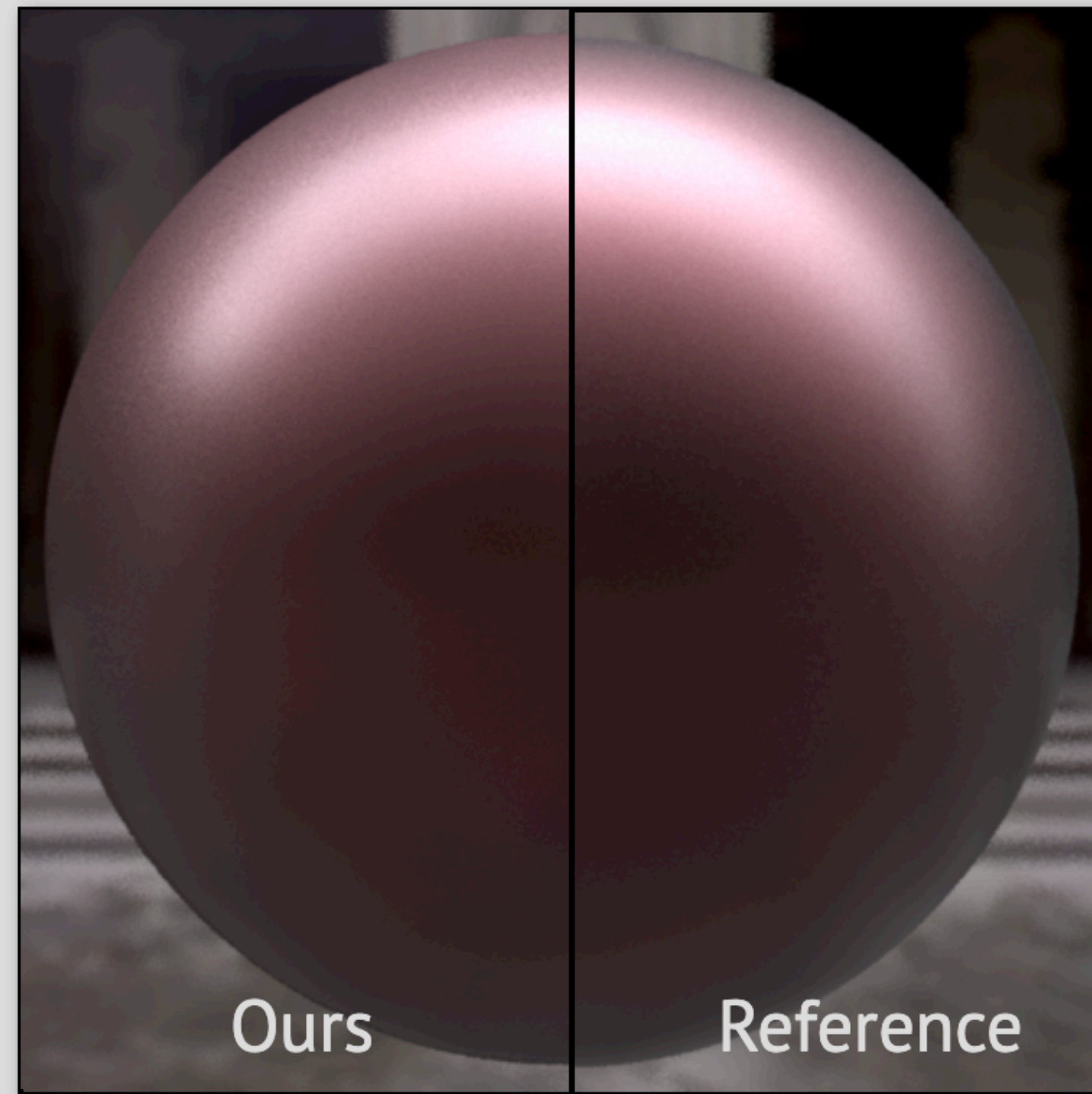
See Heitz et al. "Multiple-Scattering Microfacet BSDFs with the Smith Model"



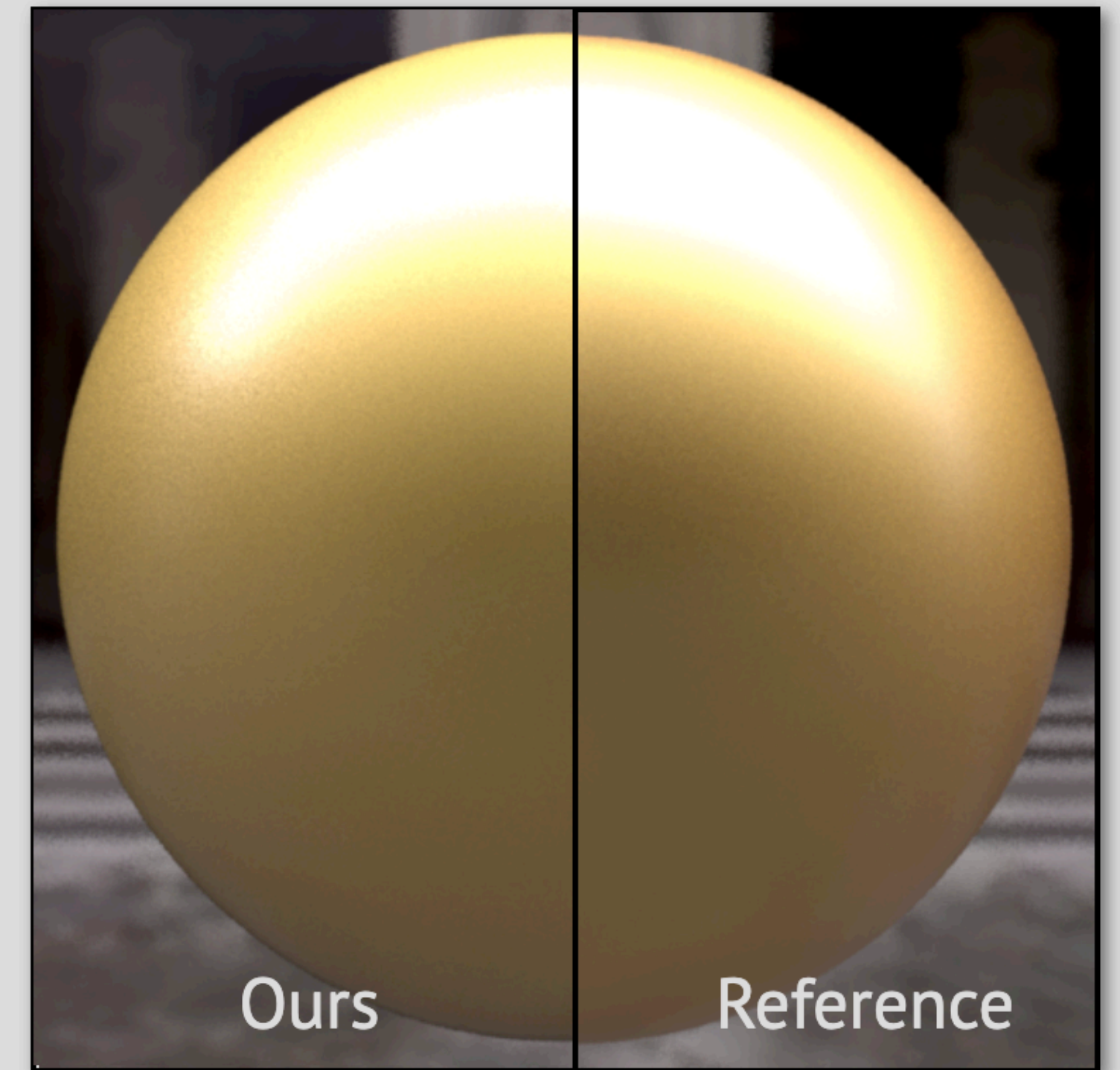
# Belcour's algorithm produces very tight approximation



Metal foil

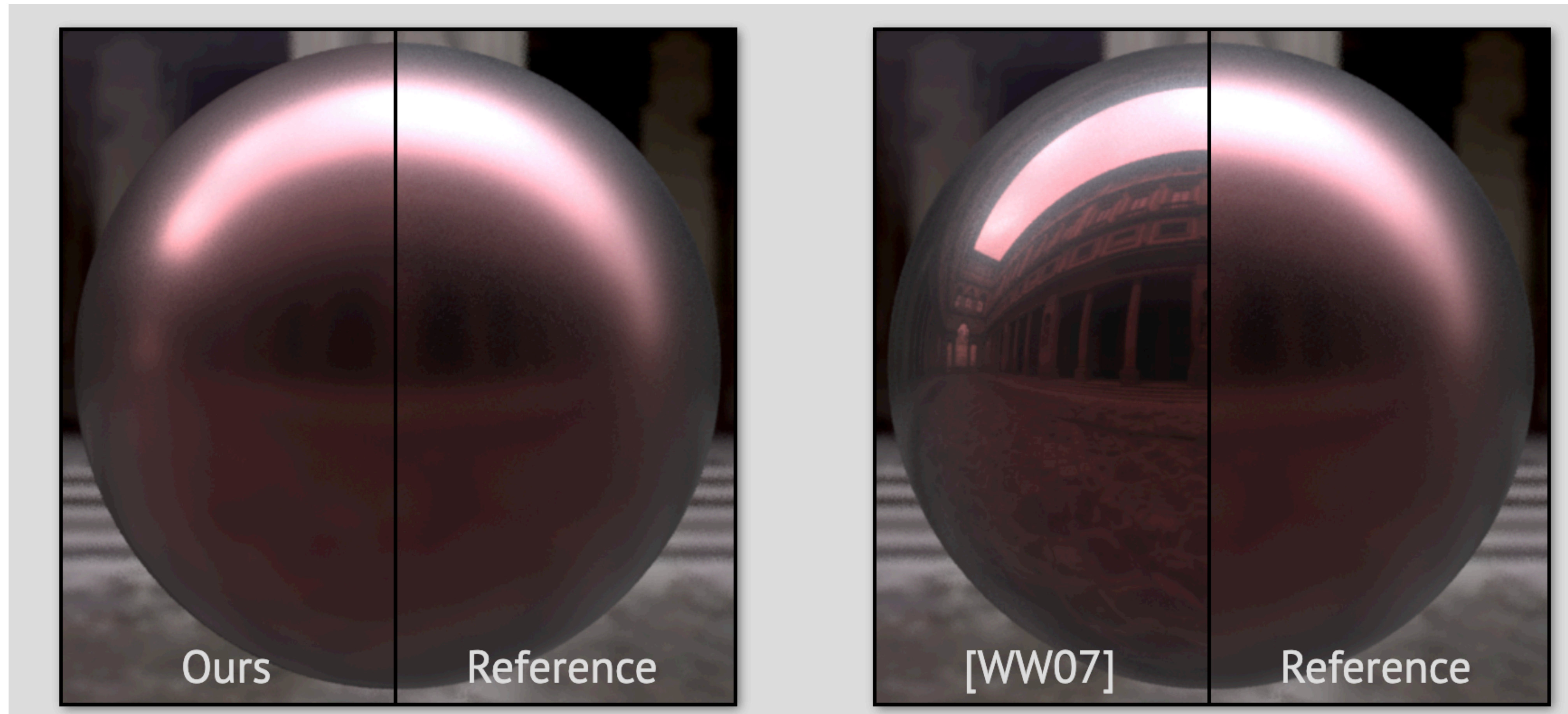


Rough metal



Gold Coated

# More accurate than a more direct blending of BSDFs



# Runs in realtime!

BEETLE scene

Material editing session

Nvidia GeForce GTX 980



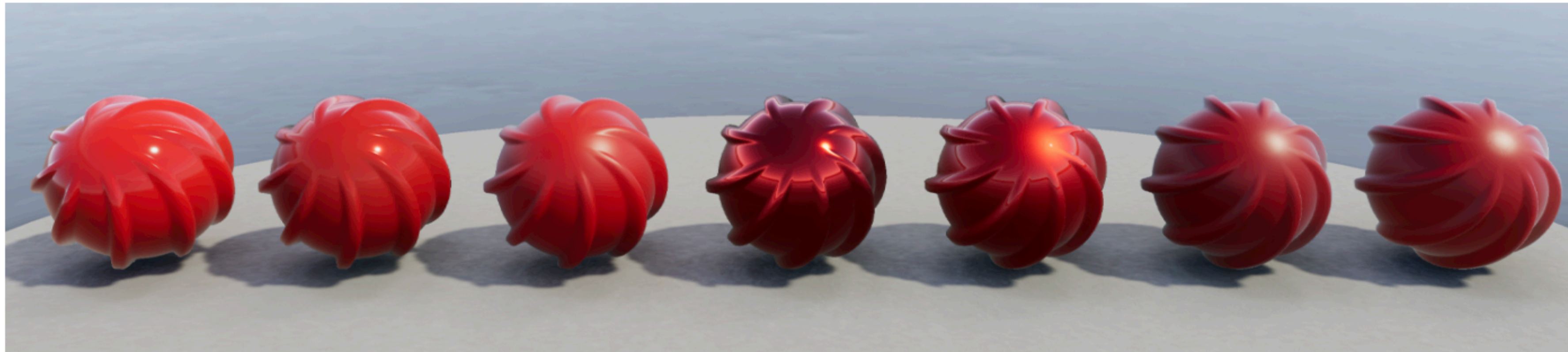
movie from Laurent Belcour

<https://belcour.github.io/blog/slides/2018-brdf-realtime-layered/slides.html#/24>

# Implemented in Unity

## StackLit

The StackLit Master Stack can render materials that are more complex than the [Lit Master Stack](#). It includes all the features available in the Lit shader and, in some cases, provides more advanced or higher quality versions. For example, it uses a more advanced form of specular occlusion and also calculates anisotropic reflections for area lights in the same way the Lit shader does for other light types. It also takes into account light interactions between two vertically stacked physical layers, along with a more complex looking general base layer.

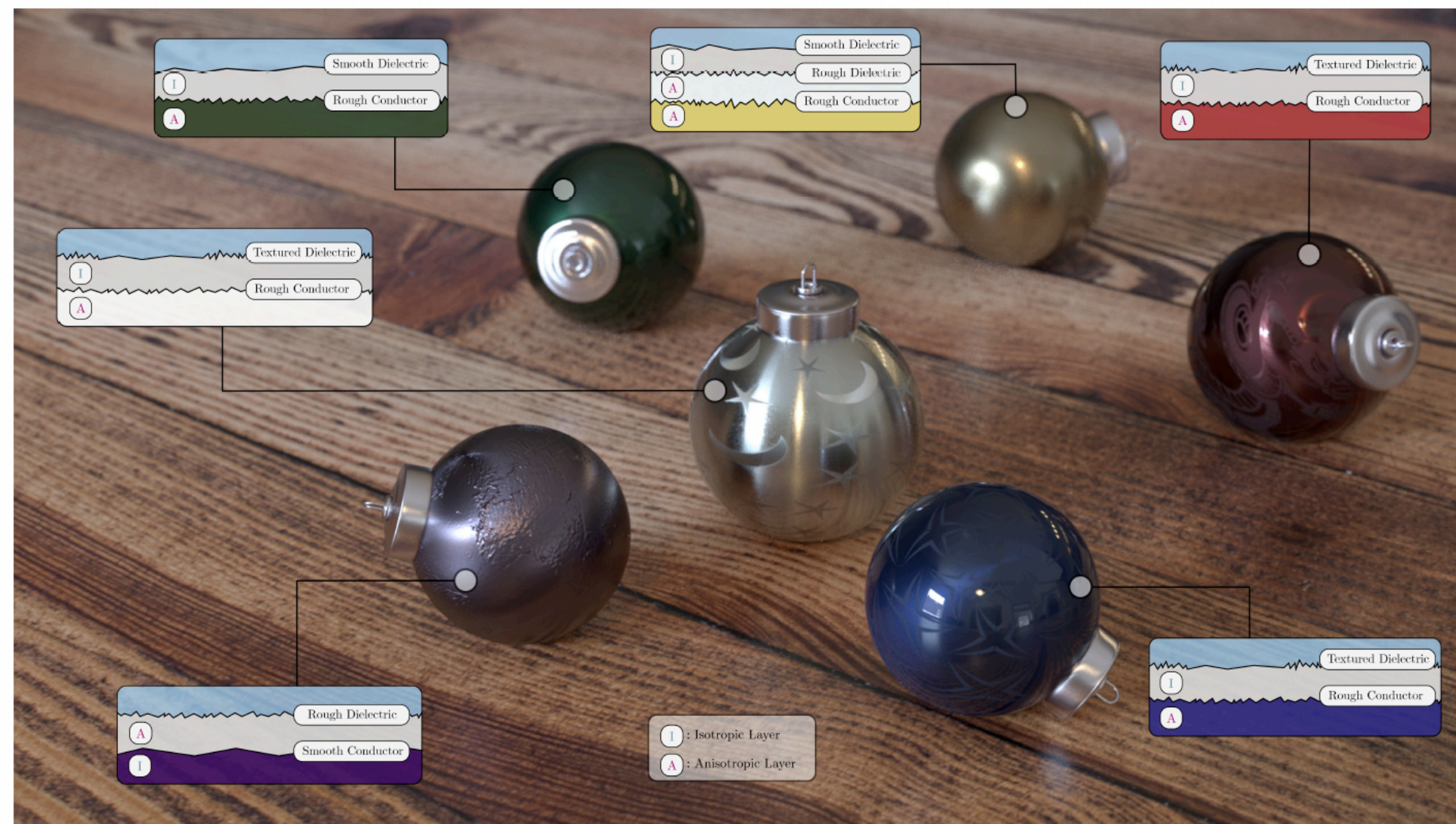


# Easily generalizable to anisotropic GGX

## Rendering Layered Materials with Anisotropic Interfaces

Philippe Weier  
Unity Technologies, EPFL

Laurent Belcour  
Unity Technologies



# Somewhat generalizable to diffuse BSDFs

## Rendering Layered Materials with Diffuse Interfaces

Heloise de Dinechin  
Unity Technologies, EPFL

Laurent Belcour  
Unity Technologies



# Next: Monte Carlo simulation

## A Comprehensive Framework for Rendering Layered Materials

Wenzel Jakob Eugene D'Eon Otto Jakob Steve Marschner

In ACM Transactions on Graphics (Proceedings of SIGGRAPH 2014)



Efficient Rendering of Layered Materials using an Atomic Decomposition with Statistical Operators

Laurent Belcour (Unity Technologies)

Published in ACM Transactions on Graphics (proc. of SIGGRAPH 2018)

[paper](#) [bib](#) [code](#) [video](#) [slides](#)



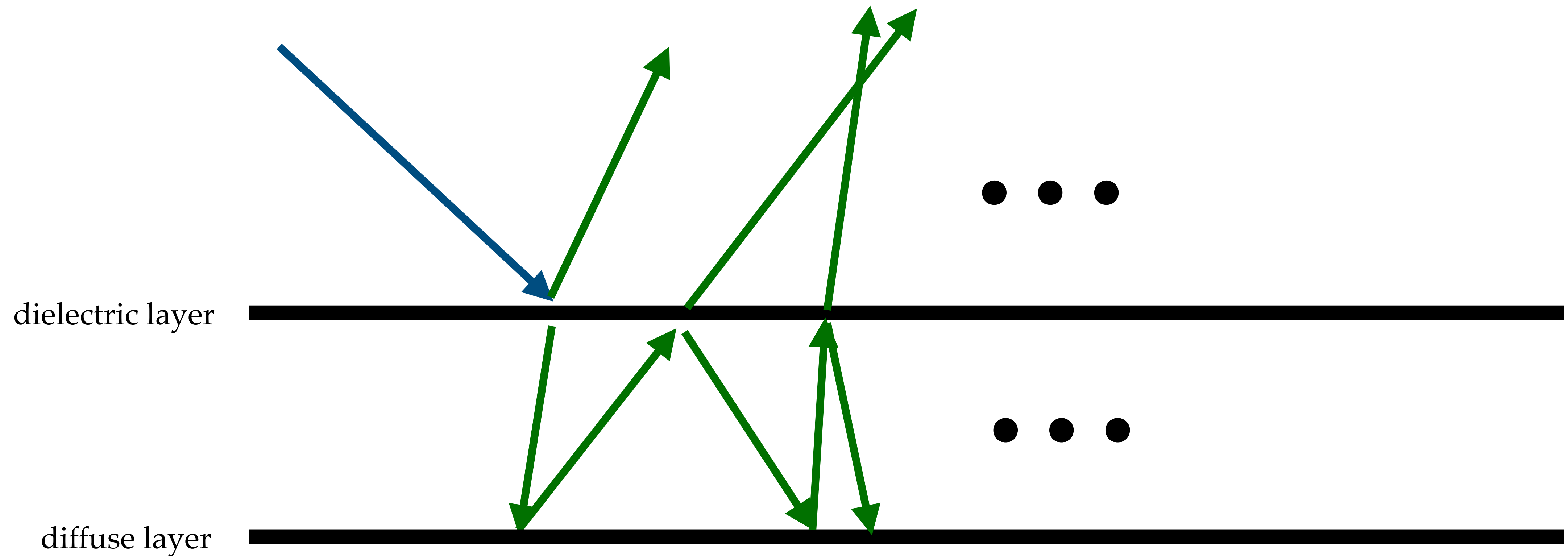
### Position-Free Monte Carlo Simulation for Arbitrary Layered BSDFs

Yu Guo<sup>1</sup>, Miloš Hašan<sup>2</sup>, Shuang Zhao<sup>1</sup>  
<sup>1</sup>University of California, Irvine <sup>2</sup>Autodesk

ACM Transactions on Graphics (SIGGRAPH Asia 2018), 37(6), 2018

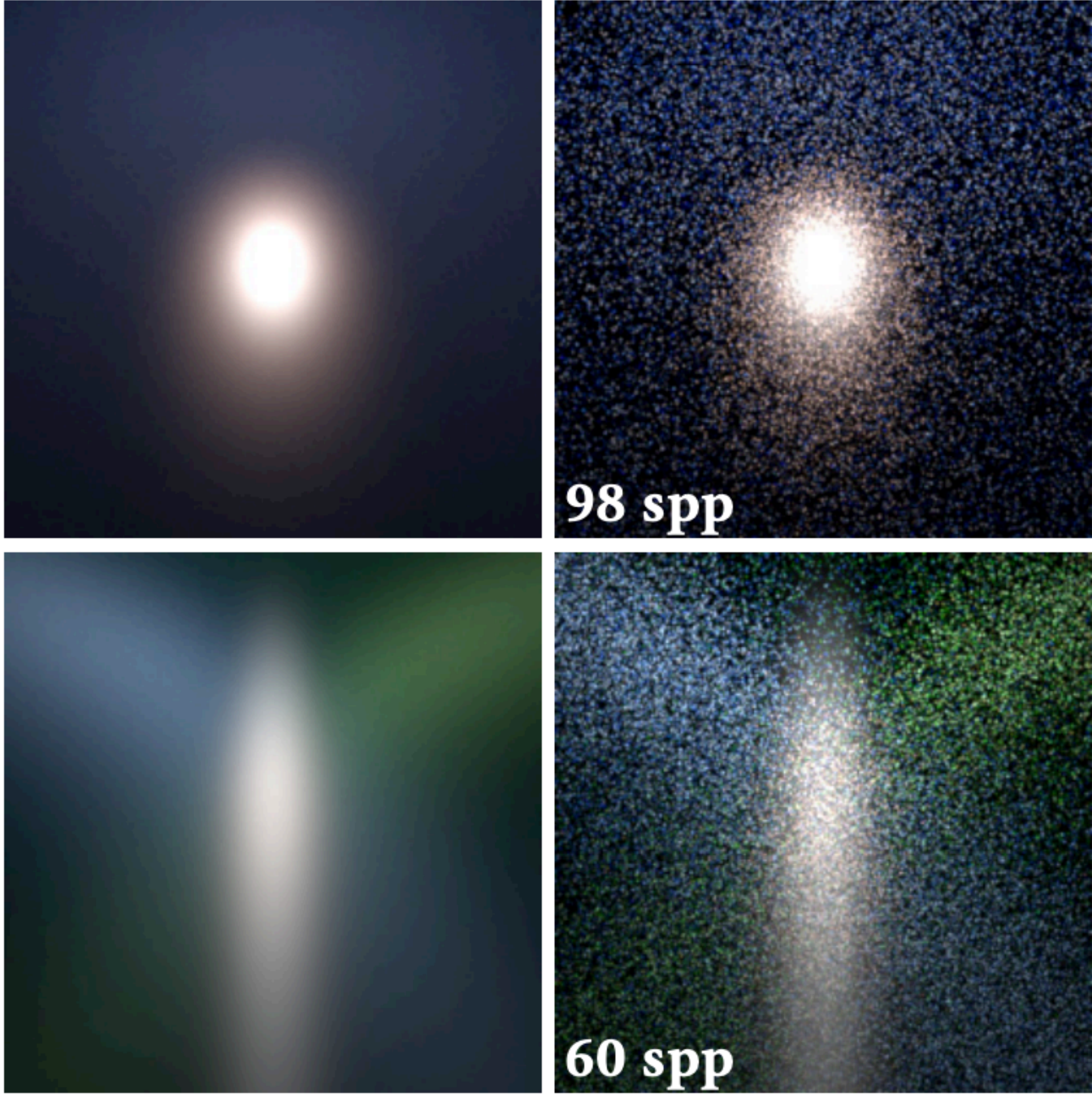


Idea: just compute the multiple scattering using Monte Carlo integration!





# Brute-force simulation is still too slow



Reference

Standard PT

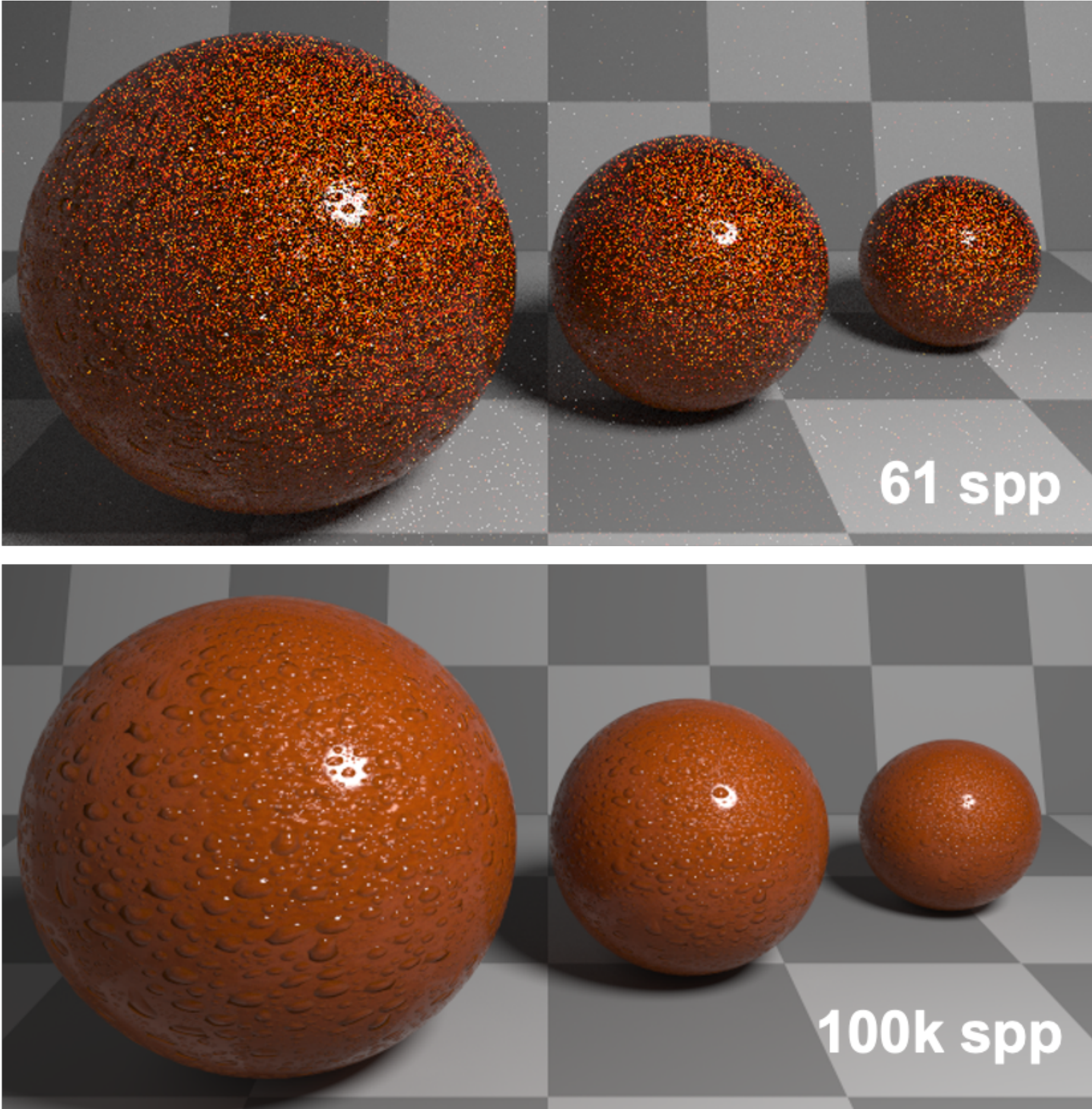
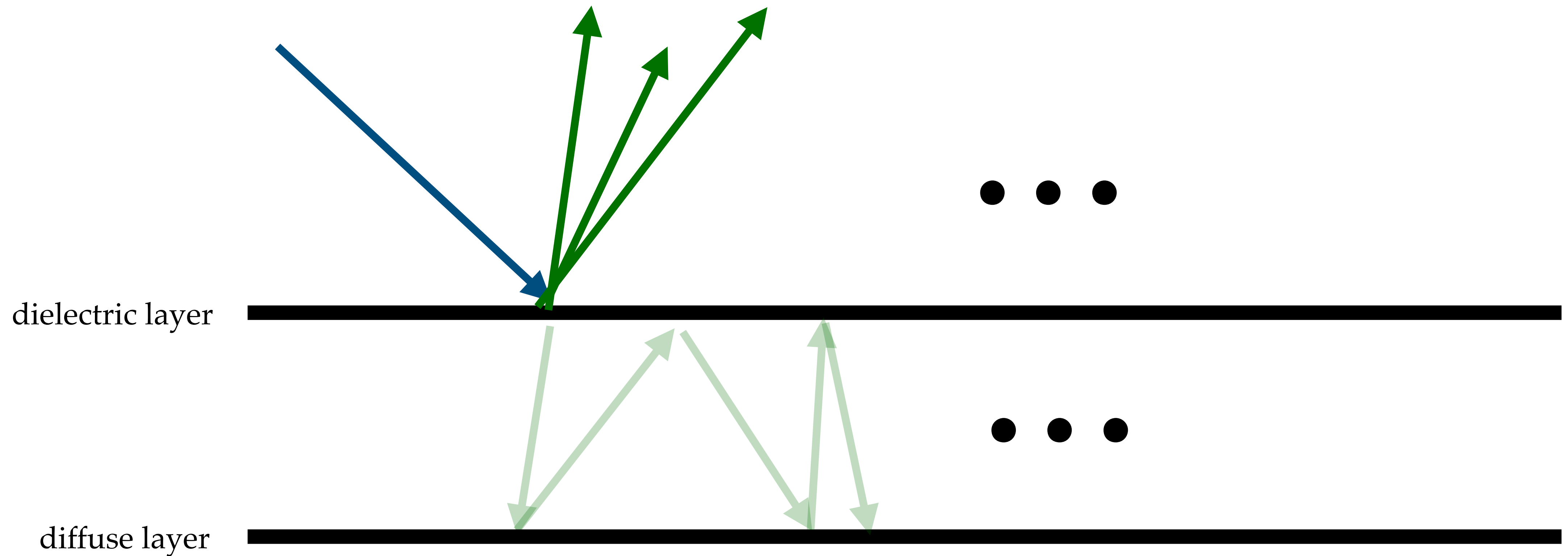


image from Yu Guo et al. & Gamboa et al.  
<https://www.ics.uci.edu/~yug10/webpage/pdf/2018TOG.pdf>  
[https://beltegeuse.github.io/research/publication/2020\\_layered/](https://beltegeuse.github.io/research/publication/2020_layered/)

# Observation:

the horizontal distance doesn't matter in a BSDF



# Idea: solve a small “1D” rendering equation inside the material

mathematically: it's a change of variable!

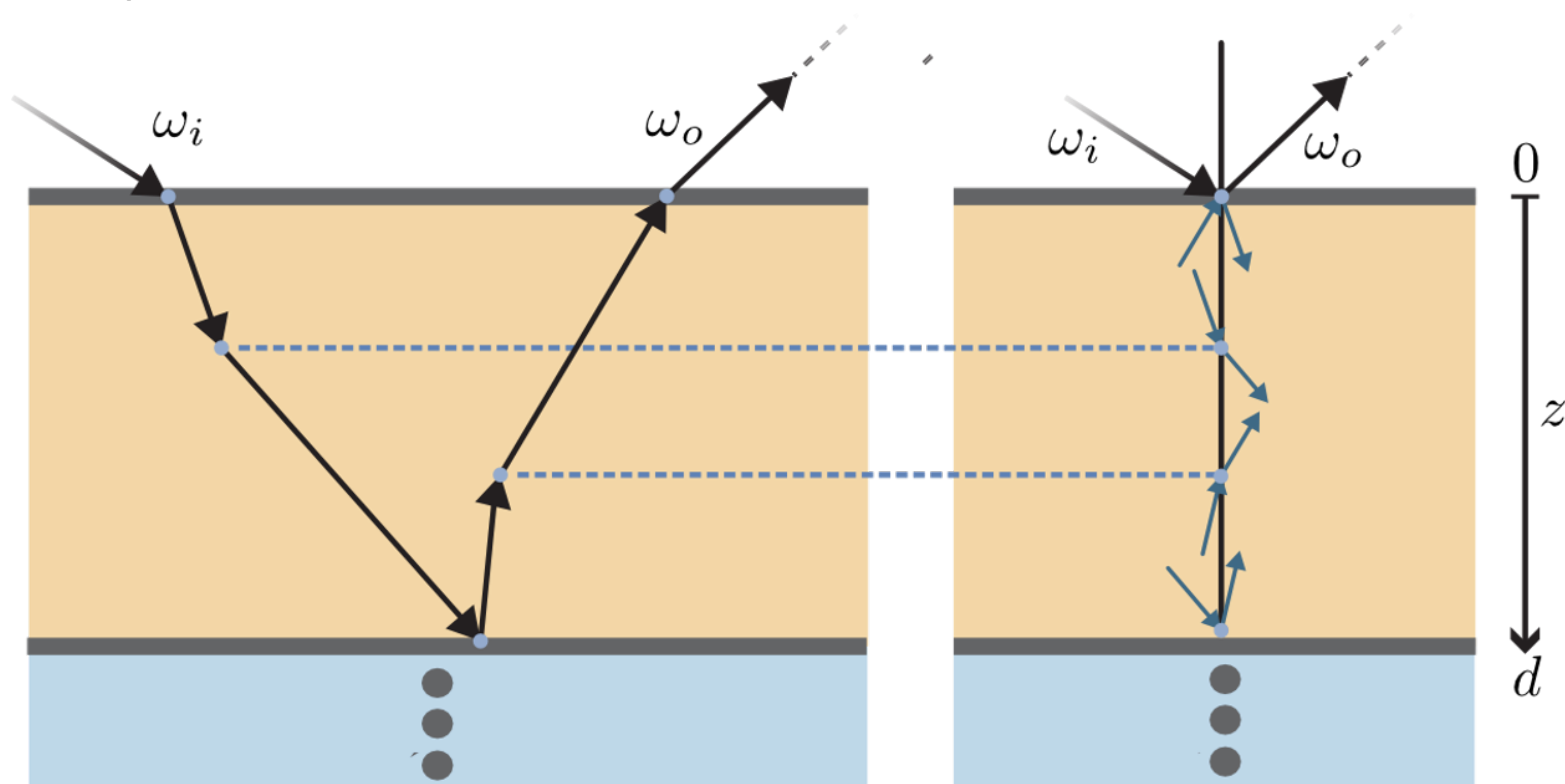
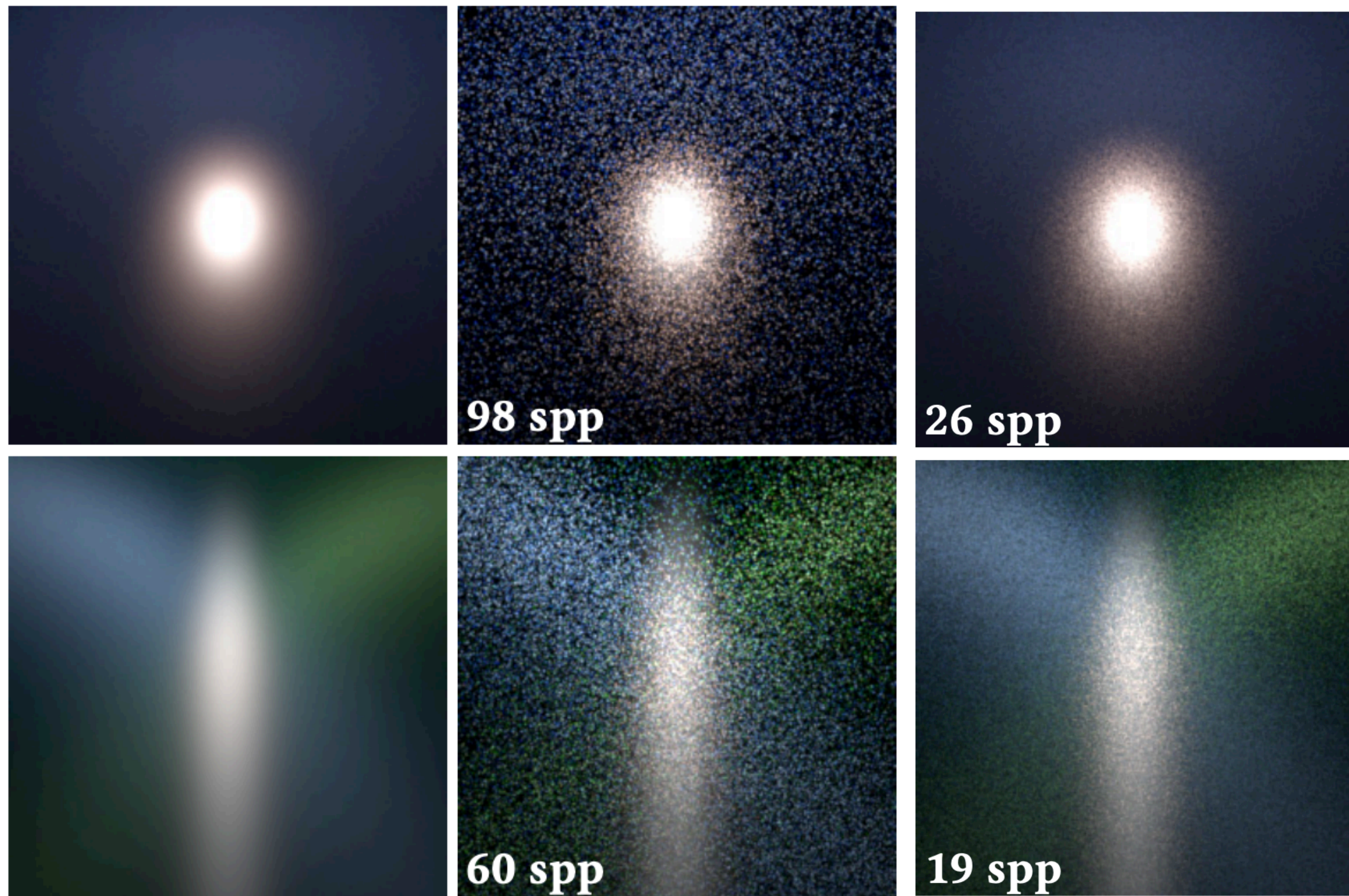


Figure from Gamboa et al.

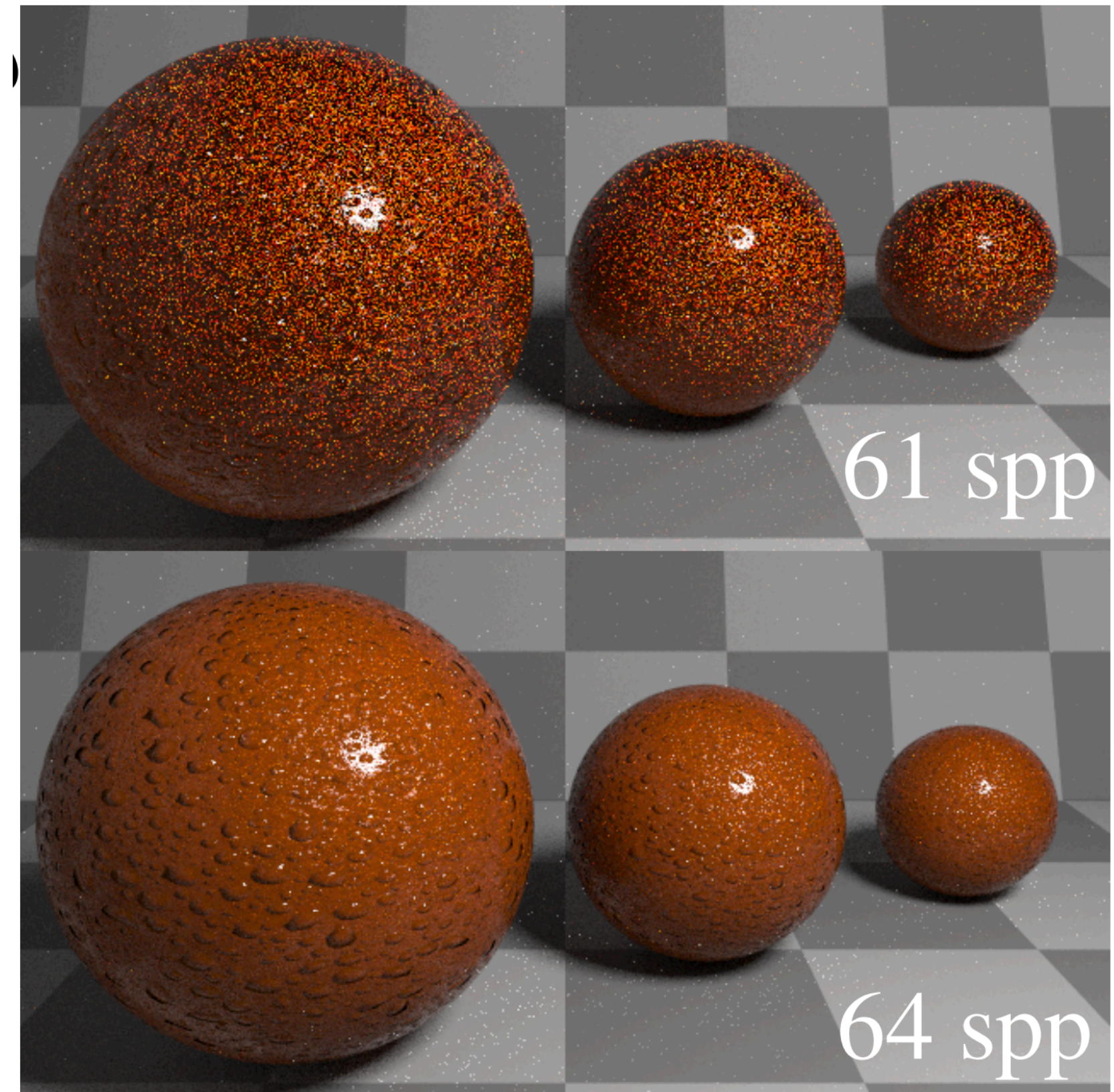
# The simplified rendering equation has much smaller variance



Reference

Standard PT

Guo et al.



Gamboa et al.

# More accurate than Belcour?

- Belcour's code doesn't directly support multi-layer high-res normal maps
- mipmapping for layered BSDF could be an interesting research topic



# Today: 3 approaches of modeling layered BSDFs

## A Comprehensive Framework for Rendering Layered Materials

Wenzel Jakob Eugene D'Eon Otto Jakob Steve Marschner

In ACM Transactions on Graphics (Proceedings of SIGGRAPH 2014)



## Efficient Rendering of Layered Materials using an Atomic Decomposition with Statistical Operators

Laurent Belcour (Unity Technologies)  
Published in ACM Transactions on Graphics (proc. of SIGGRAPH 2018)

paper TeX bib paper code video slides



**discussion:** what are the pros and cons of the three methods?  
if you want to implement one of these in your renderer,  
which one will you choose? why?



# History (in graphics)

1997

## Plane Parallel Radiance Transport for Global Illumination in Vegetation

Nelson Max  
Curtis Mobley  
Brett Keating  
En-Hua Wu

continuous adding equation!

$$\frac{dI_u(z)}{dz} = I_u(z)\tau_{uu}(z) + I_d(z)\rho_{du}(z)$$
$$-\frac{dI_d(z)}{dz} = I_u(z)\rho_{ud}(z) + I_d(z)\tau_{dd}(z)$$

2001

## An Illumination Model for a Skin Layer Bounded by Rough Surfaces

Jos Stam  
Alias | wavefront  
1218 Third Ave, 8th Floor,  
Seattle, WA 98101

2000

## Monte Carlo Evaluation Of Non-Linear Scattering Equations For Subsurface Reflection

Matt Pharr

Pat Hanrahan

Stanford University

Monte Carlo solution to the adding equation

$$\mathbf{S}_{a \rightarrow a} = \mathbf{S}_a + \mathbf{S}_a \mathbf{S}_b \mathbf{S}_a + \mathbf{S}_a \mathbf{S}_b \mathbf{S}_a \mathbf{S}_b \mathbf{S}_a + \dots \quad (3.6a)$$

$$= \sum_{n=0}^{\infty} (\mathbf{S}_a \mathbf{S}_b)^n \mathbf{S}_a \quad (3.6b)$$

$$= (\mathbf{I} - \mathbf{S}_a \mathbf{S}_b)^{-1} \mathbf{S}_a \quad (3.6c)$$

$$= \mathbf{S}_a + \mathbf{S}_a \mathbf{S}_b \mathbf{S}_{a \rightarrow a} \quad (3.6d)$$

predecessor of Wenzel's approach

```
ComputeRT:  
For  $k = 0, \dots, N$  do  
  Compute the scattering matrix  $\mathbf{M}_k$  (Appendix A)  
  Compute the reflection and transmission matrices  $\mathbf{R}_{ij}$  and  $\mathbf{T}_{ij}$  (Appendix D)  
  Compute eigenstructure of  $\mathbf{M}_k$   
  For  $i = 1, \dots, M$   
    Solve linear system for incoming direction  $\mu_i$  (Equation 15)  
    Transmit radiances out of the layer (Equation 16)  
    Set the  $i$ -th columns of  $\mathbf{R}_k$  and  $\mathbf{T}_k$   
  next  
next
```

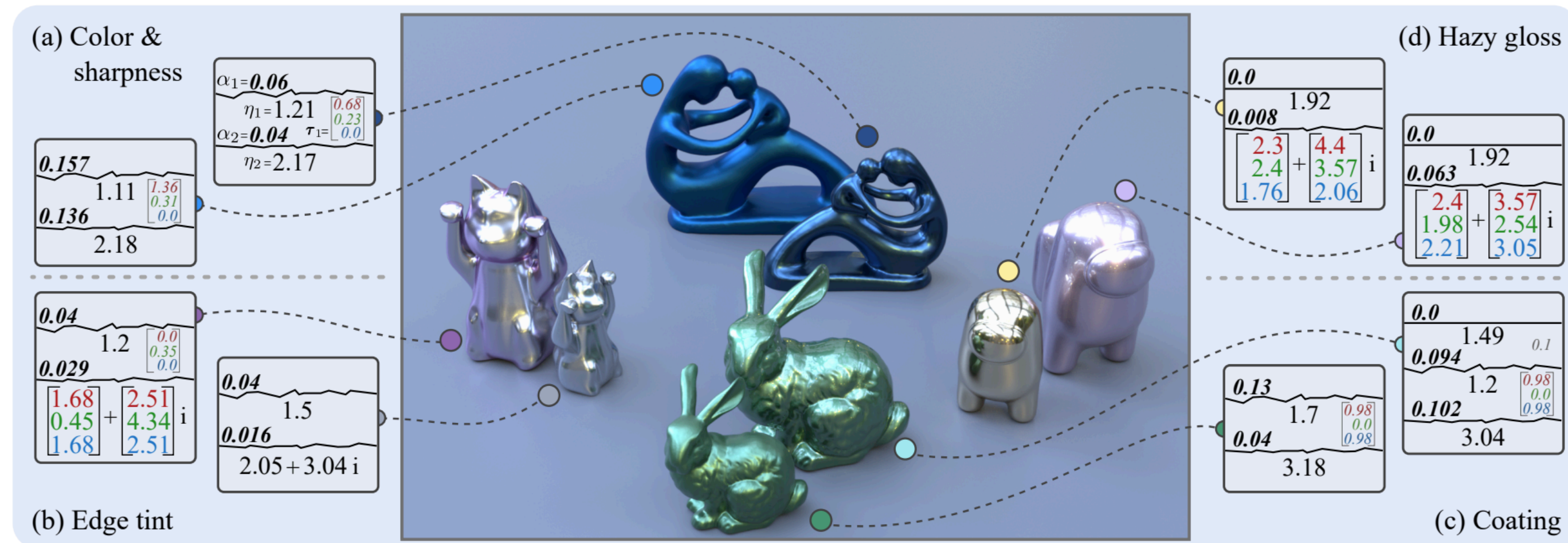
# Fun recent research about layered materials

## An Inverse Method for the Exploration of Layered Material Appearance

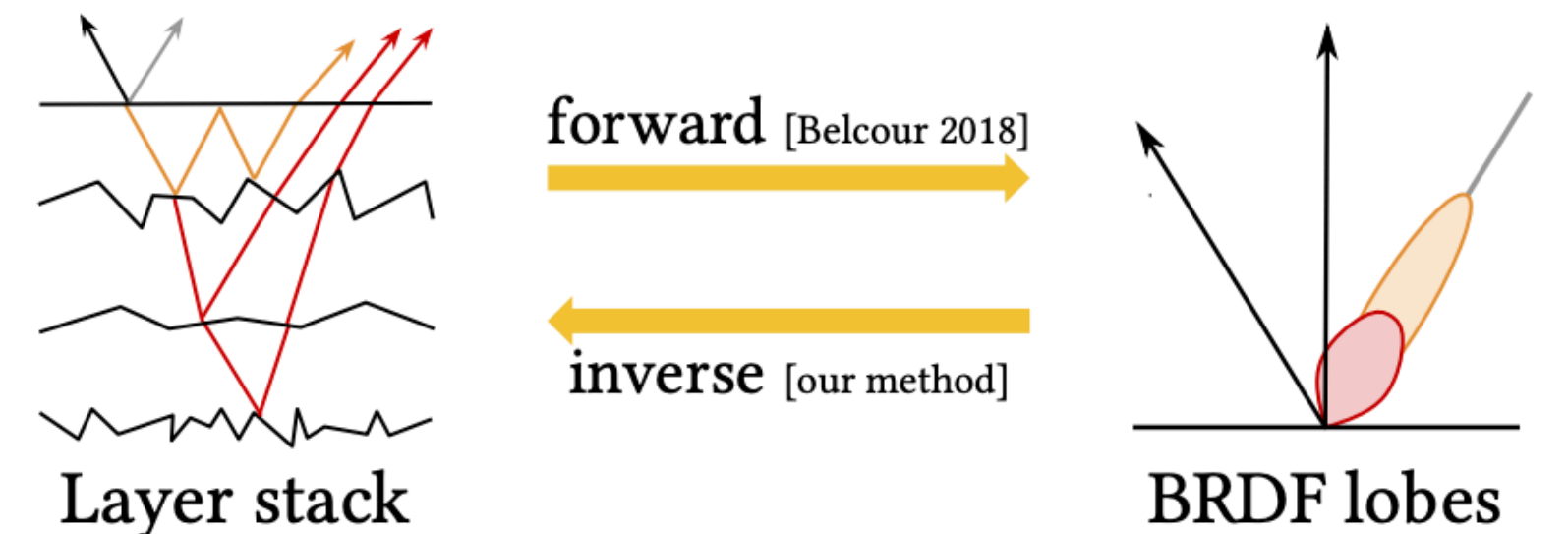
MÉGANE BATI, IOGS, Université de Bordeaux, France

PASCAL BARLA, Inria, France

ROMAIN PACANOWSKI, Inria, France



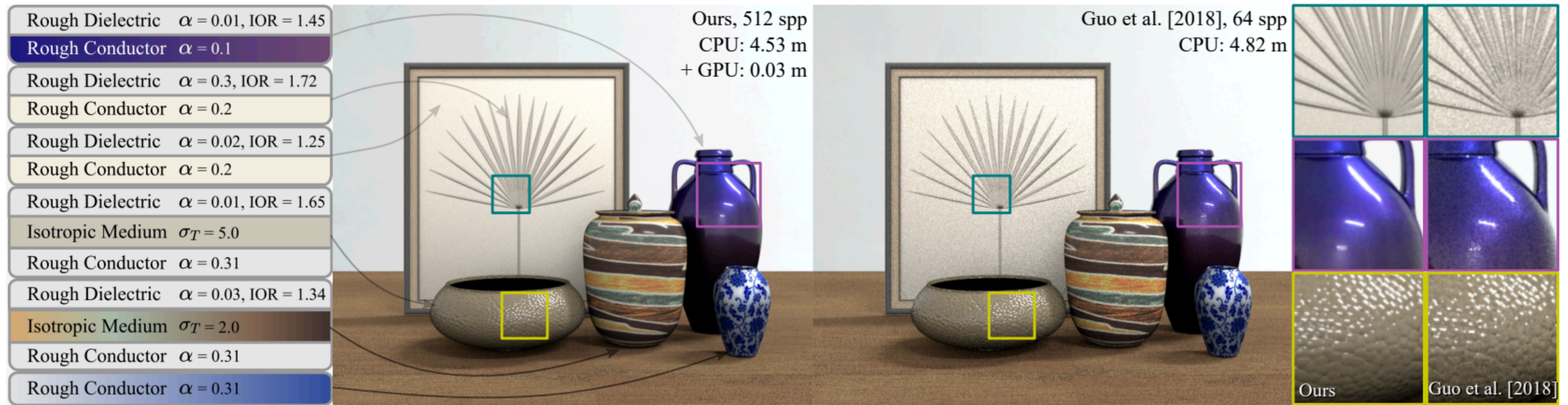
the inverse version of Belcour's algorithm





# Fun recent research about layered materials

## Neural Layered BRDFs



Jiahui Fan  
Nanjing University of Science and  
Technology  
China  
fjh@njust.edu.cn

Beibei Wang<sup>†</sup>  
Nankai University,  
Nanjing University of Science and  
Technology  
China  
beibei.wang@nankai.edu.cn

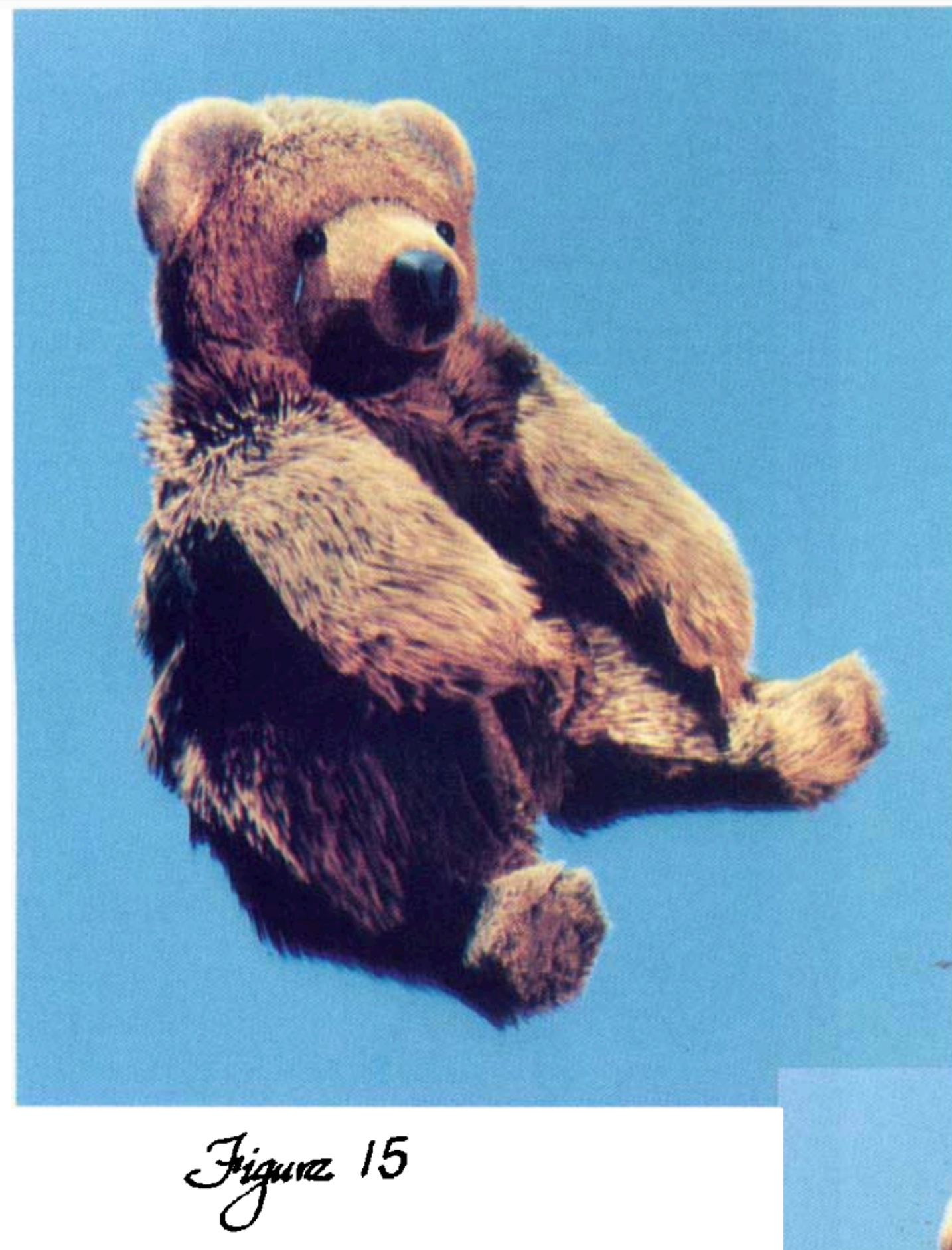
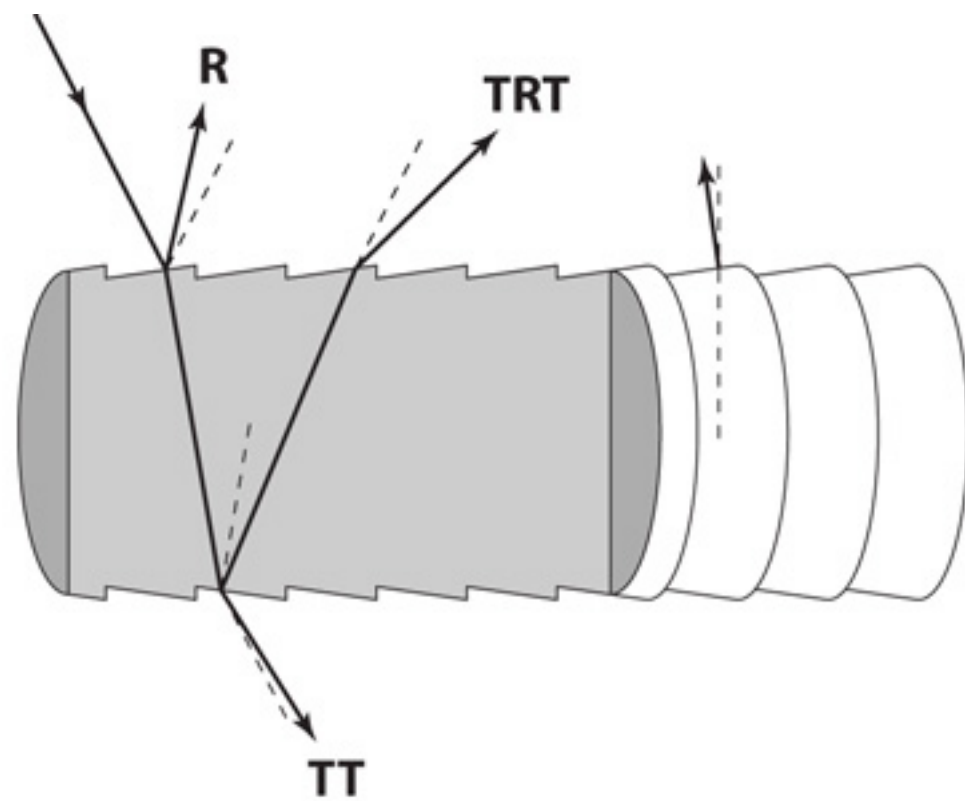
Miloš Hašan  
Adobe Research  
USA

Jian Yang<sup>†</sup>  
Nanjing University of Science and  
Technology  
China  
csyang@njust.edu.cn

Ling-Qi Yan  
University of California, Santa  
Barbara  
USA  
lingqi@cs.ucsb.edu

# Next: hair & cloth

- rendering fiber structures



*Figure 15*

