Bidirectional Scattering Distribution Functions

UCSD CSE 272
Advanced Image Synthesis

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lots of images/figures from Eric Heitz/Jonathan Dupuy/Wenzel Jakob
BSDFs describe reflection/transmission properties

\[
\int f_p(\omega, \omega') L(p', -\omega') |n_p \cdot \omega'| d\omega'
\]
BSDFs describe reflection/transmission properties

$f_p(\omega, \omega')$

beautiful illustrations from Jonathan Dupuy
BSDFs describe reflection/transmission properties

$$f_p(\omega, \omega')$$

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BSDFs describe reflection/transmission properties

\[ f_p(\omega, \omega') \]

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BSDFs describe reflection/transmission properties

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BSDFs describe reflection/transmission properties

\[ f_p(\omega, \omega') \]

beautiful illustrations from Jonathan Dupuy

Isotropic BRDFs vs anisotropic BRDFs

- isotropic BRDFs: reduces 4D BRDFs to 3D by only considering differences in azimuth angles

\[ f_r(\theta', \phi', \theta, \phi) = f_r(\theta', \theta, \phi' - \phi) \]
Isotropic BRDFs vs anisotropic BRDFs

isotropic: circular highlights

anisotropic: “directional” highlights
Reciprocity of BSDFs

**quiz 1:** why and when will this hold?

\[ f_p(\omega, \omega') = f_p(\omega', \omega) \]
Reciprocity of BSDFs

**quiz 1:** why and when will this hold?

**quiz 2:** what is the consequence of a non-reciprocal BSDF?

\[ f_p(\omega, \omega') = f_p(\omega', \omega) \]
Figure 5.2: When light enters a medium with a higher refractive index, the same light energy is squeezed into a smaller volume. This causes the radiance along each ray to increase.
Sidetrack: exceptions of reciprocity

"real" reciprocity principle:

\[
\frac{f_p(\omega, \omega')}{\eta'^2} = \frac{f_p(\omega', \omega)}{\eta^2}
\]

Figure 5.2: When light enters a medium with a higher refractive index, the same light energy is squeezed into a smaller volume. This causes the radiance along each ray to increase.
Sidetrack: exceptions of reciprocity

Figure 6.2: When absorbing media such as metals are present, the path of a light beam is not always reversible. For example, when a light beam $A_i$ is transmitted from air into some metals, there is a non-zero angle of incidence $\theta_0$ for which the beam does not change its direction of propagation (Figure (a)). However, a beam of light $B_i$, traveling in the reverse direction (from metal into air) is refracted at the surface, and follows a different path (Figure (b)).
Sidetrack: exceptions of reciprocity

- Polarized ray: electric field where the electrons only oscillate in one direction.
- Magnetic field: rotates the electric field oscillation regardless of the polarization direction.
Energy conservation of BSDFs

\[ \int f_p(\omega, \omega') \left| n_p \cdot \omega' \right| d\omega' \leq 1 \]

**quiz:** what happens if your BSDF is not energy conserving?
How to obtain a BSDF?

- we can actually measure it!

**quiz:** how would you design a device for this?

https://icons8.com/icons/set/flashlight
https://icons8.com/icons/set/camera
How to obtain a BSDF?

- we can actually measure it!
How to obtain a BSDF?

• we can actually measure it!

robot arm for BSDF measurement @ UCSD

https://cseweb.ucsd.edu/~ravir/nearfield.pdf
How to obtain a BSDF?

• we can actually measure it!

https://rgl.epfl.ch/pages/lab/pgII

the robot arm in action @ EPFL

video from Wenzel Jakob

https://rgl.epfl.ch/pages/lab/pgII
Measuring BRDFs is time/memory consuming

need to measure a 4D domain

100 samples at each dimension:
$100^4 = 100,000,000$ (100 million samples)

1 second per sample: 3 years

380 MB per wavelength

amazing illustrations & numbers from Jonathan Dupuy
Trick 1: focus on mirror reflection direction

- by applying a change of variable (again!)


amazing illustrations from Jonathan Dupuy
Trick 1: focus on mirror reflection direction

by applying a change of variable (again!)

define half-vector $H = \text{normalize}(\omega + \omega')$

when $H = n_p$

$\omega$ and $\omega'$ are mirror reflection directions

idea:
measure differences between normals/directions & the half-vector

Trick 1: focus on mirror reflection direction

- by applying a change of variable (again!)

idea:
measure differences between normals/directions & the half-vector

change of variable:
$$(\theta, \phi, \theta', \phi') \rightarrow (\theta_h, \phi_h, \theta_d, \phi_d)$$

Trick 2: focus on elevation, ignore azimuth

- assume the BSDF does not change over azimuthal angles (not always true)
- “isotropic BSDF”

amazing illustrations from Jonathan Dupuy
Trick 3: estimate the roughness


- and apply another change of variable to scale the samples!

(still need ~2 hours to measure a material after all these tricks)

amazing illustrations from Jonathan Dupuy

The MERL BSDF dataset [Matusik 2003]

most popular data
100 isotropic BRDFs

warning: not a perfect dataset!
lots of camera artifacts
(defocus/bokeh/lens flare)
MERL anisotropic extension

4 anisotropic BRDFs

renderings

satin
velvet

actual photographs

Experimental Analysis of BRDF Models

Addy Ngan, Frédéric Durand,† and Wojciech Matusik‡

MIT CSAIL
MERL
Brushed metal: common anisotropic material

https://en.wikipedia.org/wiki/Brushed_metal
EPFL material database [2018]

50 isotropic BRDFs
12 anisotropic BRDFs
(probably much higher quality than MERL)

Downsides of measured BSDFs

quiz: what are they?
Downsides of measured BSDFs

- capturing is time consuming
- very few of them
- does not support texturing

can’t support spatially varying roughness with measured BSDF
Remedy: let’s fit a model to the data!

$$f_p(\omega, \omega') = \text{some parametric function}$$
Remedy: let’s fit a model to the data!

\[ f_p(\omega, \omega') = \text{some parametric function} \]

quiz: would neural nets be a good idea? why? why not?
How to build a model:
make physical assumptions

- explain and predict the behavior of our data

$$f_p(\omega, \omega^{'}) = \text{some parametric function}$$

based on derivations from simple physical assumptions

https://en.wikipedia.org/wiki/Reductionism
**Microfacet theory**

- assumption: surfaces are made of infinitely many little mirrors (microfacets)

Figure inspired by Eric Heitz [https://jcgt.org/published/0003/02/03/](https://jcgt.org/published/0003/02/03/)
Microfacet theory

• assumption: surfaces are made of infinitely many little mirrors (microfacets)

• BSDF directions \( \omega \) and \( \omega' \) are treated as parallel rays to the microsurface
Microfacet theory

- assumption: surfaces are made of infinitely many little mirrors (microfacets)
- BSDF directions $\omega$ and $\omega'$ are treated as parallel rays to the microsurface

**quiz:** given directions $\omega$ and $\omega'$, which microfacet mirror will reflect light?
Microfacet theory

• assumption: surfaces are made of infinitely many little mirrors (microfacets)

• BSDF directions $\omega$ and $\omega'$ are treated as parallel rays to the microsurface

• given $\omega$ and $\omega'$, only microsurfaces with normal $m = H = \text{normalize}(\omega + \omega')$ will reflect light
Microfacet theory

- flat microsurfaces correspond to …?
- bumpy microsurfaces correspond to …?
Microfacet theory

- flat microsurfaces correspond to shiny surfaces
- bumpy microsurfaces correspond to …?
Microfacet theory

- flat microsurfaces correspond to shiny surfaces
- bumpy microsurfaces correspond to rough, diffusive surfaces
Describing microsurfaces using statistical distributions

- The normal distribution function $D(m)$ describes the probability density of micronormals.

- For flat microsurfaces, $D(m = n_p)$ is very high, and $D(m \neq n_p)$ is low.
Popular normal distribution function: Beckmann NDF

- assumption: microfacets are from a heightfield with Gaussian “slopes”

\[
\frac{m_x}{m_z} = \frac{\partial z}{\partial x} \sim N(0, \alpha_x^2)
\]

\[
\frac{m_y}{m_z} = \frac{\partial z}{\partial y} \sim N(0, \alpha_y^2)
\]

see “Slope Space in BRDF Theory” if you’re interested
https://www.reedbeta.com/blog/slope-space-in-brdf-theory/

Figure from Eric Heitz & Lingqi Yan et al.
https://eheitzresearch.wordpress.com/240-2/
Popular normal distribution function: Beckmann NDF

- assumption: microfacets are from a heightfield with Gaussian “slopes” \( \frac{m_y}{m_z} \) and \( \frac{m_x}{m_z} \)

\[
\begin{align*}
-\frac{m_x}{m_z} &= \frac{\partial z}{\partial x} \sim N(0,\alpha_x^2) \\
-\frac{m_y}{m_z} &= \frac{\partial z}{\partial y} \sim N(0,\alpha_y^2)
\end{align*}
\]

quiz: what does large alpha mean?

see “Slope Space in BRDF Theory” if you’re interested
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Figure from Eric Heitz & Lingqi Yan et al.
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Popular normal distribution function:
Beckmann NDF

- assumption: microfacets are from a heightfield with Gaussian “slopes”

\[
D(m) \propto \exp\left(-\frac{1}{2} \left[ \frac{m_x}{m_z} \ \frac{m_y}{m_z} \right] \begin{bmatrix} \alpha_x^2 & 0 \\ 0 & \alpha_y^2 \end{bmatrix}^{-1} \begin{bmatrix} \frac{m_x}{m_z} \\ \frac{m_y}{m_z} \end{bmatrix} \right)
\]

see “Slope Space in BRDF Theory” if you’re interested
https://www.reedbeta.com/blog/slope-space-in-brdf-theory/
Popular normal distribution function:
Trowbridge-Reitz [1975] (aka GGX [Walter 2007])

• geometric intuition: the distribution of normals of an ellipsoid

shading frame
ellipsoid
GGX NDF
(Normal Distribution Function)

brilliant figures from Eric Heitz
https://jcgt.org/published/0007/04/01/
Popular normal distribution function:
Trowbridge-Reitz [1975] (aka GGX [Walter 2007])

Ground Glass Unknown

\[
D_{GGX}(m) \propto \frac{1}{\left(1 + \frac{1}{2} \begin{bmatrix} \frac{m_x}{m_z} & \frac{m_y}{m_z} \\ \frac{m_x}{m_z} & \frac{m_y}{m_z} \end{bmatrix} \begin{bmatrix} \alpha^2_x & 0 \\ 0 & \alpha^2_y \end{bmatrix}^{-1} \begin{bmatrix} \frac{m_x}{m_z} \\ \frac{m_y}{m_z} \end{bmatrix} \right)^2}
\]

Popular normal distribution function:
Trowbridge-Reitz [1975] (aka GGX [Walter 2007])

Ground Glass Unknown

\[ D_{\text{GGX}}(m) \propto \left( \frac{1}{1 + \frac{1}{2} \left[ \begin{array}{cc} m_x & m_y \\ m_y & m_z \end{array} \right] \left[ \begin{array}{cc} \alpha_x^2 & 0 \\ 0 & \alpha_y^2 \end{array} \right]^{-1} \left[ \begin{array}{c} m_x \\ m_y \end{array} \right] } \right)^2 \]

(multivariate student-t distribution with \( \nu = 2 \rightarrow \text{heavy tailed Gaussian} \))

\( \nu = \infty \rightarrow \text{normal Gaussian} = \text{Beckmann} \)

see https://en.wikipedia.org/wiki/Student%27s_t-distribution for intuition
Beckmann vs GGX

Beckmann

quiz: what would GGX look like?
Beckmann vs GGX
Microfacets can block each other

- normal distribution function $D$ alone is not enough to determine how many mirrors are blocked
- need to specify the **microsurface geometry profile**

\[ \omega \]
Smith microsurface profile [1960]

- most popular profile in graphics
- alternative: V-cavity [Cook and Torrence 1982]
- Smith’s assumption: microsurfaces are spatially uncorrelated

Figure from Eric Heitz https://jcgt.org/published/0003/02/03/
We can compute the portion of blocked microsurfaces $\hat{G}$ under Smith’s assumption

$$\hat{G}(\omega, m) = \begin{cases} 
0 & \text{if } \omega \cdot m \leq 0 \\
\frac{\omega \cdot n_p}{\int (\omega \cdot m)D(m)dm} & \text{otherwise}
\end{cases}$$

often call the “shadowing masking term”

see “Understanding the Masking-Shadowing Function in Microfacet-based BRDFs”, Eric Heitz

https://jcgt.org/published/0003/02/03/
We can compute the portion of blocked microsurfaces $\hat{G}$ under Smith’s assumption

$$\hat{G}(\omega, m) = \begin{cases} 0 & \text{if } \omega \cdot m \leq 0 \\ \frac{\omega \cdot n_p}{\int (\omega \cdot m)D(m)dm} & \text{otherwise} \end{cases}$$

often call the “shadowing masking term”

derivation: Smith’s assumption implies

$$\hat{G}(\omega, m) = \begin{cases} 0 & \text{if } \omega \cdot m \leq 0 \\ \hat{G}'(\omega) & \text{otherwise} \end{cases}$$

we also know projected area conserves $\omega \cdot n_p = \int \hat{G}(\omega, m)D(m)\omega \cdot mdm$

see "Understanding the Masking-Shadowing Function in Microfacet-based BRDFs", Eric Heitz
https://jcgt.org/published/0003/02/03/
The microfacet BRDF:
counting **visible** micronormals at the half vector

\[ f_p(\omega, \omega') \]

\[ H = \text{normalize}(\omega + \omega') \]
The microfacet BRDF:

**counting visible micronormals at the half vector**

\[
f_p(\omega, \omega') = D(H)\hat{G}(\omega, H)\hat{G}(\omega', H)
\]

\[H = \text{normalize}(\omega + \omega')\]
The microfacet BRDF: counting **visible** micronormals at the half vector $\omega$

$$f_p(\omega, \omega')$$

$$D(H)G(\omega, \omega', H)$$

$$G(\omega, \omega', H) = \hat{G}(\omega, H)\hat{G}(\omega', H)$$

$$H = \text{normalize}(\omega + \omega')$$
The microfacet BRDF: counting **visible** micronormals at the half vector

\[ f_p(\omega, \omega') = \frac{D(H)G(\omega, \omega', H)}{4 \begin{vmatrix} \omega \cdot n_p & \omega' \cdot n_p \end{vmatrix}} \]

\[ H = \text{normalize}(\omega + \omega') \]

from analytically integrating over mirrors (lots of different Jacobians)

(again, see Heitz [https://jcgt.org/published/0003/02/03/])
The microfacet BRDF: counting \textbf{visible} micronormals at the half vector

\[ f_p(\omega, \omega') = \frac{D(H)G(\omega, \omega', H)F(\omega, H)}{4 \left| \omega \cdot n_p \right| \left| \omega' \cdot n_p \right|} \]

\[ H = \text{normalize}(\omega + \omega') \]

Fresnel equation

\[ m = H \]

\[ 1 - F \]
The Cook-Torrance-Sparrow BRDF [1967, 1982]

\[
f_p(\omega, \omega') = \frac{D(H)G(\omega, \omega', H)F(\omega, H)}{4 \left| \omega \cdot n_p \right| \left| \omega' \cdot n_p \right|}
\]

\(H = \text{normalize}(\omega + \omega')\)
The refraction extension [Walter 2007]

\[
f_p(\omega, \omega') = \begin{cases} 
\frac{D(H)G(\omega, \omega', H)F(\omega, H)}{4|\omega \cdot n_p||\omega' \cdot n_p|} & \text{if reflect} \\
\frac{D(H_r)G(\omega, \omega', H_r)(1 - F(\omega, H_r))}{\text{scary Jacobians}} & \text{if refract}
\end{cases}
\]

\[H_r = \text{normalize}(\eta\omega + \eta'\omega')\]

see http://www.graphics.cornell.edu/~bjw/microfacetbsdf.pdf for the scary Jacobian
Fresnel equation

• light as wave behaves differently for glass-like materials (dielectrics) and metal-like materials (conductors)

\[ 1 - F \approx 0 \]

real number \( \eta, \eta' \)

complex number \( \eta, \eta' \)
For metals, the complex index of refraction varies with wavelength.

(a) Measured copper material (the default), rendered using 30 spectral samples between 360 and 830 nm

(b) Measured gold material (Listing 17)

Images from Wenzel Jakob
https://www.mitsuba-renderer.org/releases/current/documentation.pdf
Fresnel equation for metal-like interfaces is scary & unintuitive

- also we don’t have a lot of spectral data

\[
a^2 = \frac{1}{2n^2} \left( \sqrt{(n_i^2 - k_i^2 - n_i^2 \sin^2 \theta)^2 + 4n_i^2k_i^2} - n_i^2 - k_i^2 - n_i^2 \sin^2 \theta \right)
\]
\[
b^2 = \frac{1}{2n^2} \left( \sqrt{(n_i^2 - k_i^2 - n_i^2 \sin^2 \theta)^2 + 4n_i^2k_i^2} - n_i^2 + k_i^2 + n_i^2 \sin^2 \theta \right)
\]
\[
R_s = \frac{a^2 + b^2 - 2n \cos \theta \cos \theta + \cos^2 \theta}{a^2 + b^2 + 2n \cos \theta + \cos^2 \theta}
\]
\[
R_p = \frac{R_s \cos^2 \theta (a^2 + b^2) - 2n \cos \theta \sin^2 \theta \cos^2 \theta}{a^2 + b^2 + 2n \cos \theta + \cos^2 \theta}
\]

using \(a^2\) and \(b^2\) with \(\eta\) and \(\eta'\) as follow the give same result:

\[
a^2 = \frac{1}{2} \left( \sqrt{(\eta^2 - \eta_k^2 - \sin^2 \theta)^2 + 4\eta^2\eta_k^2} + \eta^2 - \eta_k^2 - \sin^2 \theta \right)
\]
\[
b^2 = \frac{1}{2} \left( \sqrt{(\eta^2 - \eta_k^2 - \sin^2 \theta)^2 + 4\eta^2\eta_k^2} - \eta^2 + \eta_k^2 + \sin^2 \theta \right)
\]

Derivation from conductor Fresnel equation can be found in [5] (p.111) and use

\[
(n_t - ik_t) \cos \theta_t = (n_t - ik_t) \sqrt{1 - \left(\frac{m}{m - ik_t} \sin \theta \right)^2} = \sqrt{(n_t - ik_t)^2 - n_t^2 \sin^2 \theta}
\]
\[
a - ib = (n_t - ik_t)^2 - n_t^2 \sin^2 \theta)
\]

In practice there is some simplification possible:

\[
a^2 + b^2 = \sqrt{(\eta^2 - \eta_k^2 - \sin^2 \theta)^2 + 4\eta^2\eta_k^2}
\]
\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]
\[
R_p = \frac{R_s \cos^2 \theta (a^2 + b^2) - 2n \cos \theta \sin^2 \theta \cos \theta}{a^2 + b^2 + 2n \cos \theta + \cos^2 \theta}
\]
Graphics people use Schlick’s approximation

\[ F \approx F_0 + (1 - F_0)(1 - \cos \theta)^5 \]

\[ F_0 = \left( \frac{\eta - \eta'}{\eta + \eta'} \right)^2 \]

for real index of refraction

small \( \cos \theta \), \( F \sim 1 \)

large \( \cos \theta \), \( F \sim F_0 \)
Graphics people use Schlick’s approximation for metals

\[ F \approx \text{color} + (1 - \text{color})(1 - \cos \theta)^5 \]

metal becomes colorless/white at grazing angle

highly recommend “Some Thoughts on the Fresnel Term”
https://www.youtube.com/watch?v=kEcDbI7eS0w
Cook-Torrance-Sparrow BSDF fits well to MERL measured data!

measured (nickel material)  

Cook-Torrance-Sparrow

from Ngan et al. “Experimental Analysis of BRDF Models”

http://people.csail.mit.edu/addy/research/ngan05_brdf_eval.pdf
Limitations of microfacet models

• from the EPFL dataset

- strongly anisotropic but “hazy”
- iridescence?
- ???
  (butterfly wings)

awesome material images from Jonathan Dupuy & Wenzel Jakob

https://rgl.epfl.ch/pages/lab/pgII
Multiple scattering

- Cook-Torrance-Sparrow BSDF ignores multiple bounces inside the microsurfaces
Multiple scattering

• ignoring multiple bounces lead to energy loss, esp. at high roughness

images from Heitz et al.
https://eheitzresearch.wordpress.com/240-2/
Remedy: simulate multiple bounces inside the microsurfaces

See Heitz et al. “Multiple-Scattering Microfacet BSDFs with the Smith Model”
Remedy: simulate multiple bounces inside the microsurfaces

• or not, see “Misunderstanding multiscattering” by Angelo Pesce
  https://c0de517e.blogspot.com/2019/08/misunderstanding-multiscattering.html

See Heitz et al. “Multiple-Scattering Microfacet BSDFs with the Smith Model”
Next time: Uber BSDF

- one BSDF to rule them all?

next Monday is MLK, so see you next Wednesday!