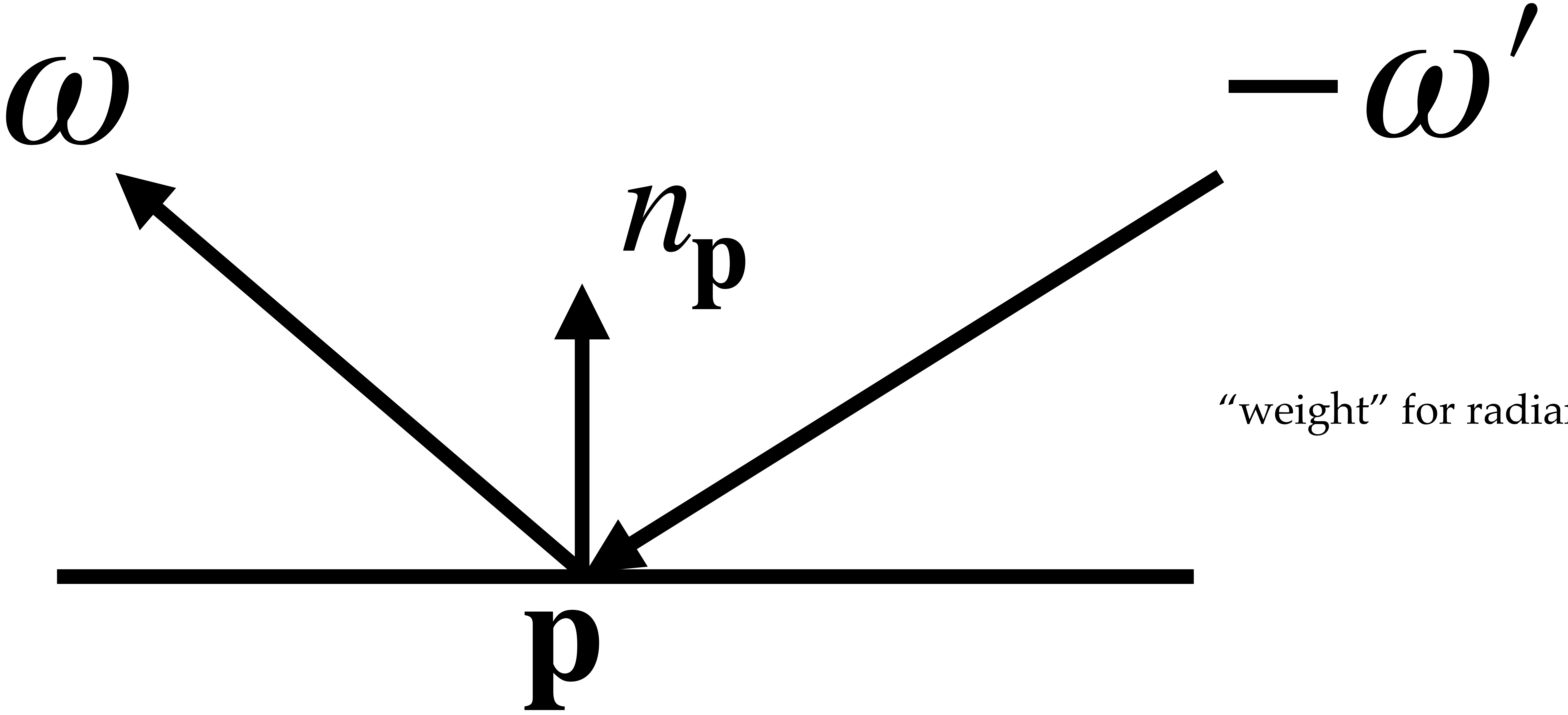
Bidirectional Scattering Distribution Functions

UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li

lots of images/figures from Eric Heitz/Jonathan Dupuy/Wenzel Jakob

BSDFs describe reflection/transmission properties

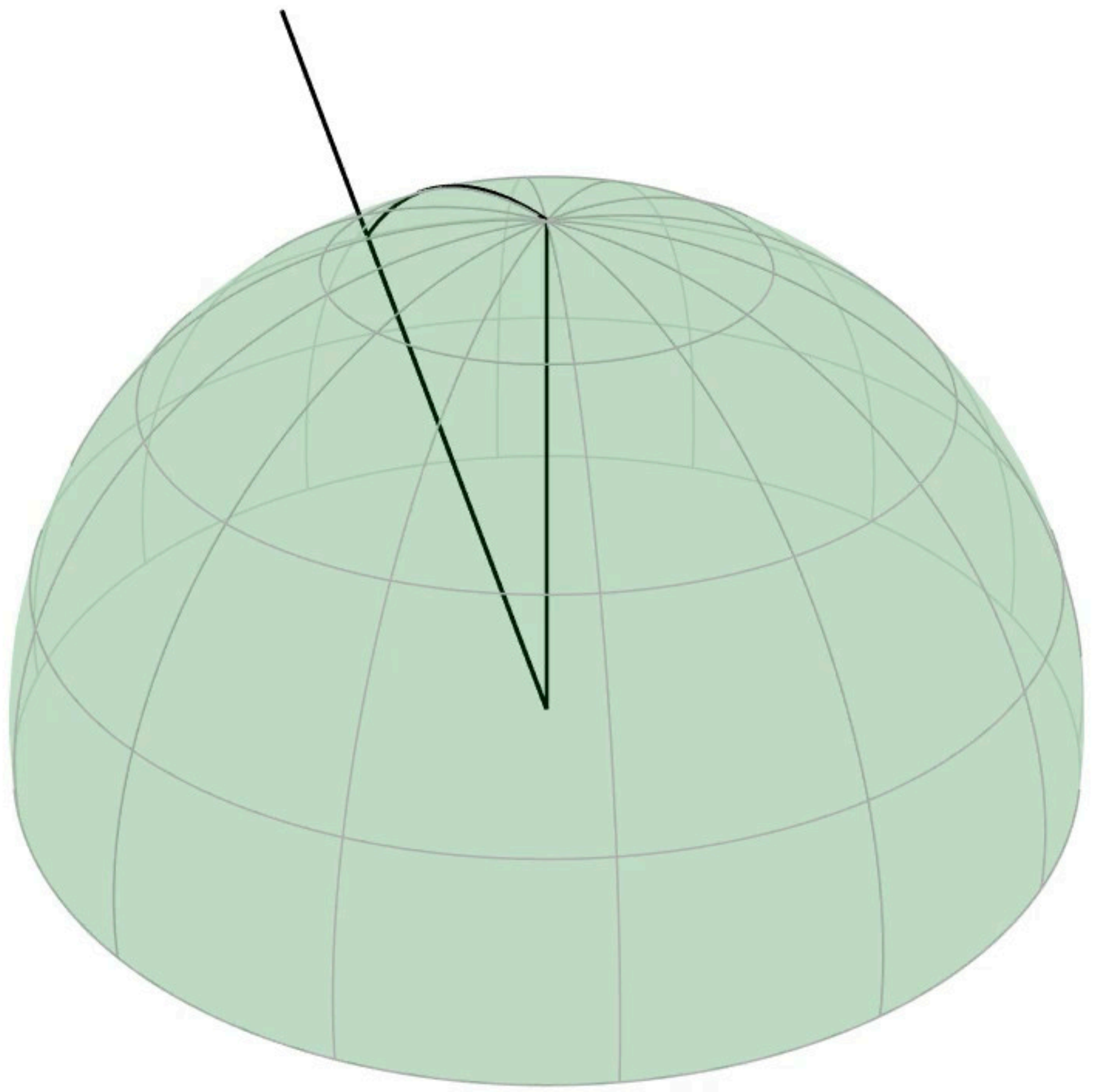


"weight" for radiance L

$$\int f_{\mathbf{p}}(\omega, \omega') L(\mathbf{p}', -\omega') \, | n_{\mathbf{p}} \cdot \omega' | \, \mathrm{d}\omega'$$

BSDFs describe reflection/transmission properties

ω

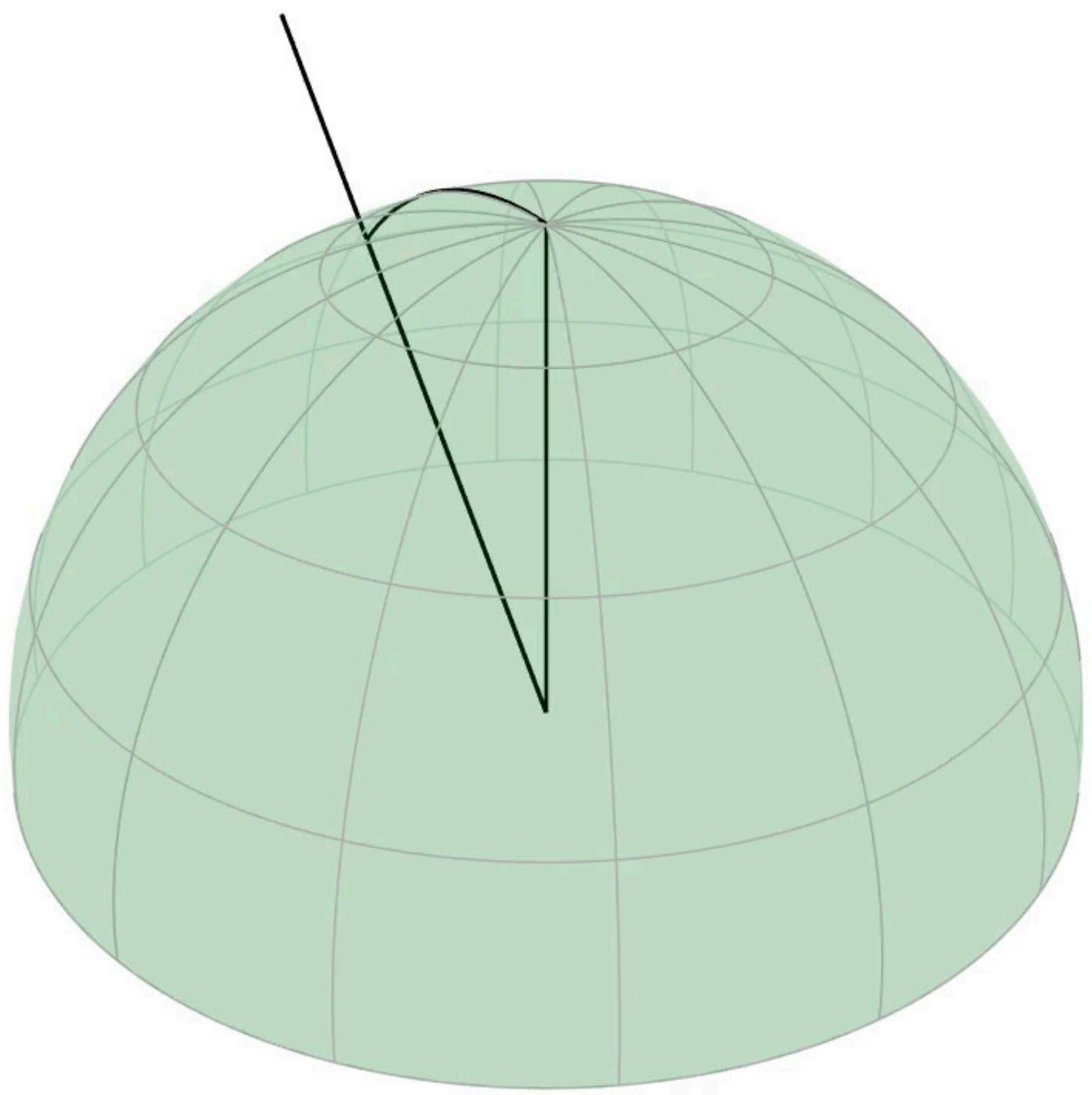


$$f_{\mathbf{p}}(\omega, \omega')$$

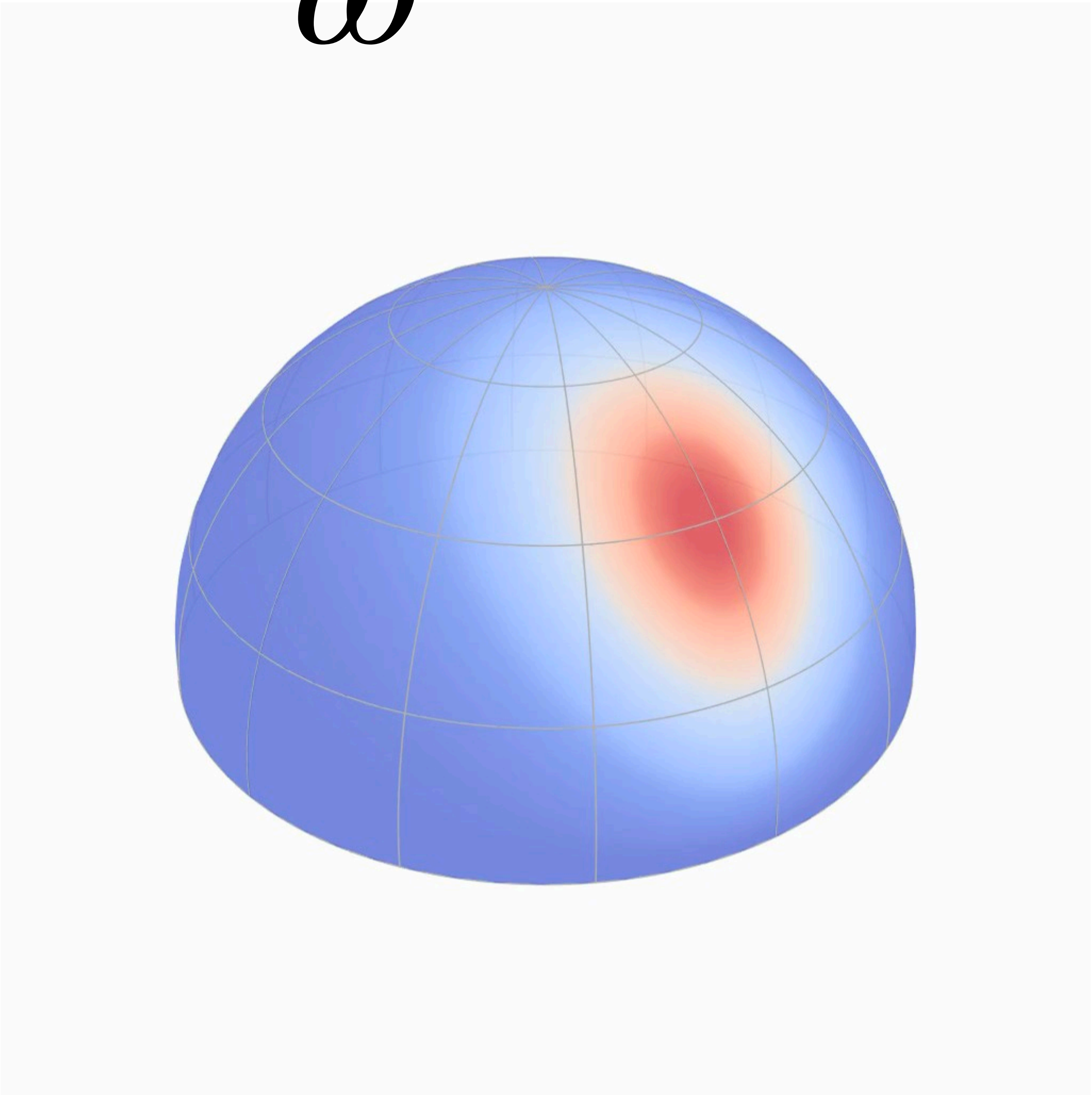
beautiful illustrations from Jonathan Dupuy
http://onrendering.com/data/papers/powitacq/slides/powitacq.html

BSDFs describe reflection/transmission properties

ω



ω'



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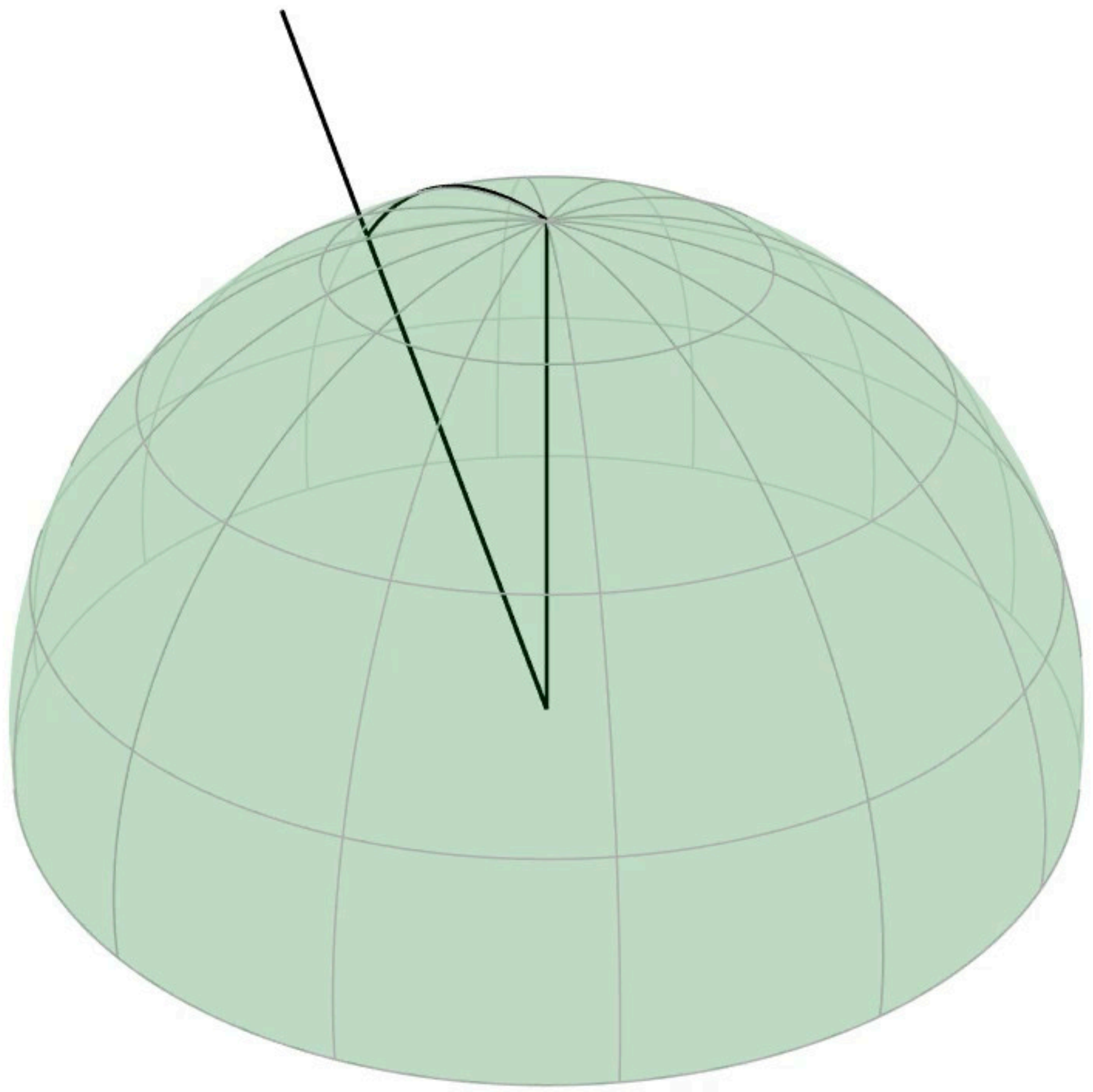
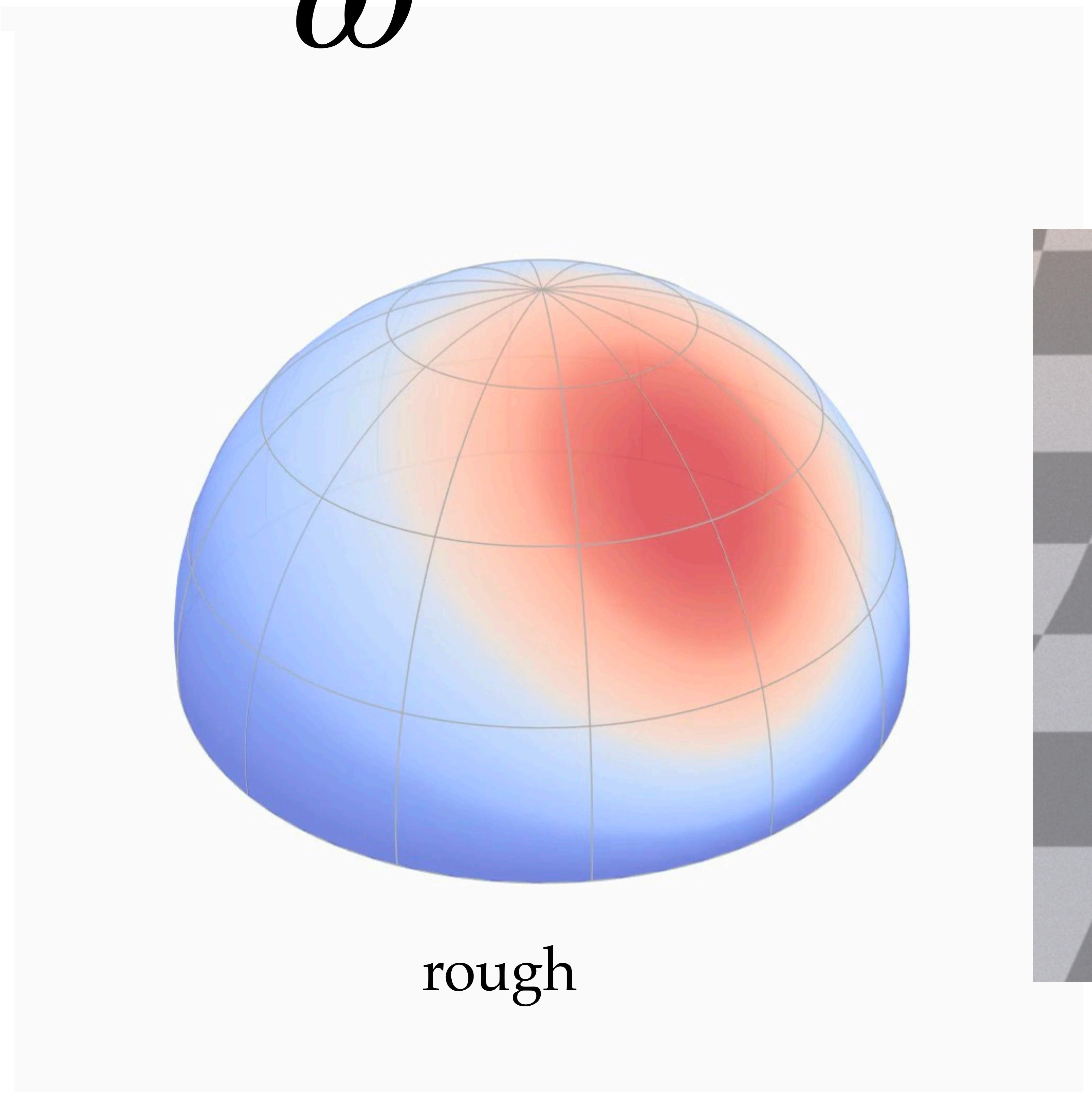
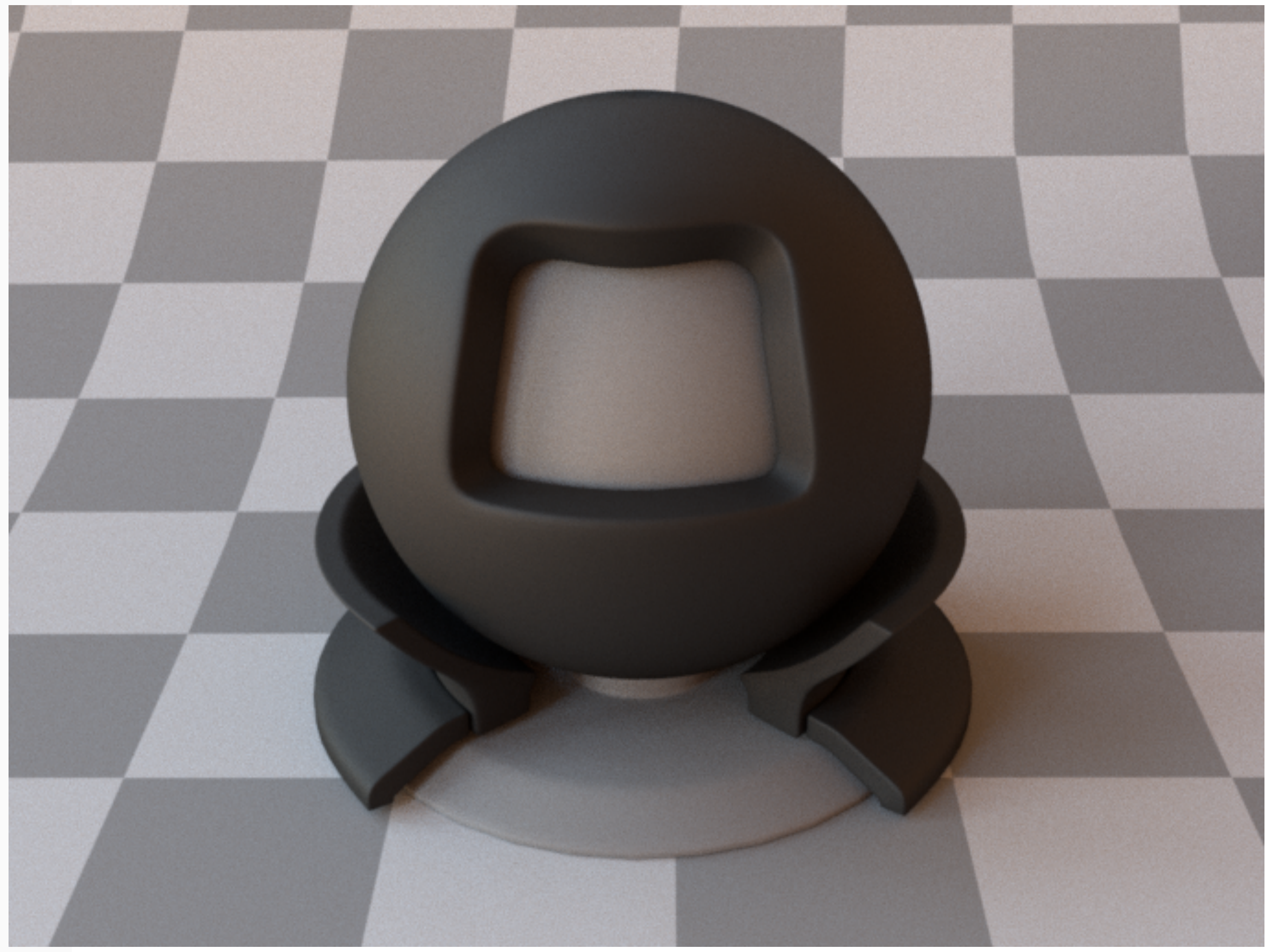
BSDFs describe reflection/transmission properties

ω ω'







$$f_{\mathbf{p}}(\omega, \omega')$$

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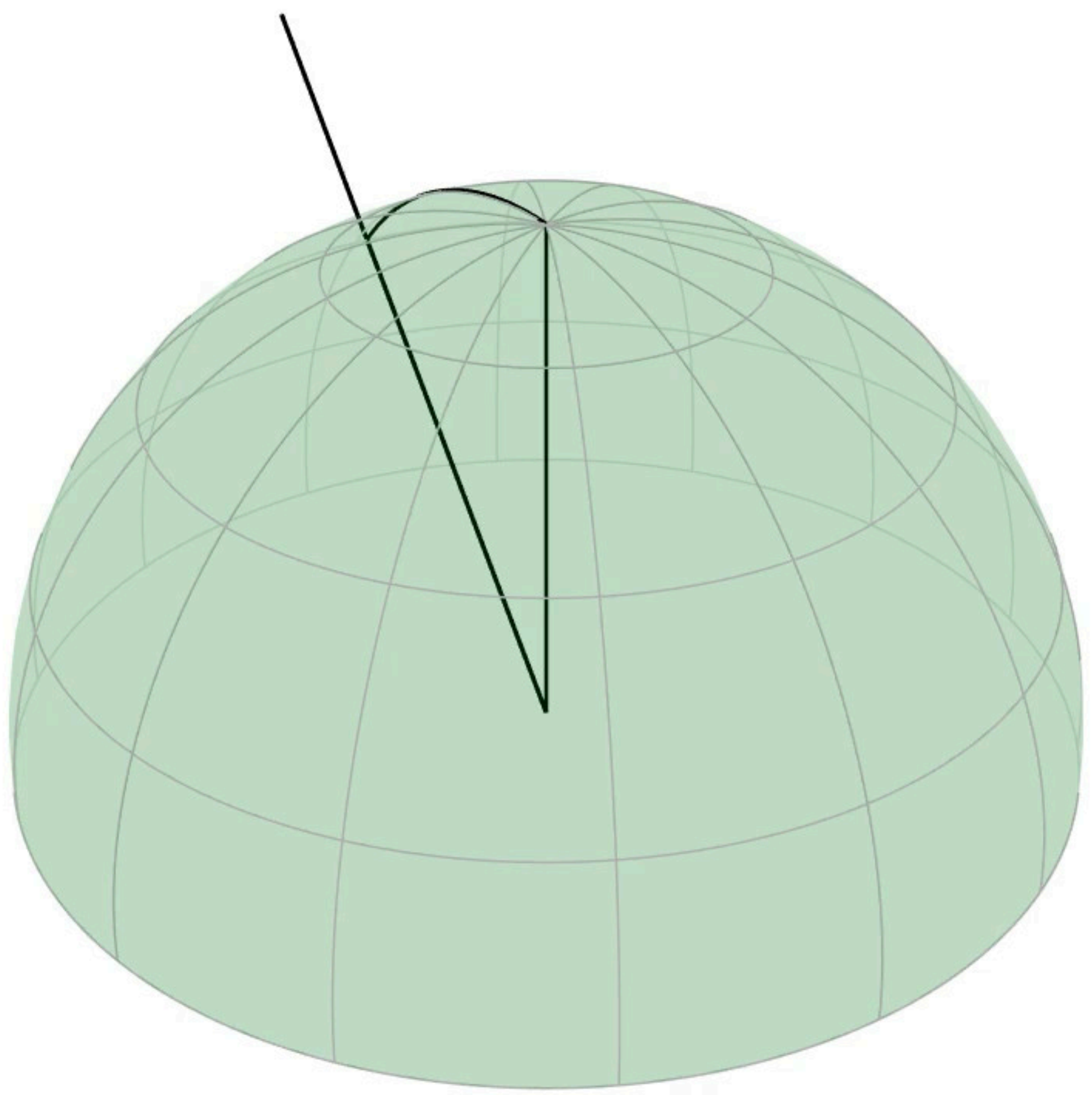
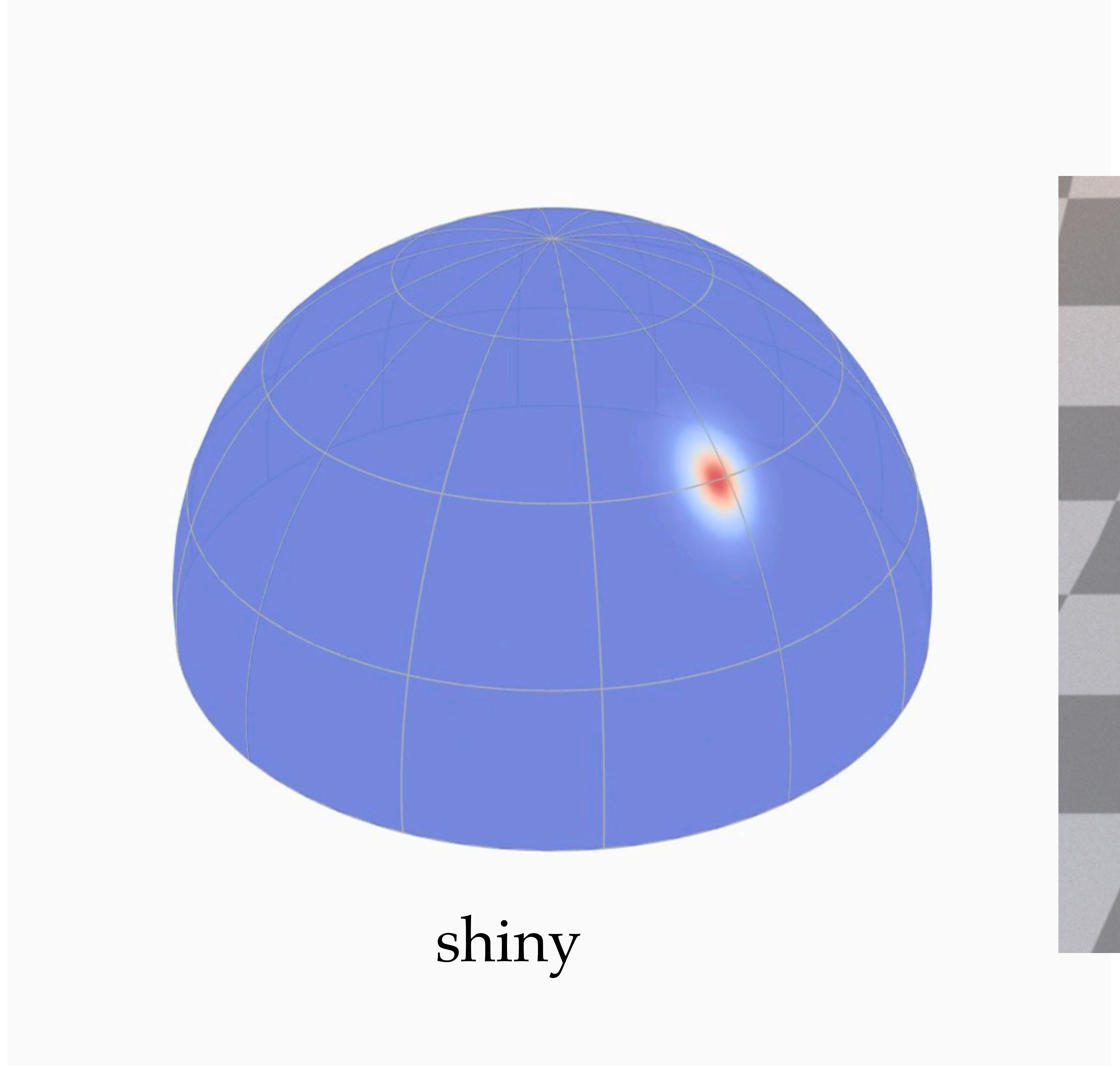
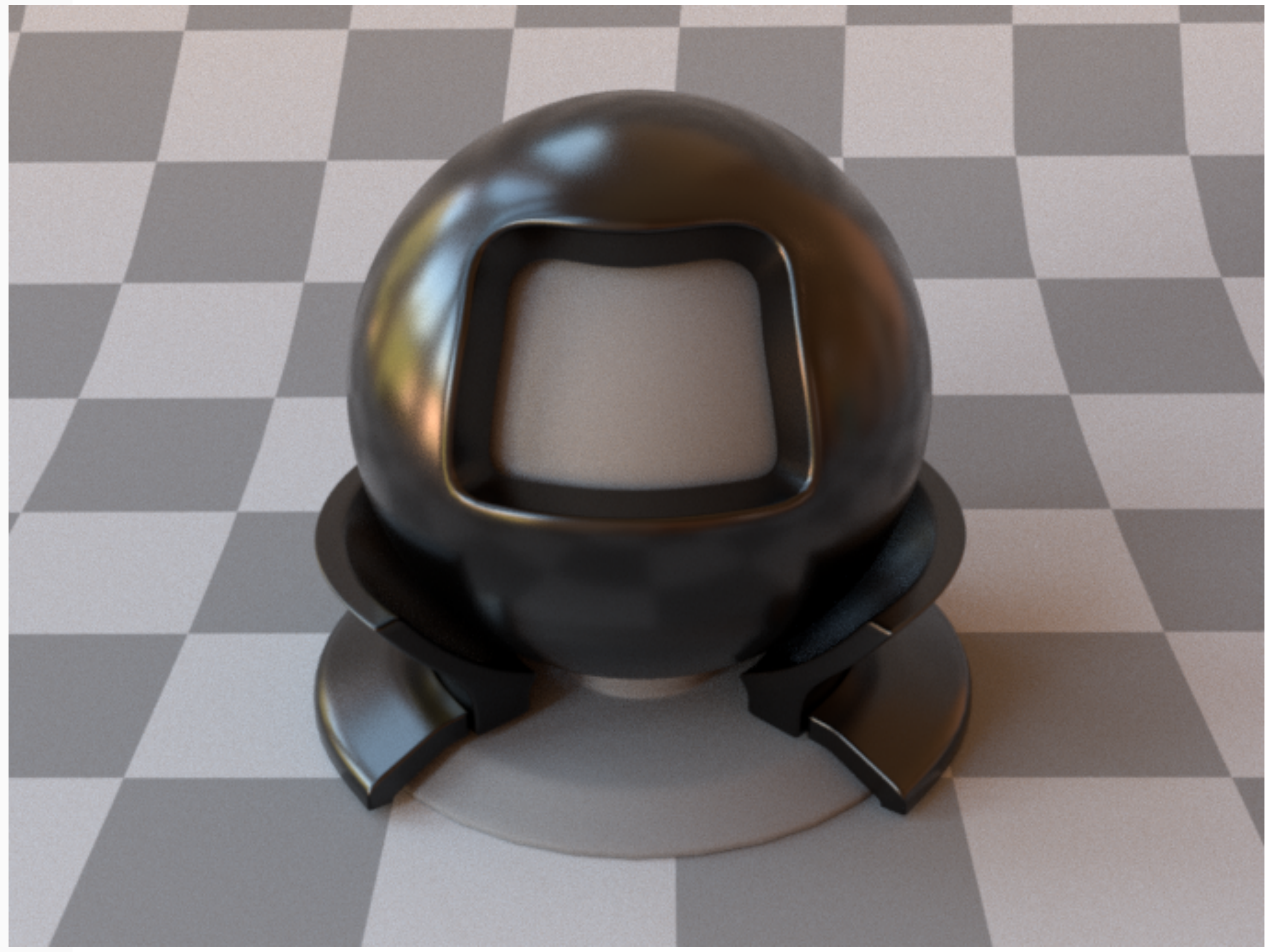
BSDFs describe reflection/transmission properties

ω



ω'





$$f_{\mathbf{p}}(\omega, \omega')$$

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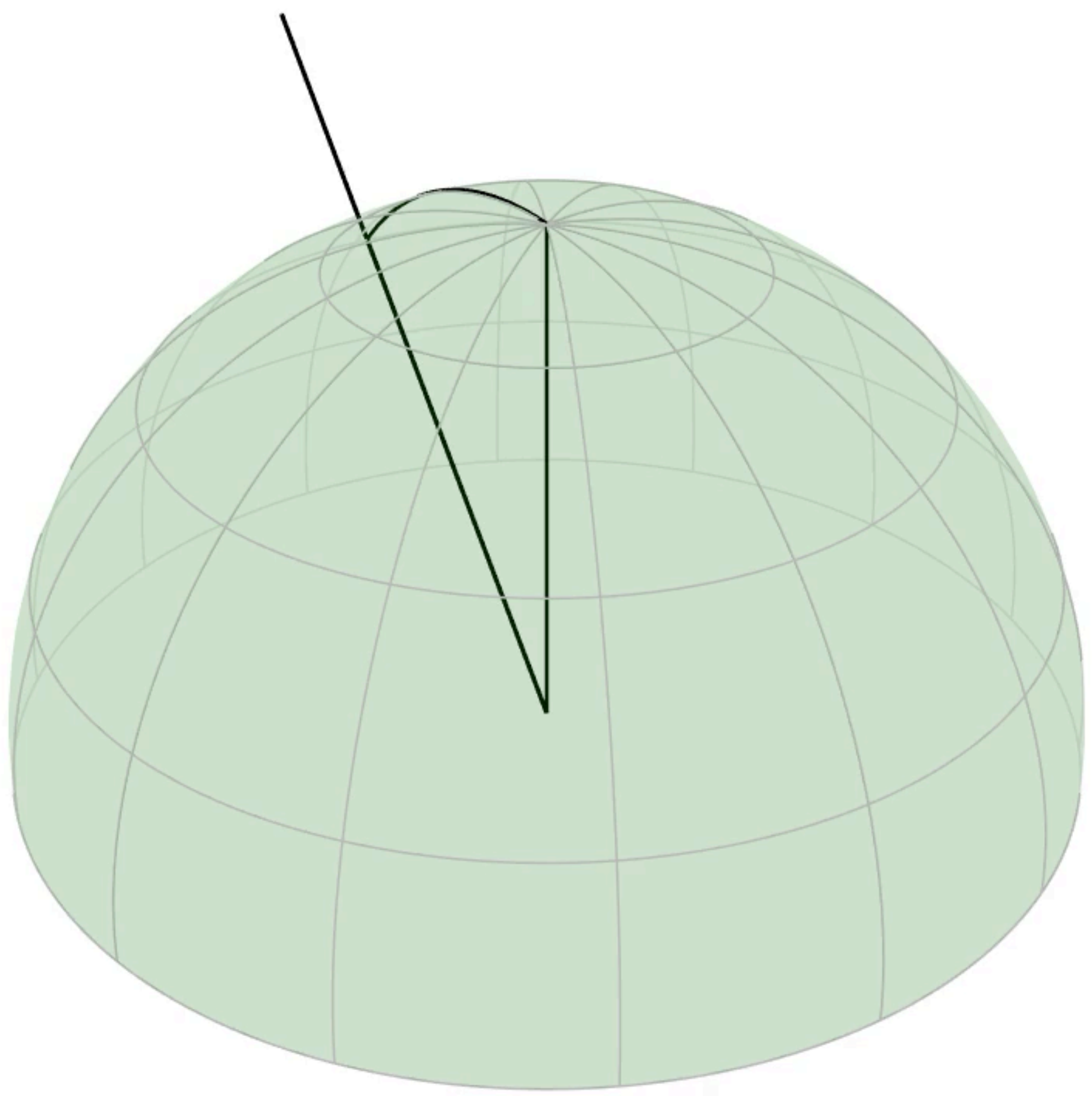
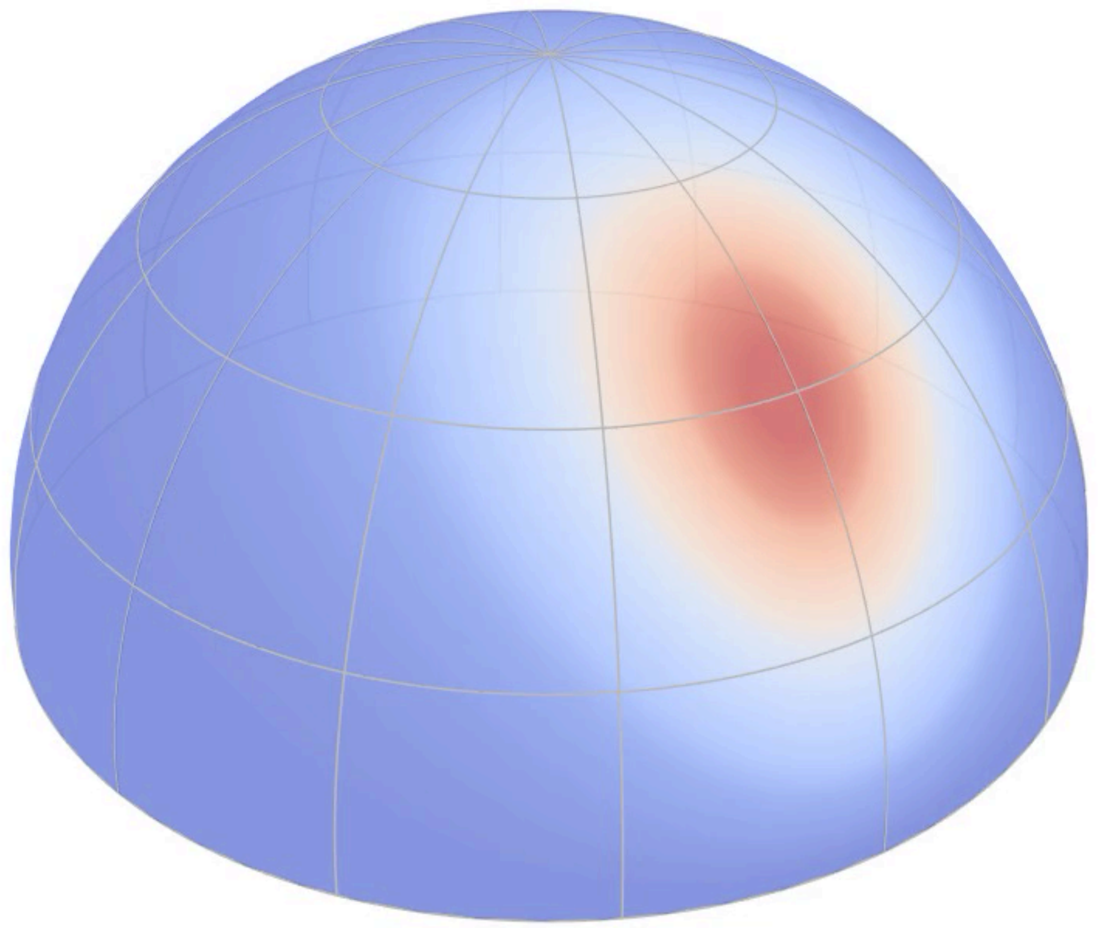
BSDFs describe reflection/transmission properties

ω $\qquad\qquad$ ω'





$$f_{\mathbf{p}}(\omega, \omega')$$

beautiful illustrations from Jonathan Dupuy
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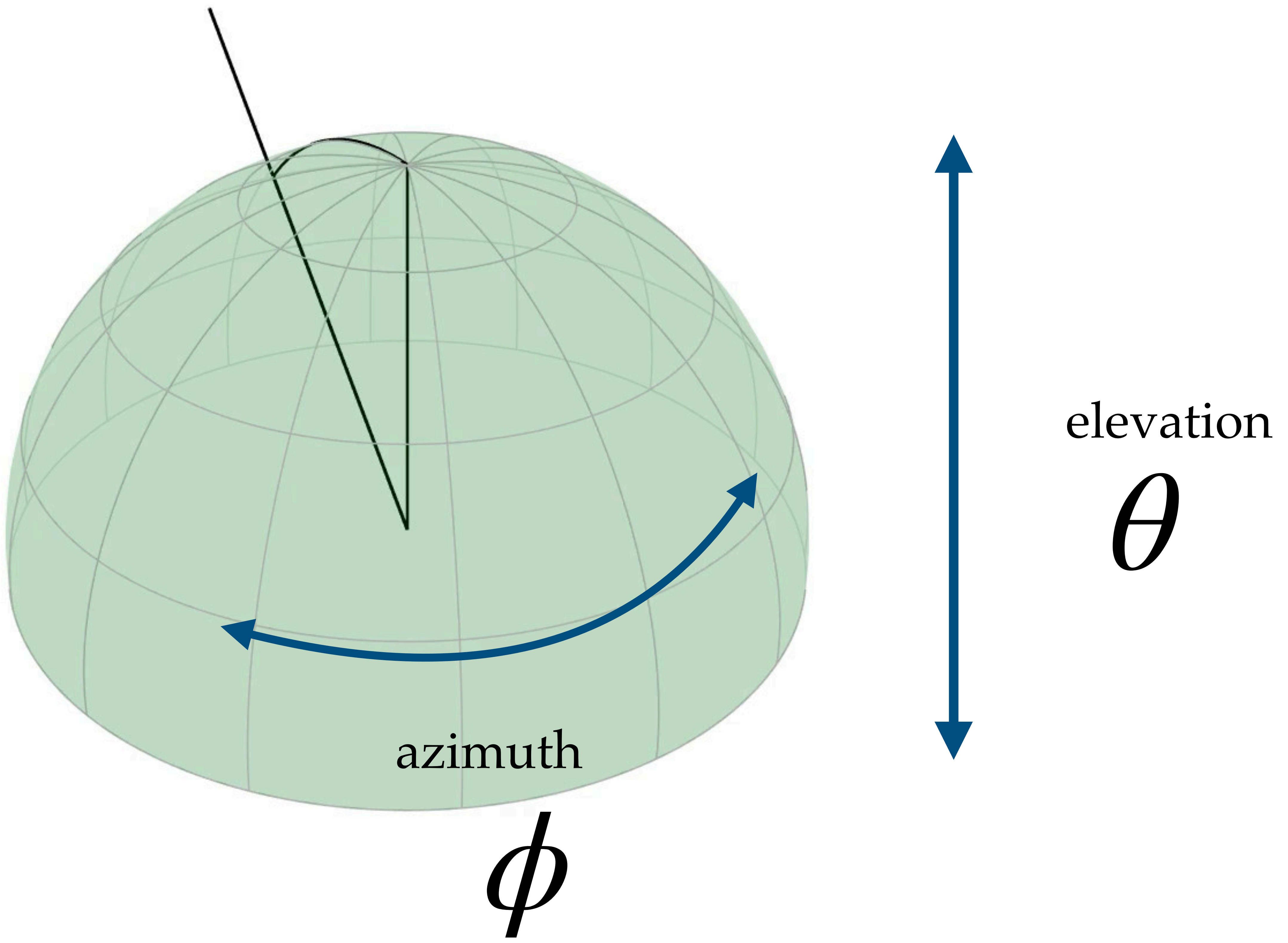
Isotropic BRDFs vs anisotropic BRDFs

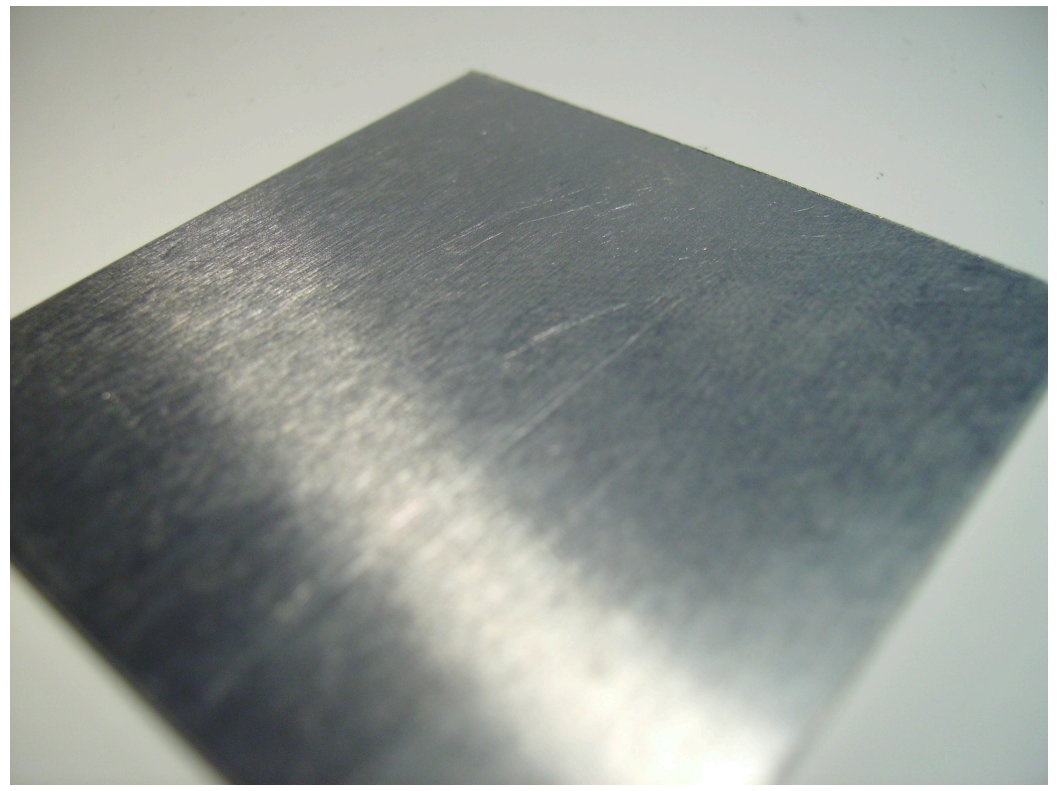
- isotropic BRDFs: reduces 4D BRDFs to 3D by only considering differences in azimuth angles

$$f_r(\theta', \phi', \theta, \phi) = f_r(\theta', \theta, \phi' - \phi)$$



Isotropic BRDFs vs anisotropic BRDFs

isotropic: circular highlights

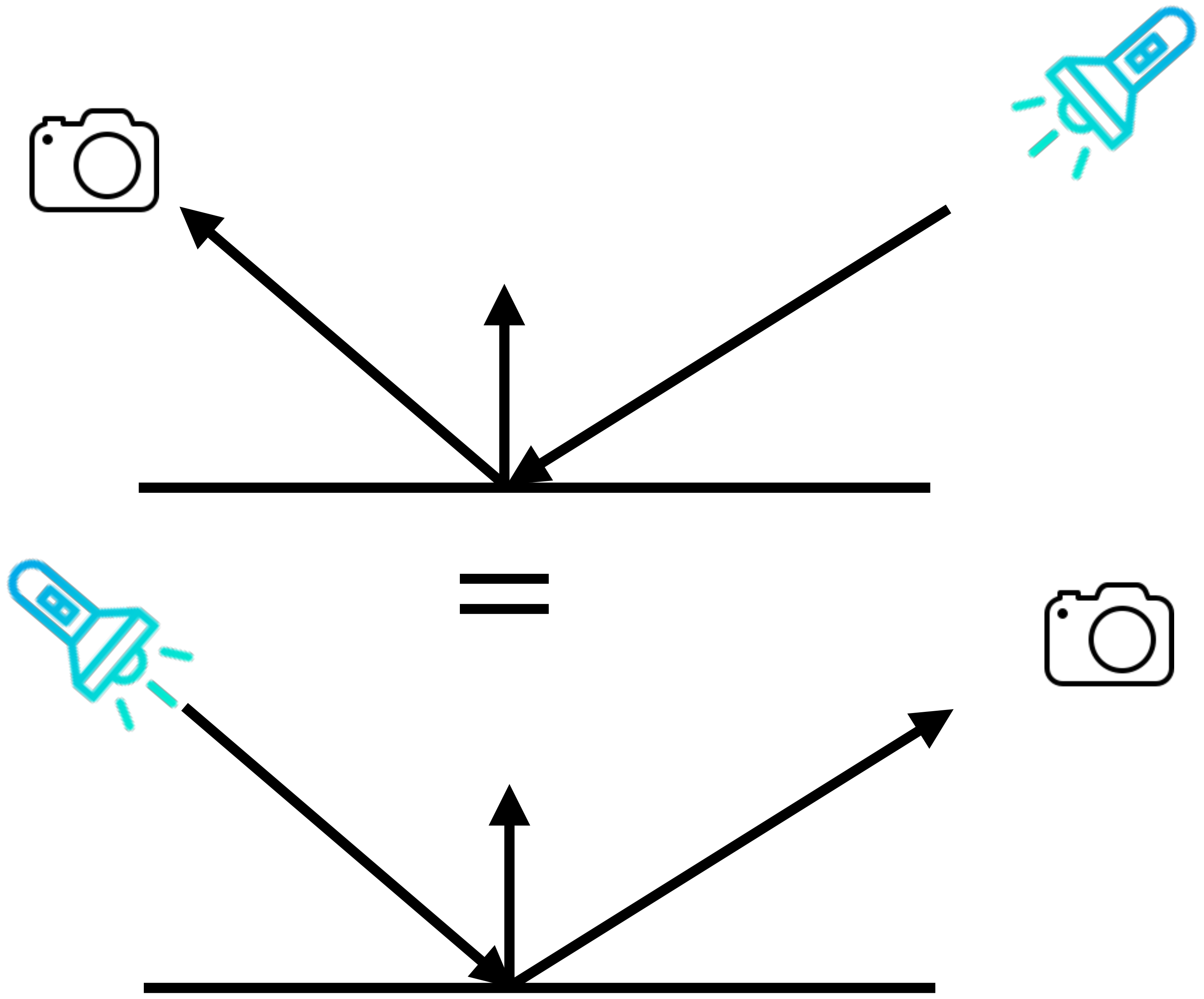


anisotropic: “directional” highlights

Reciprocity of BSDFs

quiz 1: why and when will this hold?

$$f_{\mathbf{p}}(\omega, \omega') = f_{\mathbf{p}}(\omega', \omega)$$



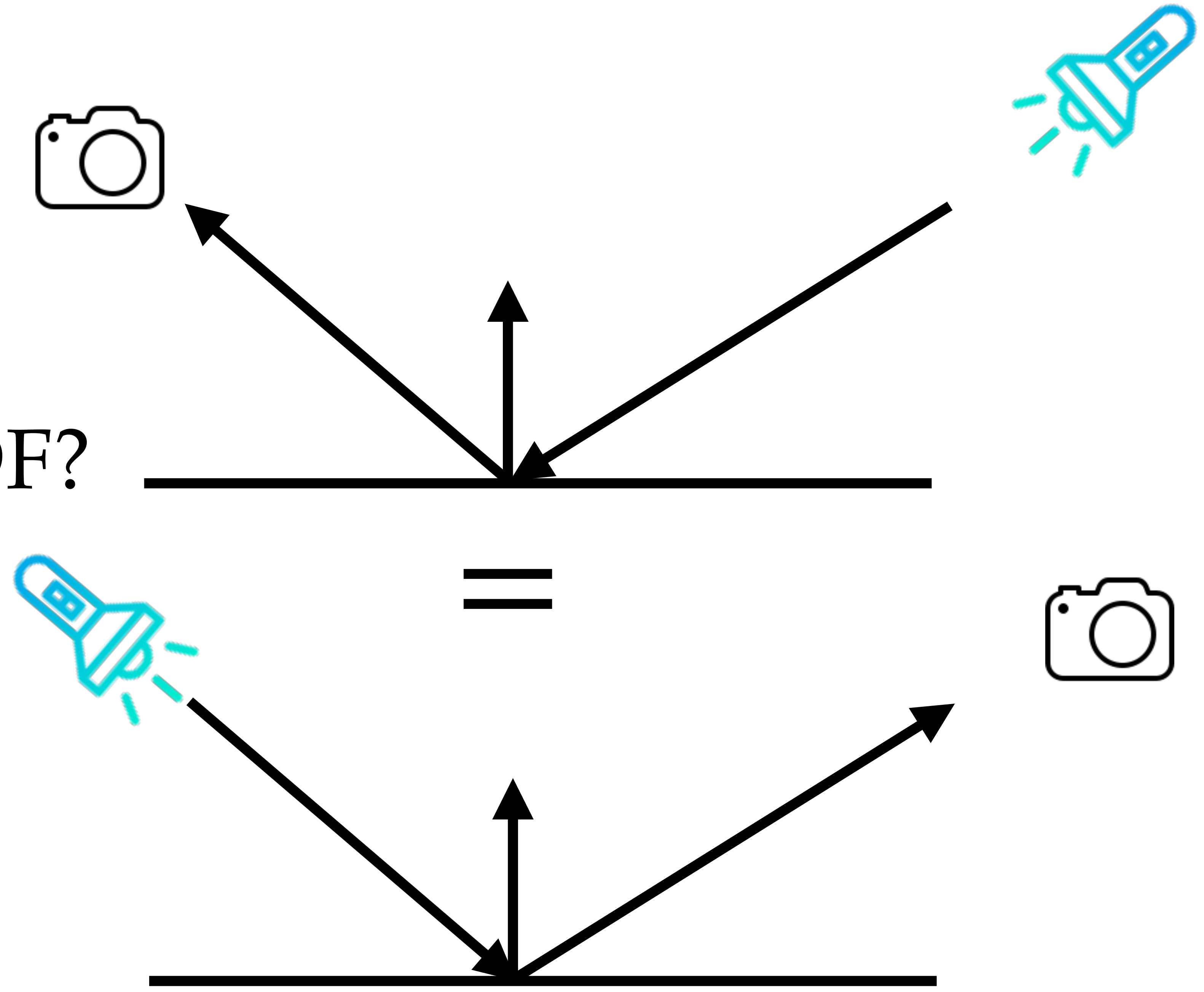
https://icons8.com/icons/set/flashlight
https://icons8.com/icons/set/camera

Reciprocity of BSDFs

quiz 1: why and when will this hold?

quiz 2: what is the consequence of a non-reciprocal BSDF?

$$f_{\mathbf{p}}(\omega, \omega') = f_{\mathbf{p}}(\omega', \omega)$$



https://icons8.com/icons/set/flashlight
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Sidetrack: exceptions of reciprocity

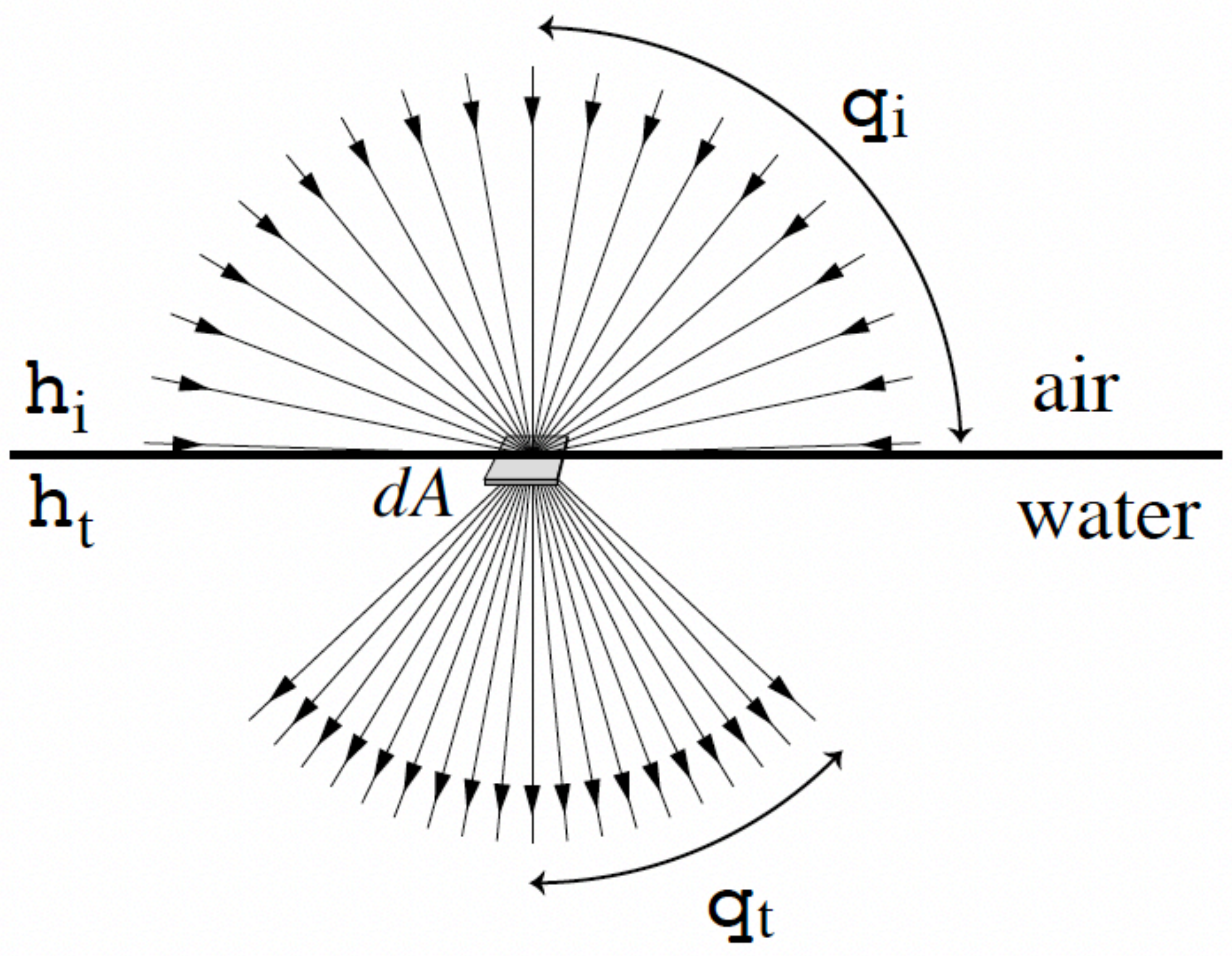

Figure 5.2: When light enters a medium with a higher refractive index, the same light energy is squeezed into a smaller volume. This causes the radiance along each ray to increase.

Eric Veach

ROBUST MONTE CARLO METHODS
FOR LIGHT TRANSPORT SIMULATION

Sidetrack: exceptions of reciprocity

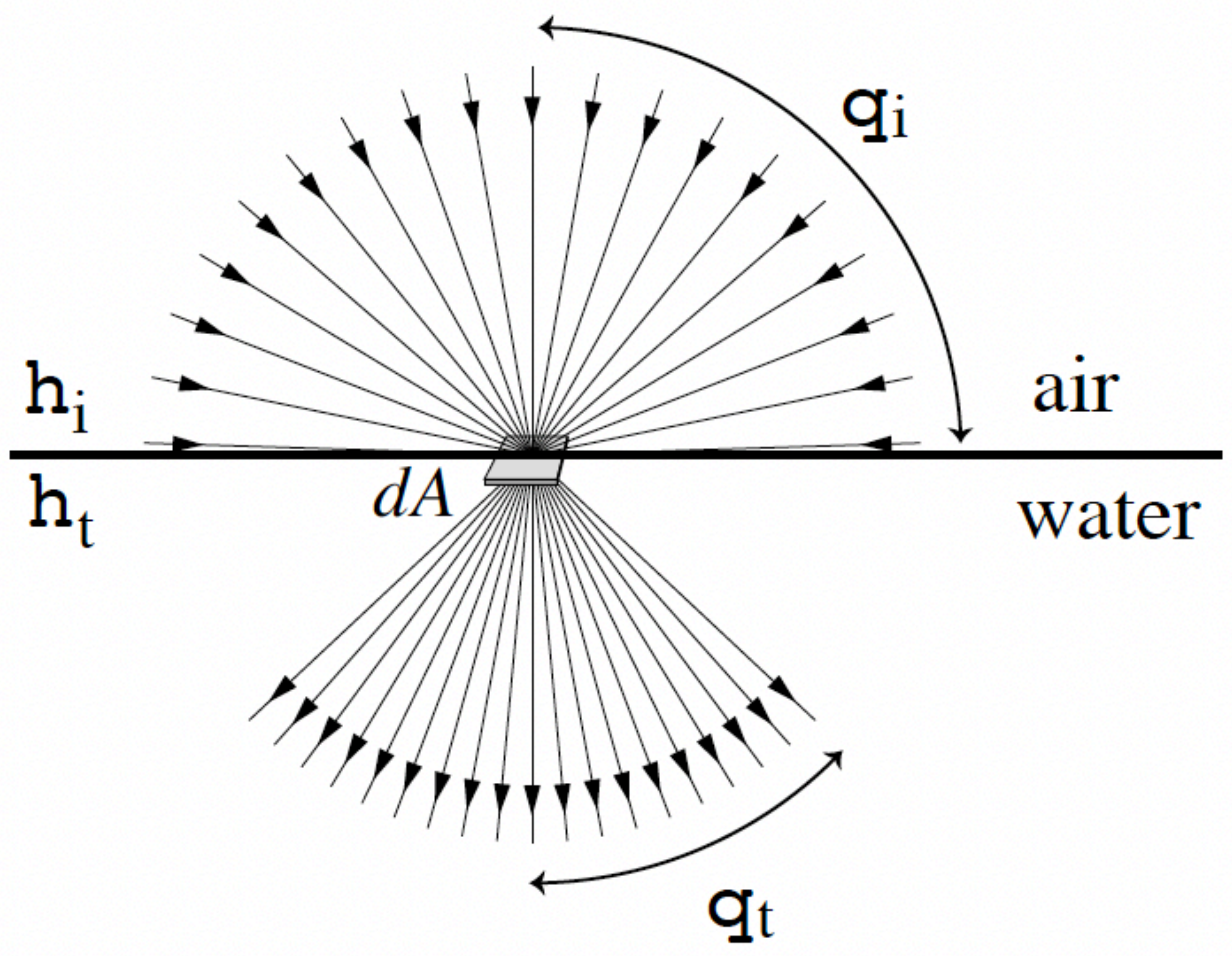

Figure 5.2: When light enters a medium with a higher refractive index, the same light energy is squeezed into a smaller volume. This causes the radiance along each ray to increase.

"real" reciprocity principle:

$$\frac{f_{\mathbf{p}}(\omega, \omega')}{\eta'^2} = \frac{f_{\mathbf{p}}(\omega', \omega)}{\eta^2}$$

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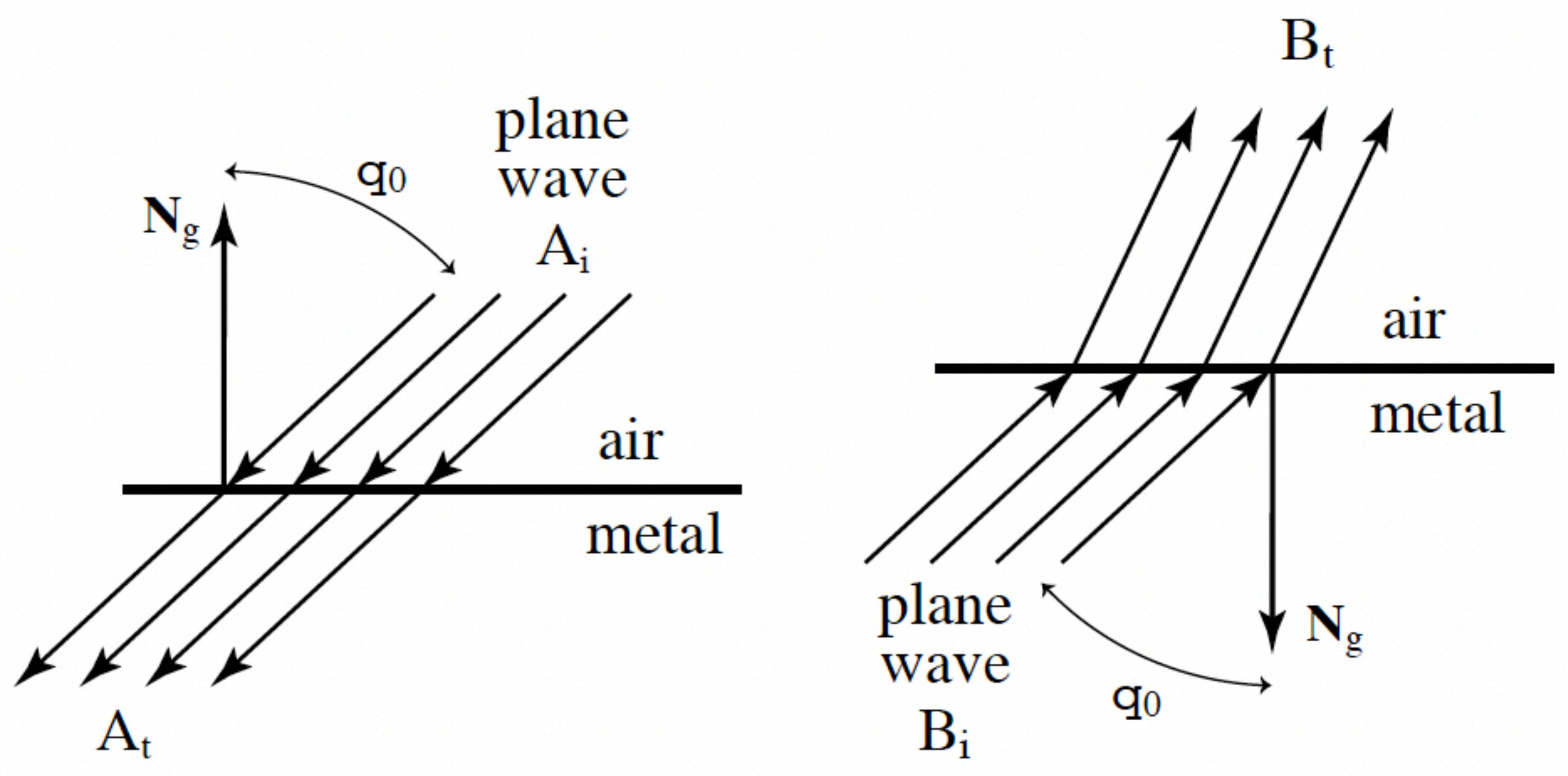
Sidetrack: exceptions of reciprocity



(a) Air to metal **(b)** Metal to air

Figure 6.2: When absorbing media such as metals are present, the path of a light beam is not always reversible. For example, when a light beam A_i is transmitted from air into some metals, there is a non-zero angle of incidence θ_0 for which the beam does not change its direction of propagation (Figure (a)). However, a beam of light B_i traveling in the reverse direction (from metal into air) is refracted at the surface, and follows a different path (Figure (b)).

absorbing media breaks reciprocity

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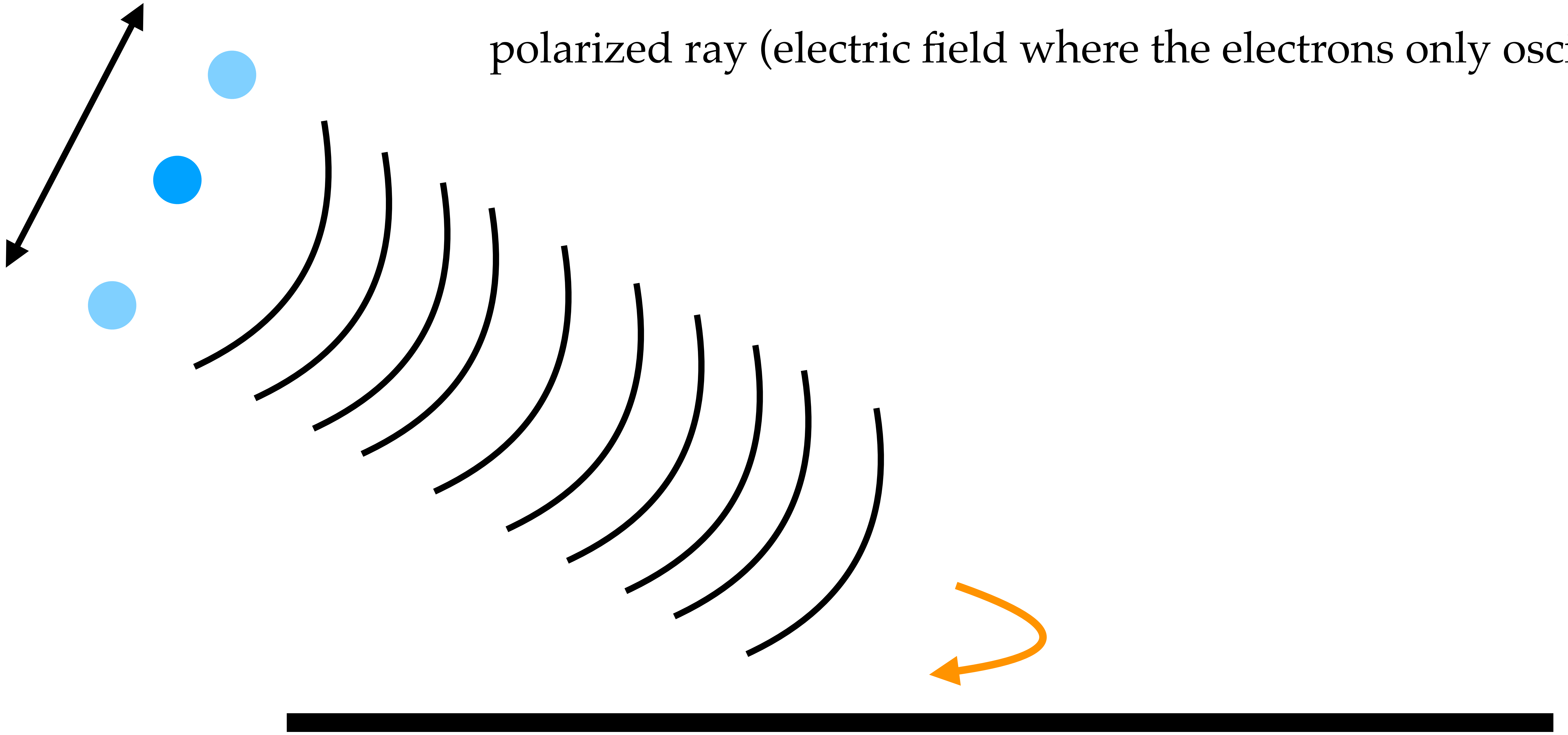
ROBUST MONTE CARLO METHODS
FOR LIGHT TRANSPORT SIMULATION

Sidetrack: exceptions of reciprocity

polarized ray (electric field where the electrons only oscillate in one direction)



magnetic field
(rotates the electric field oscillation regardless of the polarization direction)

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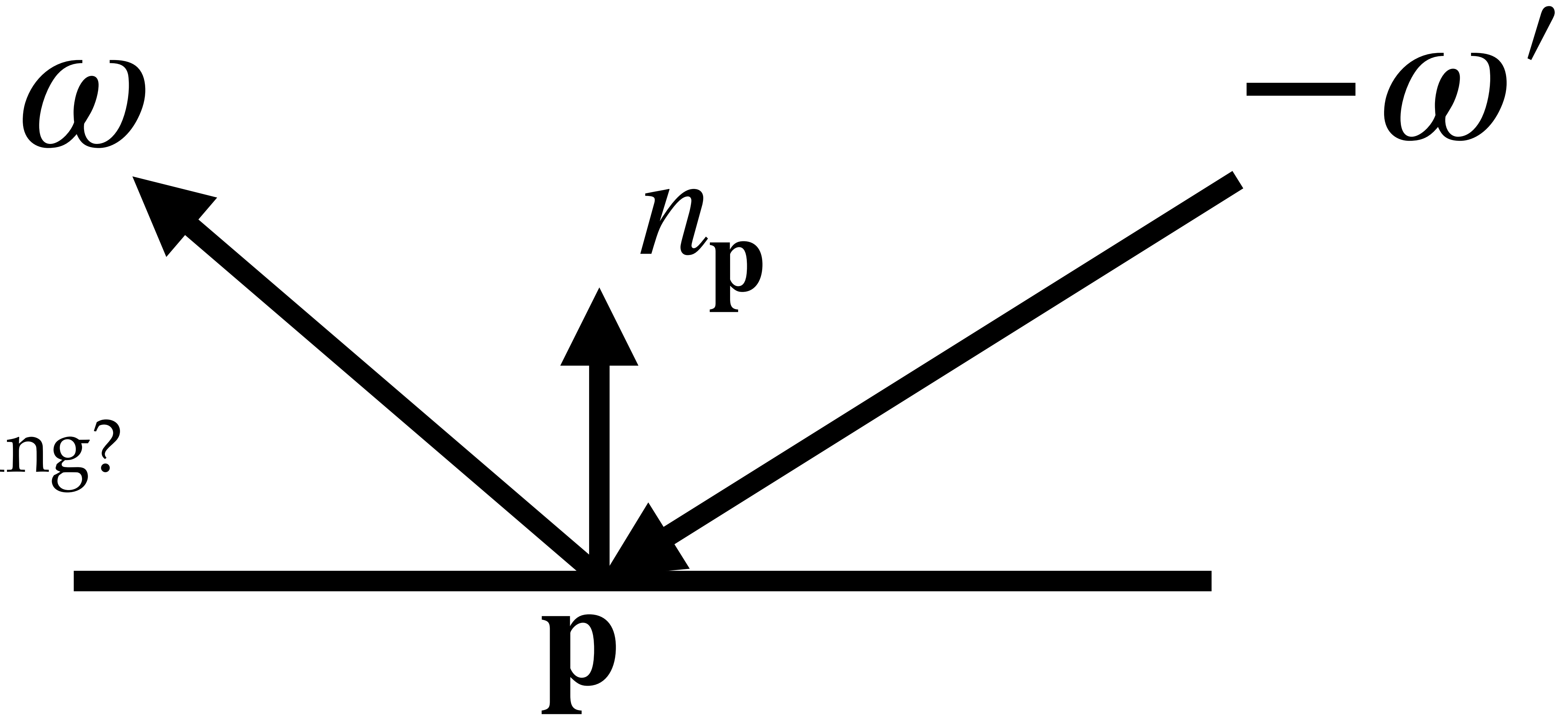
ROBUST MONTE CARLO METHODS
FOR LIGHT TRANSPORT SIMULATION

Energy conservation of BSDFs

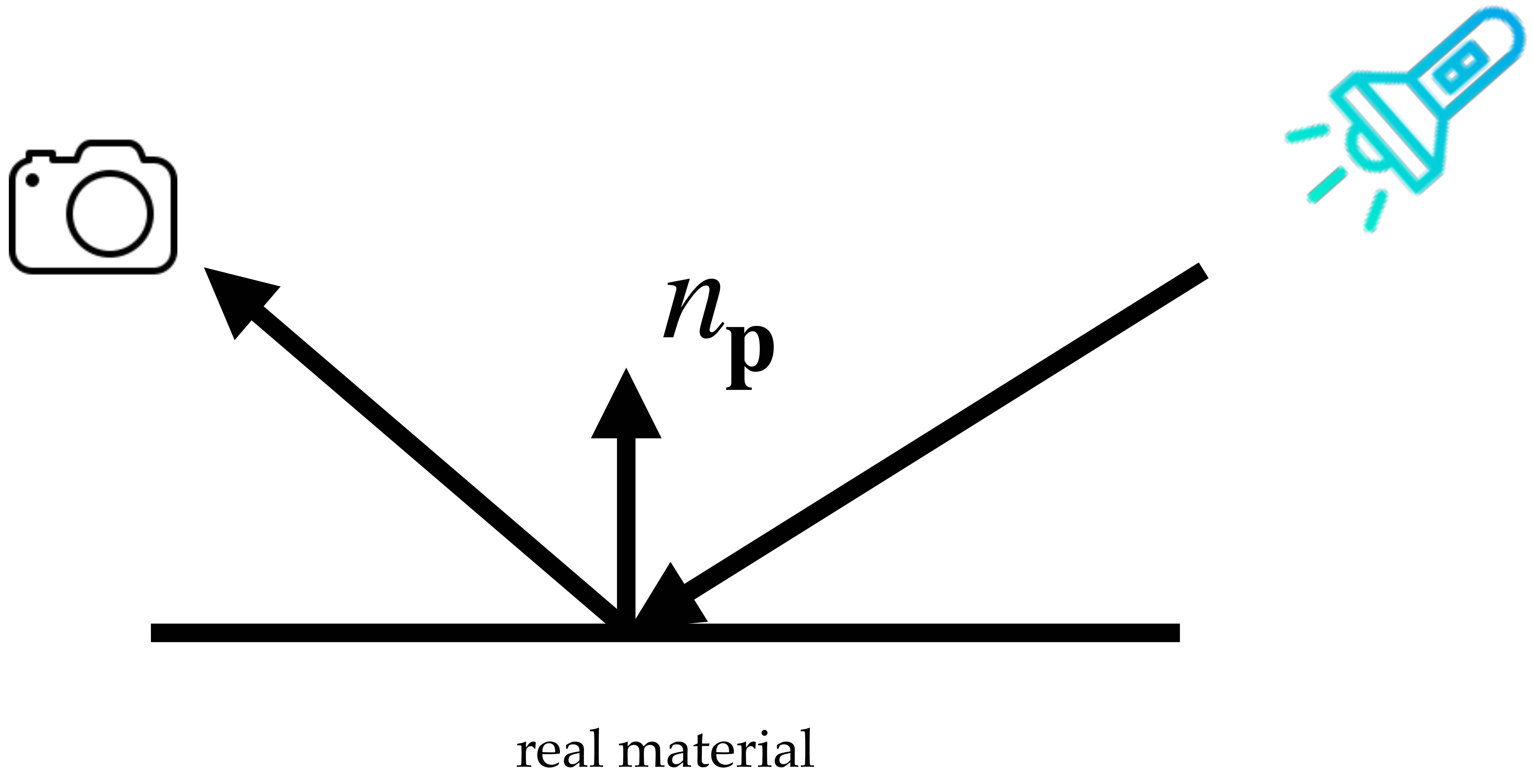
$$\int f_{\mathbf{p}}(\omega, \omega') \left| n_{\mathbf{p}} \cdot \omega' \right| \mathrm{d}\omega' \leq 1$$



quiz: what happens if your BSDF is not energy conserving?

How to obtain a BSDF?

- we can actually measure it!



quiz: how would you design a device for this?

https://icons8.com/icons/set/flashlight
https://icons8.com/icons/set/camera

How to obtain a BSDF?

- we can actually measure it!

A Data-Driven Reflectance Model

Wojciech Matusik [*] Hanspeter Pfister [†] Matt Brand[†] Leonard McMillan [‡]

How to obtain a BSDF?

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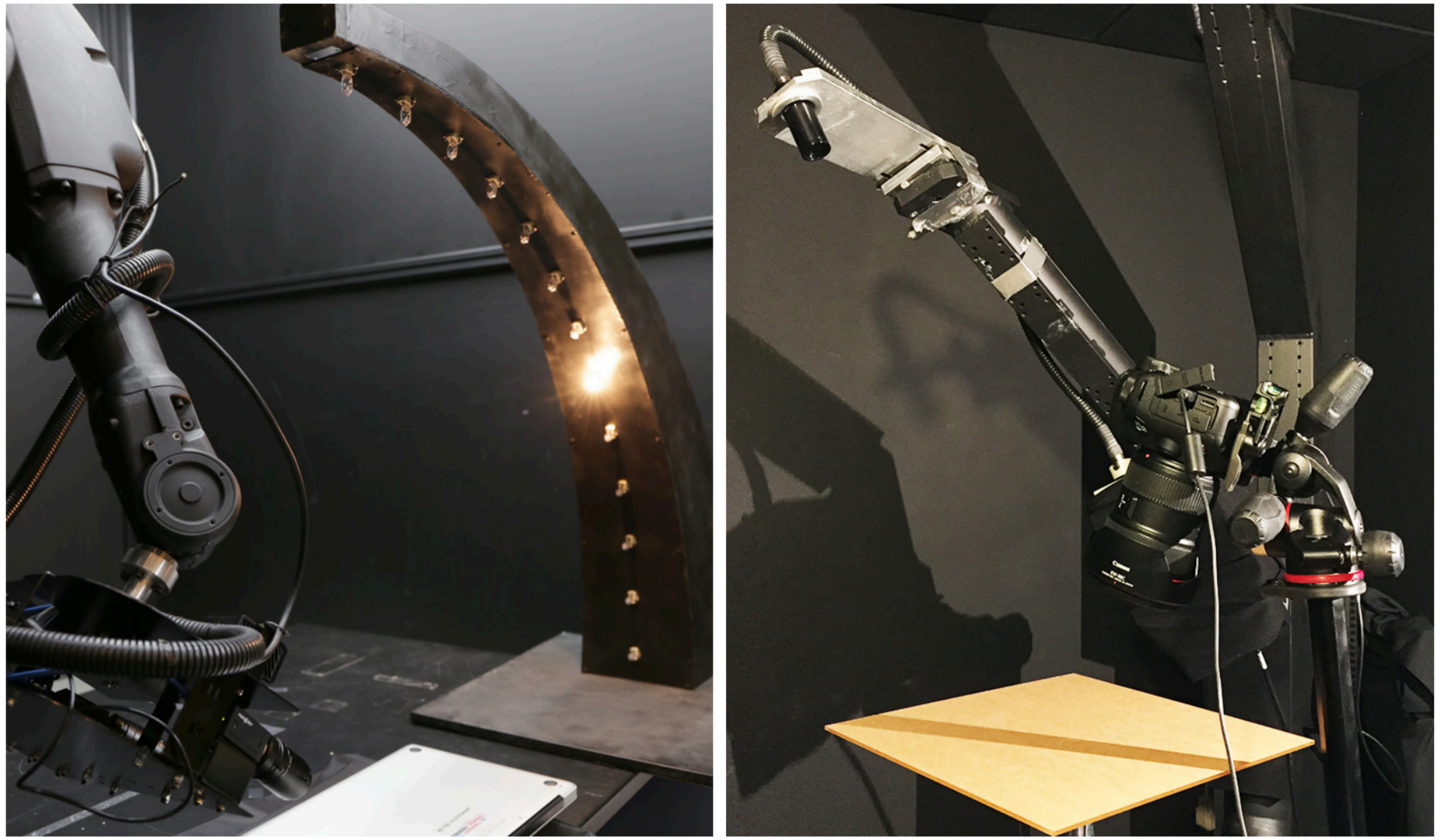
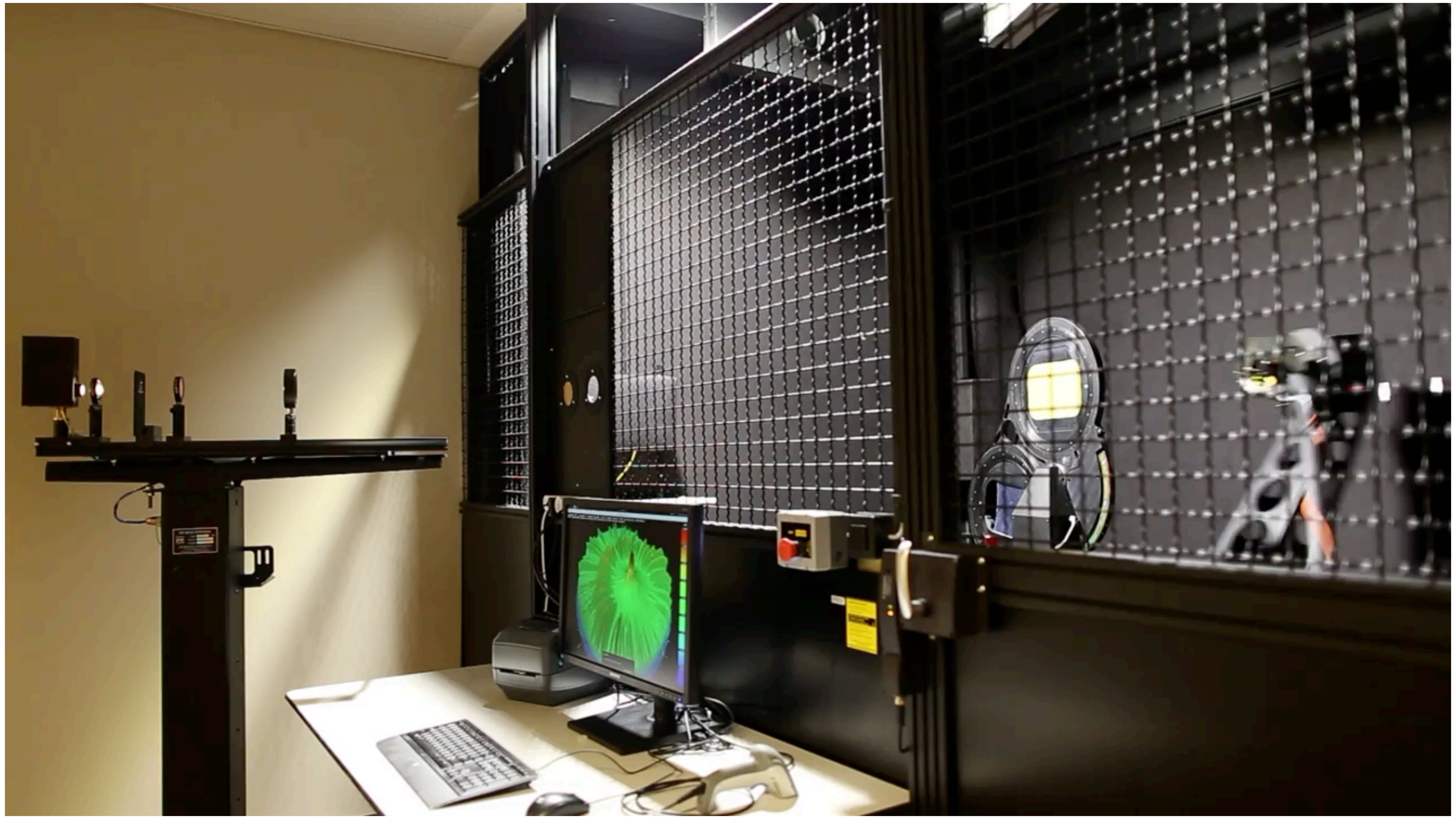


robot arm for BSDF measurement @ UCSD

https://cseweb.ucsd.edu/~ravir/nearfield.pdf

How to obtain a BSDF?

- we can actually measure it!

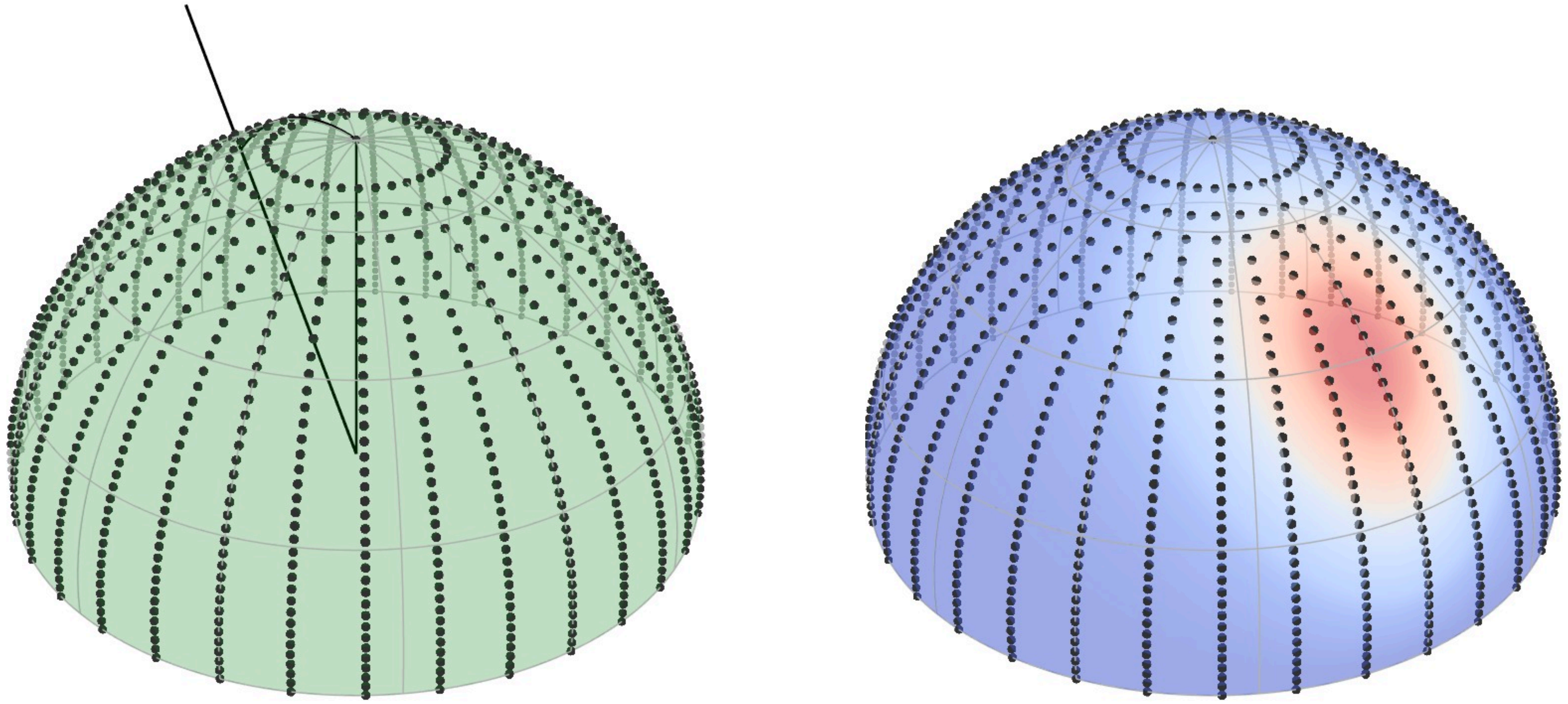


the robot arm in action @ EPFL

video from Wenzel Jakob

https://rgl.epfl.ch/pages/lab/pgII

Measuring BRDFs is time/memory consuming



need to measure a 4D domain

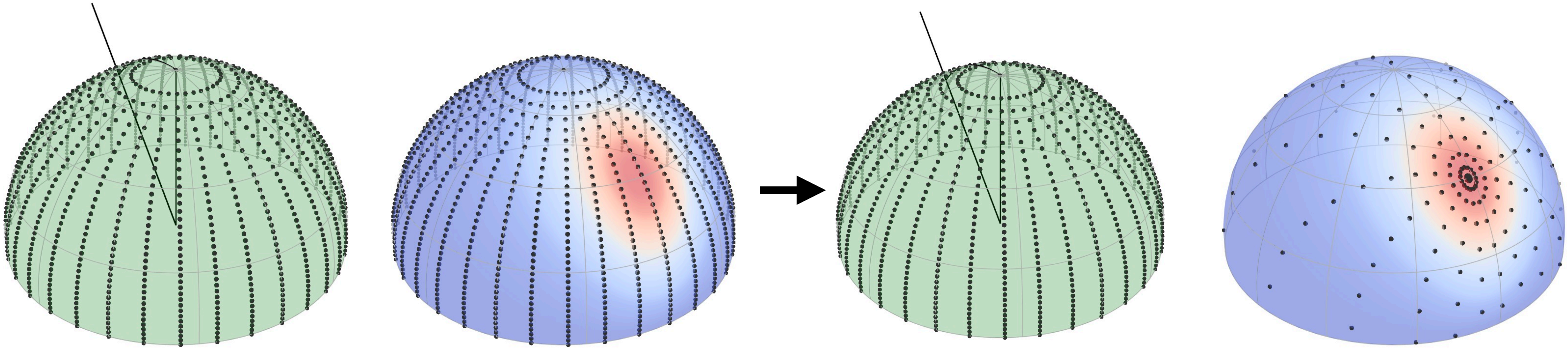
100 samples at each dimension:
100^4 = 100,000,000 (100 million samples)
1 second per sample: 3 years
380 MB per wavelength

amazing illustrations & numbers from Jonathan Dupuy
http://onrendering.com/data/papers/powitacq/slides/powitacq.html

Trick 1: focus on mirror reflection direction

Szymon Rusinkiewicz, "A New Change of Variables for Efficient BRDF Representation", 1998

- by applying a change of variable (again!)



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Trick 1: focus on mirror reflection direction

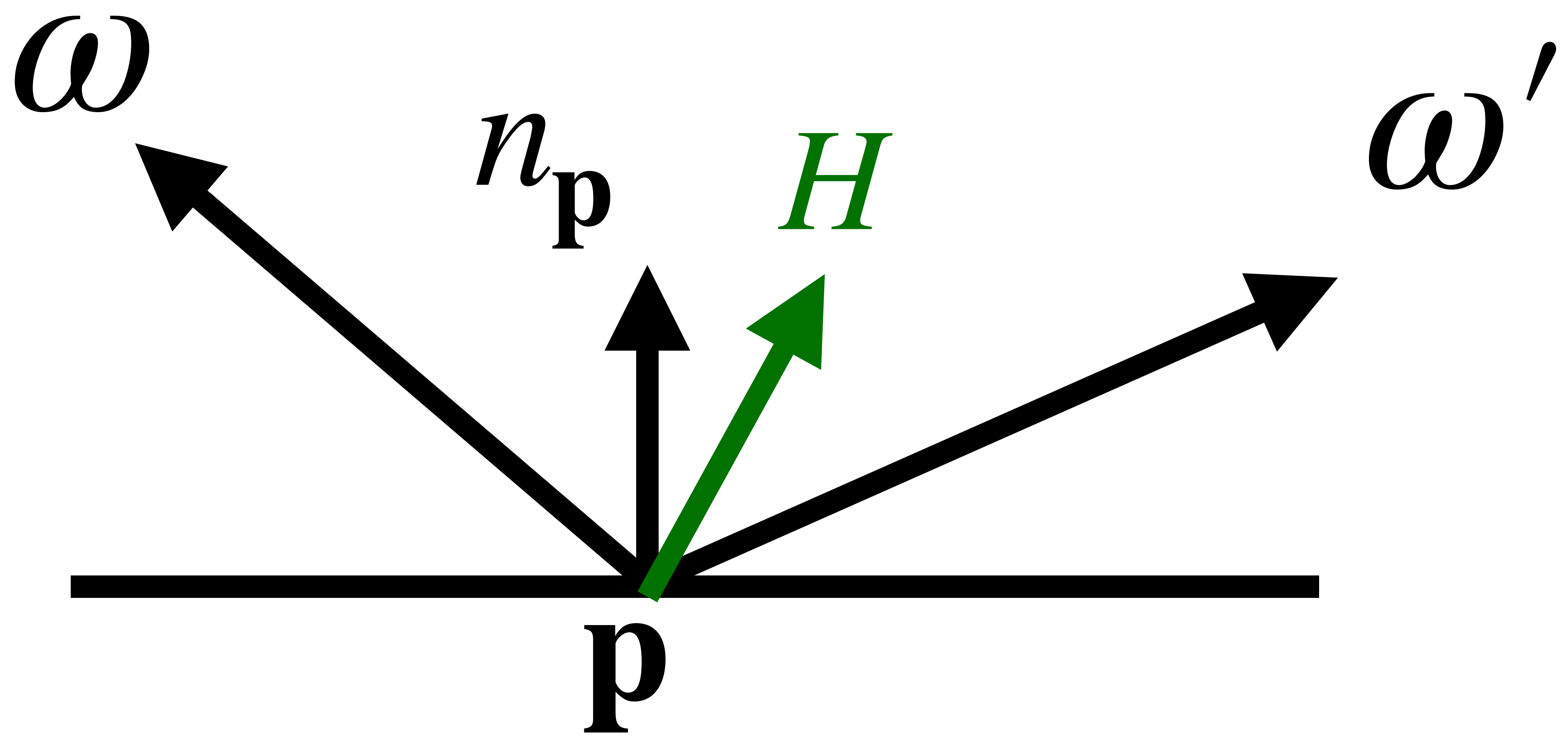
Szymon Rusinkiewicz, "A New Change of Variables for Efficient BRDF Representation", 1998

- by applying a change of variable (again!)

when $H = n_{\mathbf{p}}$
ω and ω' are mirror reflection directions

define **half-vector** $H = \text{normalize}(\omega + \omega')$

idea:
measure differences between normals/directions
& the half-vector



Trick 1: focus on mirror reflection direction

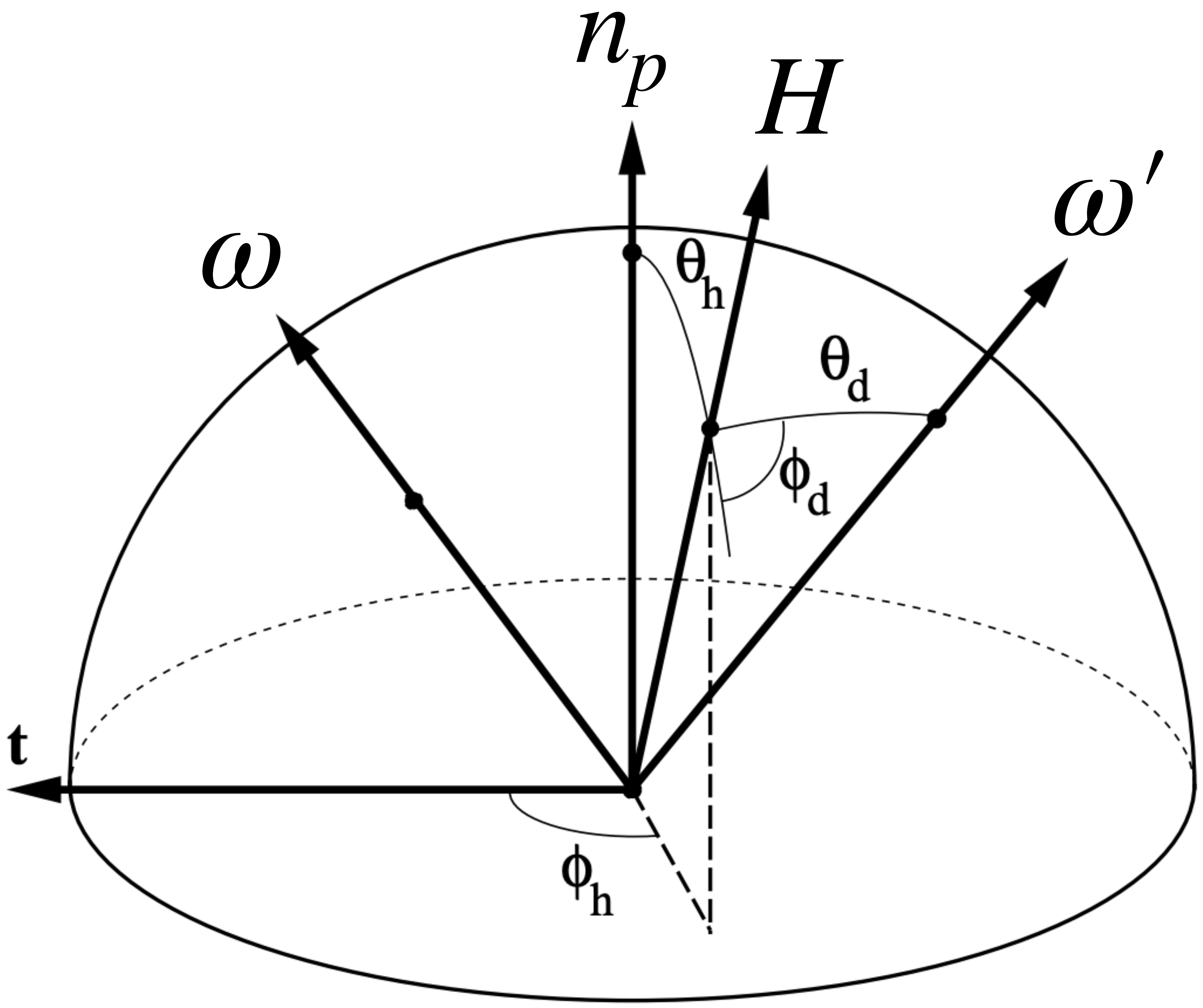
Szymon Rusinkiewicz, "A New Change of Variables for Efficient BRDF Representation", 1998

- by applying a change of variable (again!)

idea:
measure differences between normals/directions
& the half-vector

change of variable:

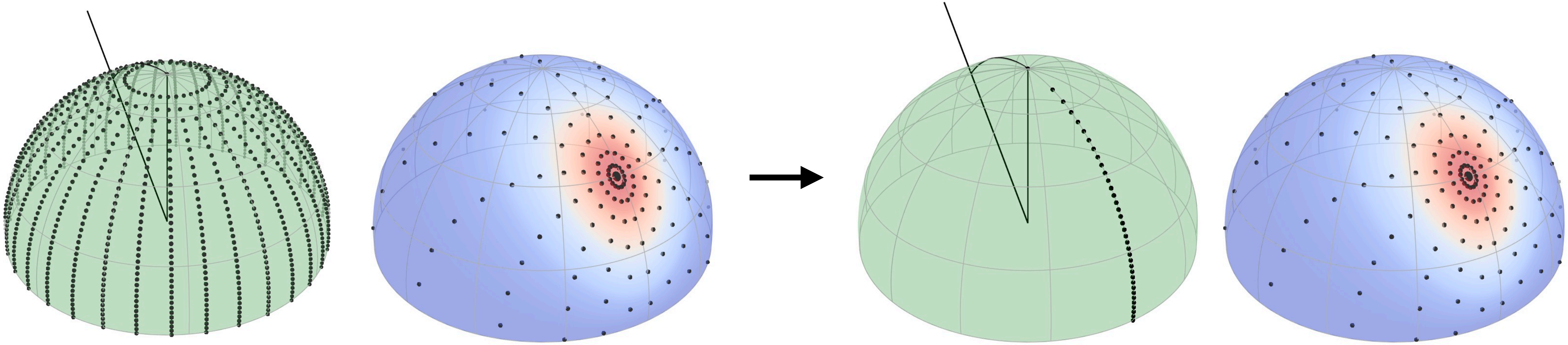
$$(\theta, \phi, \theta', \phi') \rightarrow (\theta_h, \phi_h, \theta_d, \phi_d)$$



https://www.cs.princeton.edu/~smr/papers/brdf_change_of_variables/brdf_change_of_variables.pdf

Trick 2: focus on elevation, ignore azimuth

- assume the BSDF does not change over azimuthal angles (not always true)
  - "isotropic BSDF"



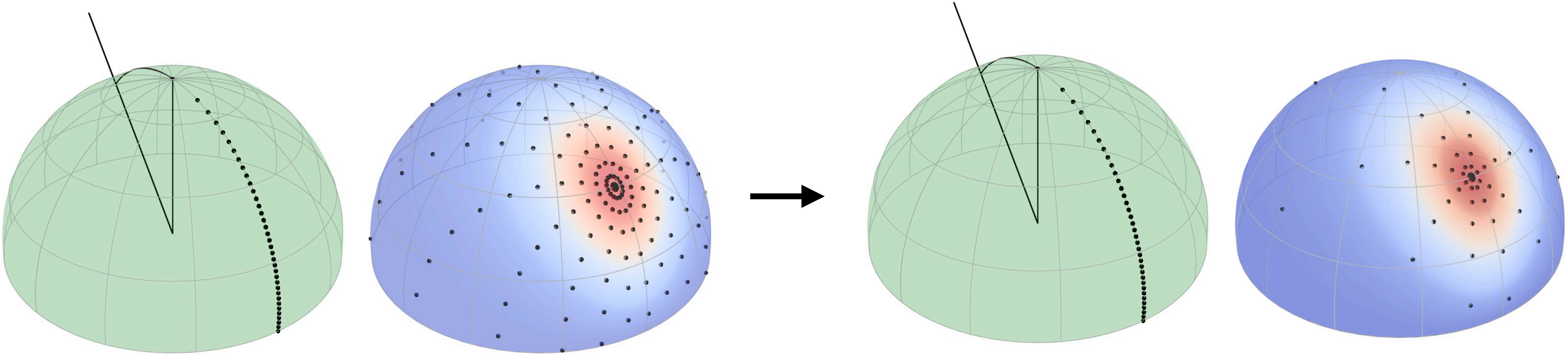
amazing illustrations from Jonathan Dupuy
http://onrendering.com/data/papers/powitacq/slides/powitacq.html

Trick 3: estimate the roughness

Jonathan Dupuy and Wenzel Jakob, "An Adaptive Parameterization for Efficient Material Acquisition and Rendering", 2018

- and apply another change of variable to scale the samples!

(still need ~2 hours to measure a material after all these tricks)



amazing illustrations from Jonathan Dupuy
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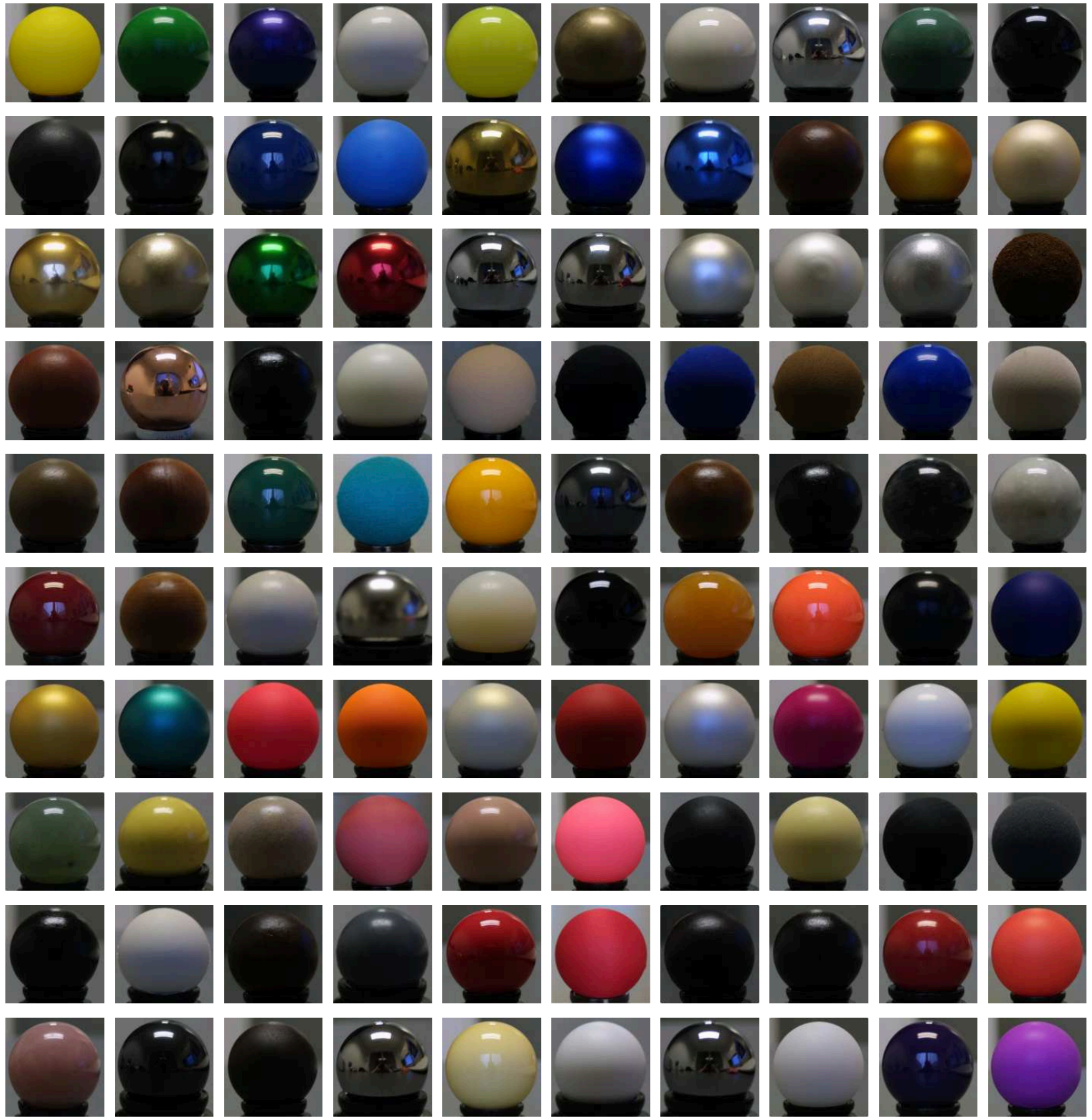
The MERL BSDF dataset [Matusik 2003]



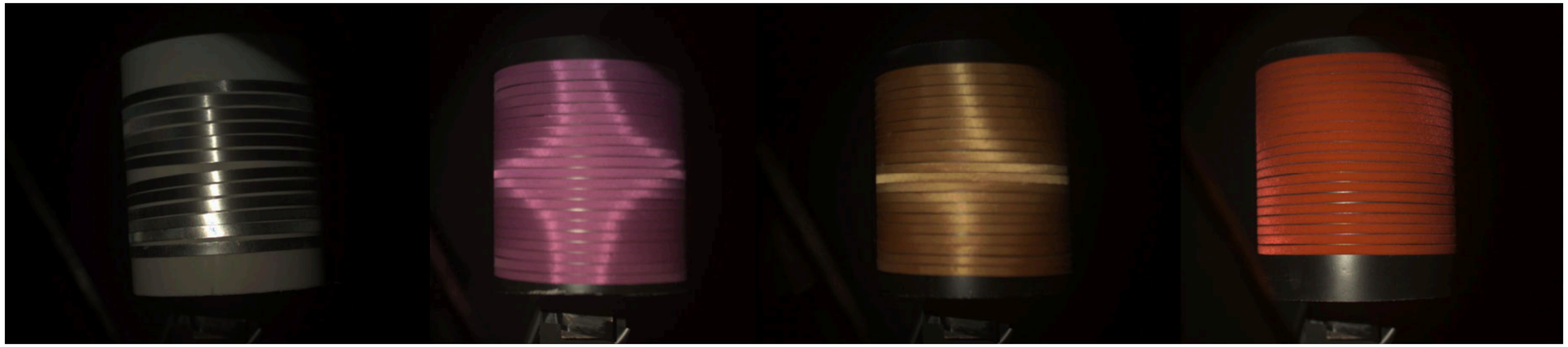
most popular data
100 isotropic BRDFs

warning: not a perfect dataset!
lots of camera artifacts
(defocus/bokeh/lens flare)

A Data-Driven Reflectance Model

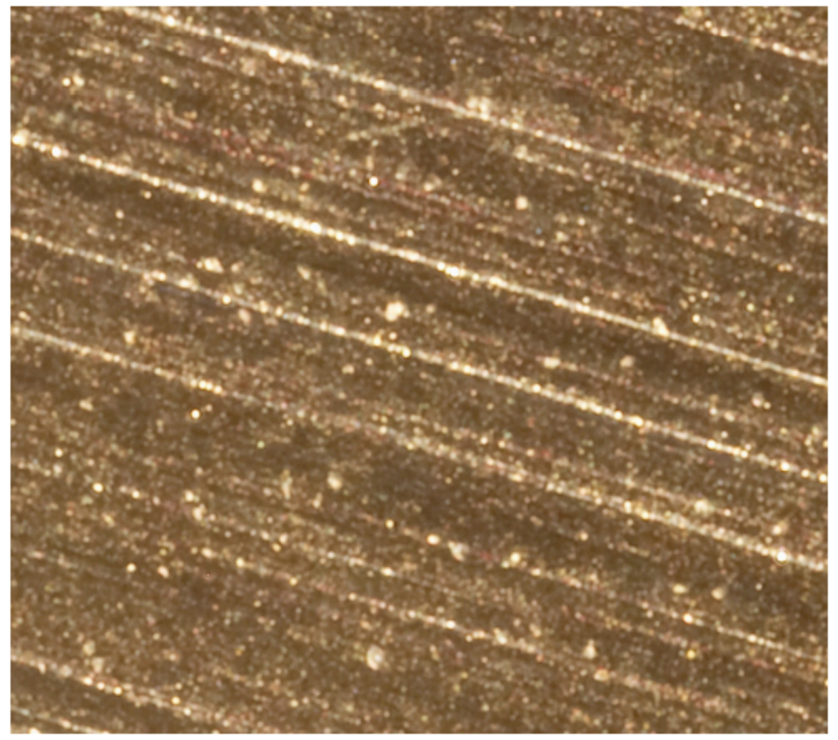
Wojciech Matusik * Hanspeter Pfister † Matt Brand† Leonard McMillan ‡

MERL anisotropic extension

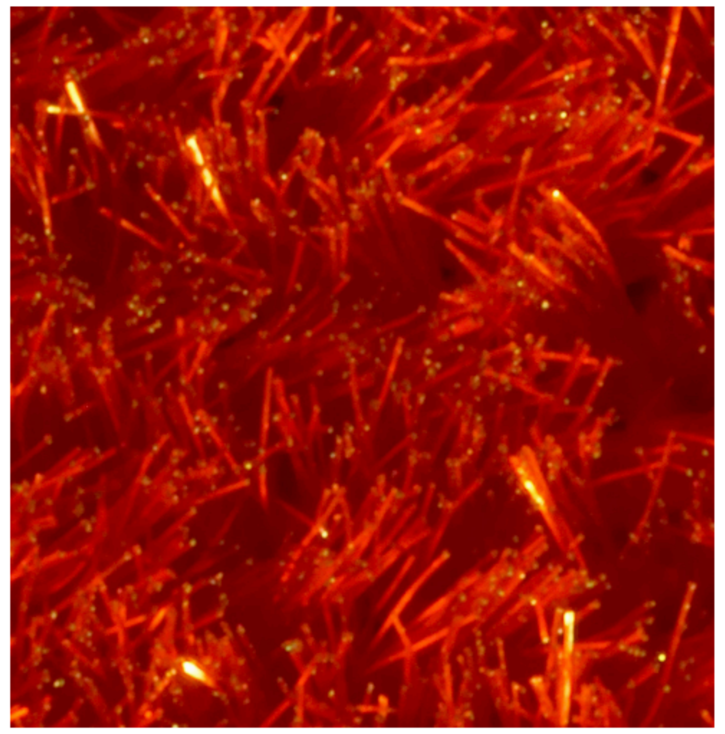


renderings

4 anisotropic BRDFs



satin



velvet

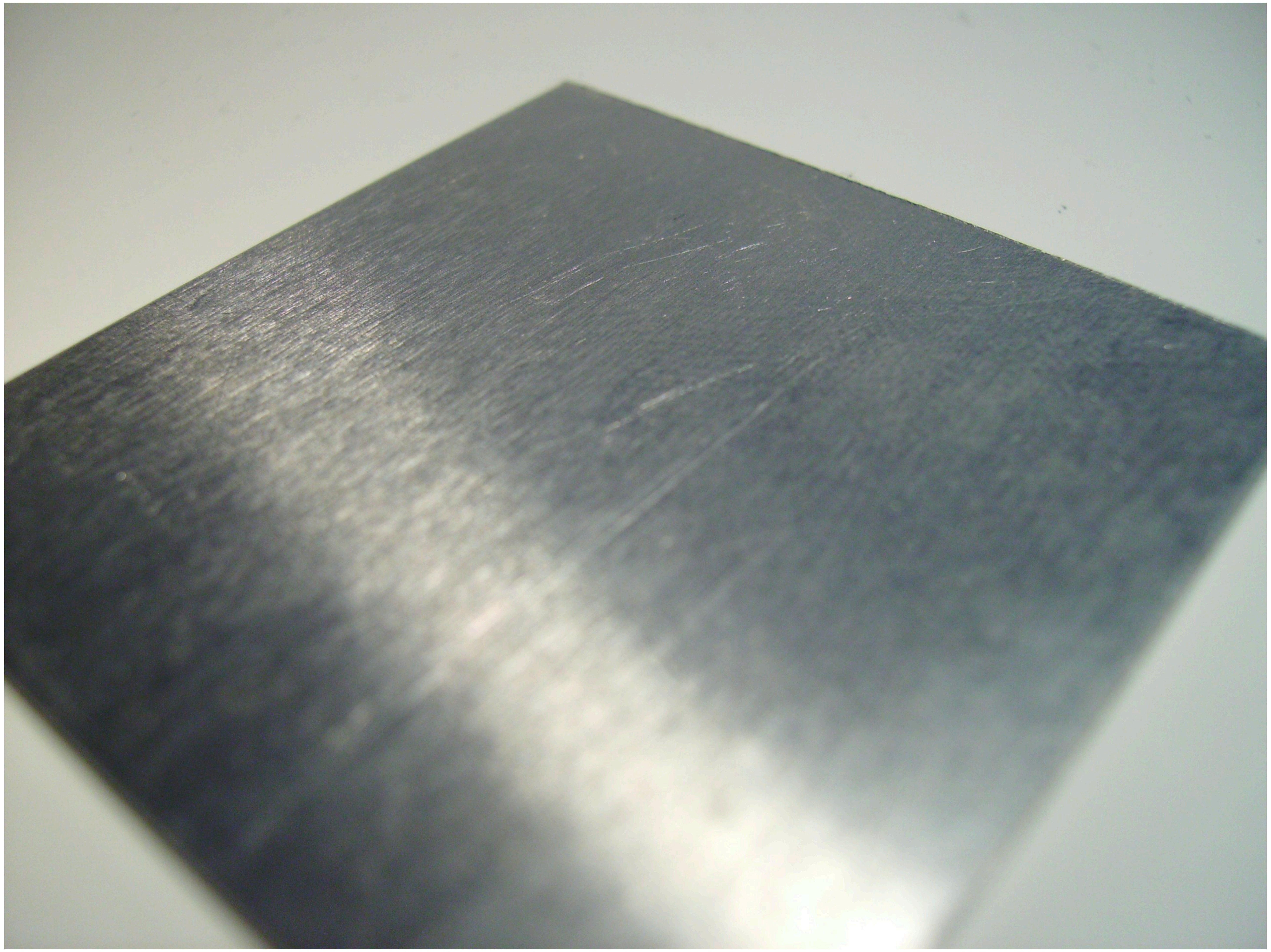
actual photographs

Experimental Analysis of BRDF Models

Addy Ngan, Frédo Durand,[†] and Wojciech Matusik[‡]

MIT CSAIL MERL

Brushed metal: common anisotropic material



real photograph

https://en.wikipedia.org/wiki/Brushed_metal

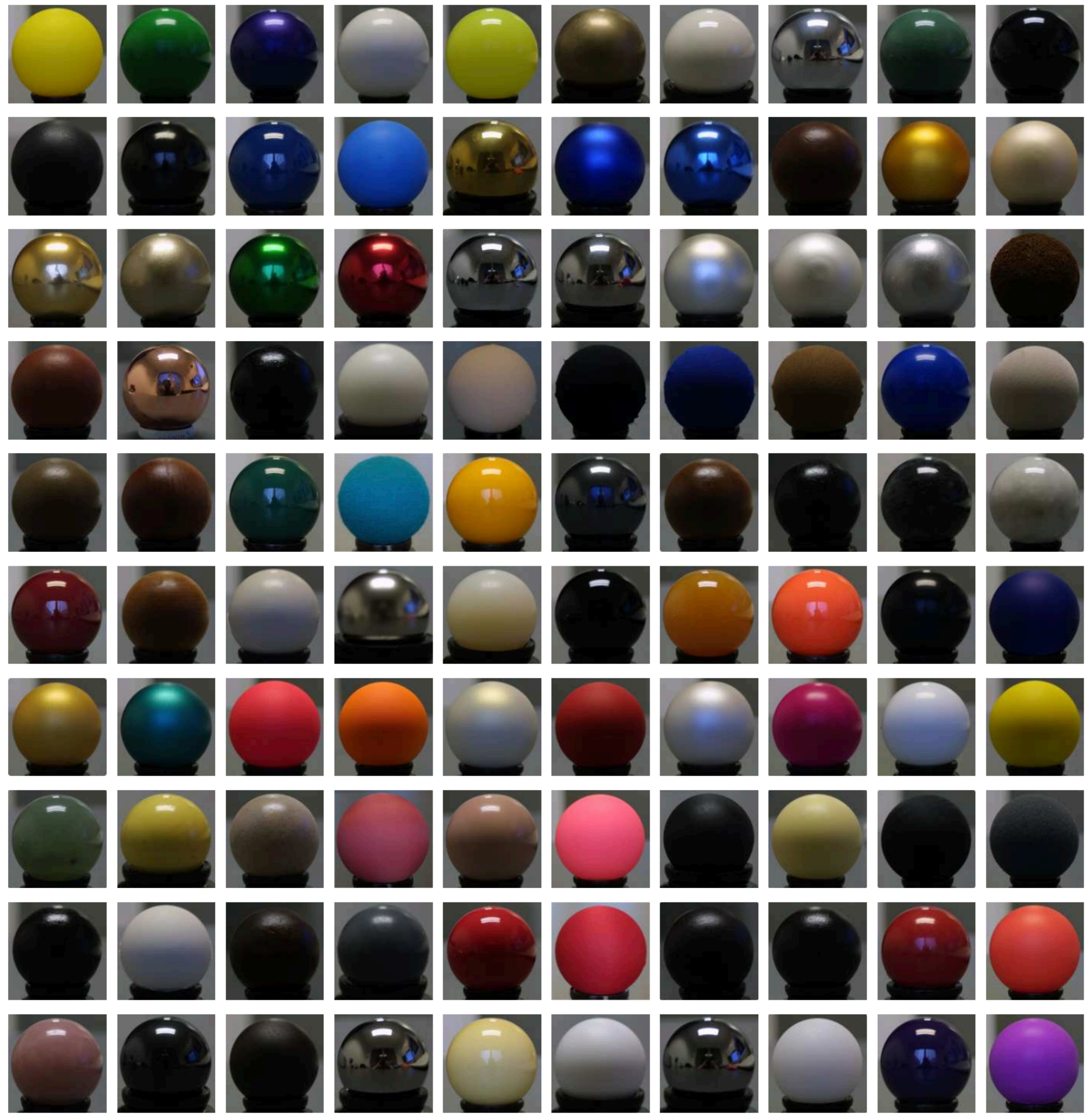
EPFL material database [2018]

50 isotropic BRDFs
12 anisotropic BRDFs
(probably much higher quality than MERL)

http://onrendering.com/data/papers/powitacq/slides/powitacq.html

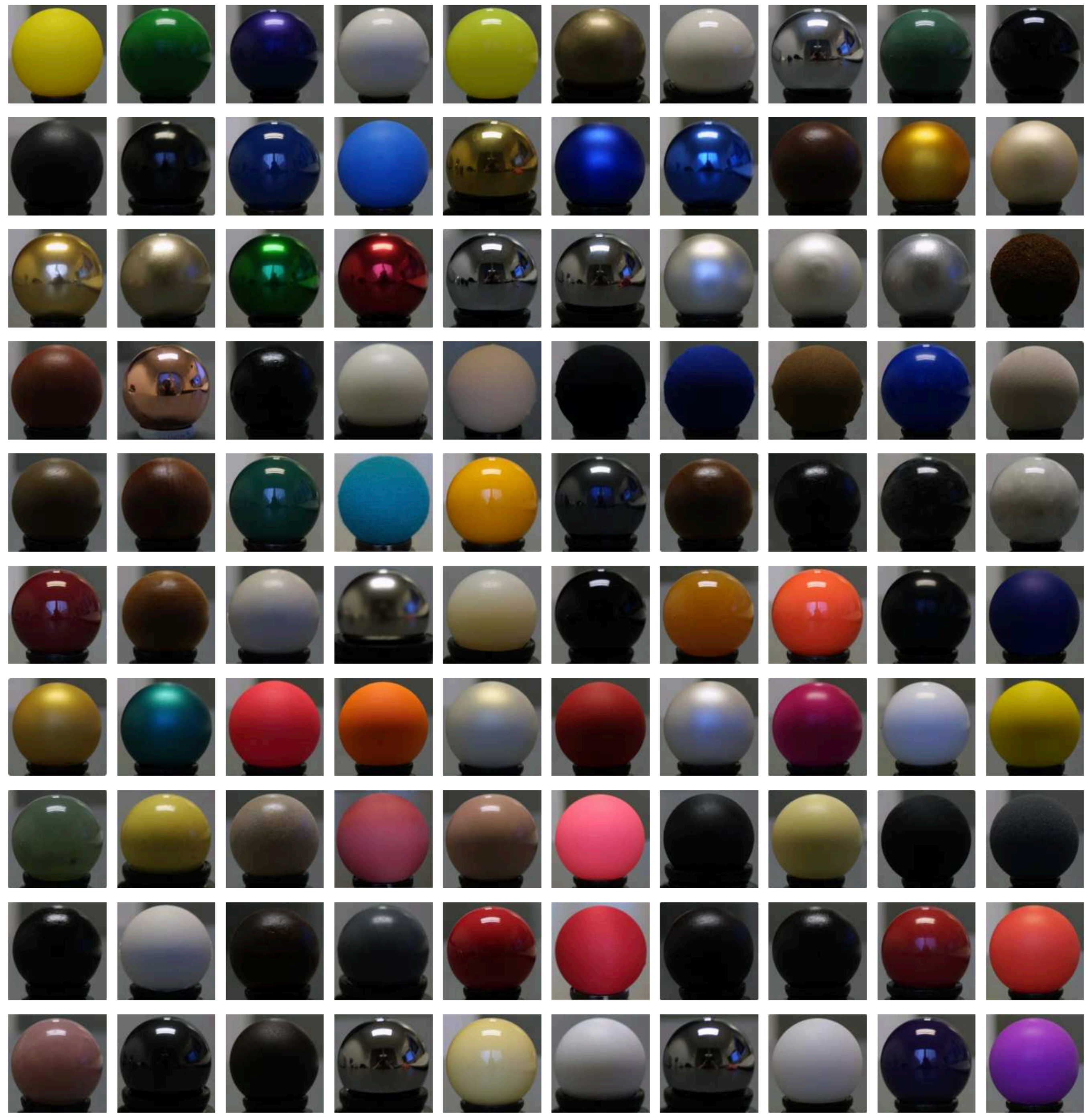
Downsides of measured BSDFs

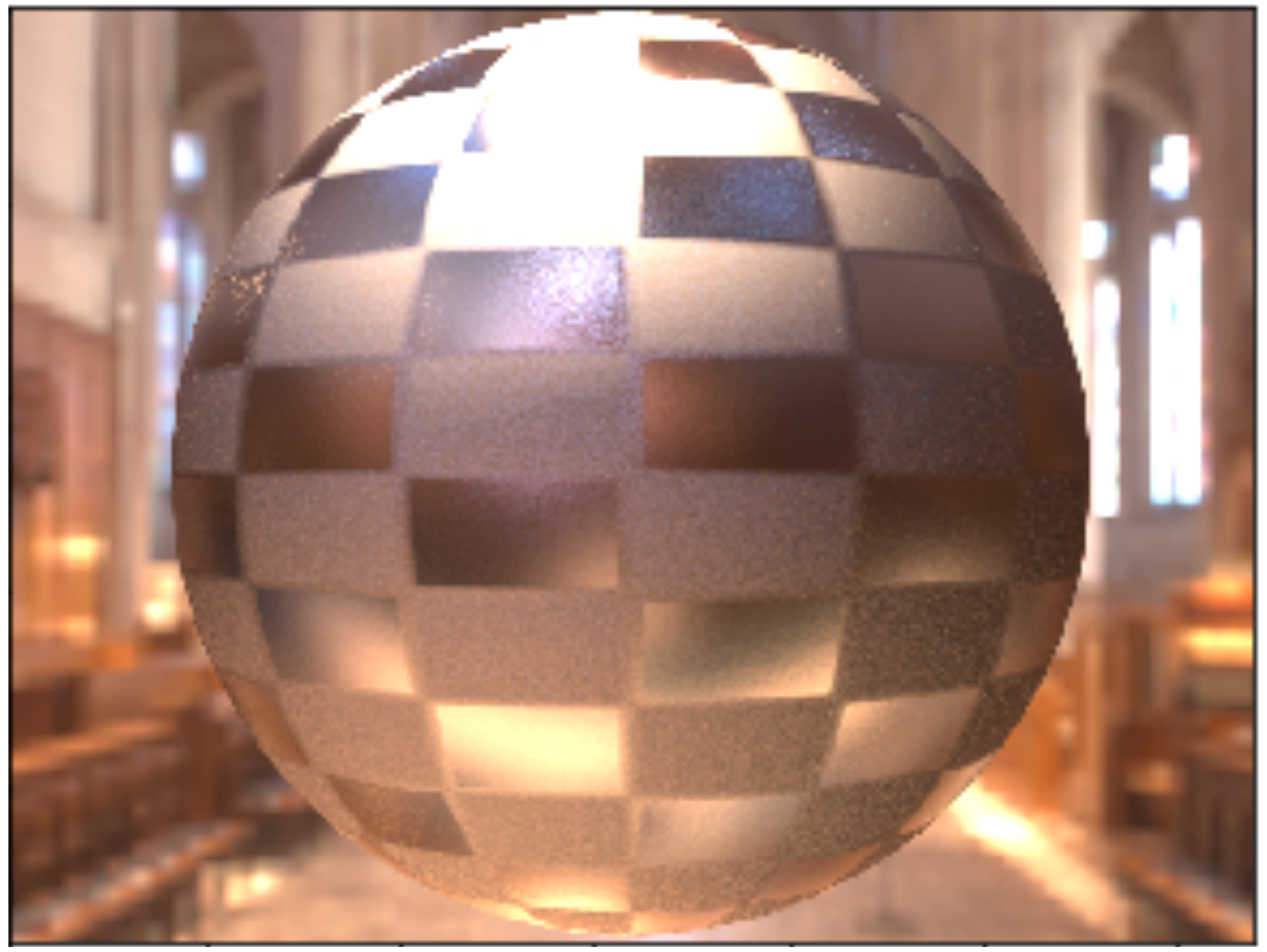
quiz: what are they?



Downsides of measured BSDFs

- capturing is time consuming
  - very few of them
- does not support texturing

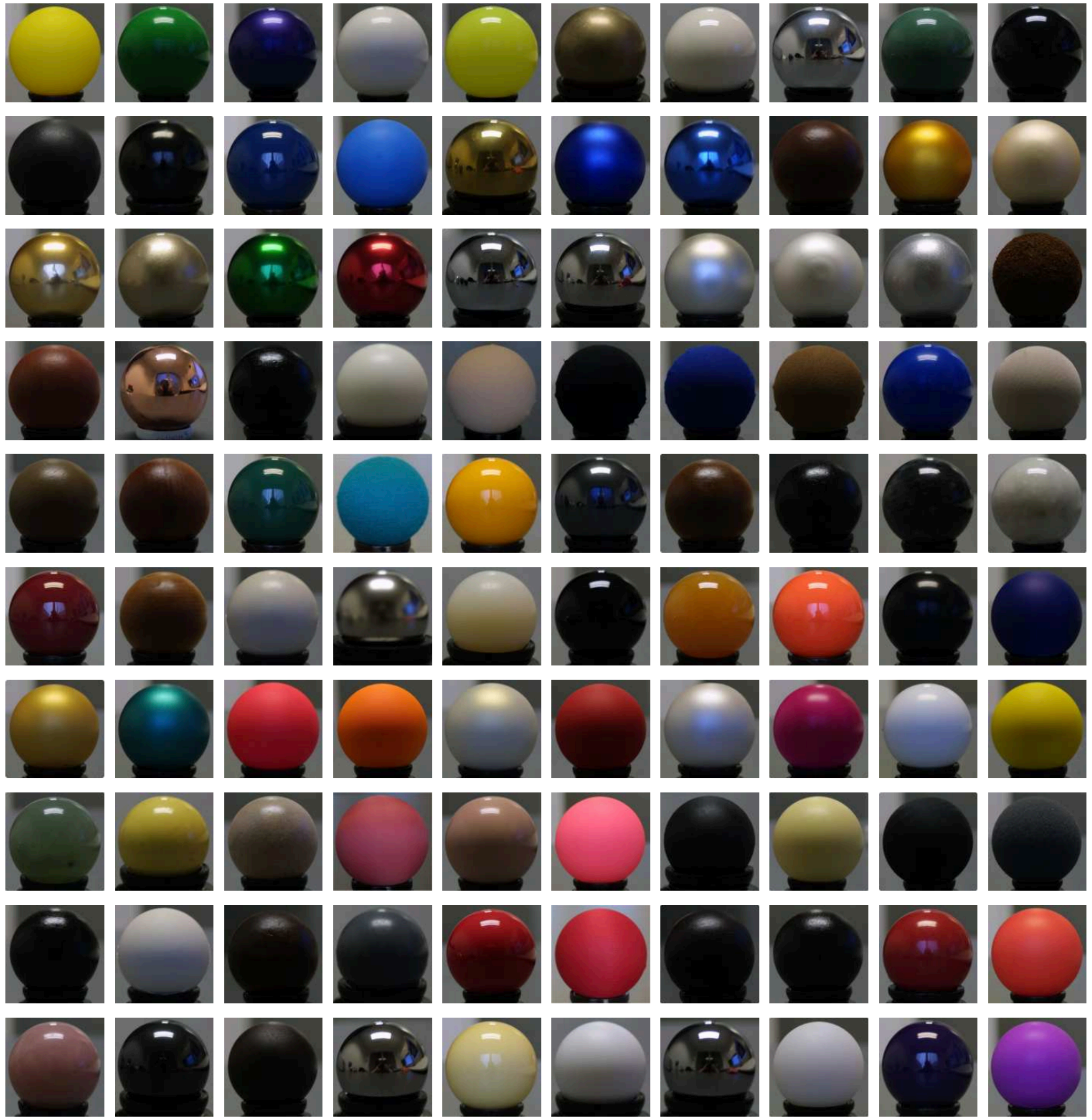




can't support spatially varying roughness with measured BSDF

Remedy: let's fit a model to the data!

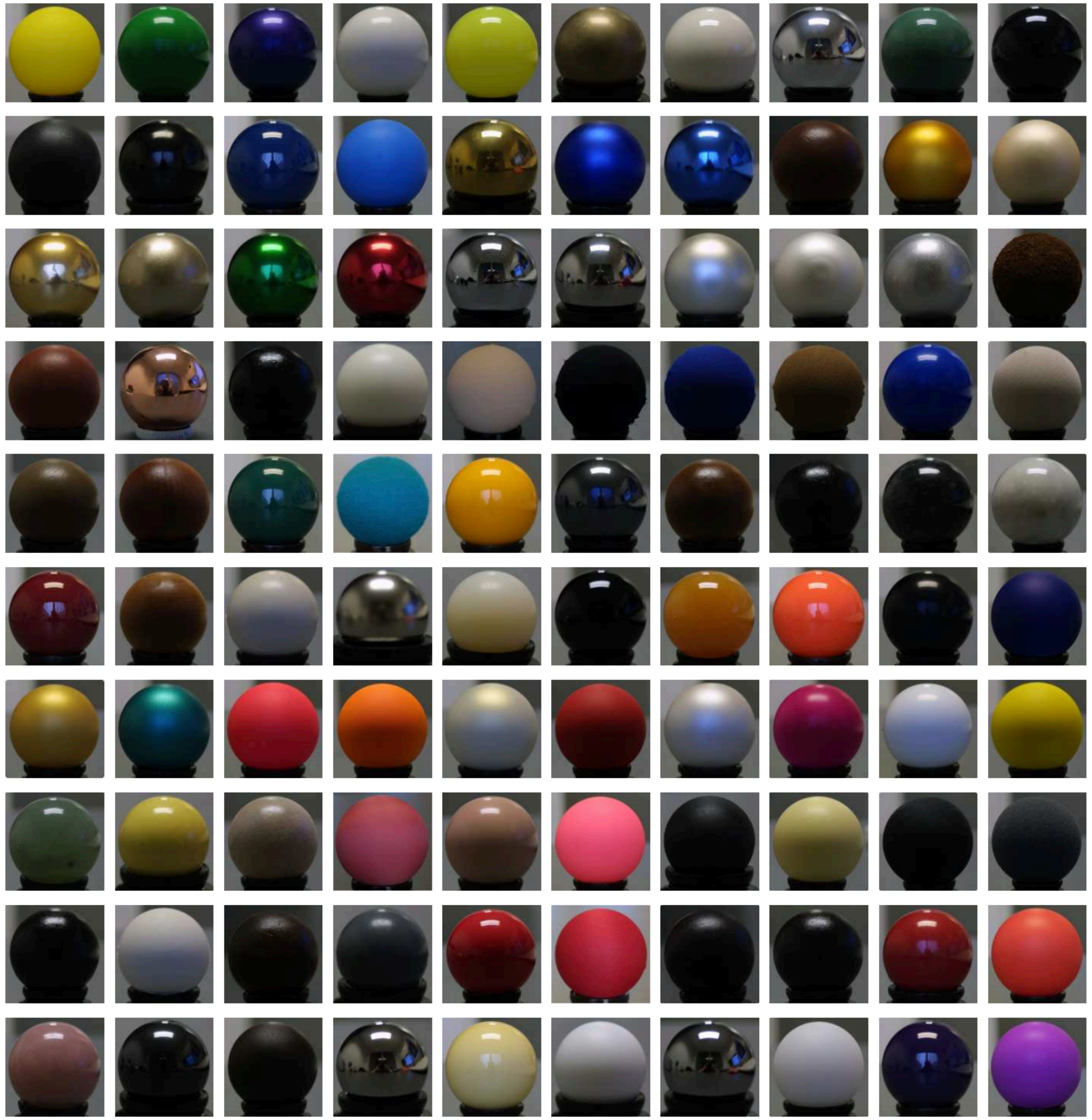
$f_{\mathbf{p}}(\omega, \omega') = \text{some parametric function}$



Remedy: let's fit a model to the data!

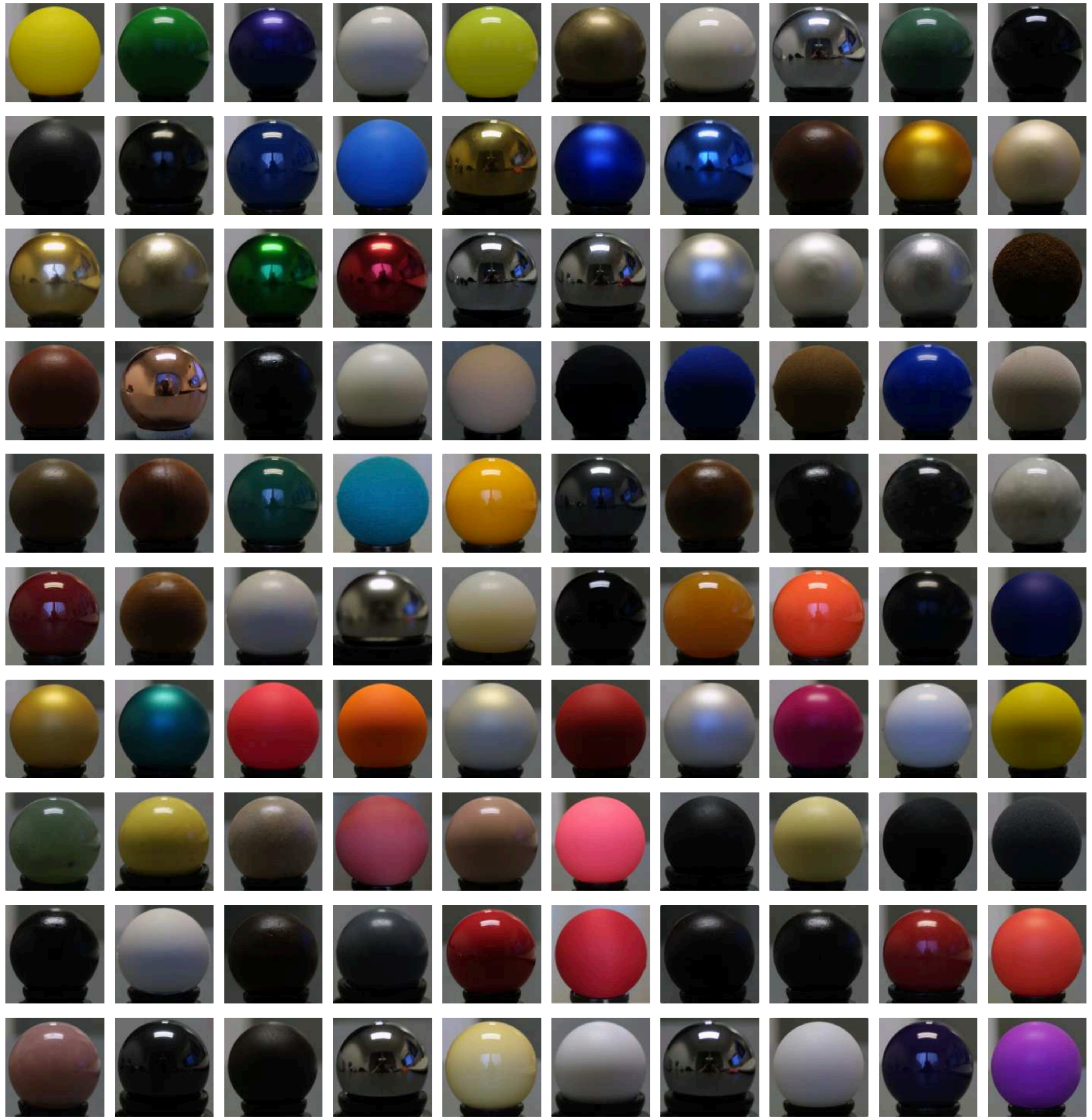
$$f_{\mathbf{p}}(\omega, \omega') = \text{some parametric function}$$

quiz: would neural nets be a good idea? why? why not?



How to build a model: make physical assumptions

- **explain** and **predict** the behavior of our data



$$f_{\mathbf{p}}(\omega, \omega') = \text{some parametric function}$$

based on derivations from simple physical assumptions

https://en.wikipedia.org/wiki/Reductionism

Microfacet theory

- assumption: surfaces are made of infinitely many little mirrors (microfacets)

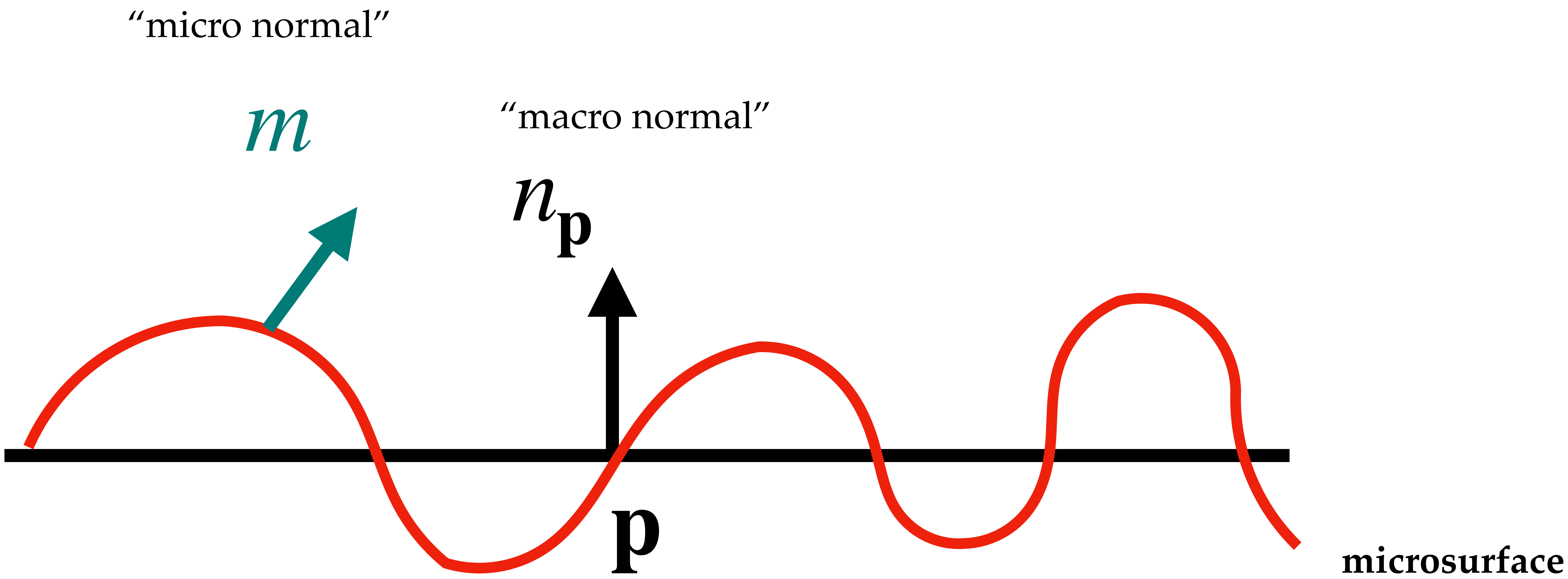
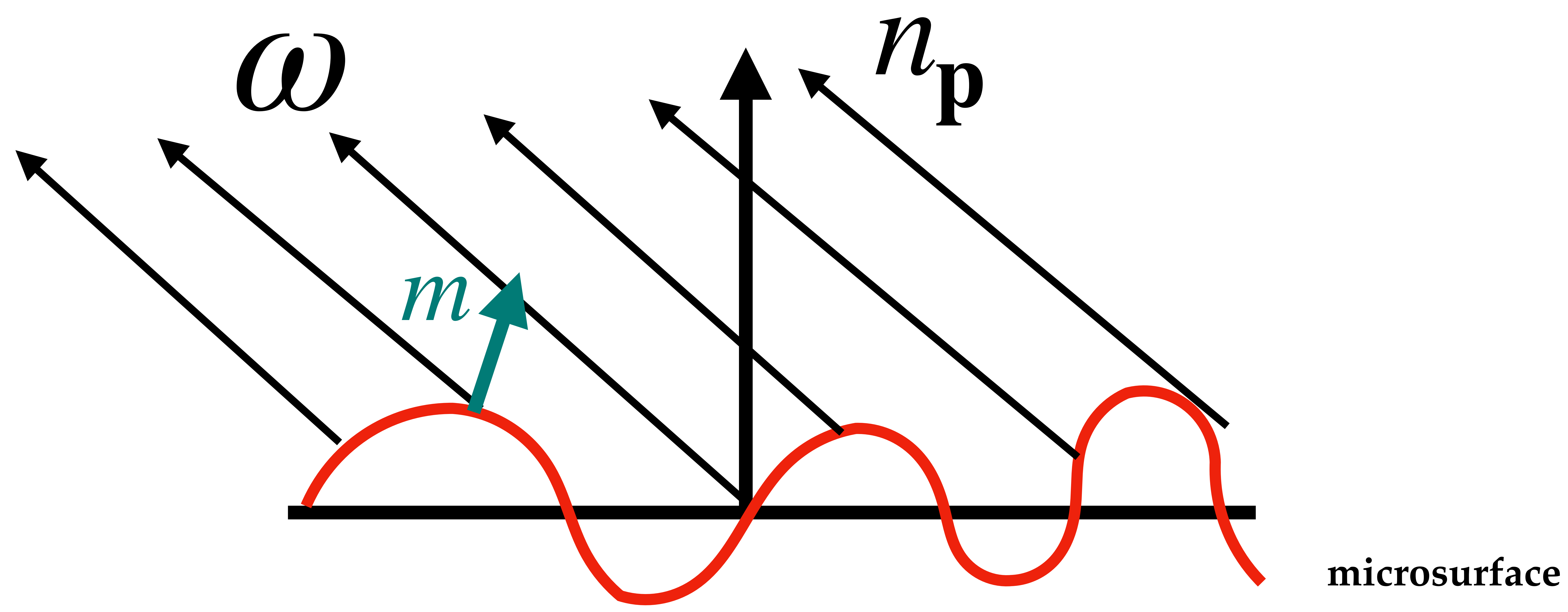


Figure inspired by Eric Heitz https://jcgt.org/published/0003/02/03/

Microfacet theory

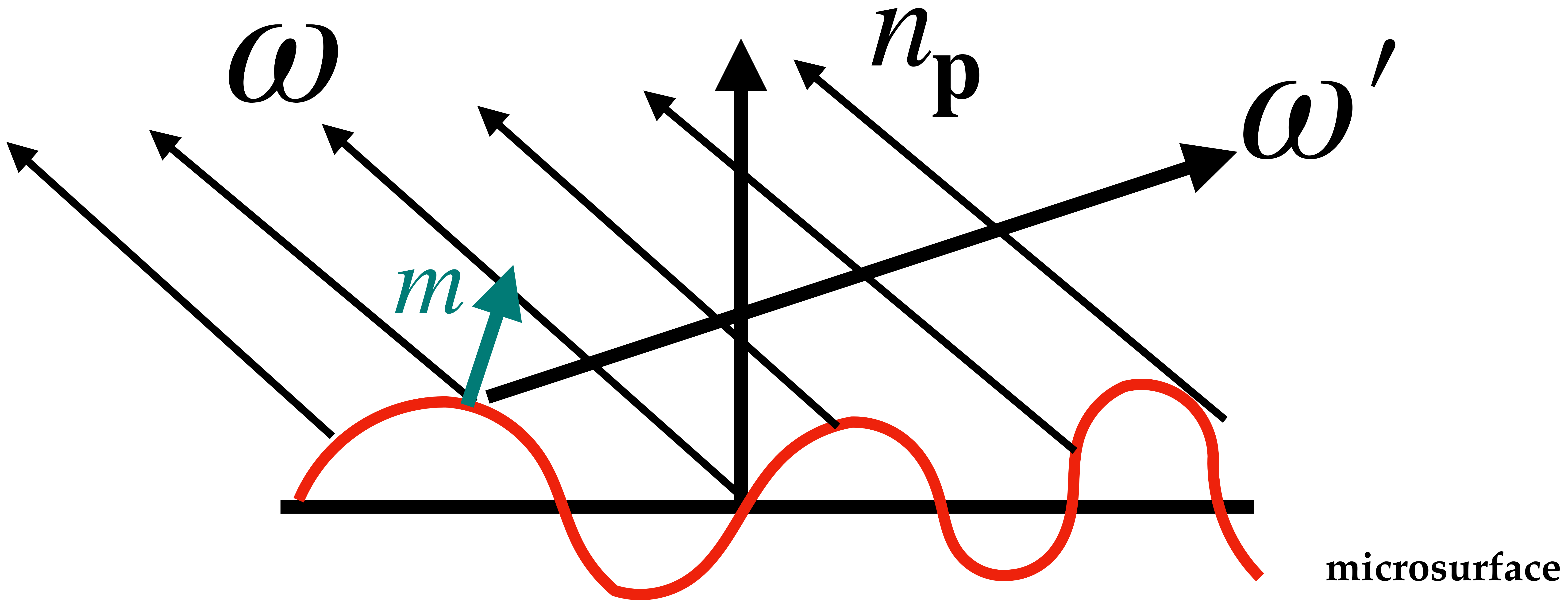
- assumption: surfaces are made of infinitely many little mirrors (microfacets)
- BSDF directions ω and ω' are treated as parallel rays to the microsurface



Microfacet theory

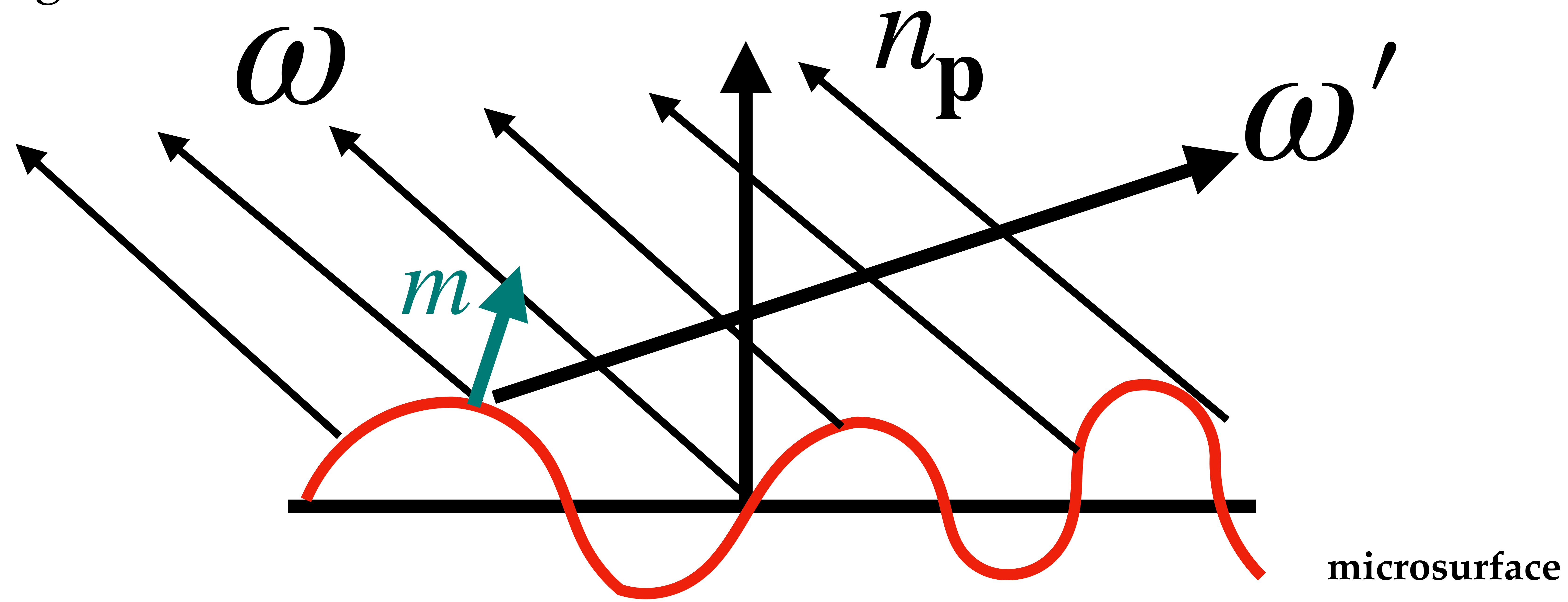
- assumption: surfaces are made of infinitely many little mirrors (microfacets)
- BSDF directions ω and ω' are treated as parallel rays to the microsurface

quiz: given directions ω and ω', which microfacet mirror will reflect light?



Microfacet theory

- assumption: surfaces are made of infinitely many little mirrors (microfacets)
- BSDF directions ω and ω' are treated as parallel rays to the microsurface
  - given ω and ω', only microsurfaces with normal $m = H = \text{normalize}(\omega + \omega')$ will reflect light



Microfacet theory

- flat microsurfaces correspond to …?
- bumpy microsurfaces correspond to …?

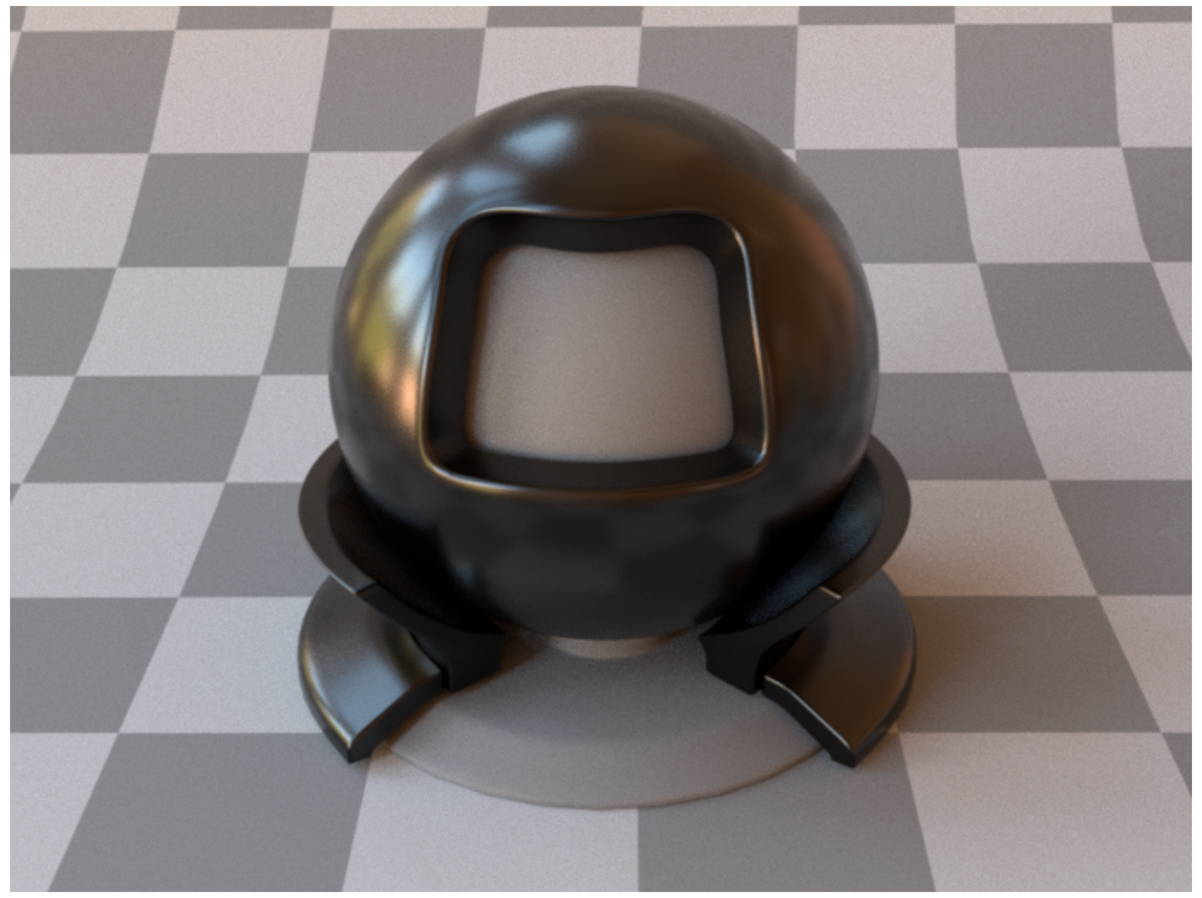
flat microsurface

bumpy microsurface

Microfacet theory

- flat microsurfaces correspond to shiny surfaces
- bumpy microsurfaces correspond to …?

flat microsurface



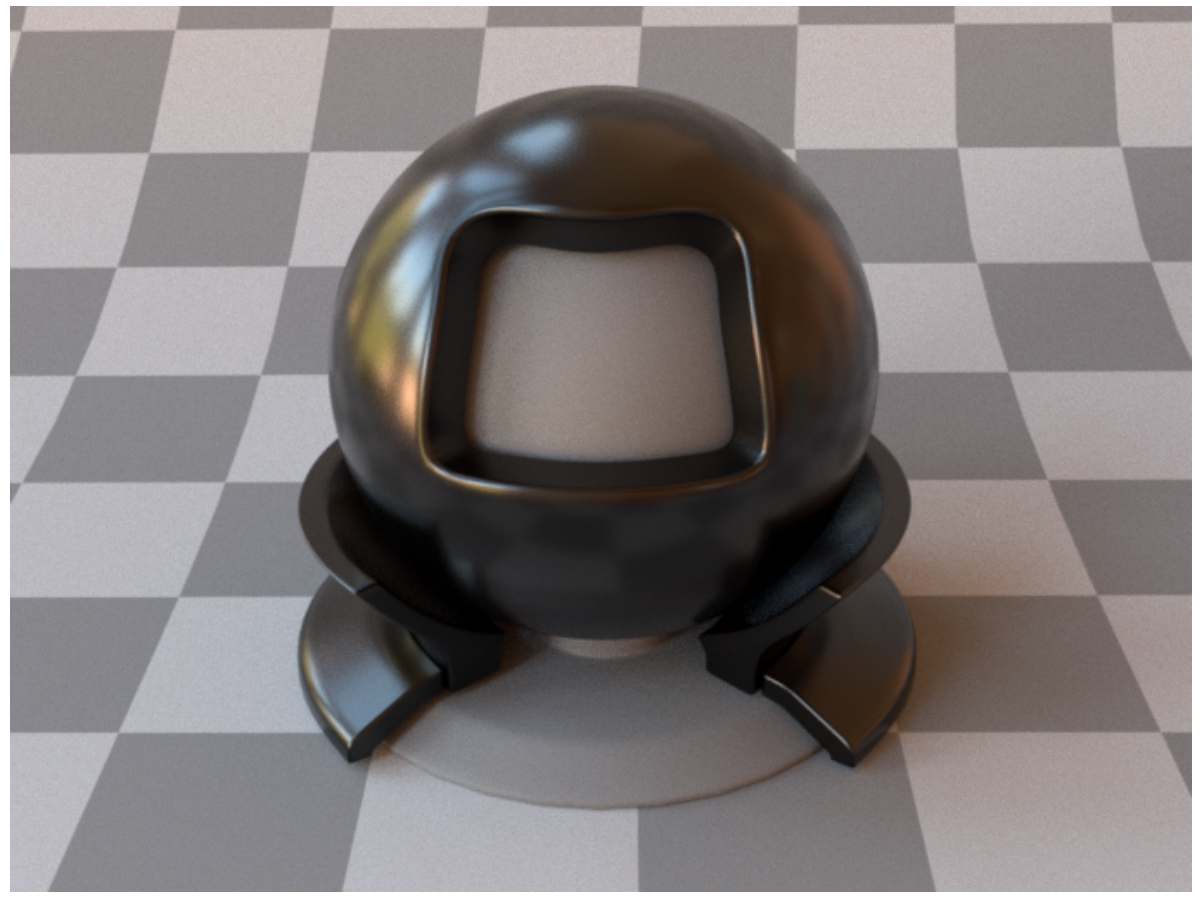
bumpy microsurface

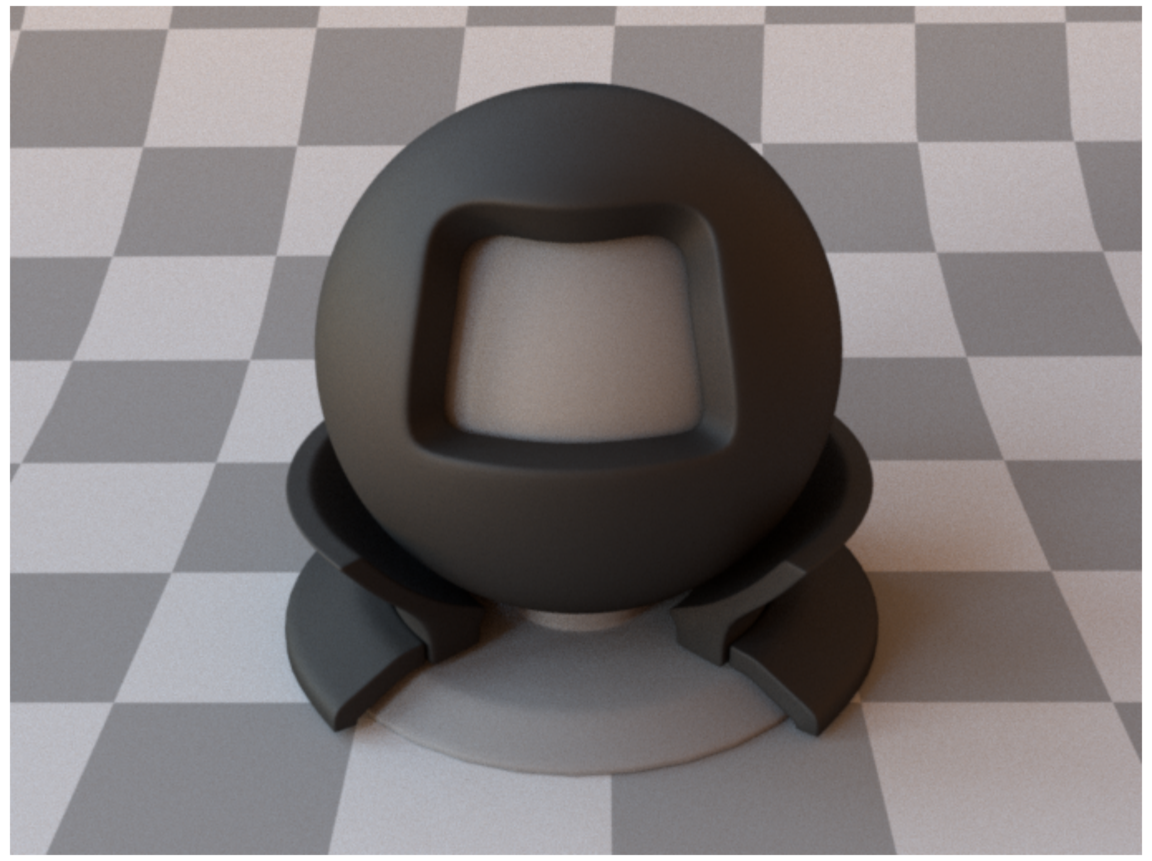
Microfacet theory

- flat microsurfaces correspond to shiny surfaces
- bumpy microsurfaces correspond to rough, diffusive surfaces

flat microsurface

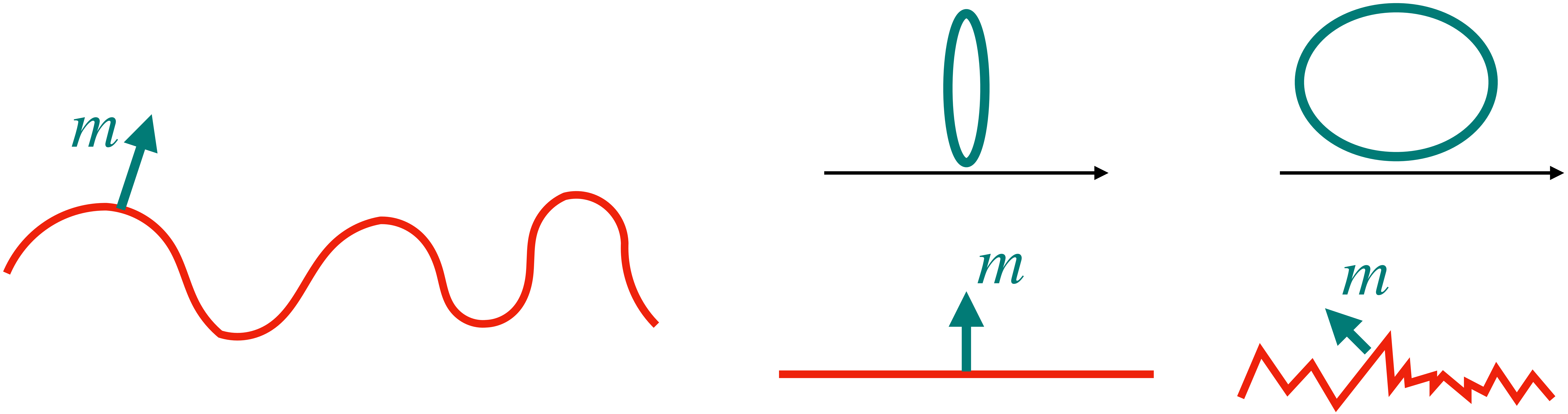
bumpy microsurface





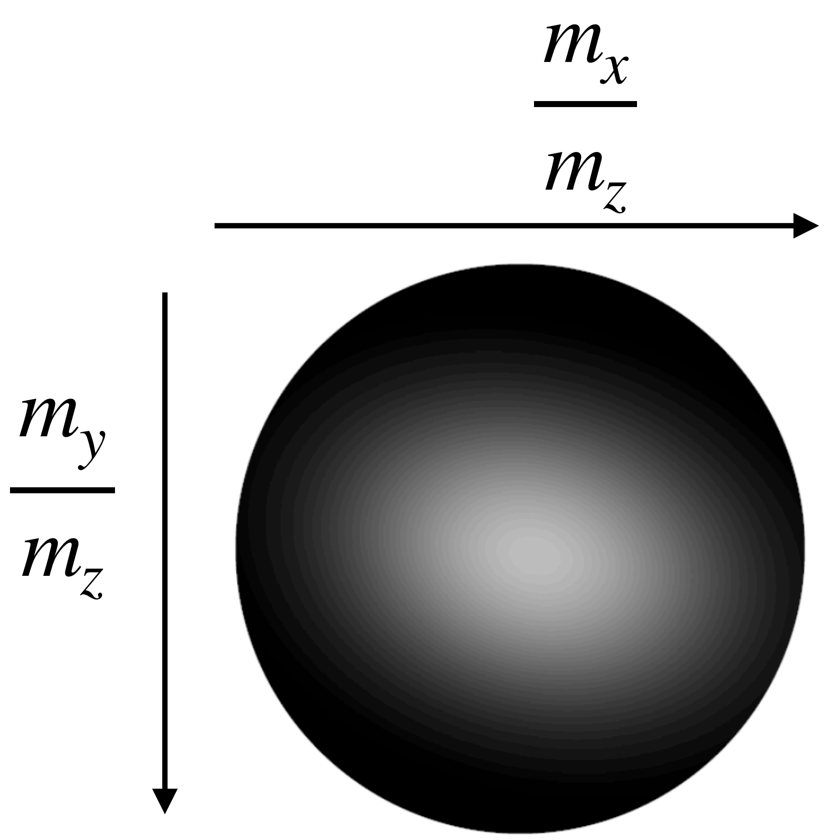
Describing microsurfaces using statistical distributions

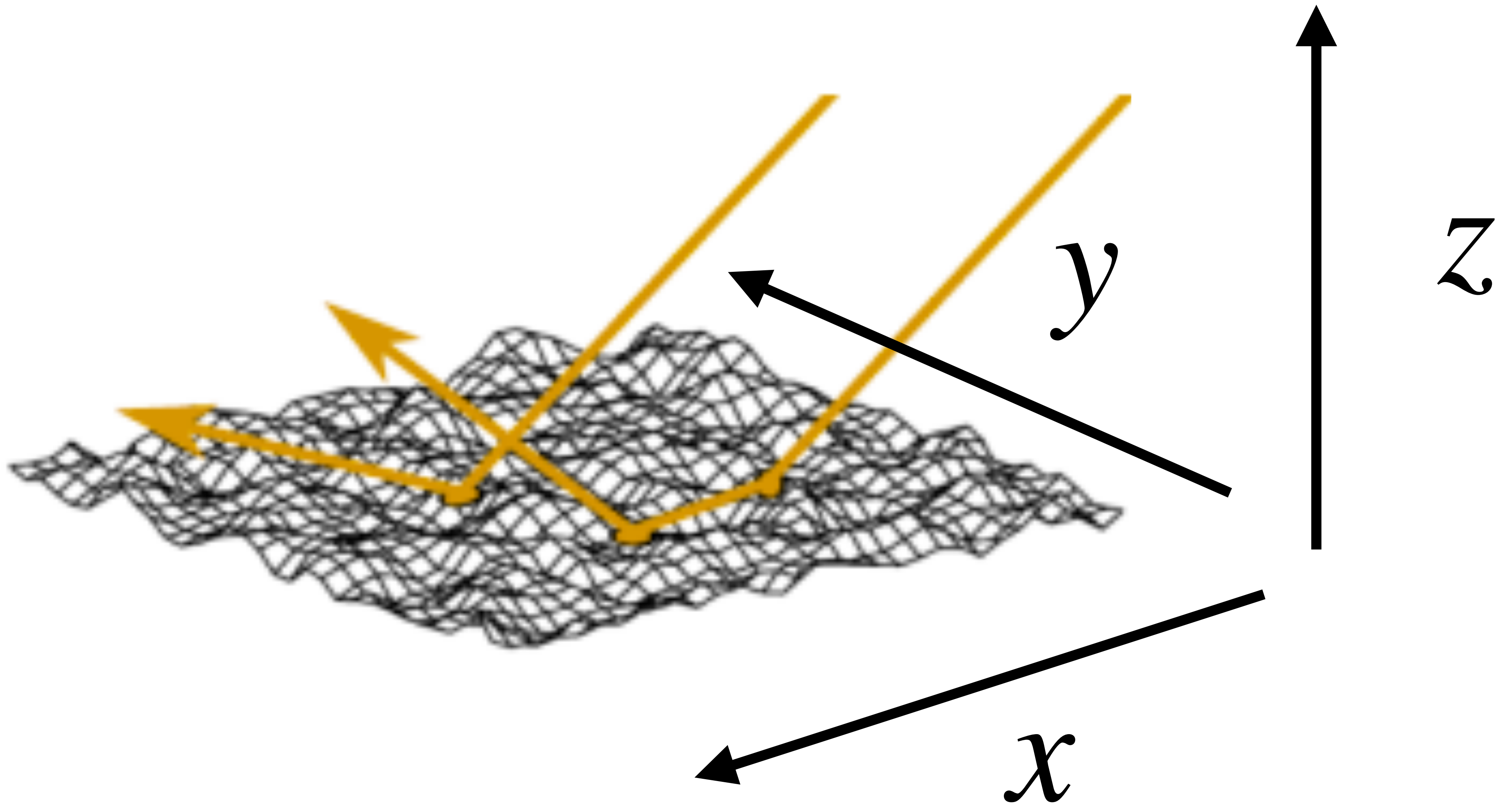
- the **normal distribution function** $D(m)$ describes the probability density of micronormals
  - for flat microsurfaces, $D(m = n_{\mathbf{p}})$ is very high, and $D(m \neq n_{\mathbf{p}})$ is low



Popular normal distribution function: Beckmann NDF

- assumption: microfacets are from a heightfield with Gaussian "slopes"





$$-\frac{m_x}{m_z} = \frac{\partial z}{\partial x} \sim N(0,\alpha_x^2)$$

$$-\frac{m_y}{m_z} = \frac{\partial z}{\partial y} \sim N(0,\alpha_y^2)$$

see "Slope Space in BRDF Theory" if you're interested
https://www.reedbeta.com/blog/slope-space-in-brdf-theory/

Figure from Eric Heitz & Lingqi Yan et al.
https://eheitzresearch.wordpress.com/240-2/
https://sites.cs.ucsb.edu/~lingqi/publications/paper_glints.pdf

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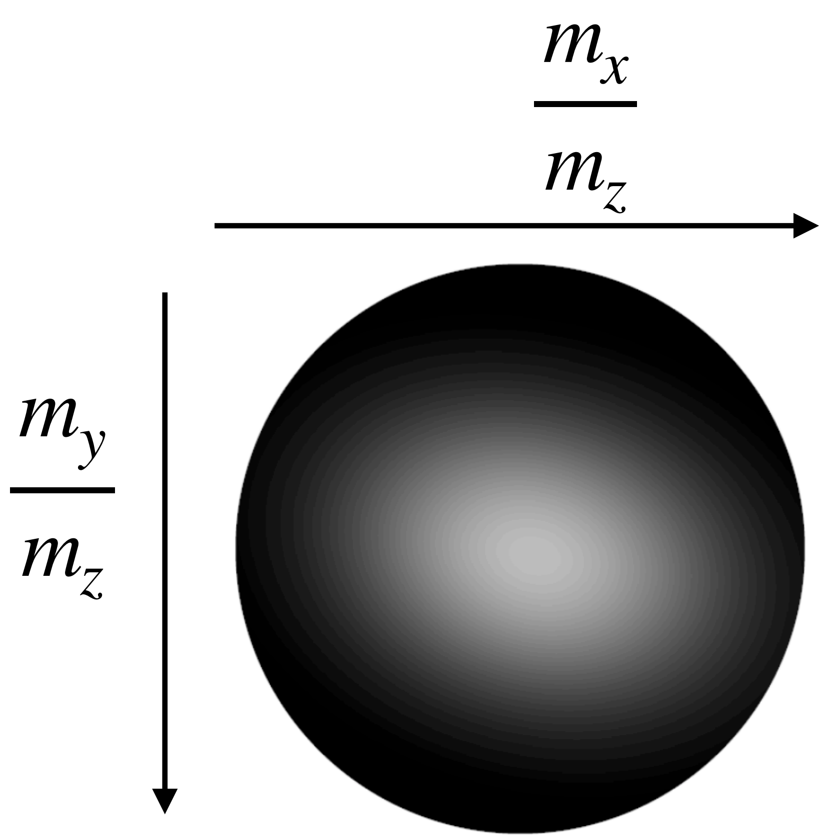
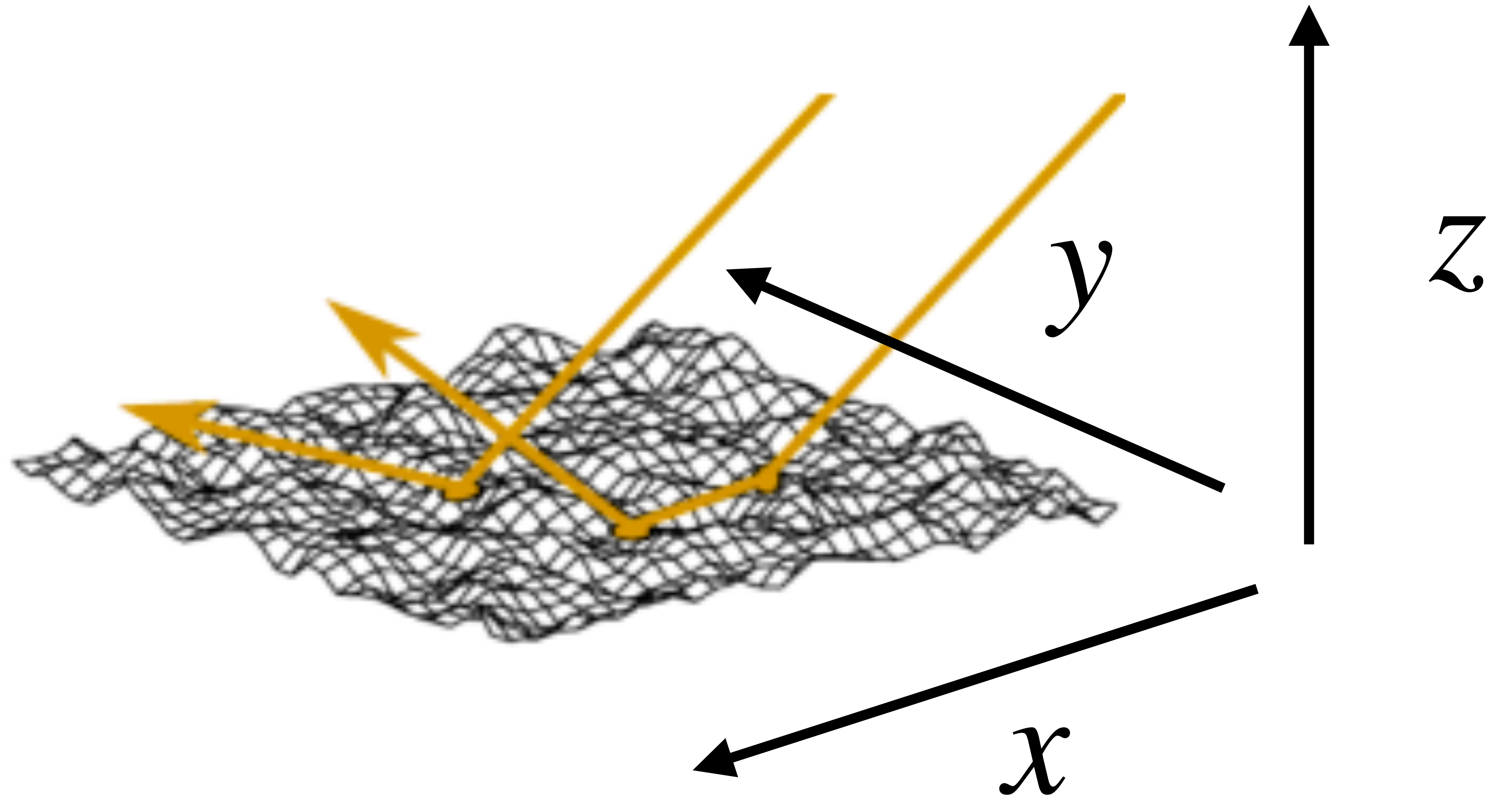


quiz: what does large alpha mean?



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Popular normal distribution function: Beckmann NDF

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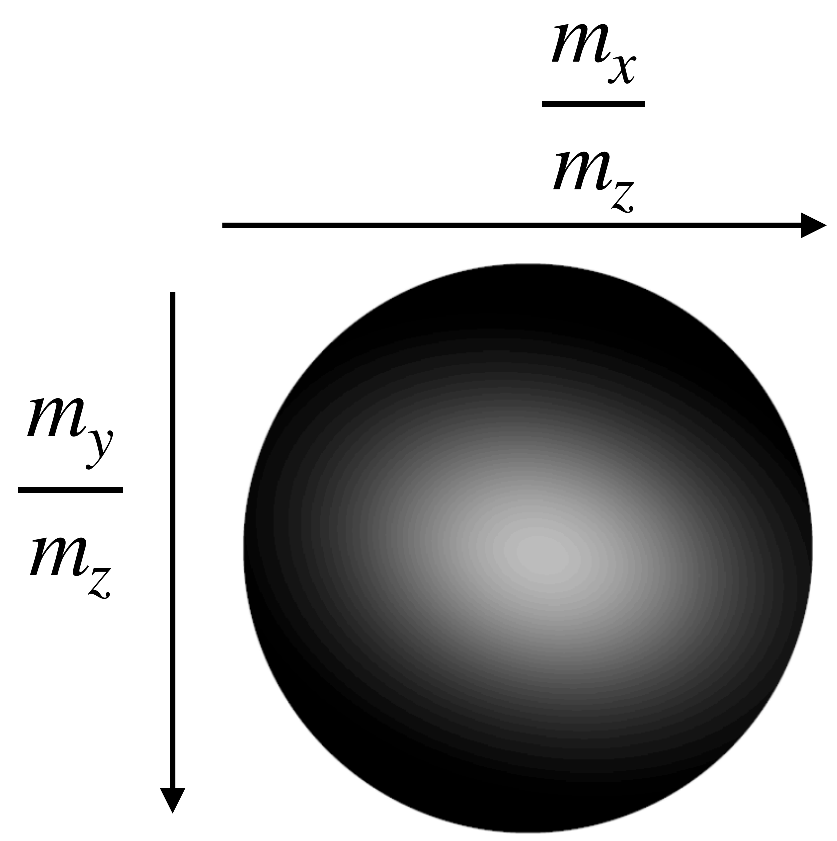


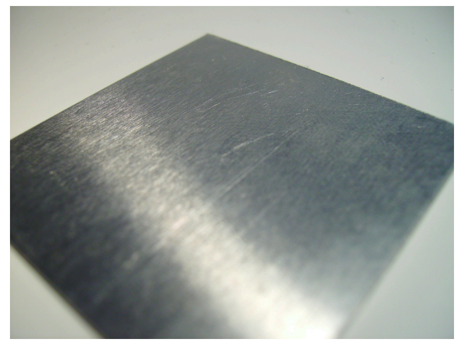
$$D(m) \propto \exp\left(-\frac{1}{2} \begin{bmatrix} \frac{m_x}{m_z} & \frac{m_y}{m_z} \end{bmatrix} \begin{bmatrix} \alpha_x^2 & 0 \\ 0 & \alpha_y^2 \end{bmatrix}^{-1} \begin{bmatrix} \frac{m_x}{m_z} \\ \frac{m_y}{m_z} \end{bmatrix} \right)$$

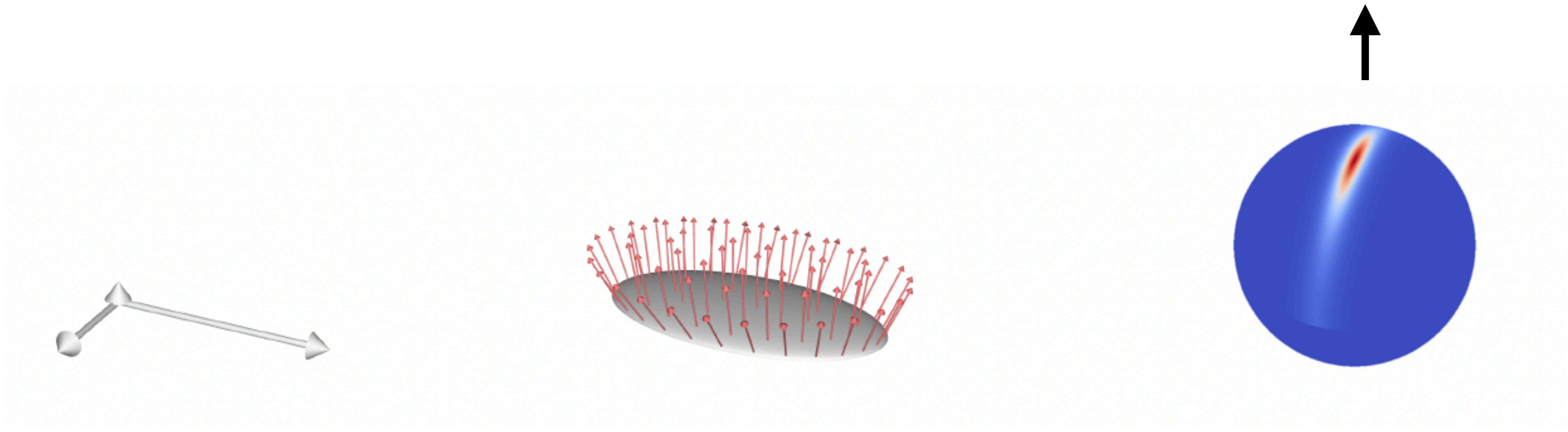
see "Slope Space in BRDF Theory" if you're interested
https://www.reedbeta.com/blog/slope-space-in-brdf-theory/

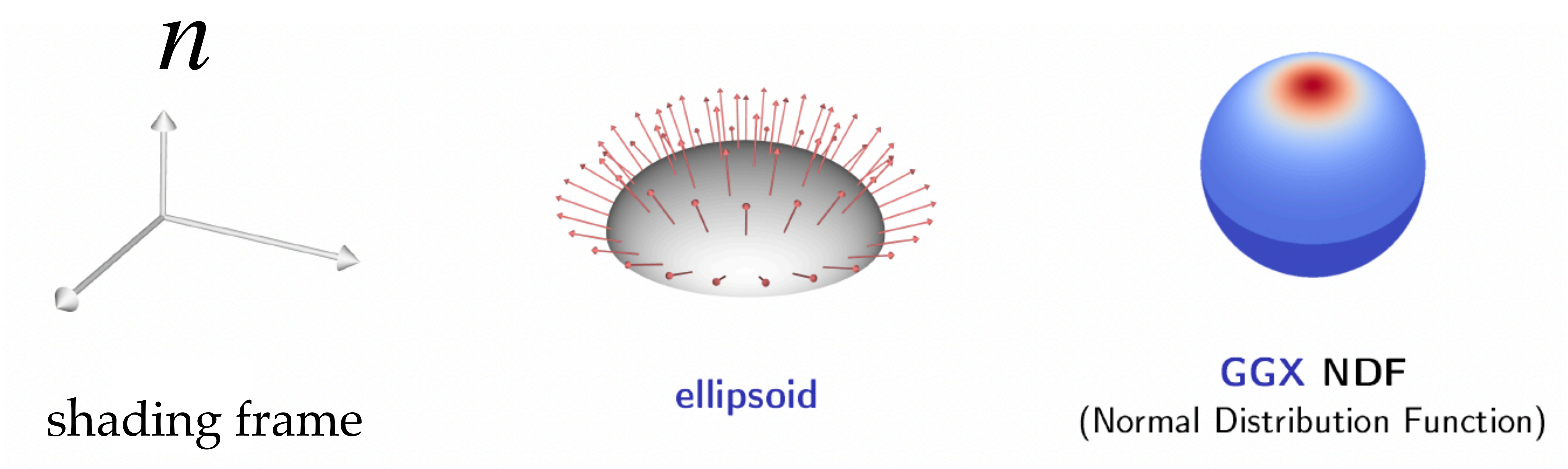
Popular normal distribution function: Trowbridge-Reitz [1975] (aka GGX [Walter 2007])

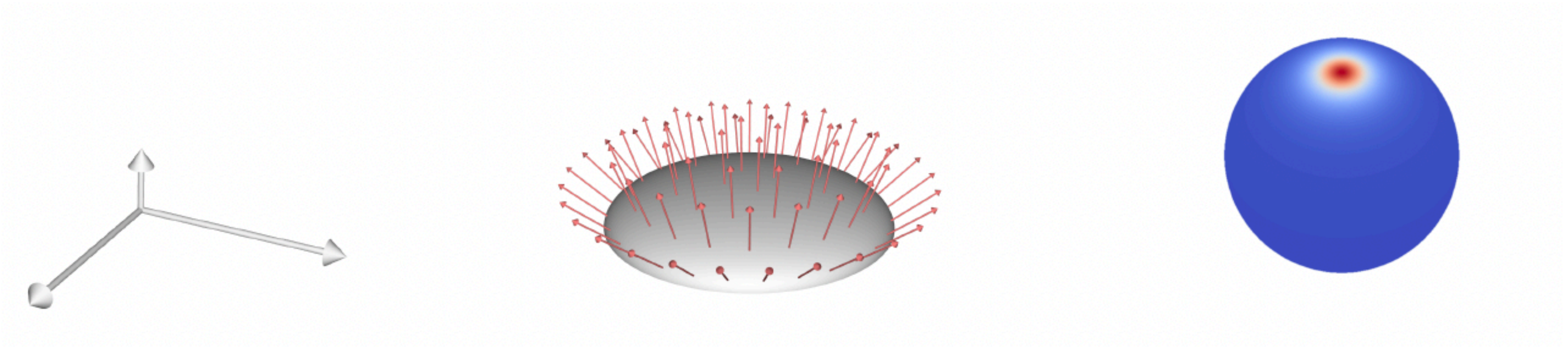
- geometric intuition: the distribution of normals of an ellipsoid

Ground Glass Unknown









brilliant figures from Eric Heitz
https://jcgt.org/published/0007/04/01/

Popular normal distribution function: Trowbridge-Reitz [1975] (aka GGX [Walter 2007])

Ground Glass Unknown

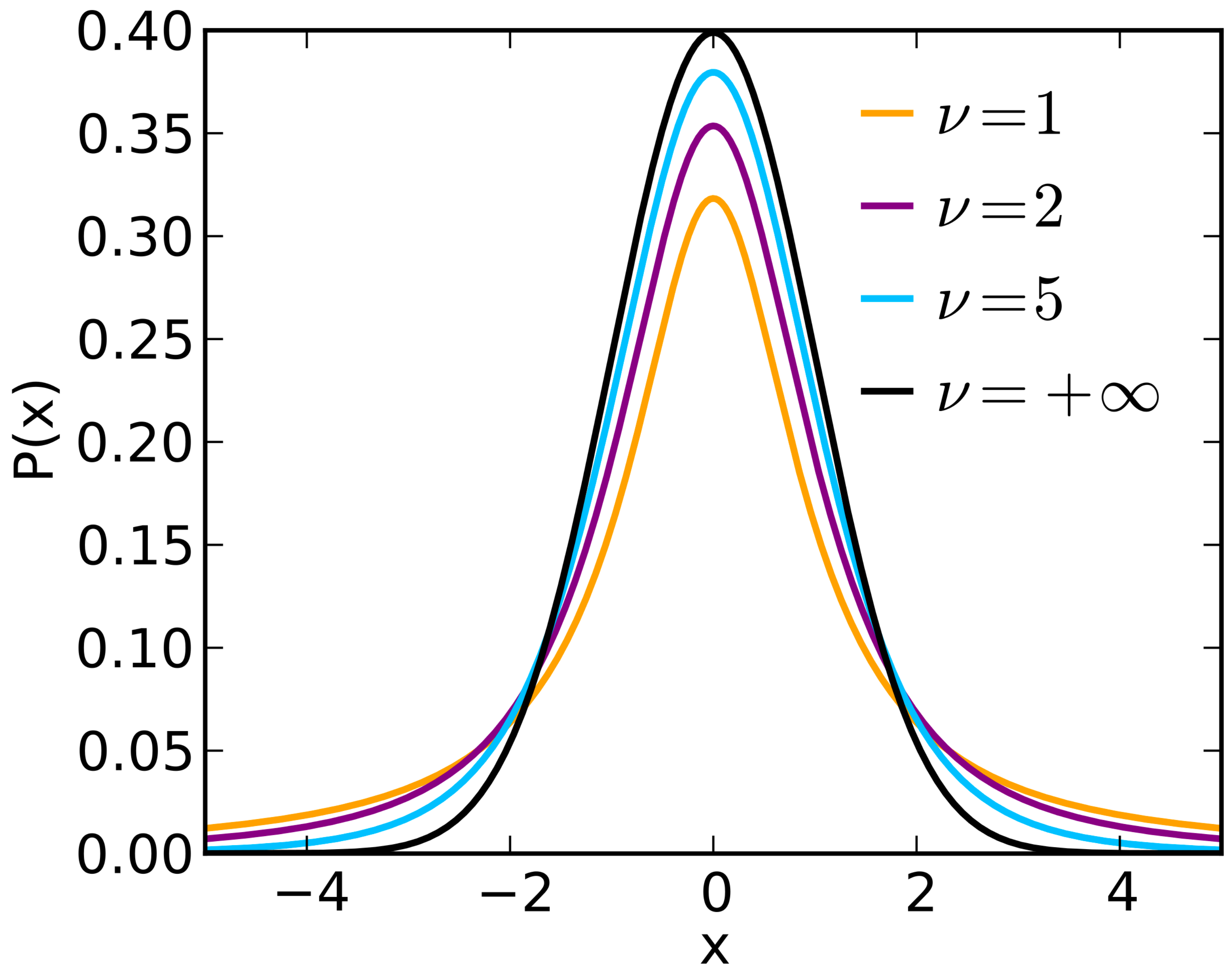
$$D_{\mathrm{GGX}}(m) \propto \frac{1}{\left(1 + \frac{1}{2}\begin{bmatrix}\frac{m_x}{m_z} & \frac{m_y}{m_z}\end{bmatrix}\begin{bmatrix}\alpha_x^2 & 0 \\ 0 & \alpha_y^2\end{bmatrix}^{-1}\begin{bmatrix}\frac{m_x}{m_z} \\ \frac{m_y}{m_z}\end{bmatrix}\right)^2}$$

Popular normal distribution function: Trowbridge-Reitz [1975] (aka GGX [Walter 2007])

Ground Glass Unknown

(multivariate student-t distribution with $\nu = 2$ —> heavy tailed Gaussian)

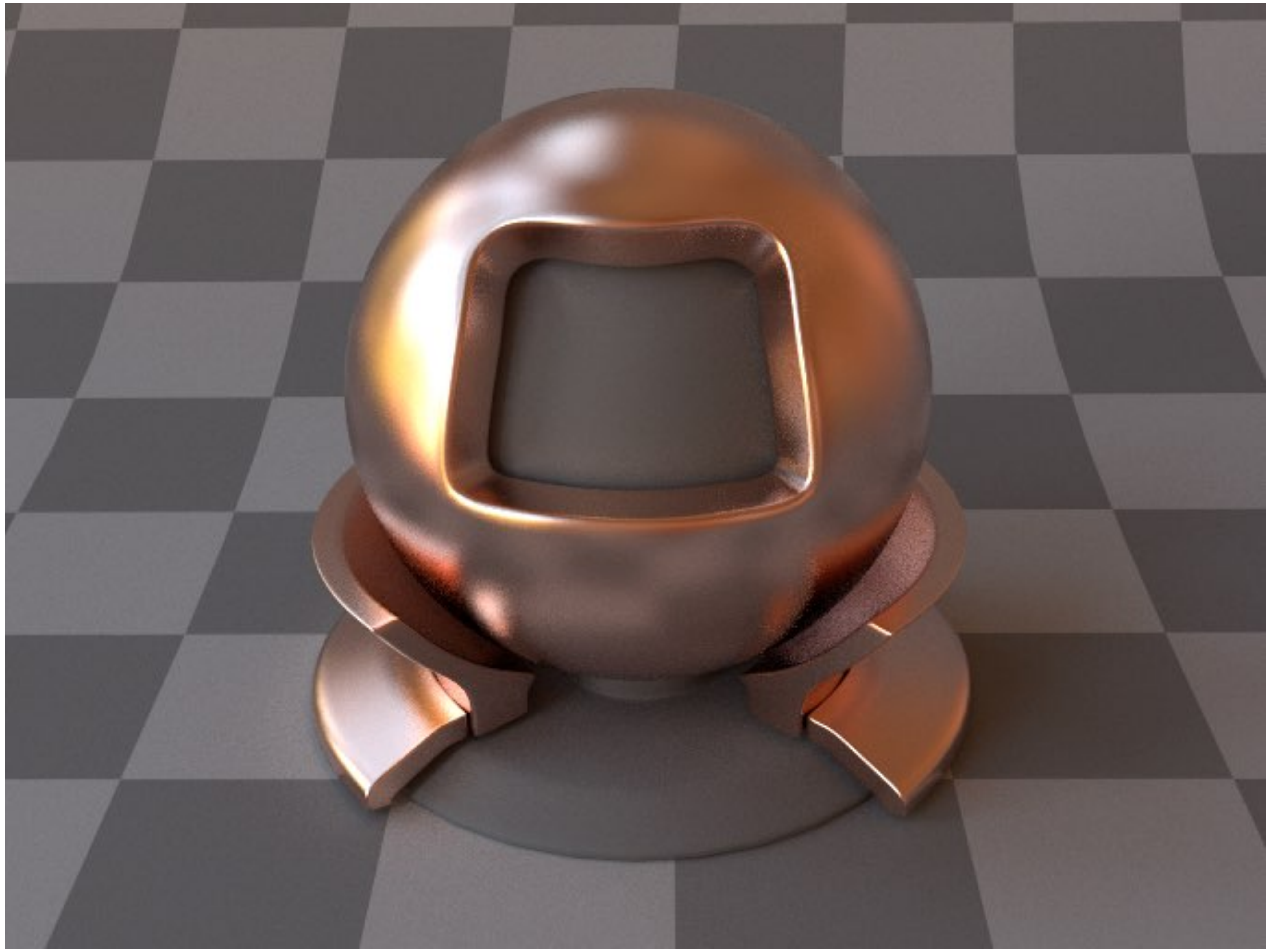
$$D_{\mathrm{GGX}}(m) \propto \frac{1}{\left(1 + \frac{1}{2}\begin{bmatrix}\frac{m_x}{m_z} & \frac{m_y}{m_z}\end{bmatrix}\begin{bmatrix}\alpha_x^2 & 0 \\ 0 & \alpha_y^2\end{bmatrix}^{-1}\begin{bmatrix}\frac{m_x}{m_z} \\ \frac{m_y}{m_z}\end{bmatrix}\right)^2}$$



$\nu = \infty$ -> normal Gaussian = Beckmann

see https://en.wikipedia.org/wiki/Student%27s_t-distribution for intuition

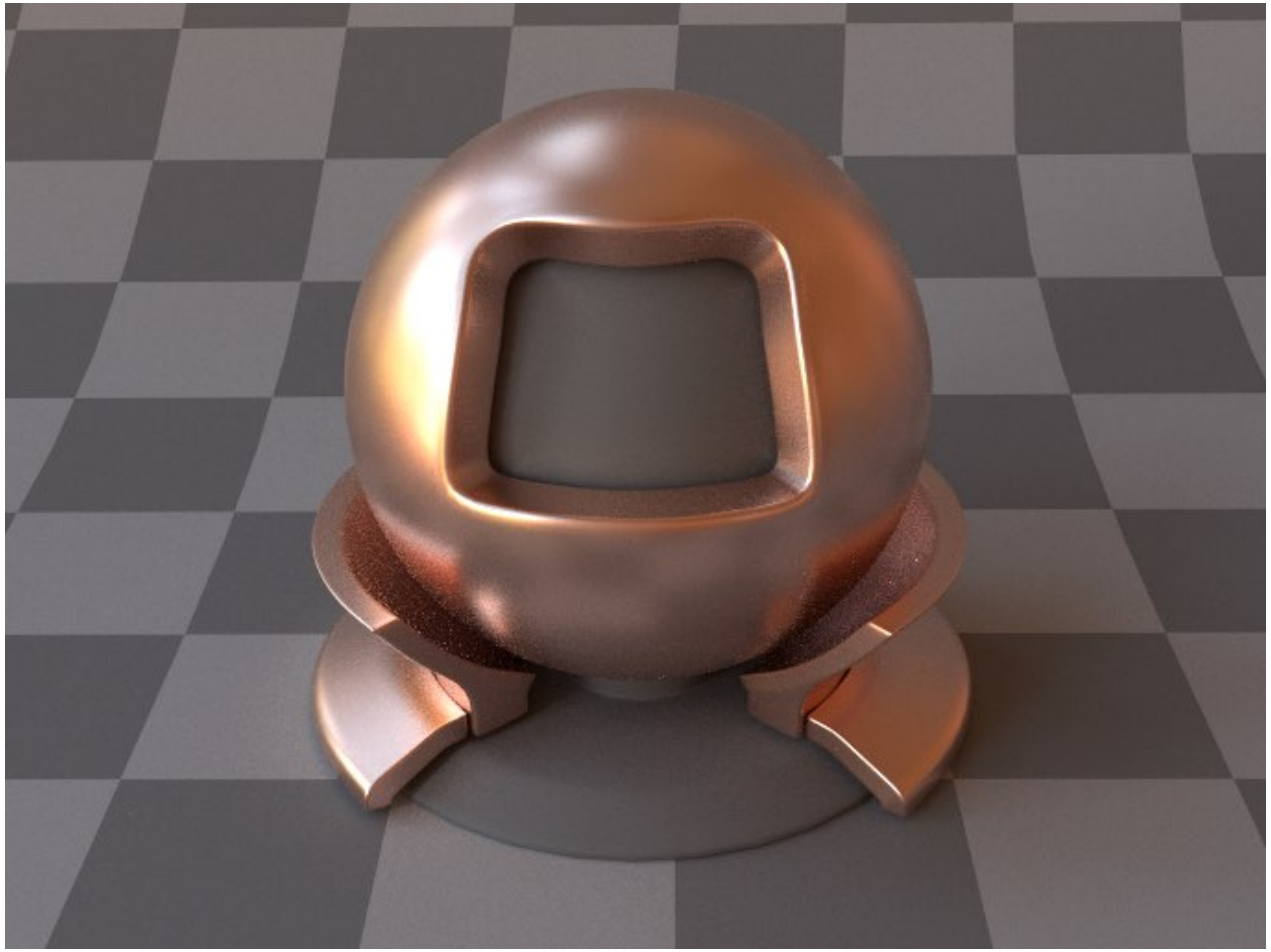
Beckmann vs GGX



Beckmann

quiz: what would GGX look like?

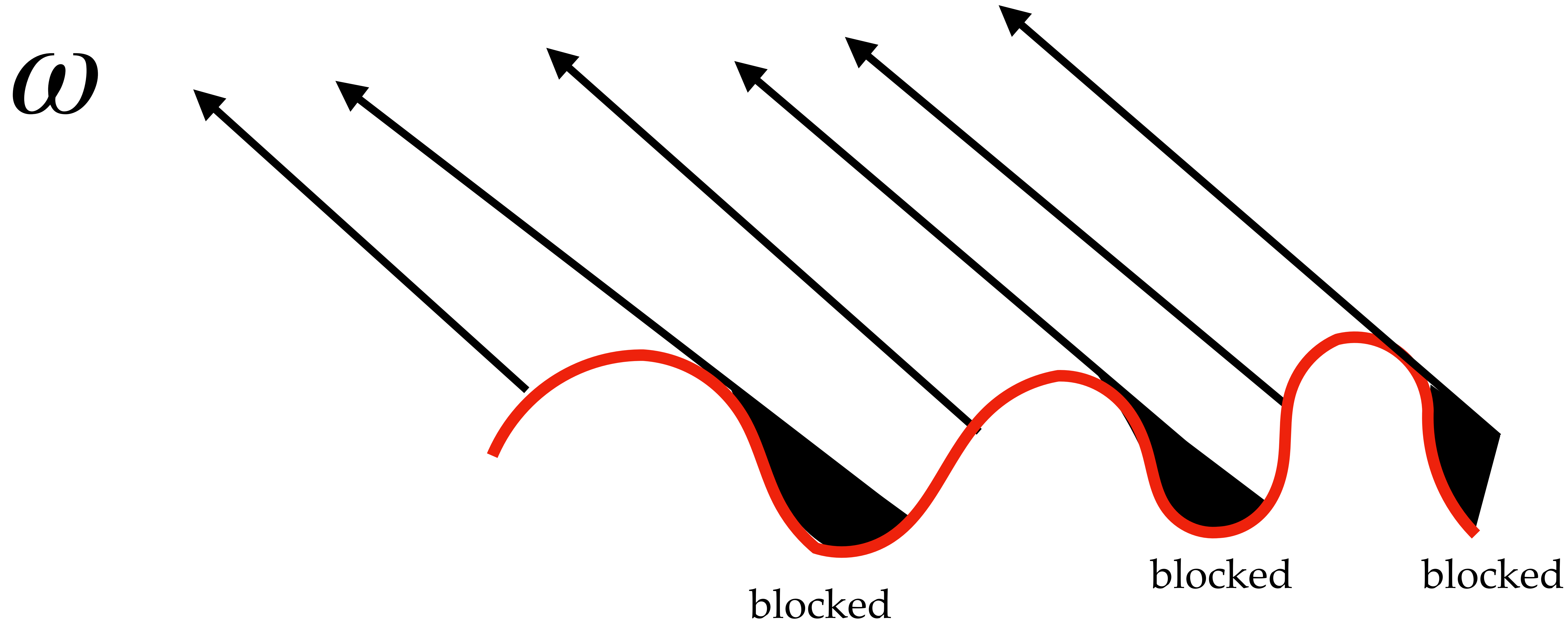
Beckmann vs GGX



GGX

Microfacets can block each other

- normal distribution function D alone is not enough to determine how many mirrors are blocked
- need to specify the **microsurface geometry profile**



Smith microsurface profile [1960]

- most popular profile in graphics
  - alternative: V-cavity [Cook and Torrence 1982]
- Smith's assumption: microsurfaces are **spatially uncorrelated**

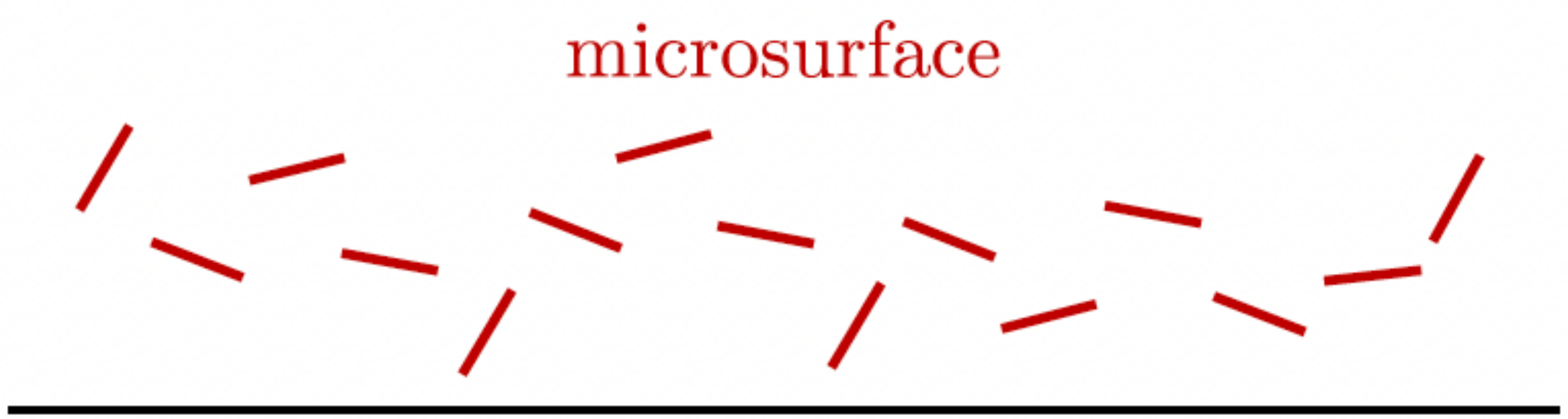
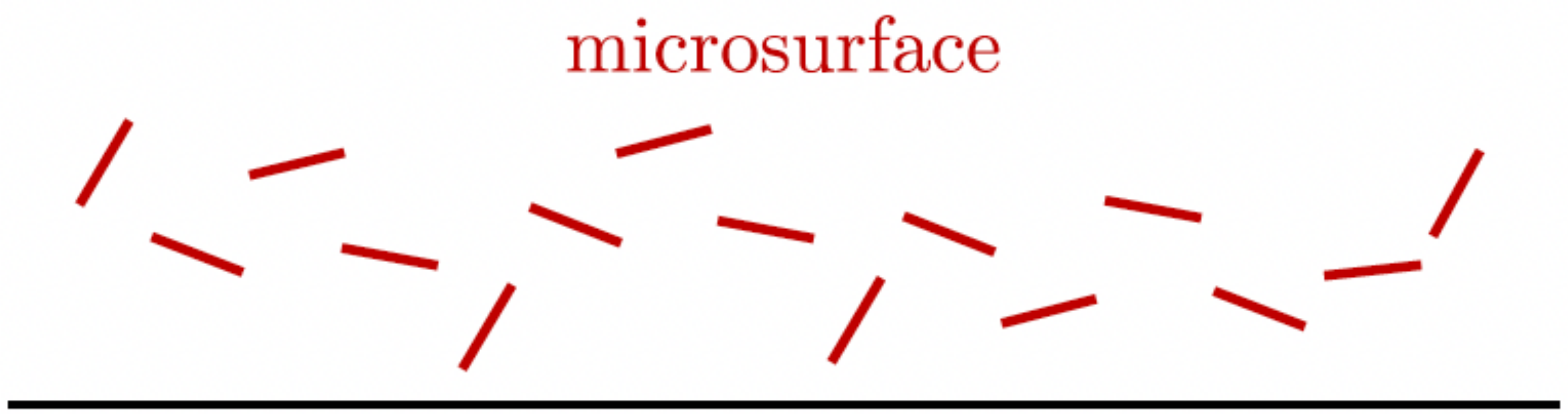


Figure from Eric Heitz https://jcgt.org/published/0003/02/03/

We can compute the portion of blocked microsurfaces $\hat{G}$ under Smith's assumption

$$\hat{G}(\omega, m) = \begin{cases} 0 & \text{if } \omega \cdot m \leq 0 \\ \dfrac{\omega \cdot n_{\mathbf{p}}}{\int (\omega \cdot m) D(m) \mathrm{d}m} & \text{otherwise} \end{cases}$$

often call the "shadowing masking term"



see "Understanding the Masking-Shadowing Function in Microfacet-based BRDFs", Eric Heitz
https://jcgt.org/published/0003/02/03/

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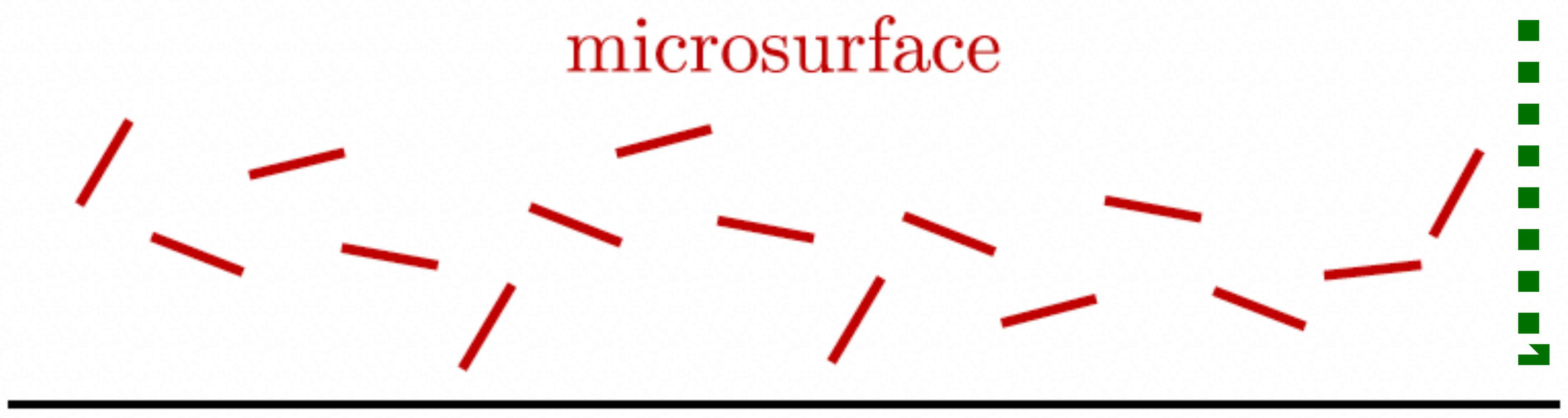


derivation: Smith's assumption implies (read at home)

$$\hat{G}(w, m) = \begin{cases} 0 & \text{if } \omega \cdot m \leq 0 \\ \hat{G}'(\omega) & \text{otherwise} \end{cases}$$

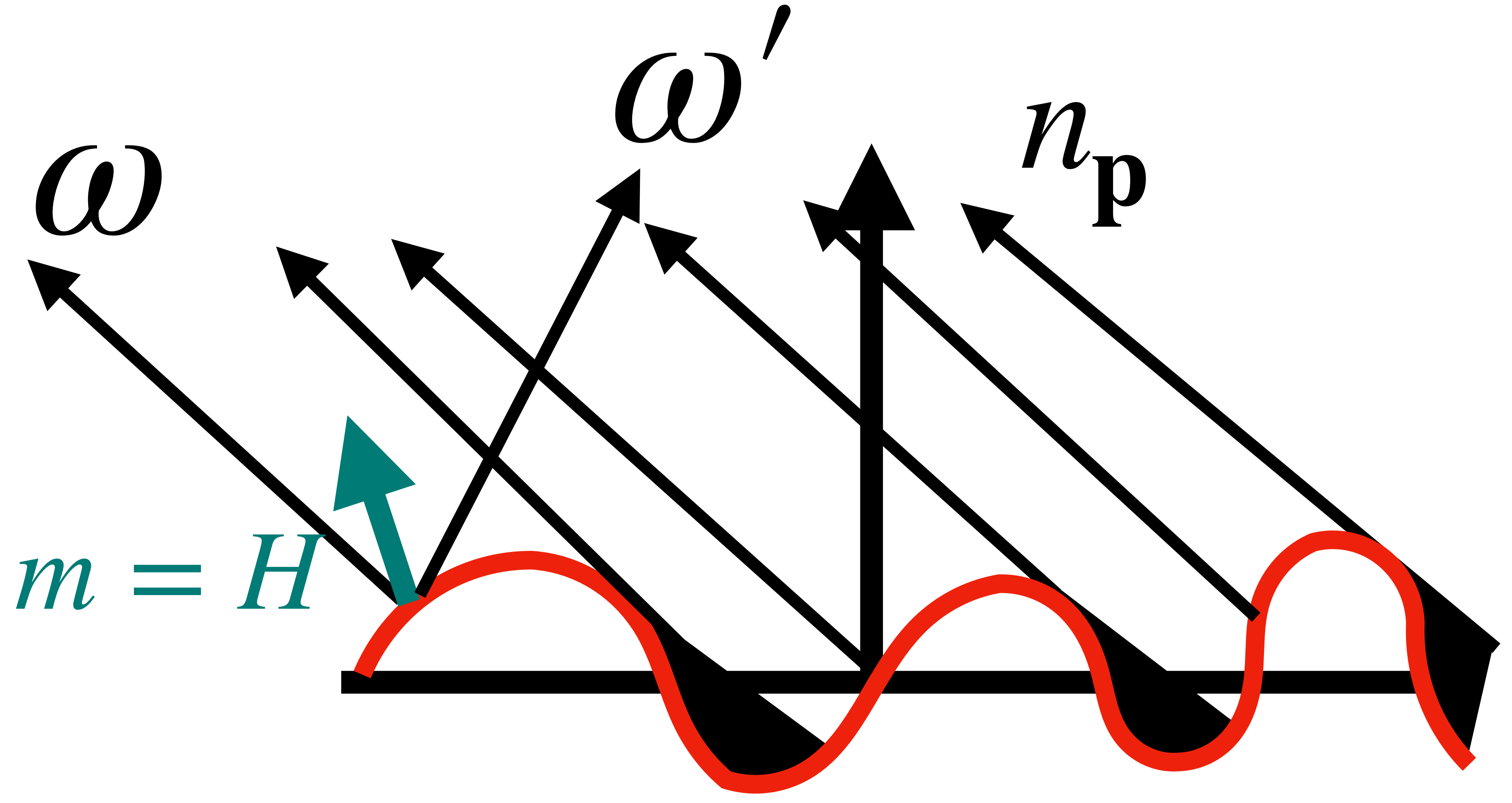
we also know projected area conserves $\omega \cdot n_{\mathbf{p}} = \int \hat{G}(\omega, m) D(m) \omega \cdot m \mathrm{d}m$

see "Understanding the Masking-Shadowing Function in Microfacet-based BRDFs", Eric Heitz
https://jcgt.org/published/0003/02/03/

The microfacet BRDF: counting **visible** micronormals at the half vector

$$f_{\mathbf{p}}(\omega, \omega')$$

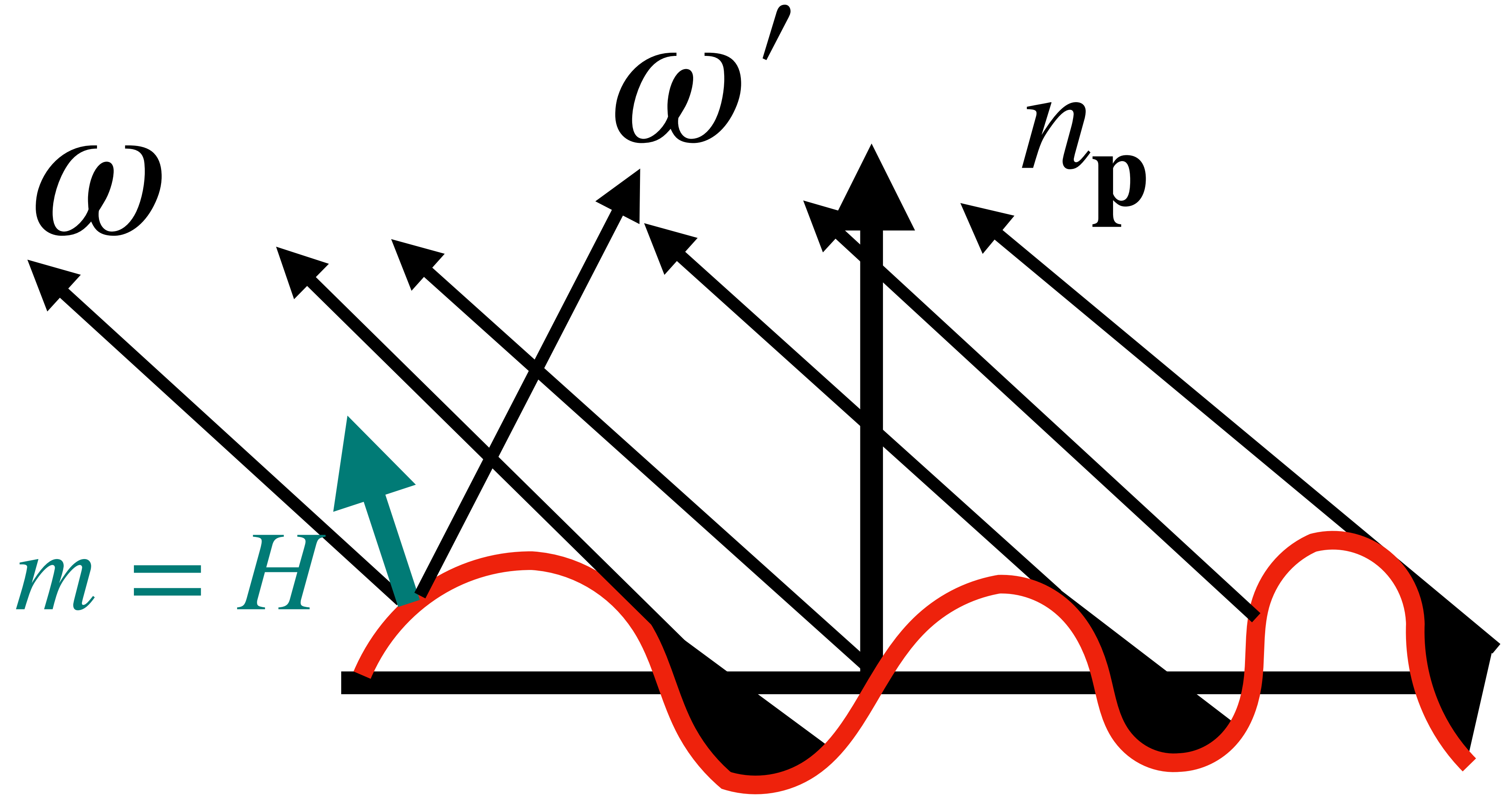
$H = \text{normalize}(\omega + \omega')$



The microfacet BRDF: counting **visible** micronormals at the half vector

$$f_{\mathbf{p}}(\omega, \omega') \qquad D(H)\hat{G}(\omega, H)\hat{G}(\omega', H)$$

$H = \text{normalize}(\omega + \omega')$



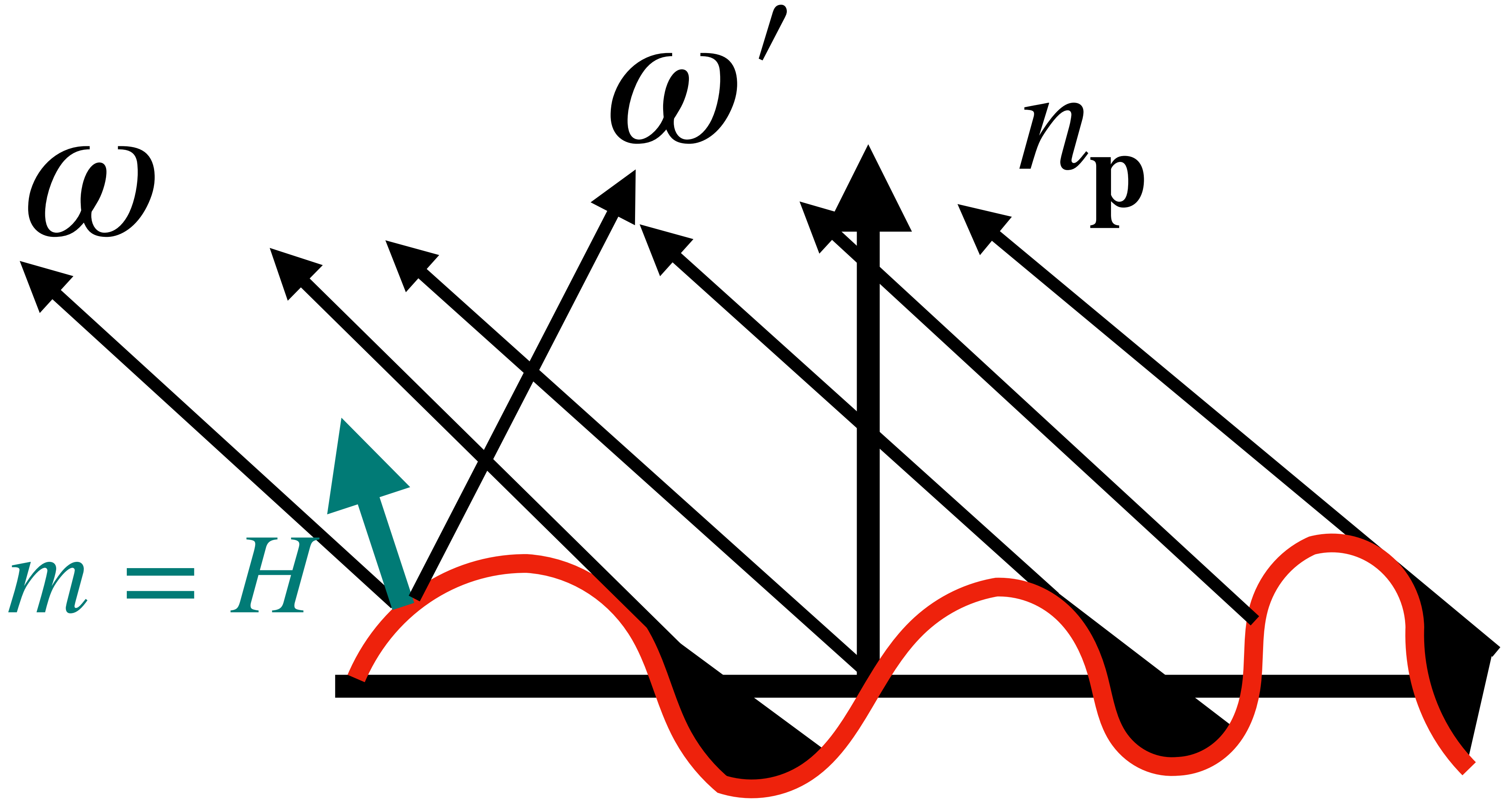
The microfacet BRDF: counting **visible** micronormals at the half vector

$$D(H)G(\omega, \omega', H)$$

$$G(\omega, \omega', H) = \hat{G}(\omega, H)\hat{G}(\omega', H)$$

$$f_{\mathbf{p}}(\omega, \omega')$$

$$H = \text{normalize}(\omega + \omega')$$



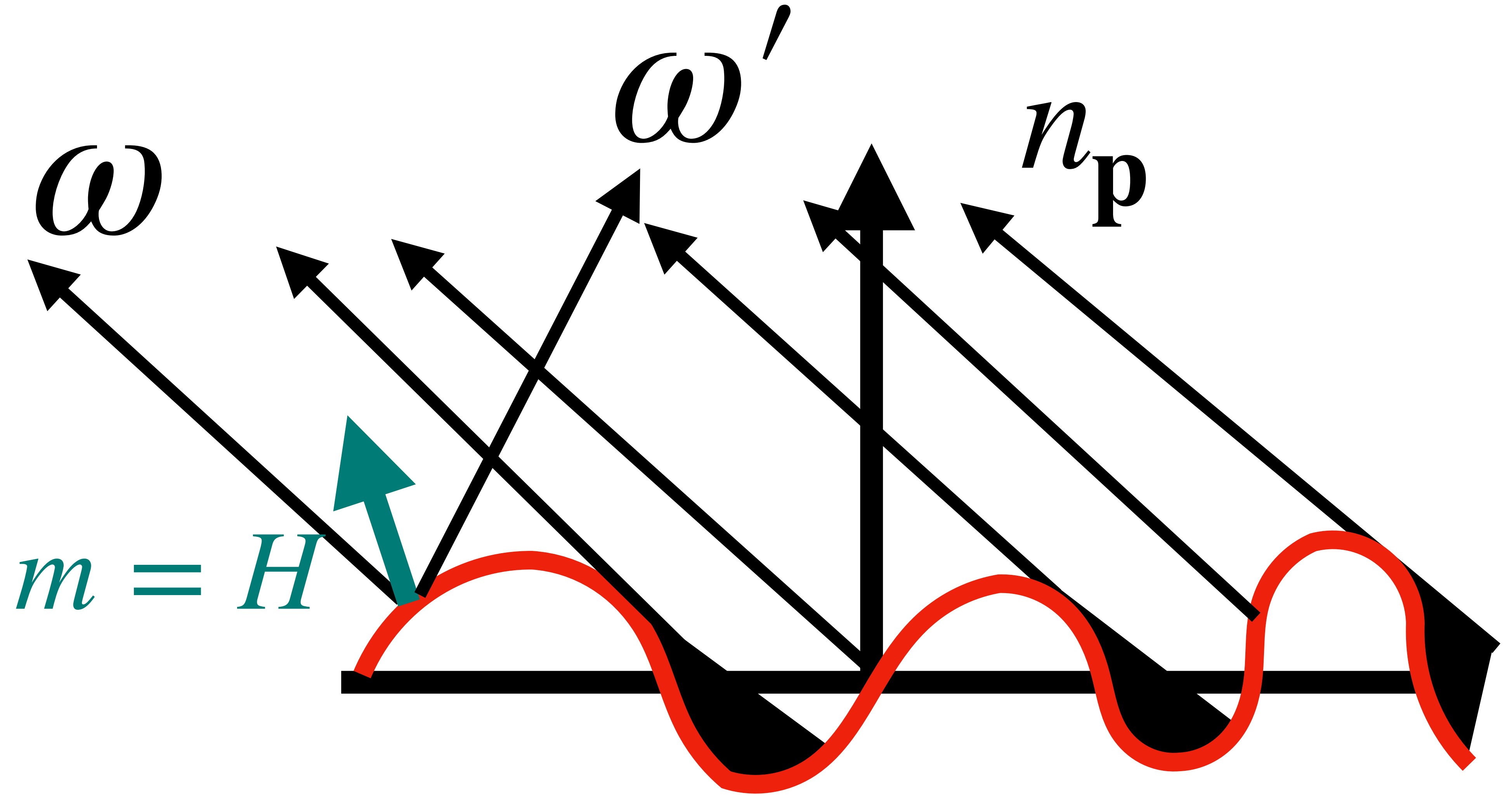
The microfacet BRDF: counting **visible** micronormals at the half vector

$$f_{\mathbf{p}}(\omega, \omega') \qquad \frac{D(H)G(\omega, \omega', H)}{4\left|\omega \cdot n_{\mathbf{p}}\right|\left|\omega' \cdot n_{\mathbf{p}}\right|}$$

$H = \text{normalize}(\omega + \omega')$

from analytically integrating over mirrors (lots of different Jacobians)

(again, see Heitz https://jcgt.org/published/0003/02/03/)

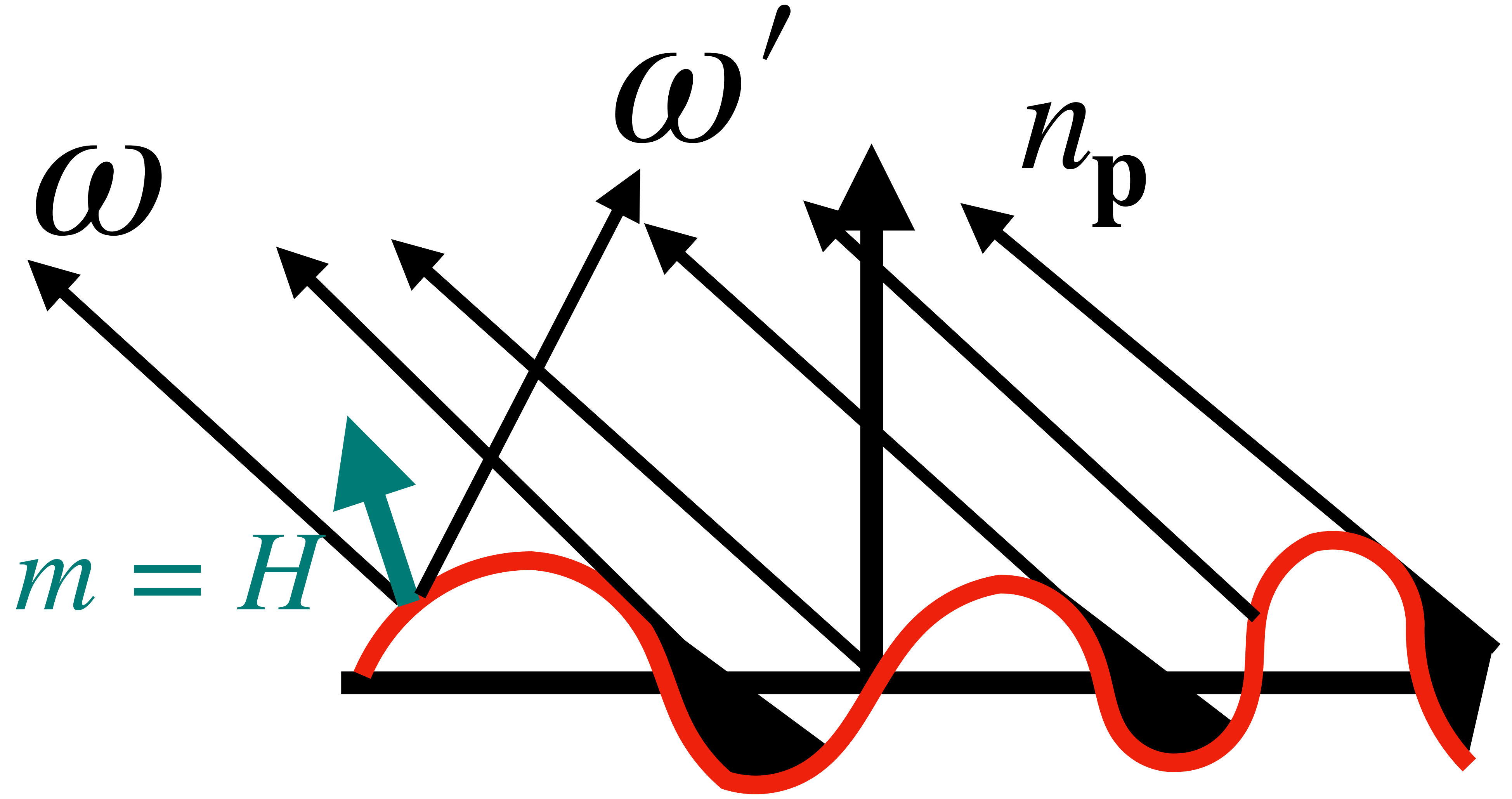


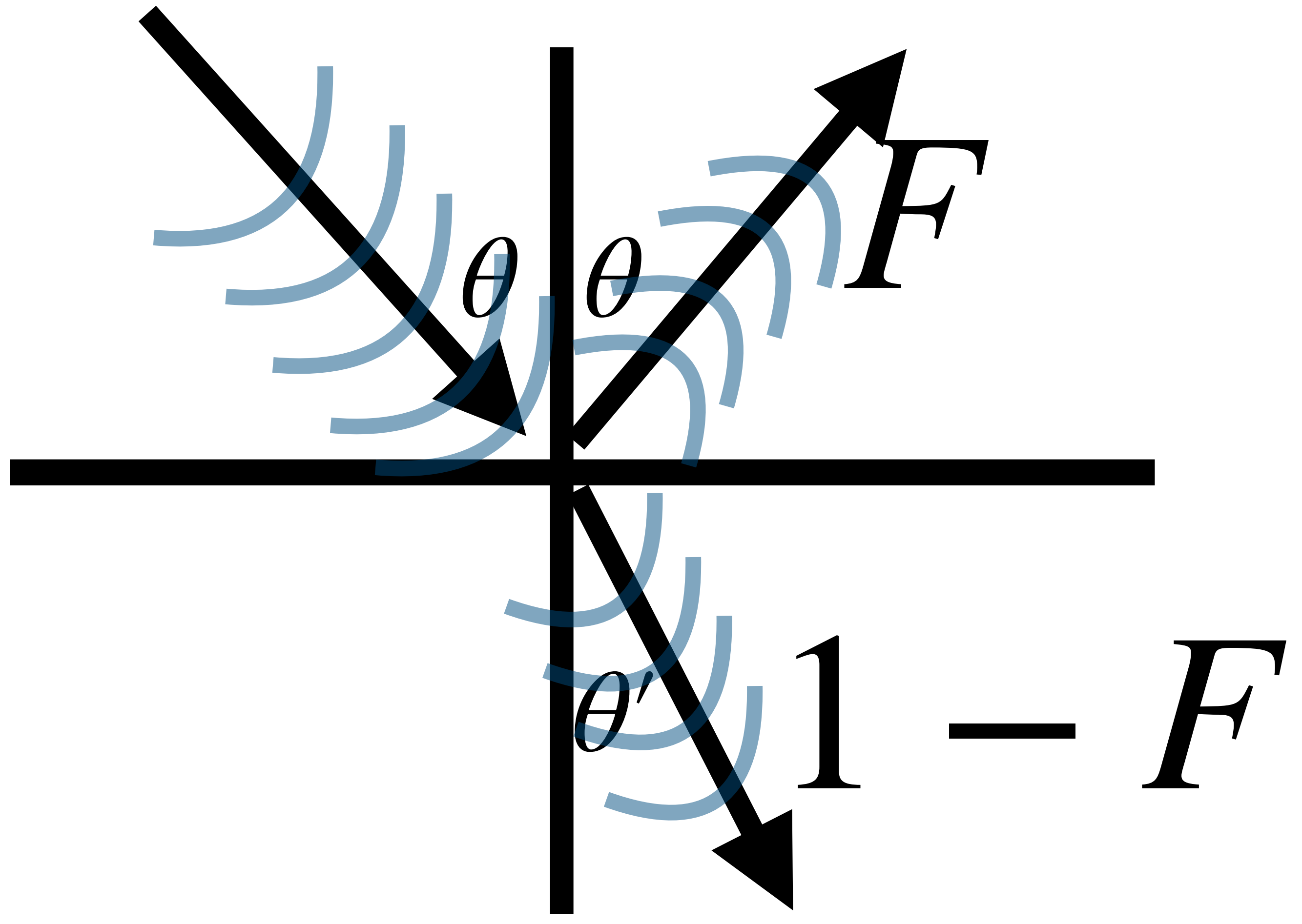
The microfacet BRDF: counting **visible** micronormals at the half vector

$$f_{\mathbf{p}}(\omega, \omega') = \frac{D(H)G(\omega, \omega', H)F(\omega, H)}{4\left|\omega \cdot n_{\mathbf{p}}\right|\left|\omega' \cdot n_{\mathbf{p}}\right|}$$

Fresnel equation

$H = \text{normalize}(\omega + \omega')$

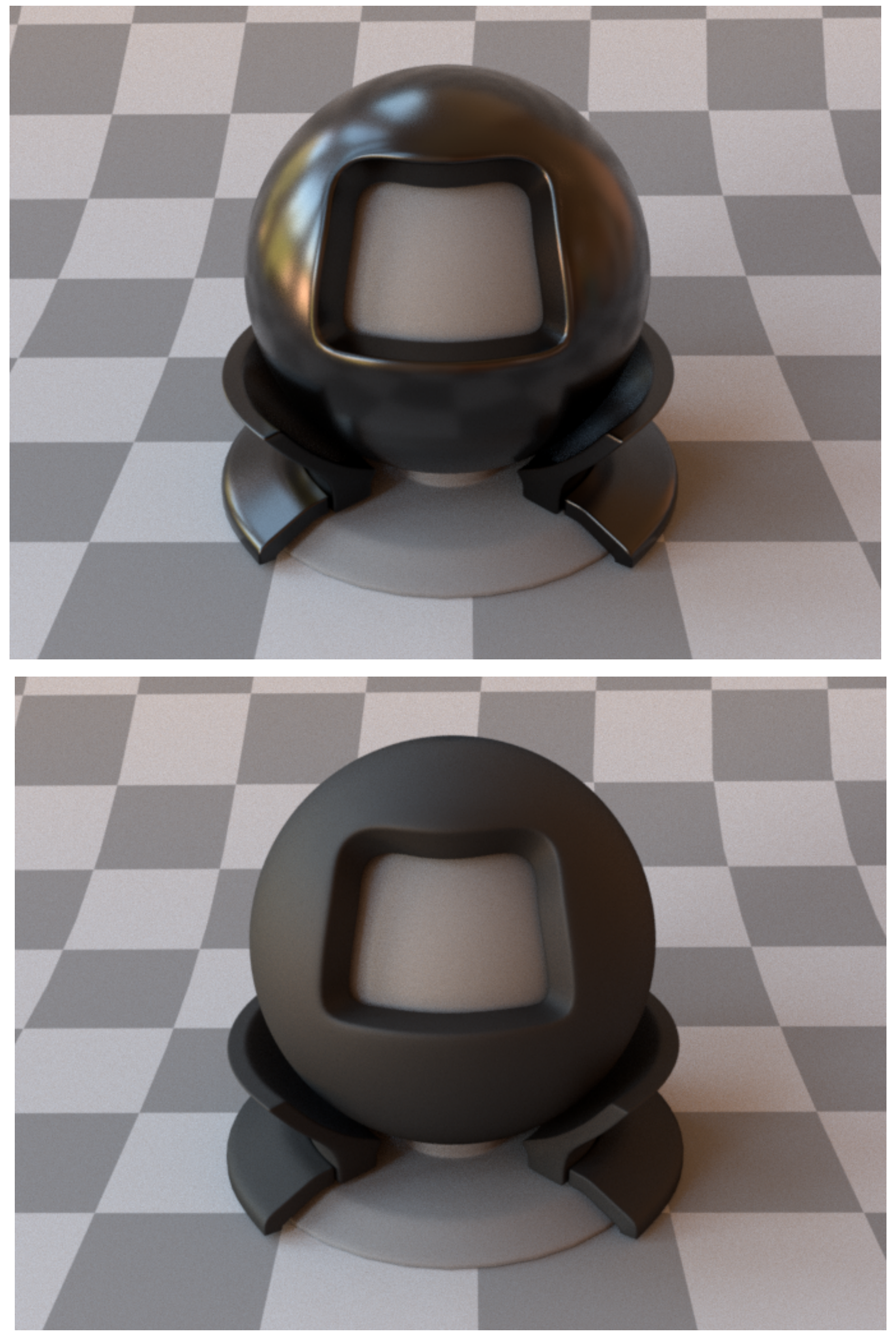


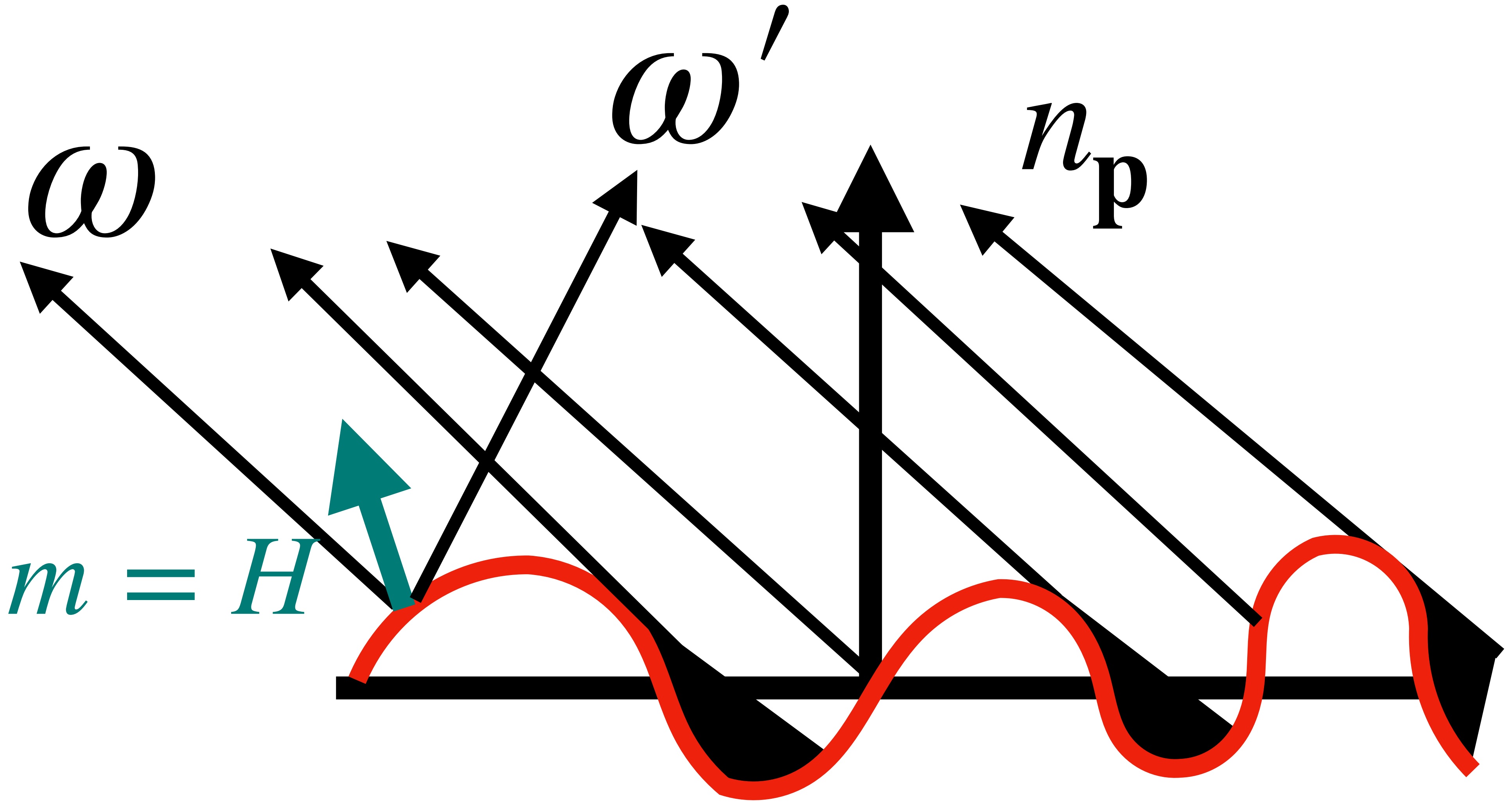


The Cook-Torrance-Sparrow BRDF [1967, 1982]

$$f_{\mathbf{p}}(\omega, \omega') = \frac{D(H)G(\omega, \omega', H)F(\omega, H)}{4\left|\omega \cdot n_{\mathbf{p}}\right|\left|\omega' \cdot n_{\mathbf{p}}\right|}$$

$H = \text{normalize}(\omega + \omega')$



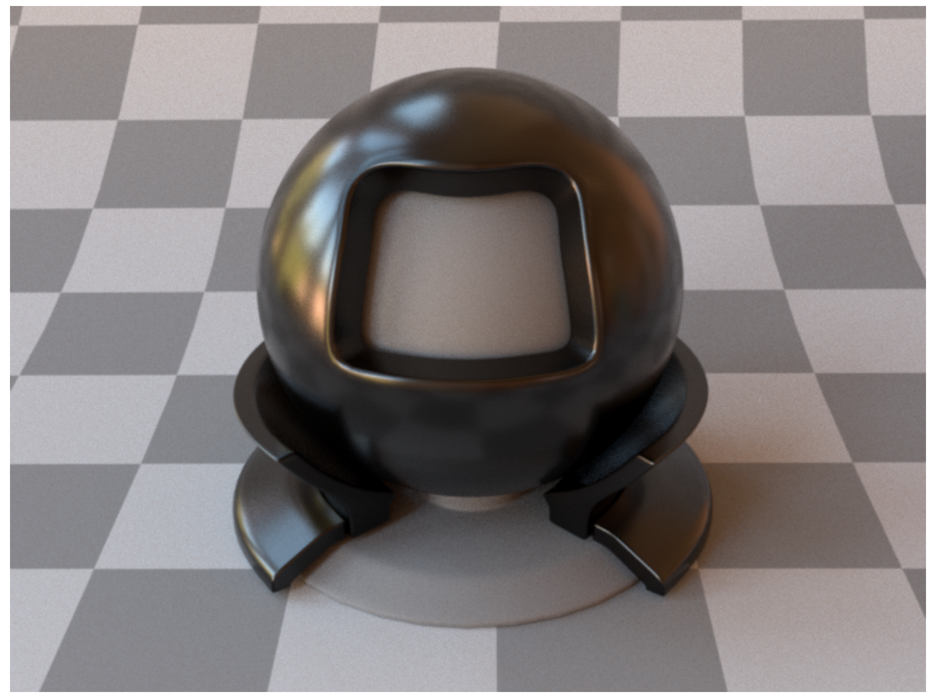


A Reflectance Model for Computer Graphics

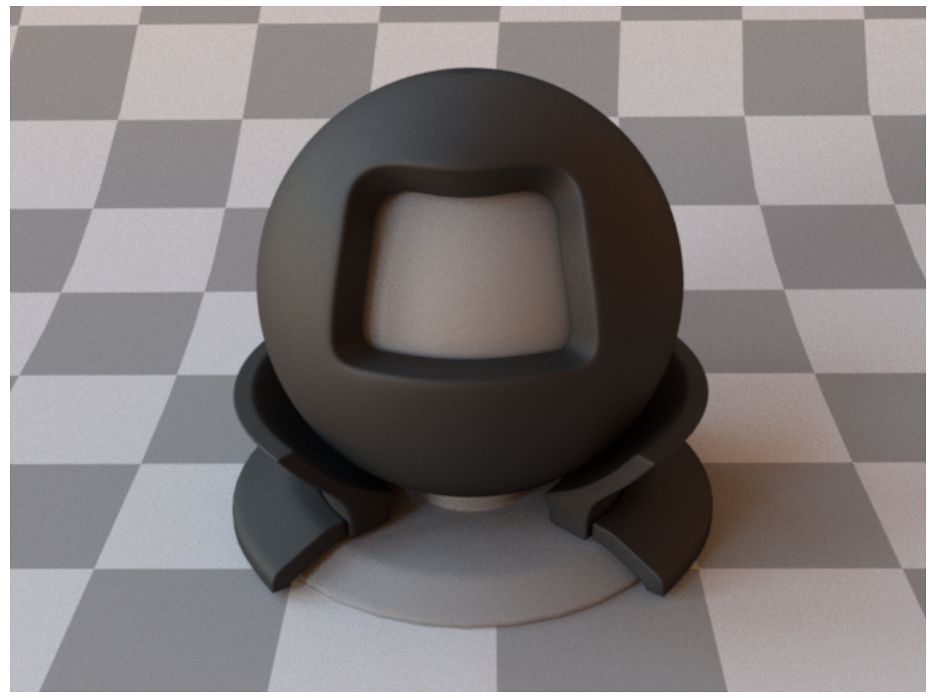
ROBERT L. COOK
Lucasfilm Ltd.
and
KENNETH E. TORRANCE
Cornell University

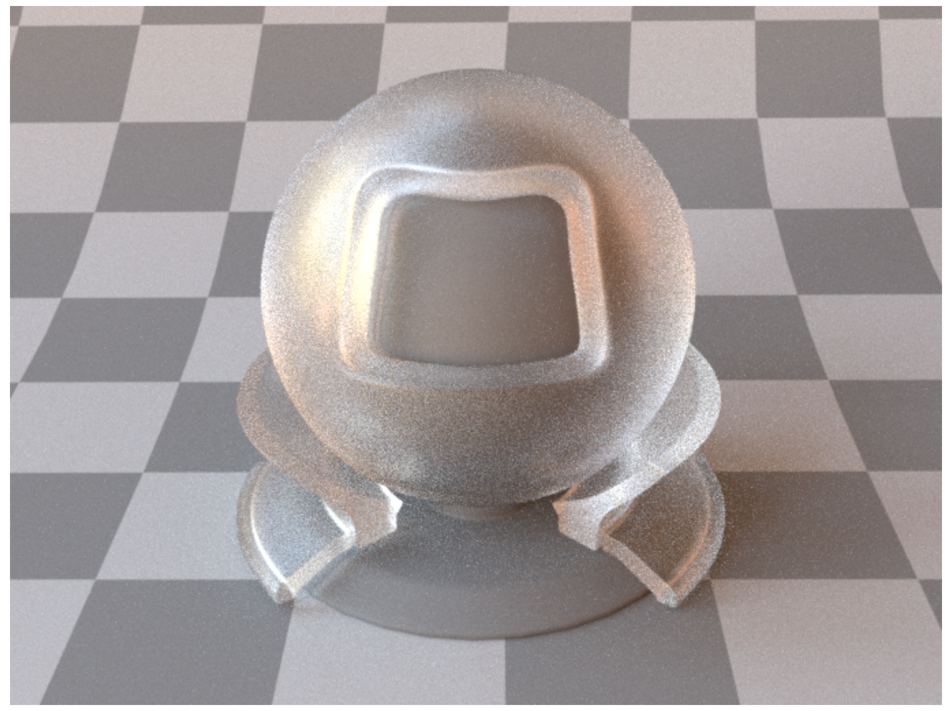
The refraction extension [Walter 2007]

$$f_{\mathbf{p}}(\omega, \omega') = \begin{cases} \dfrac{D(H)G(\omega, \omega', H)F(\omega, H)}{4\left|\omega \cdot n_{\mathbf{p}}\right|\left|\omega' \cdot n_{\mathbf{p}}\right|} & \text{if reflect} \\ \dfrac{D(H_r)G(\omega, \omega', H_r)(1 - F(\omega, H_r))}{\text{scary Jacobians}} & \text{if refract} \end{cases}$$



$$H_r = \text{normalize}(\eta\omega + \eta'\omega')$$

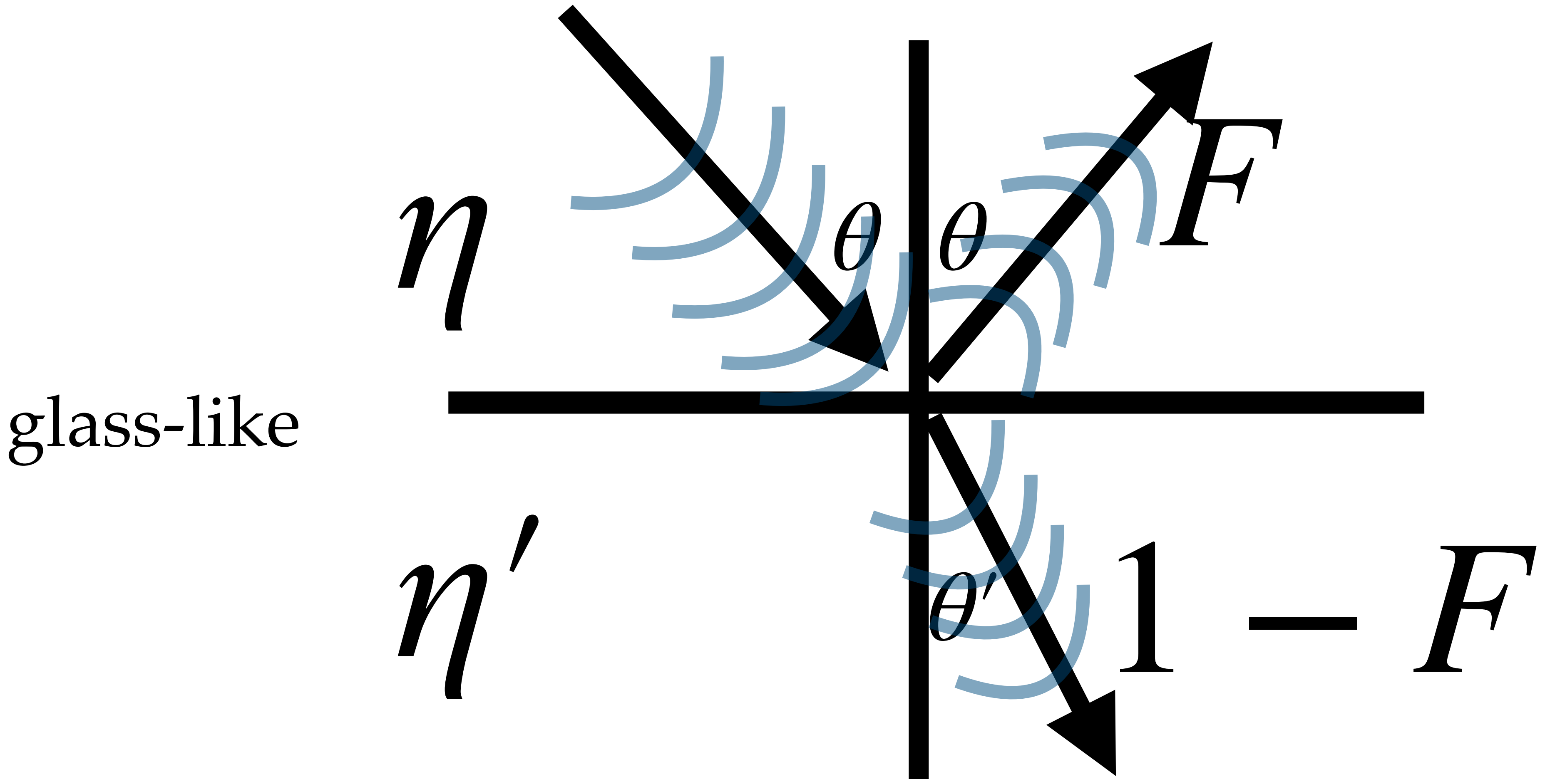




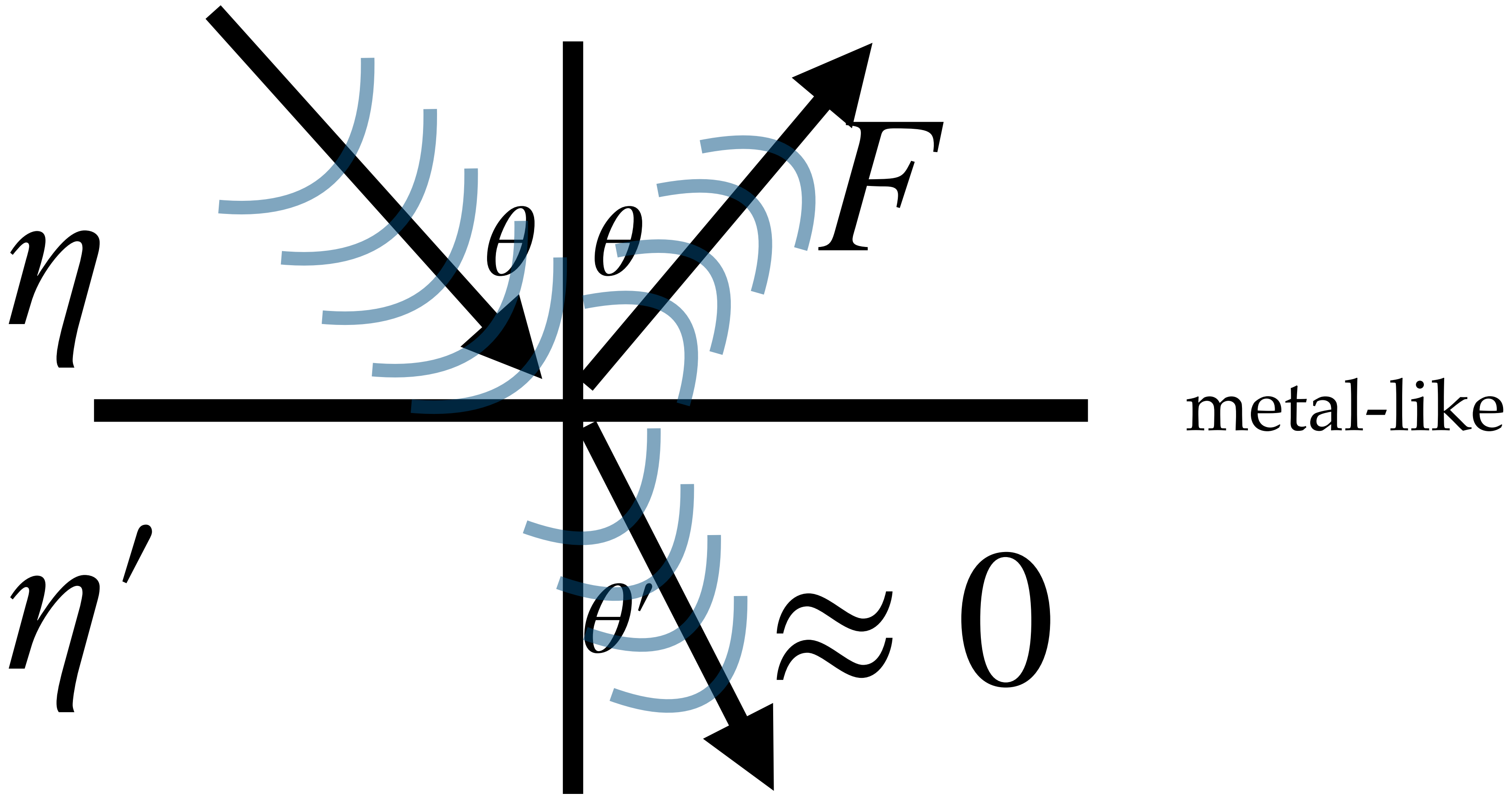
see http://www.graphics.cornell.edu/~bjw/microfacetbsdf.pdf
for the scary Jacobian

Fresnel equation

- light as wave behaves differently for glass-like materials (dielectrics) and metal-like materials (conductors)

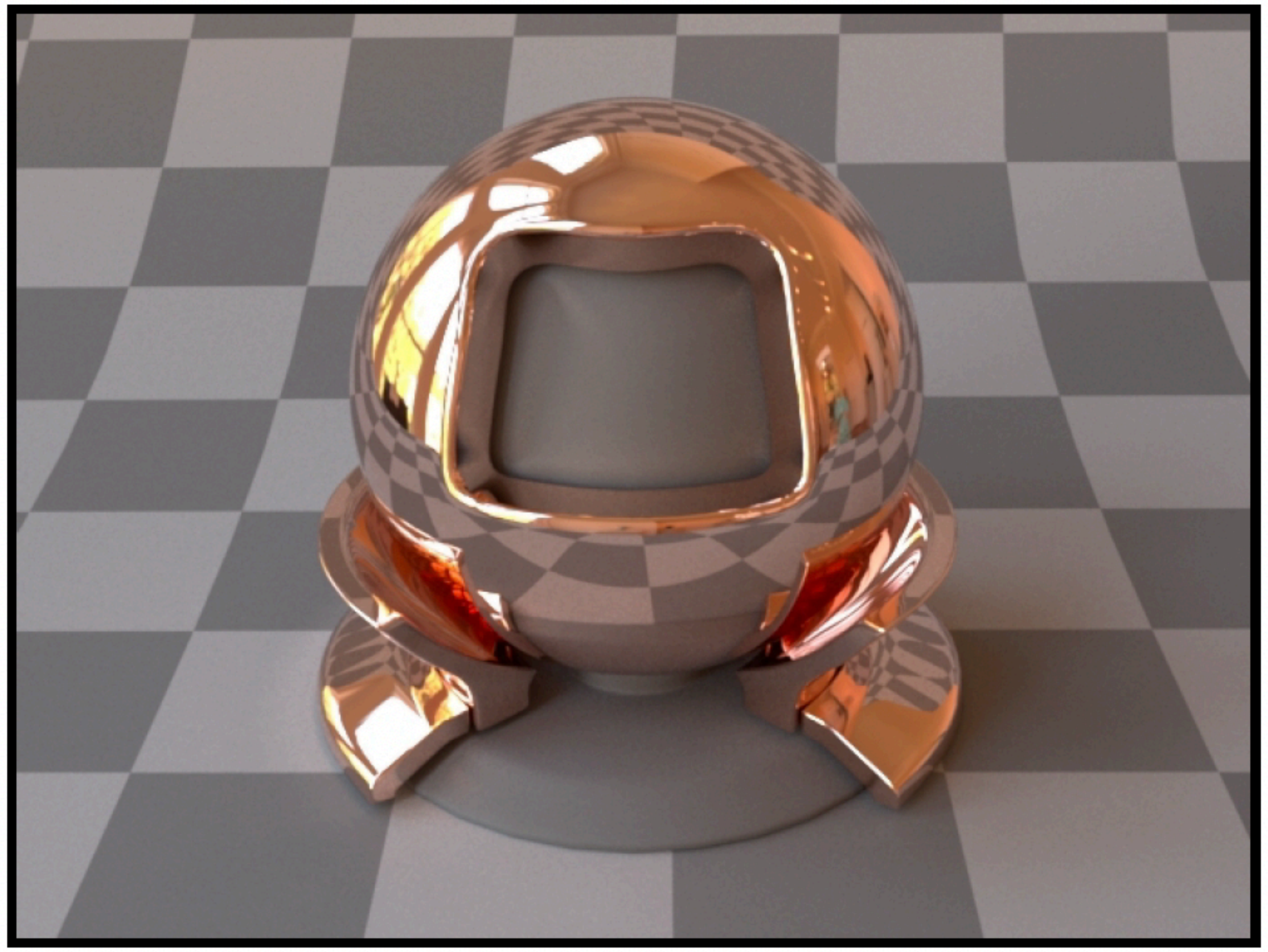


real number η, η'

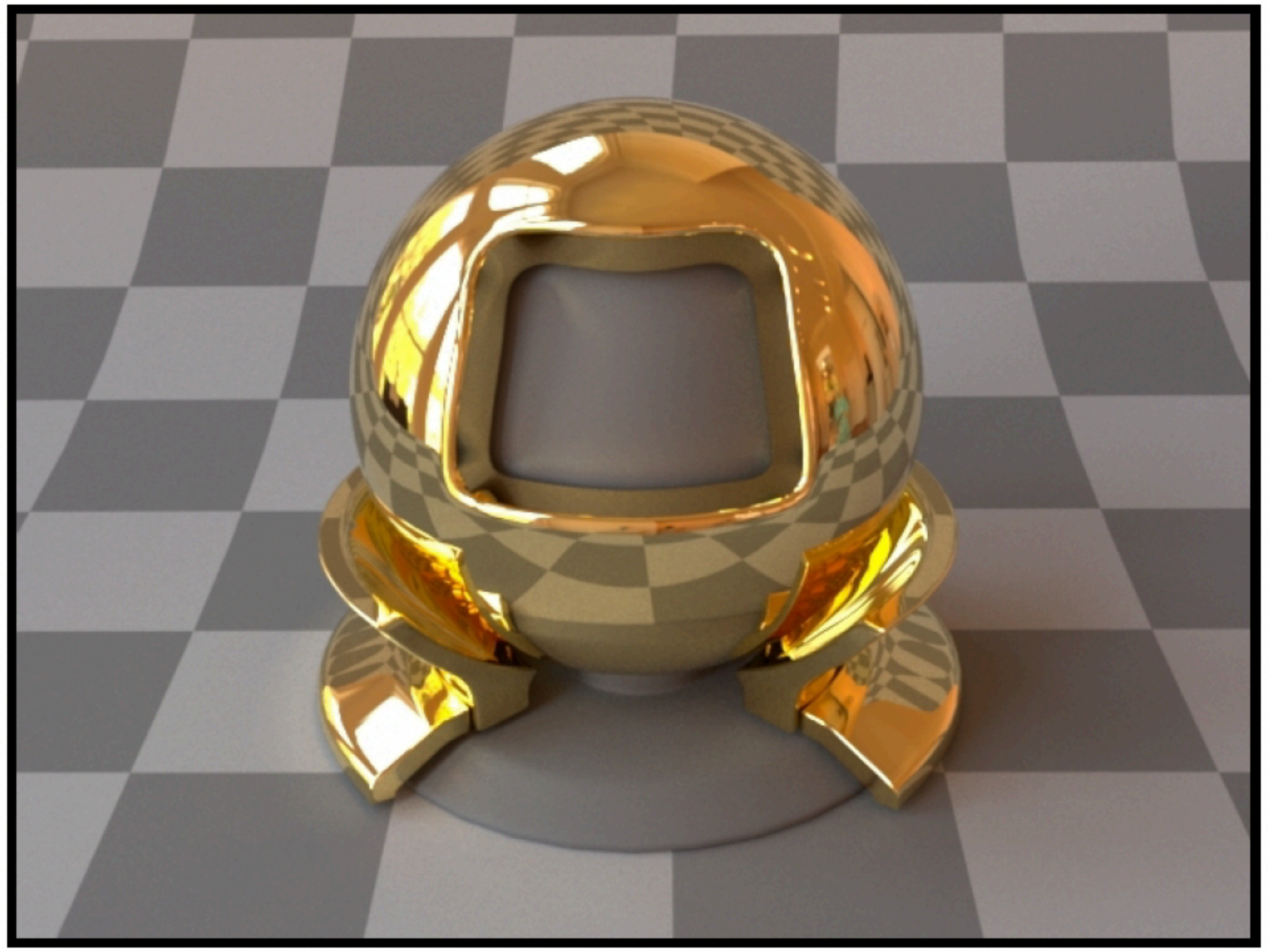


complex number η, η'

For metals, the complex index of refraction varies with wavelength



(a) Measured copper material (the default), rendered using 30 spectral samples between 360 and 830*nm*

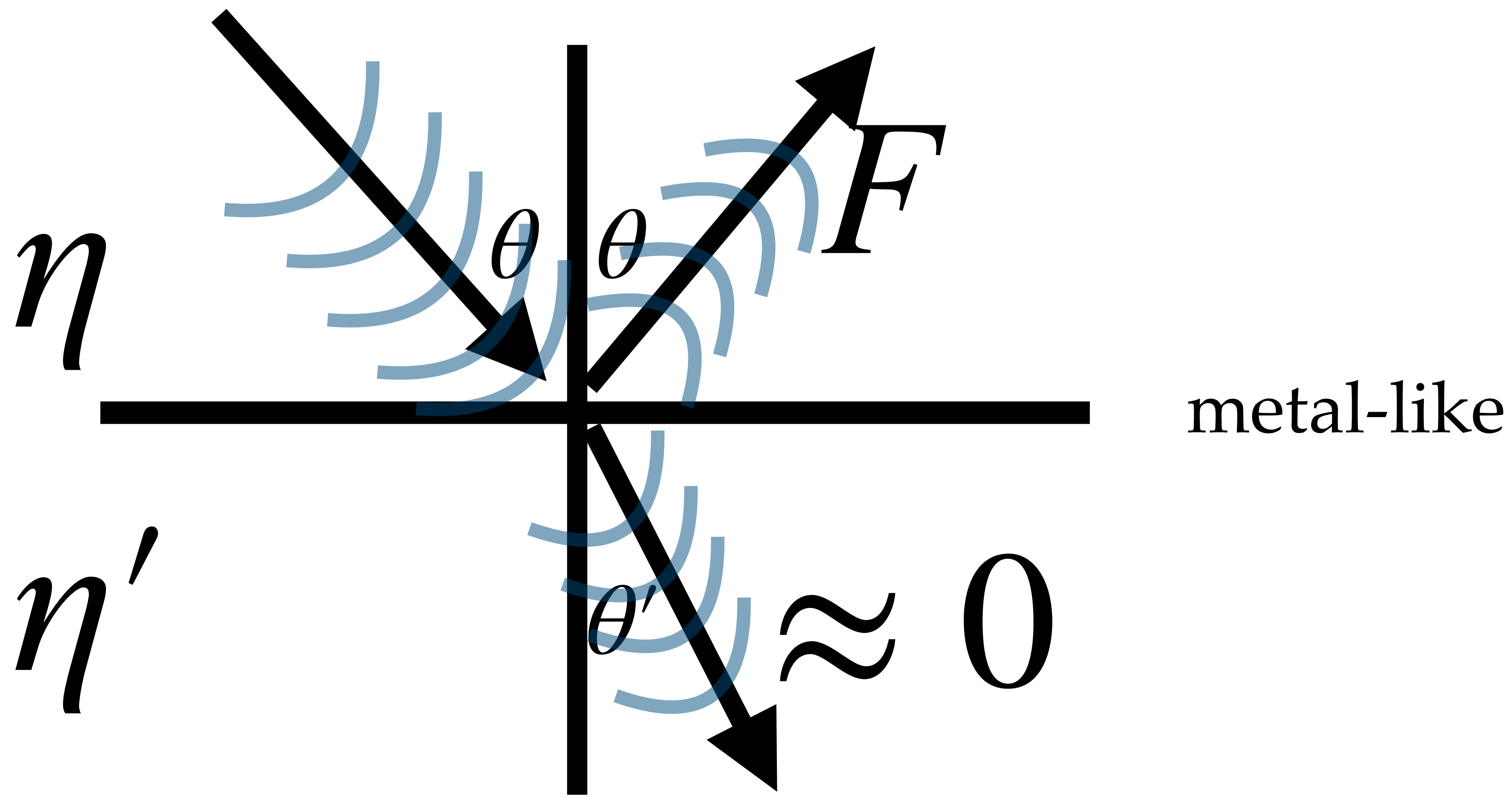


(b) Measured gold material (Listing 17)

images from Wenzel Jakob
https://www.mitsuba-renderer.org/releases/current/documentation.pdf

Fresnel equation for metal-like interfaces is scary & unintuitive

- also we don't have a lot of spectral data



complex number η, η'

$$a^2 = \frac{1}{2n_i^2}\left(\sqrt{(n_t^2 - k_t^2 - n_i^2\sin^2\theta)^2 + 4n_t^2k_t^2} + n_t^2 - k_t^2 - n_i^2\sin^2\theta\right)$$

$$b^2 = \frac{1}{2n_i^2}\left(\sqrt{(n_t^2 - k_t^2 - n_i^2\sin^2\theta)^2 + 4n_t^2k_t^2} - n_t^2 + k_t^2 + n_i^2\sin^2\theta\right)$$

$$R_s = \frac{a^2+b^2-2a\cos\theta+\cos^2\theta}{a^2+b^2+2a\cos\theta+\cos^2\theta}$$

$$R_p = R_s\frac{a^2+b^2-2a\sin\theta\tan\theta+\sin^2\theta\tan^2\theta}{a^2+b^2+2a\sin\theta\tan\theta+\sin^2\theta\tan^2\theta}$$

using a^2 and b^2 with η and η_k as follow give the same result:

$$a^2 = \frac{1}{2}\left(\sqrt{(\eta^2 - \eta_k^2 - \sin^2\theta)^2 + 4\eta^2\eta_k^2} + \eta^2 - \eta_k^2 - \sin^2\theta\right)$$

$$b^2 = \frac{1}{2}\left(\sqrt{(\eta^2 - \eta_k^2 - \sin^2\theta)^2 + 4\eta^2\eta_k^2} - \eta^2 + \eta_k^2 + \sin^2\theta\right)$$

Derivation from conductor Fresnel equation can be found in [5] (p.111) and use

$$(n_t - \mathbf{i}k_t)\cos\theta_t = (n_t - \mathbf{i}k_t)\sqrt{1 - \left(\frac{n_i}{(n_t - \mathbf{i}k_t)}\sin\theta\right)^2} = \sqrt{(n_t - \mathbf{i}k_t)^2 - n_i^2\sin^2\theta}$$

$$a - \mathbf{i}b = \sqrt{(n_t - \mathbf{i}k_t)^2 - n_i^2\sin^2\theta)}$$

In practice there is some simplification possible:

$$a^2 + b^2 = \sqrt{(\eta^2 - \eta_k^2 - \sin^2\theta)^2 + 4\eta^2\eta_k^2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

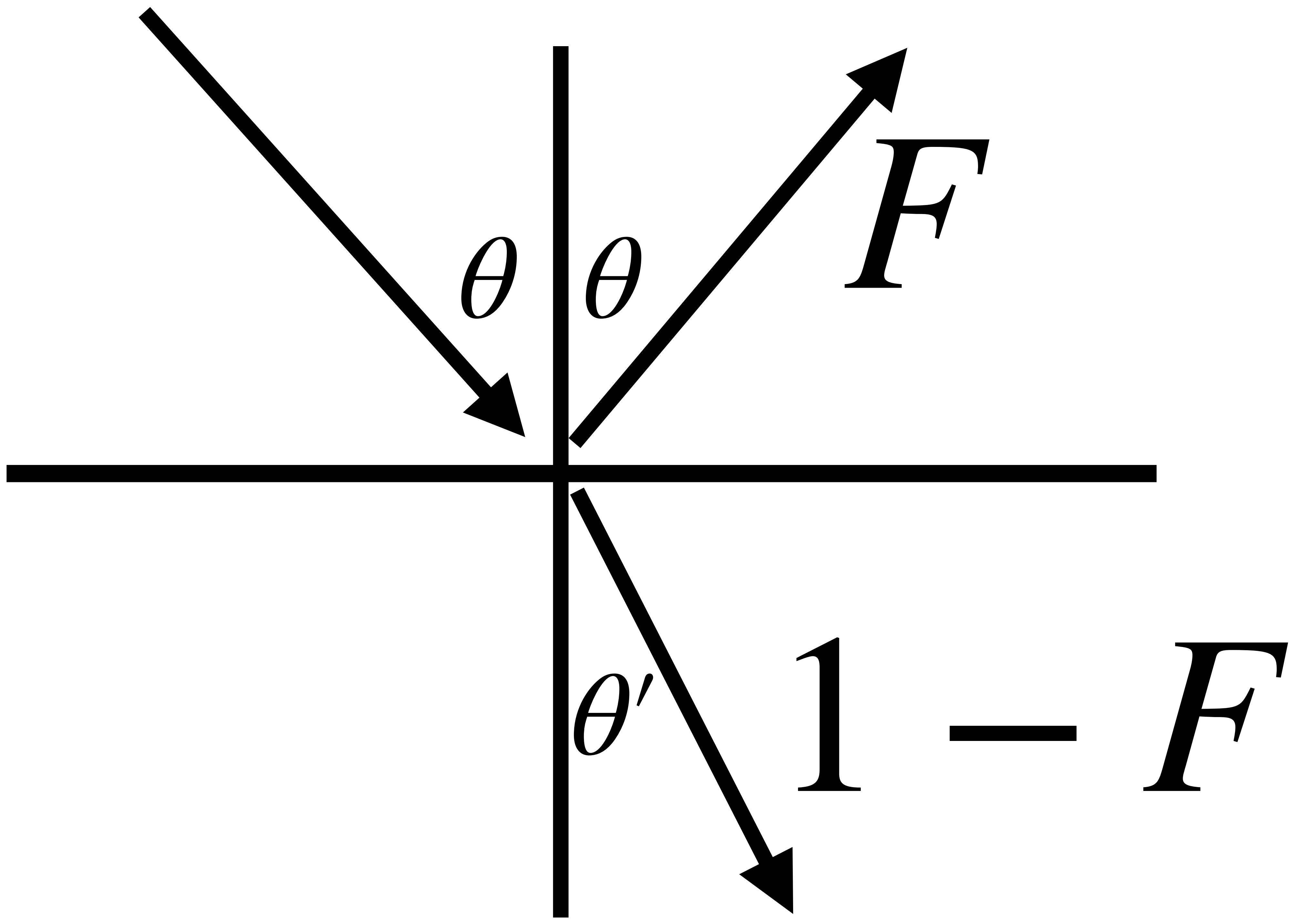
$$R_p = R_s\frac{\cos^2\theta(a^2+b^2)-2a\cos\theta\sin^2\theta+\sin^4\theta}{\cos^2\theta(a^2+b^2)+2a\cos\theta\sin^2\theta+\sin^4\theta}$$

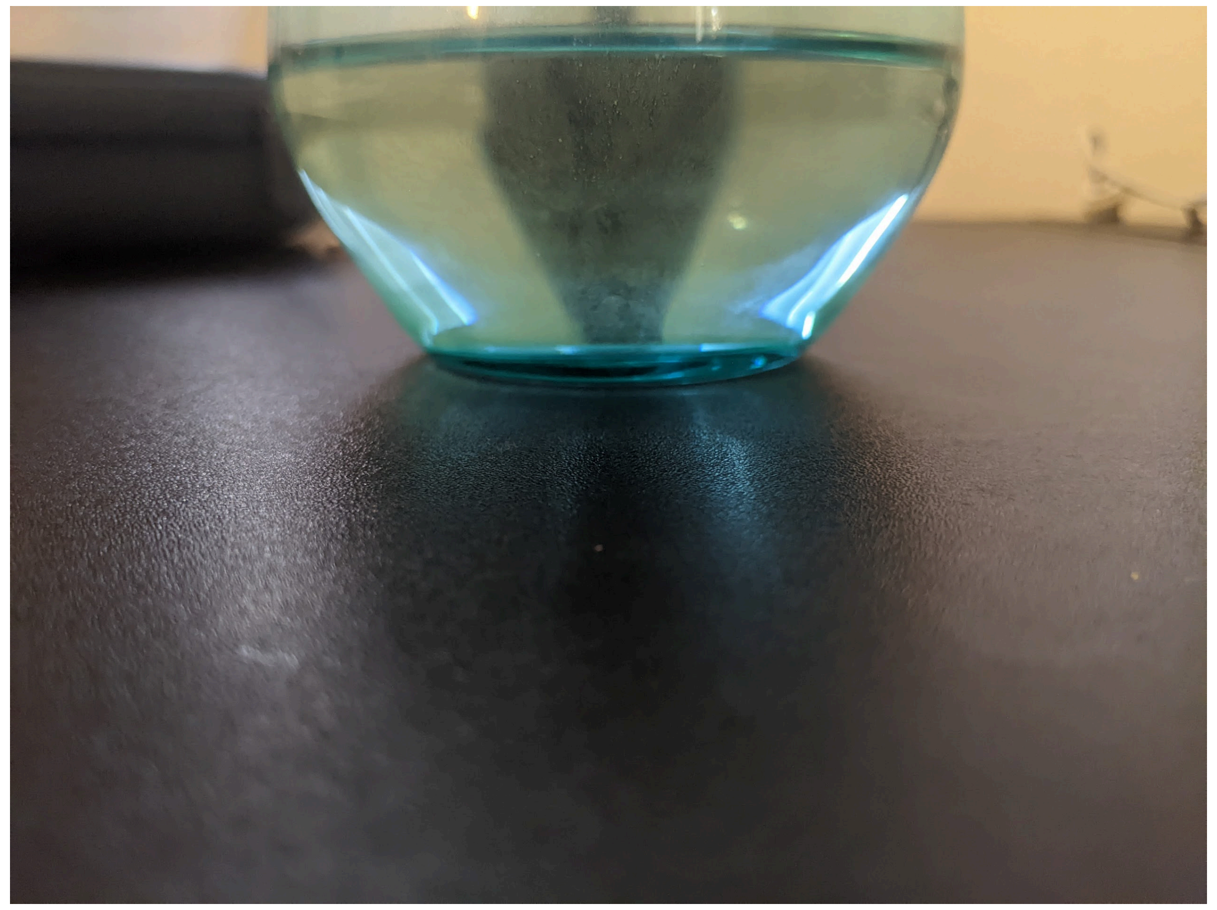
Graphics people use Schlick's approximation

$$F \approx F_0 + (1 - F_0)(1 - \cos\theta)^5$$

$$F_0 = \left(\frac{\eta - \eta'}{\eta + \eta'}\right)^2$$

for real index of refraction





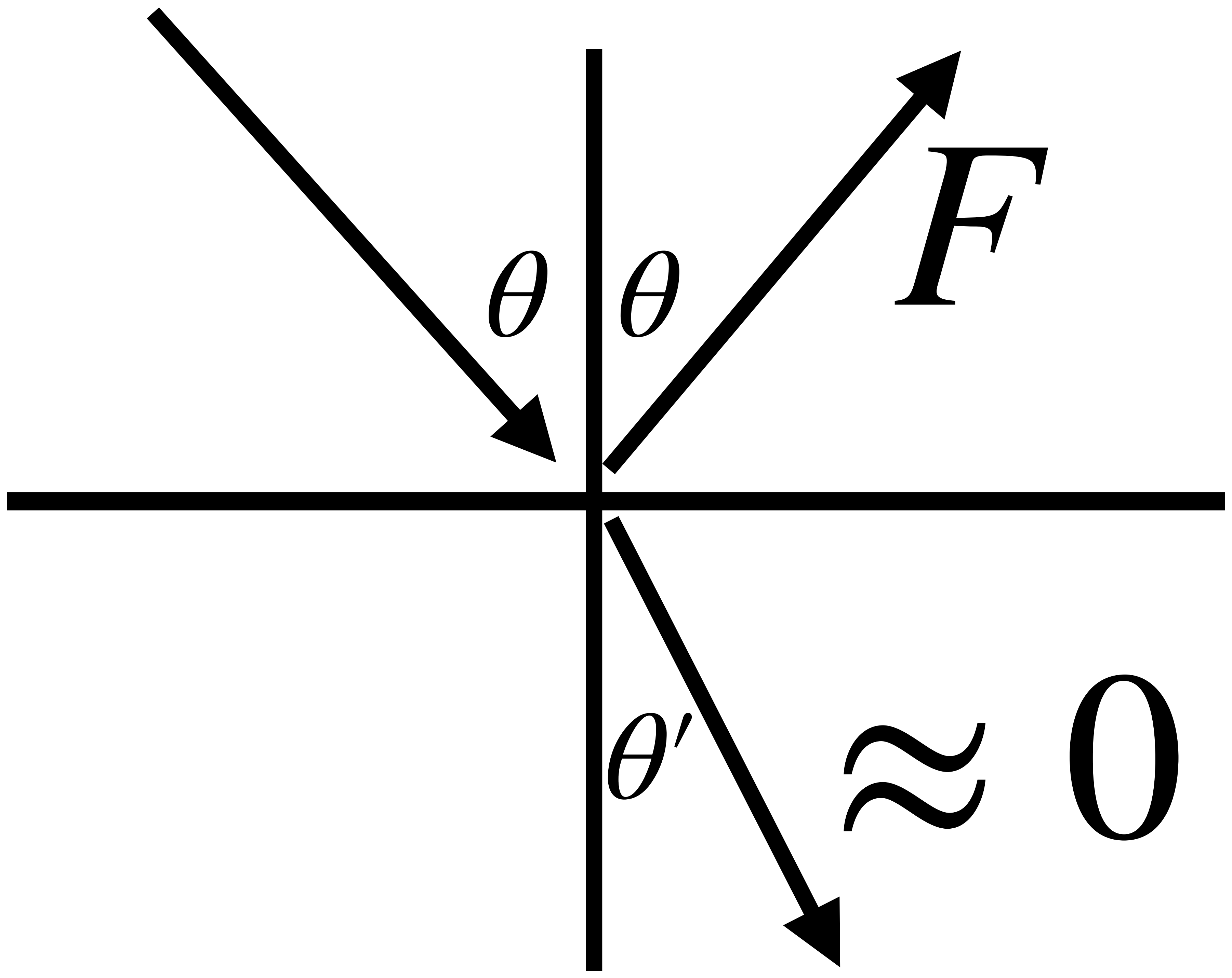
small cos θ, F ~ 1

large cos θ, F ~ F0

Graphics people use Schlick's approximation

for metals

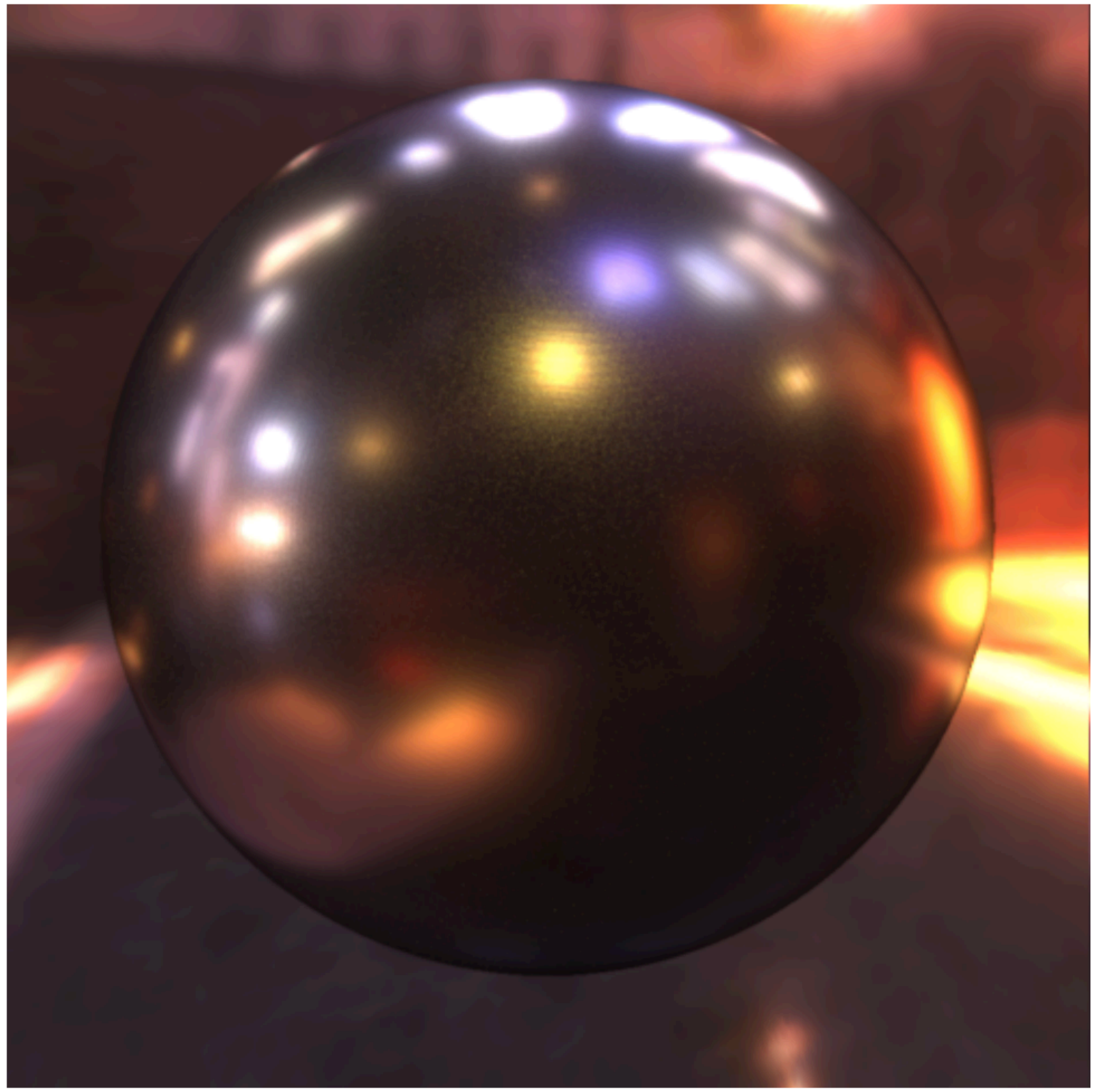
$$F \approx \text{color} + (1 - \text{color})(1 - \cos\theta)^5$$



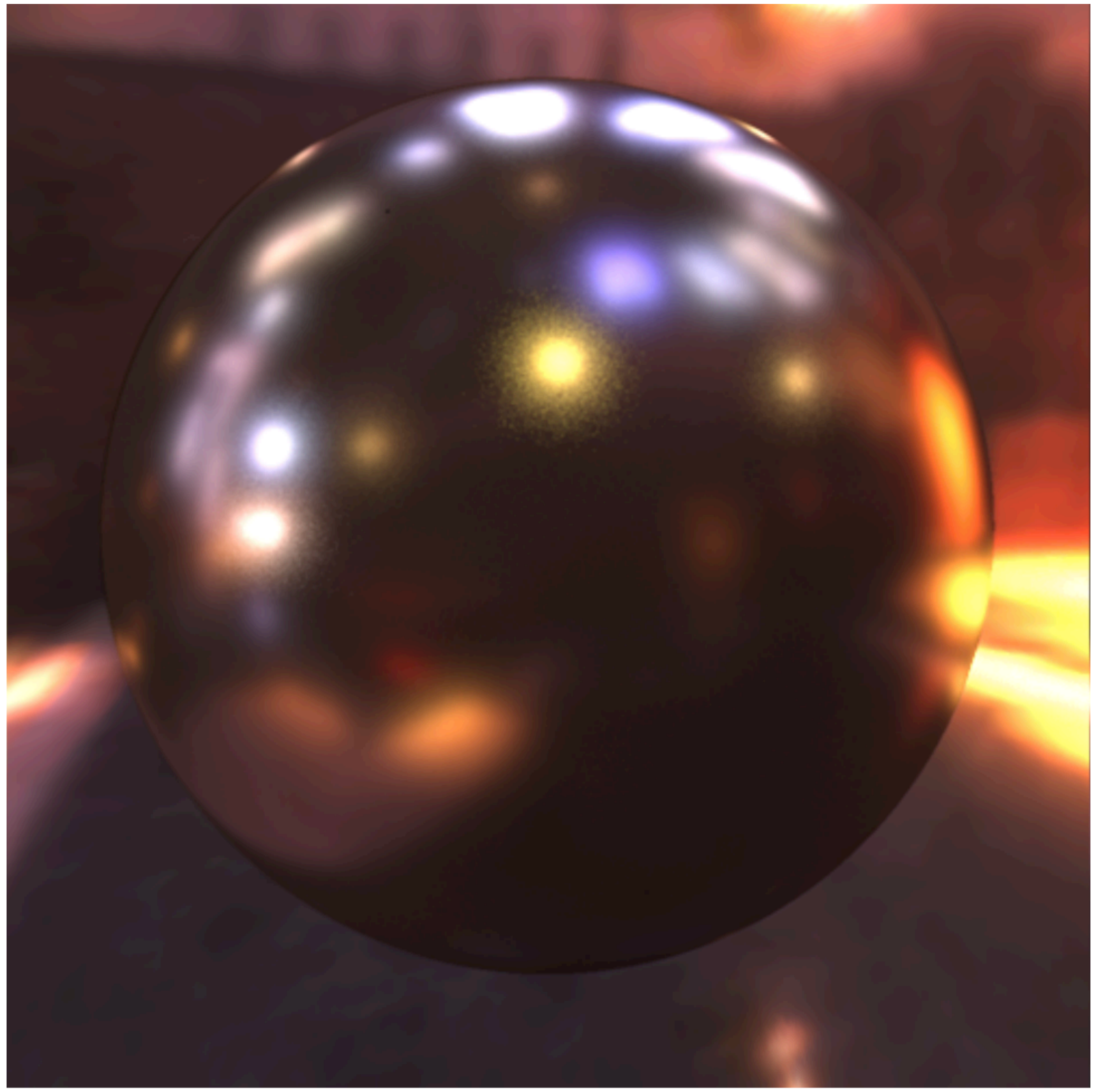
metal becomes colorless/white at grazing angle

highly recommend "Some Thoughts on the Fresnel Term"
https://www.youtube.com/watch?v=kEcDbl7eS0w

Cook-Torrance-Sparrow BSDF fits well to MERL measured data!



measured (nickel material)

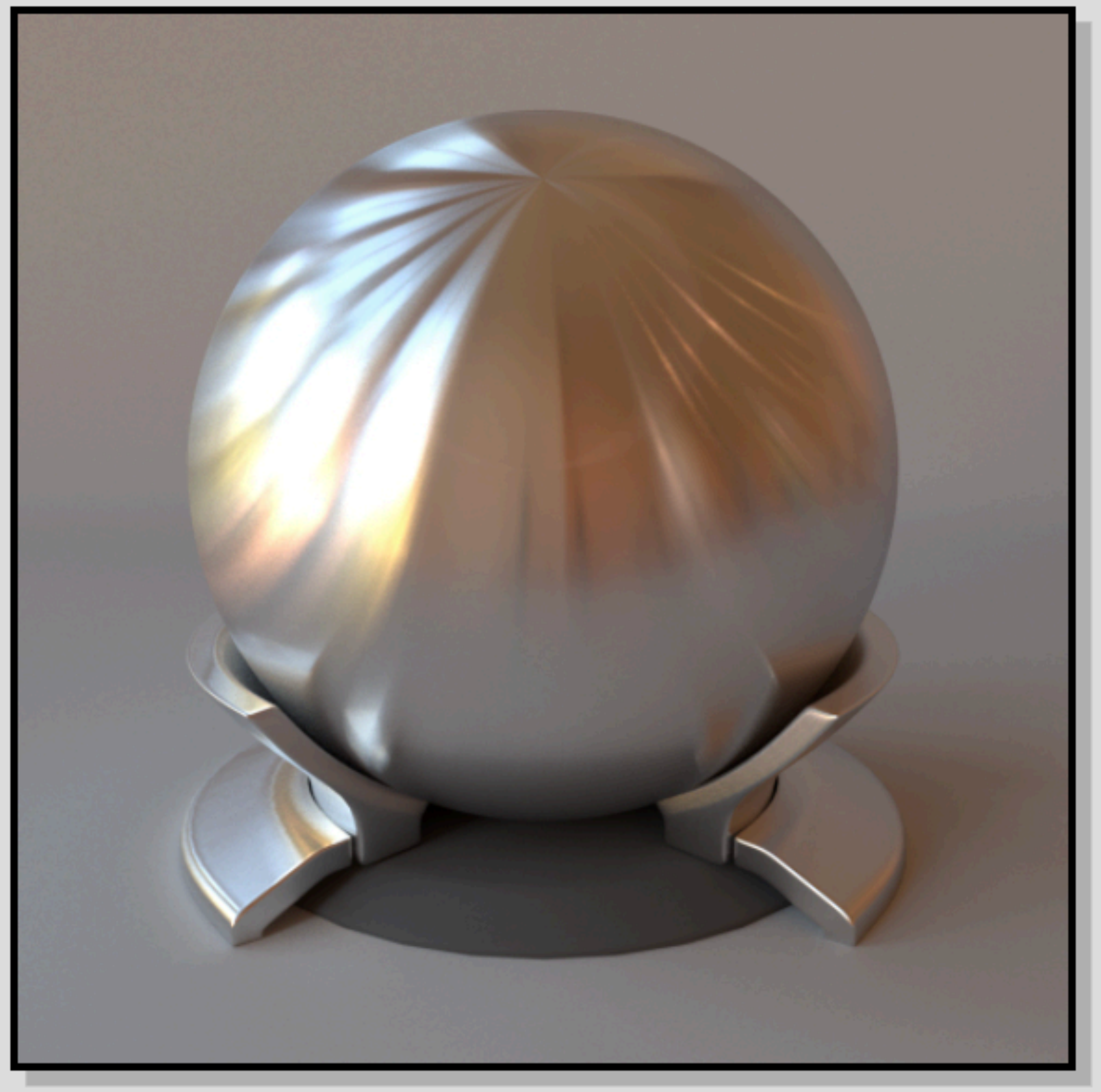


Cook-Torrance-Sparrow

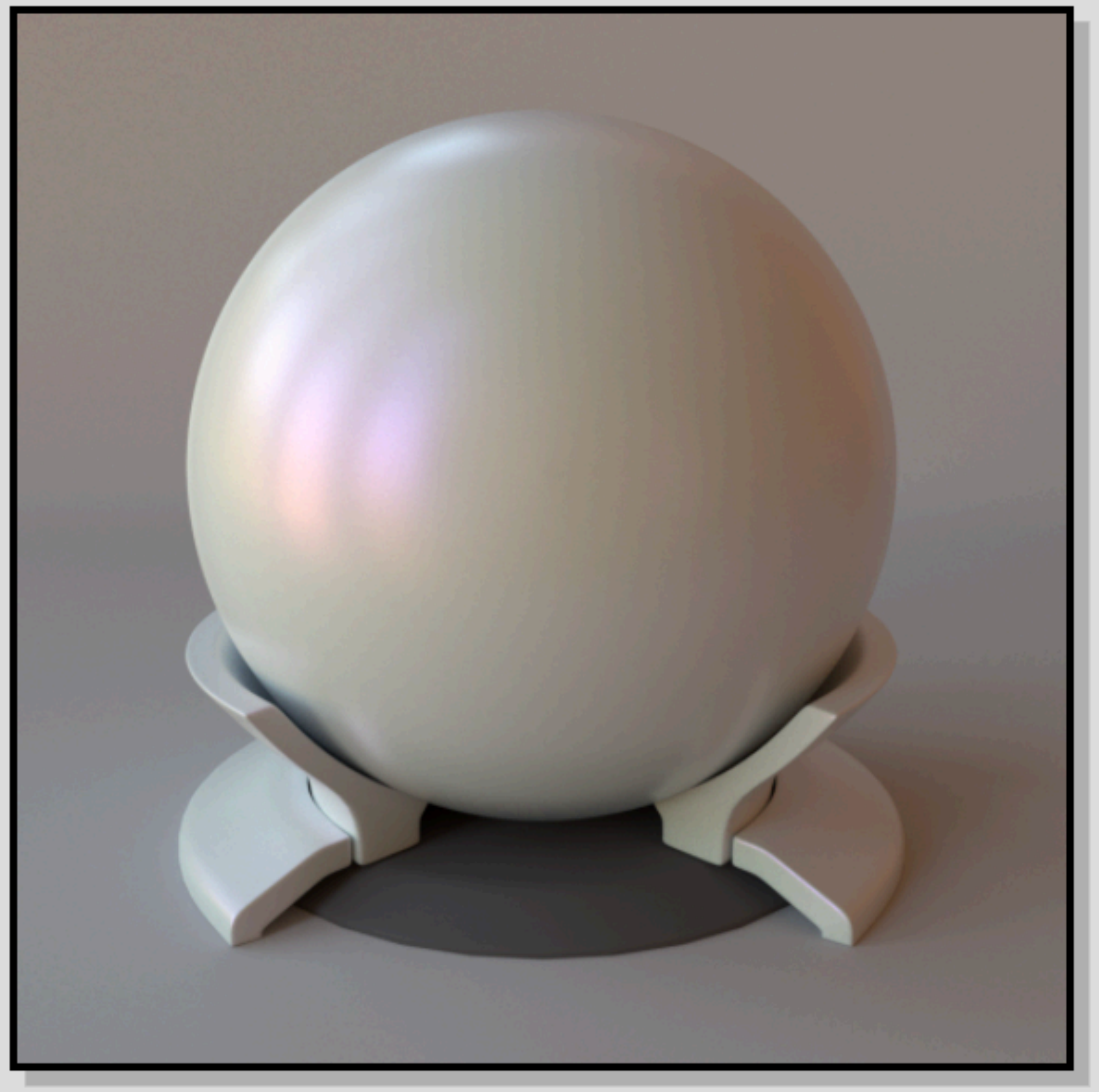
from Ngan et al. “Experimental Analysis of BRDF Models"
http://people.csail.mit.edu/addy/research/ngan05_brdf_eval.pdf

Limitations of microfacet models

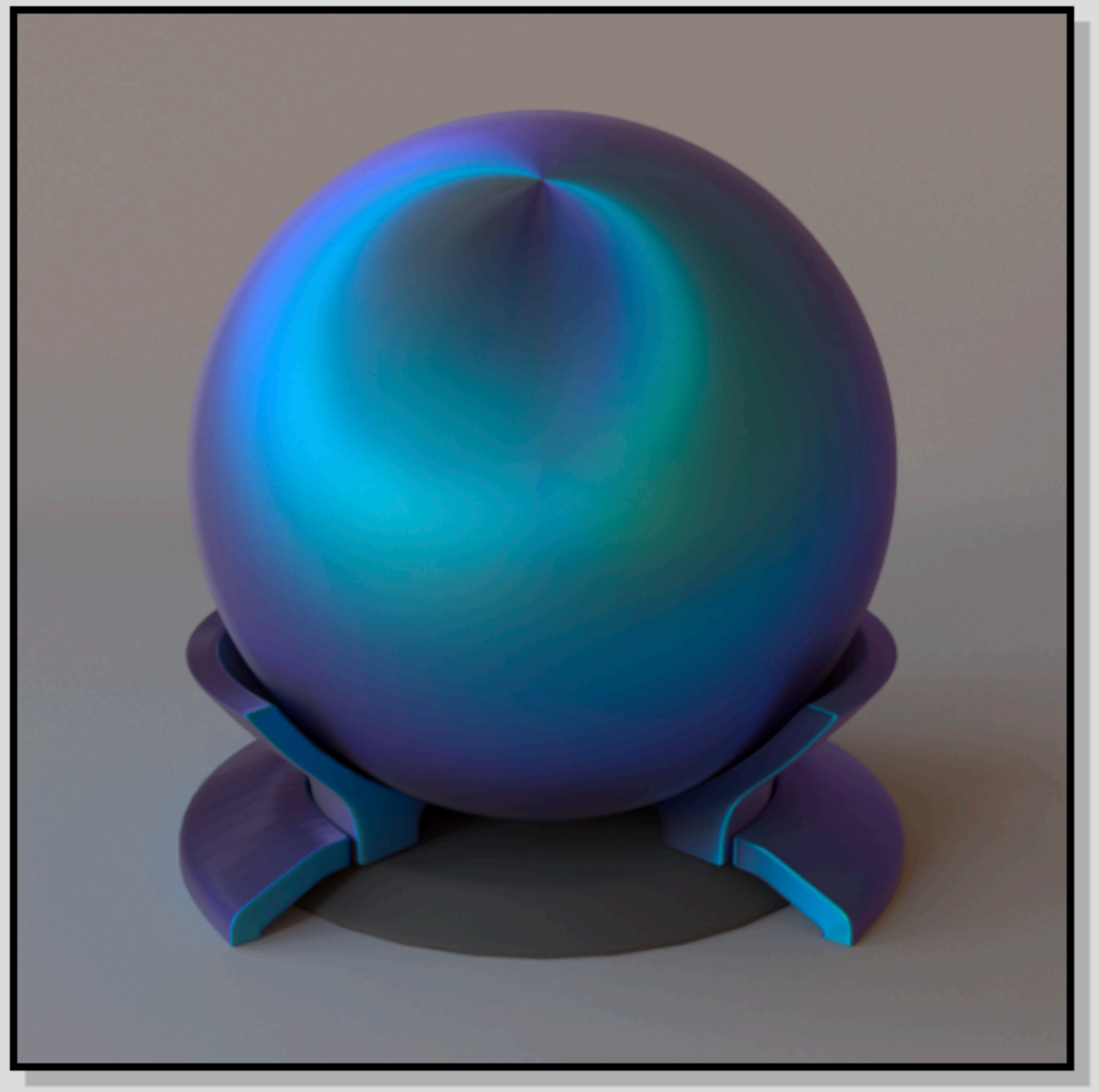
- from the EPFL dataset


aniso_brushed_aluminium

strongly anisotropic but "hazy"


satin_purple

iridescence?


aniso_morpho_melenaus

???
(butterfly wings)

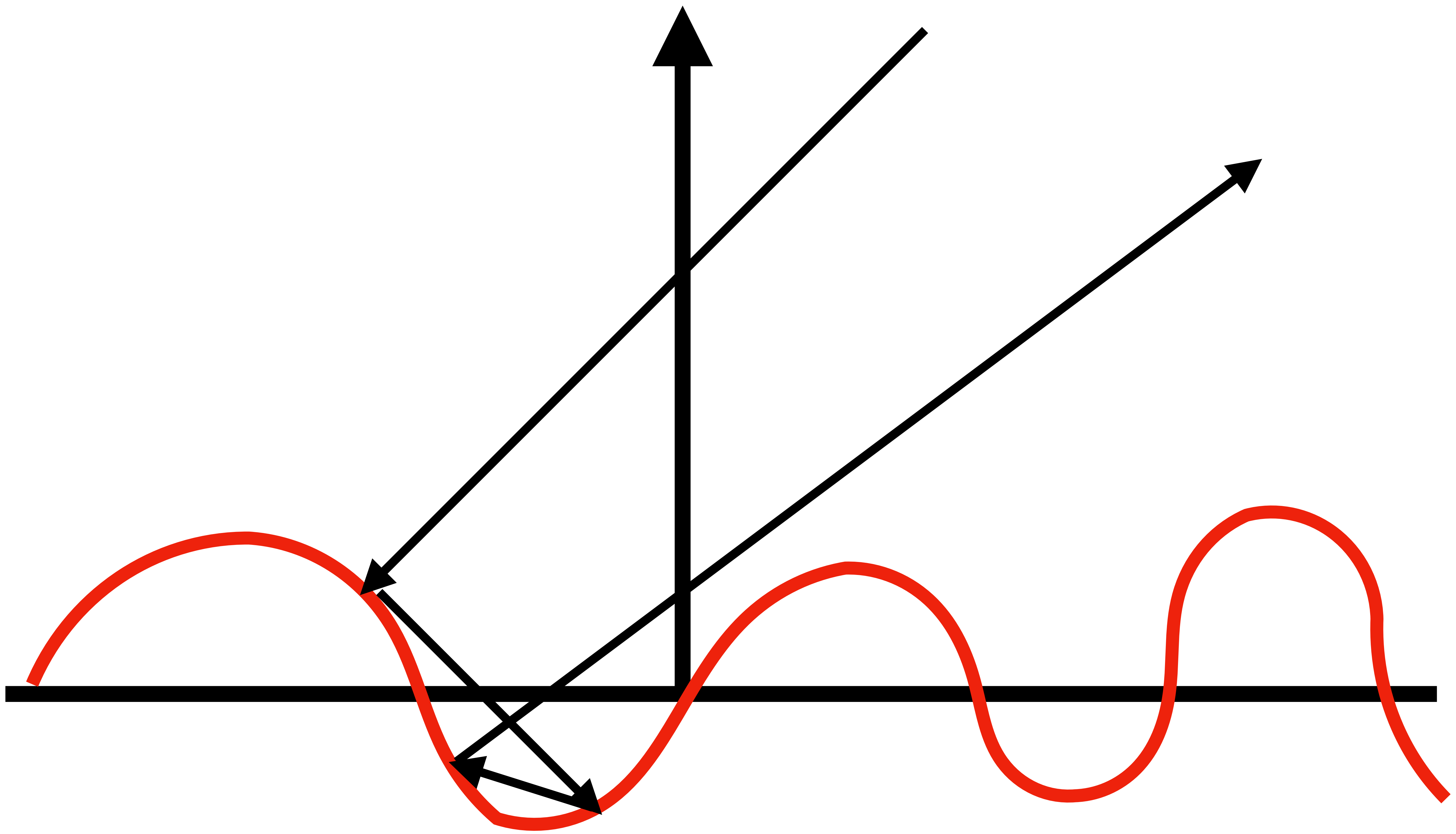
awesome material images from Jonathan Dupuy & Wenzel Jakob

http://onrendering.com/data/papers/powitacq/slides/powitacq.html

https://rgl.epfl.ch/pages/lab/pgII

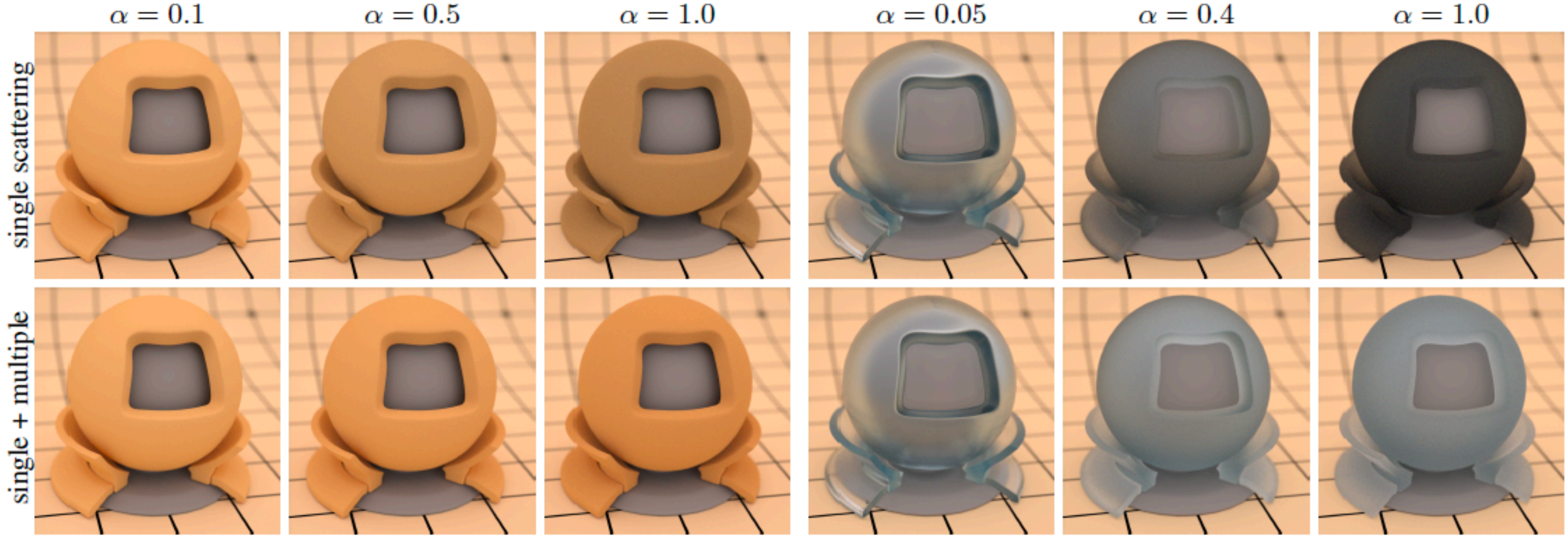
Multiple scattering

- Cook-Torrance-Sparrow BSDF ignores multiple bounces inside the microsurfaces



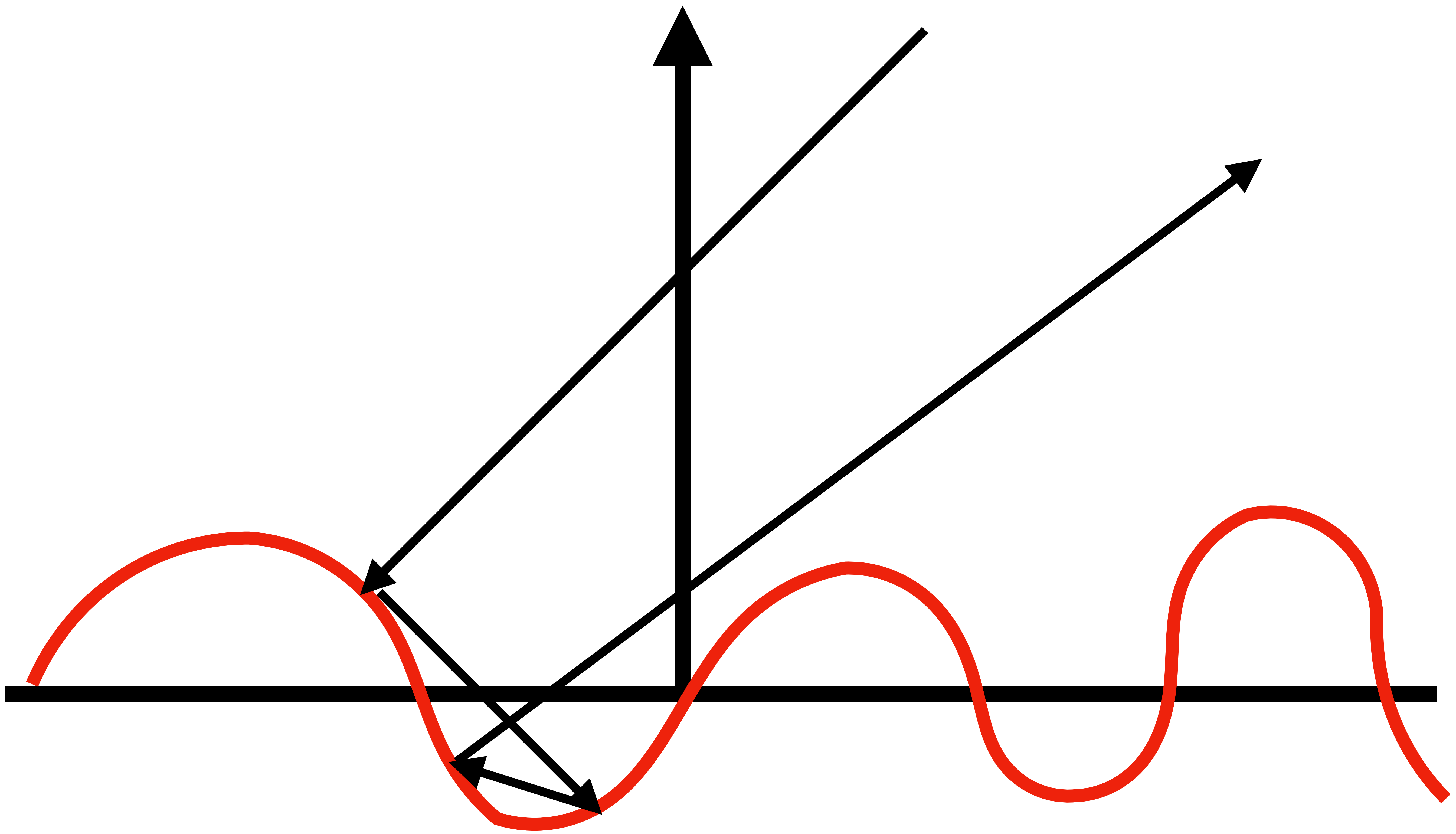
Multiple scattering

- ignoring multiple bounces lead to energy loss, esp. at high roughness



images from Heitz et al.
https://eheitzresearch.wordpress.com/240-2/

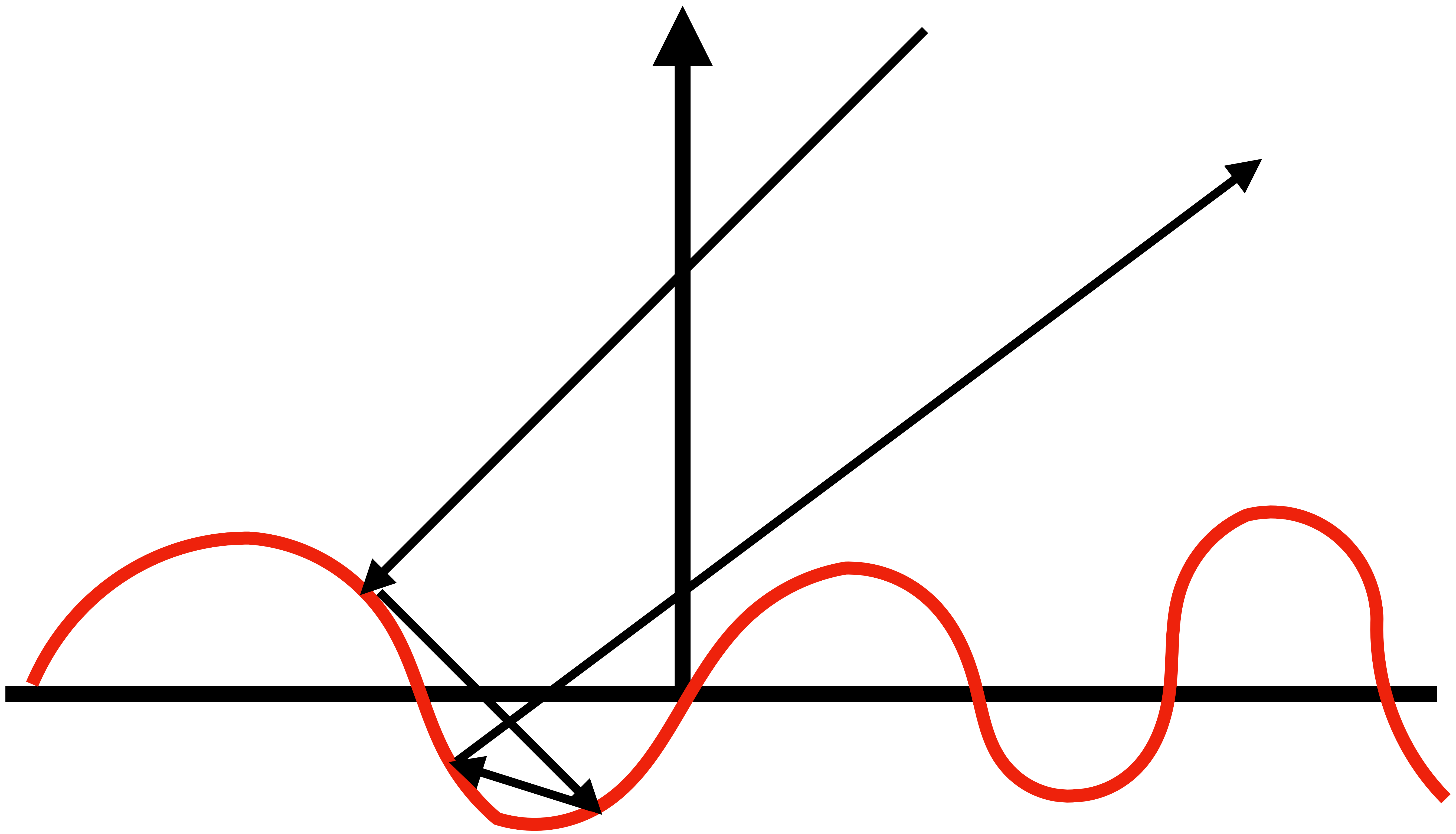
Remedy: simulate multiple bounces inside the microsurfaces



See Heitz et al. "Multiple-Scattering Microfacet BSDFs with the Smith Model"

Remedy: simulate multiple bounces inside the microsurfaces

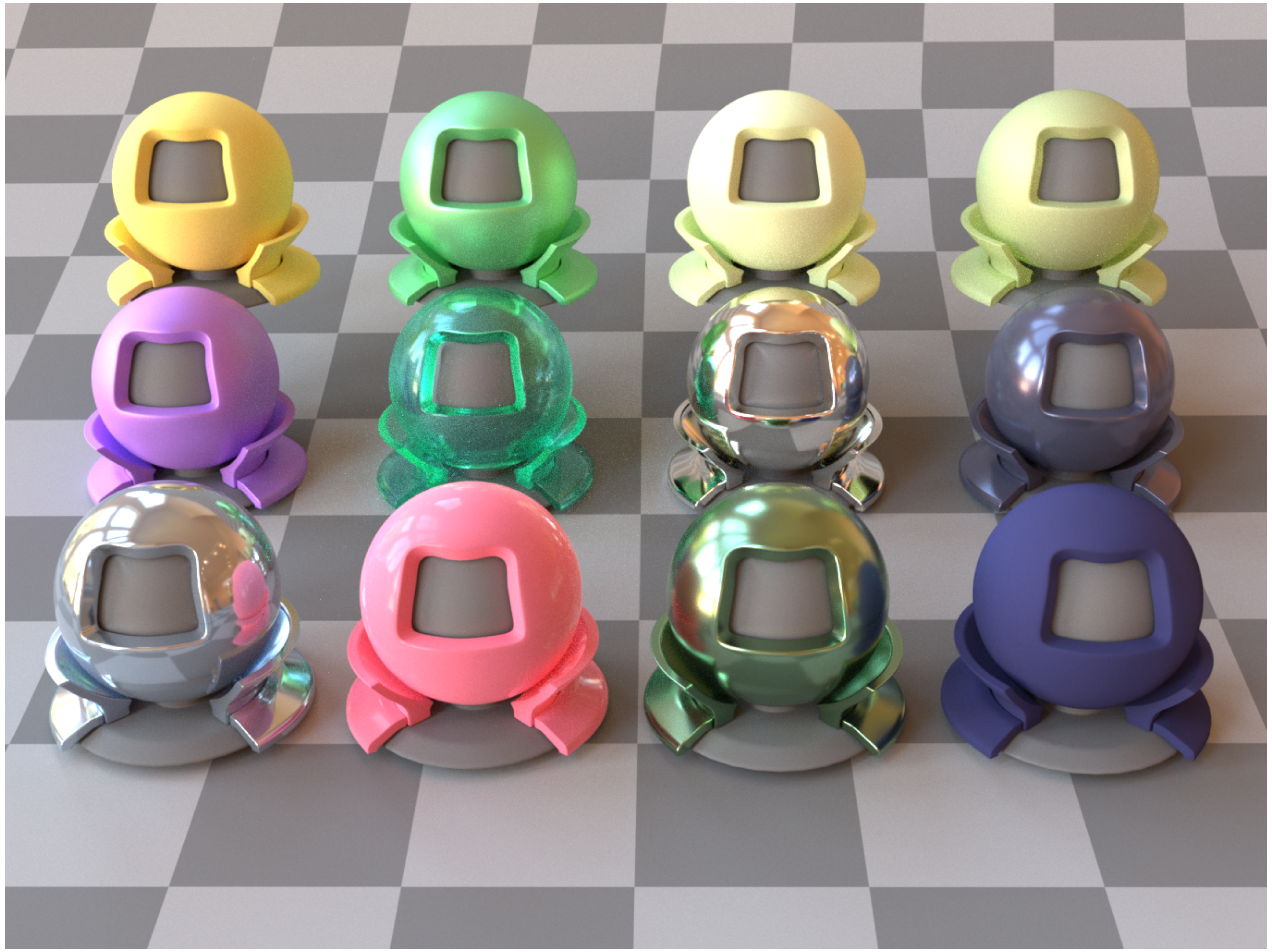
- or not, see “Misunderstanding multiscattering” by Angelo Pesce
  https://c0de517e.blogspot.com/2019/08/misunderstanding-multiscattering.html



See Heitz et al. “Multiple-Scattering Microfacet BSDFs with the Smith Model”

Next time: Uber BSDF

- one BSDF to rule them all?



next Monday is MLK, so see you next Wednesday!