Photon Mapping

UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li
SDS light paths

pinhole camera

mirror (specular)

diffuse

mirror (specular)

point light
Idea 1: allow "near miss"

pinhole camera

mirror (specular)
diffuse

mirror (specular)

point light
Idea 2: share light subpaths among different pixels
Photon mapping

1. trace random light subpaths
Photon mapping

1. trace random light subpaths

2. store **photons** on diffuse surfaces
Photon mapping

1. trace random light subpaths

2. store **photons** on diffuse surfaces
Photon mapping

1. trace random light subpaths

2. store photons on diffuse surfaces

3. trace random camera subpaths
Photon mapping

1. trace random light subpaths
2. store **photons** on diffuse surfaces
3. trace random camera subpaths
4. reconstruct path contribution from photons
Photon mapping

1. trace random light subpaths
2. store photons on diffuse surfaces
3. trace random camera subpaths
4. reconstruct path contribution from photons

Bidirectional Photon Mapping

Jiří Vorba
Supervised by: Jaroslav Křivánek
Charles University, Prague
Photon mapping

1. trace random light subpaths
2. store photons on diffuse surfaces
3. trace random camera subpaths
4. reconstruct path contribution from photons
Math formulation: blurring path contribution

\[
\int_{\text{light paths}} f(\vec{x}) d\vec{x} \rightarrow \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x_2') f(\vec{x}') d\vec{x} dx_2'
\]

\(k\): convolution kernel

e.g. a disk kernel \(\frac{1}{\pi r^2}\)
Math formulation: blurring path contribution

\[ \int_{\text{light paths}} f(\bar{x}) d\bar{x} \rightarrow \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x_2') f(\bar{x}') d\bar{x} dx_2' \]

\( k \): convolution kernel

e.g. a disk kernel \( \frac{1}{\pi r^2} \)
Sidetrack: blurring an integrand does *not* necessarily change its integral!

recall: integration = taking DC in frequency domain

\[ \int f(x) \, dx = \hat{f}(0) \]

blurring = multiply the DCs in frequency domain

\[ \int \int k(x, y) f(x) \, dx \, dy = \hat{f}(0) \hat{k}(0) \]

as long as \( \hat{k}(0) = 1 \), the integral is preserved!
Photon mapping: estimating the blurring integral using camera subpaths & light subpaths

\[ \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x'_2)f(\bar{x}')d\bar{x}dx'_2 \approx \frac{k(x_2, x'_2)f(\bar{x}')}{p(x_0 \rightarrow x_1 \rightarrow x_2)p(x_4 \rightarrow x_3 \rightarrow x'_2)} \]
Photon mapping: estimating the blurring integral using camera subpaths & light subpaths

\[ \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x'_2)f(\vec{x}') \, d\vec{x} \, dx'_2 \approx \frac{k(x_2, x'_2)f(\vec{x}')}{p(x_0 \rightarrow x_1 \rightarrow x_2)p(x_4 \rightarrow x_3 \rightarrow x'_2)} \]

crucial detail: which BSDF/geometry term should we use?
- \( G(x_1 \rightarrow x_2)\rho(x_1 \rightarrow x_2 \rightarrow x_3)G(x_2 \rightarrow x_3) \)
- \( G(x_1 \rightarrow x'_2)\rho(x_1 \rightarrow x'_2 \rightarrow x_3)G(x'_2 \rightarrow x_3) \)
- \( G(x_1 \rightarrow x_2)\rho(x_1 \rightarrow x'_2 \rightarrow x_3)G(x'_2 \rightarrow x_3) \)
- \( G(x_1 \rightarrow x_2)\rho(x_1 \rightarrow x_2 \rightarrow x_3)G(x'_2 \rightarrow x_3) \)
Photon mapping: estimating the blurring integral using camera subpaths & light subpaths

\[ \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x'_2) f(\bar{x}') d\bar{x} dx'_2 \approx \frac{k(x_2, x'_2) f(\bar{x}')}{p(x_0 \to x_1 \to x_2) p(x_4 \to x_3 \to x'_2)} \]

crucial detail:
which BSDF/geometry term should we use?

- \(G(x_1 \to x_2) \rho(x_1 \to x_2 \to x_3) G(x_2 \to x_3)\)
- \(G(x_1 \to x'_2) \rho(x_1 \to x'_2 \to x_3) G(x'_2 \to x_3)\)
- \(G(x_1 \to x_2) \rho(x_1 \to x'_2 \to x_3) G(x'_2 \to x_3)\)
- \(G(x_1 \to x_2) \rho(x_1 \to x_2 \to x_3) G(x'_2 \to x_3)\)

for now, let's use \(G(x_1 \to x_2) \rho(x_1 \to x_2 \to x_3) G(x'_2 \to x_3)\)
Density estimation interpretation of photon mapping

- reconstructing radiance at position $x$ using randomly sampled photons at position $x_i$

$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} k(x_i, x) \gamma_i$$

important:
$N = \text{all photons, not just photon nearby to } x!$
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

normalize kernel s.t. \( x' - x \)
is constrained to a unit circle

\[ \frac{1}{r^2} \int k(t)dt = 1 \quad \int tk(t)dt = 0 \]
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

\[ \text{bias} = E \left[ \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \right] - L(x, \omega) \]

normalize kernel s.t. \( x' - x \)

is constrained to a unit circle

\[ \frac{1}{r^2} \int k(t) dt = 1 \quad \int tk(t) dt = 0 \]
Bias-variance analysis of photon mapping

\[ \text{bias} = E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega) \]

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

normalize kernel s.t. \( x' - x \)
is constrained to a unit circle

\[ \frac{1}{r^2} \int k(t) dt = 1 \quad \int tk(t) dt = 0 \]
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

bias = \[ E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega) \]

\[ \frac{1}{r^2} \int k(t) dt = 1 \quad \int tk(t) dt = 0 \]

normalize kernel s.t. \( x' - x \) is constrained to a unit circle
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

\[ bias = E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega) \]

\[ E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] = \frac{1}{r^2} \int k(t)p(x + rt)dt \]

normalize kernel s.t. \( x' - x \)

is constrained to a unit circle

\[ \frac{1}{r^2} \int k(t)dt = 1 \]
\[ \int tk(t)dt = 0 \]
Bias-variance analysis of photon mapping

$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i$$

bias = $$E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega)$$

$$E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] = \frac{1}{r^2} \int k(t)p(x + rt)dt \quad t = \frac{X - x}{r}$$

normalize kernel s.t. $$x' - x$$ is constrained to a unit circle

$$\frac{1}{r^2} \int k(t)dt = 1 \quad \int tk(t)dt = 0$$

$$p(x + rt) \approx p(x) + rt \nabla p(x) + r^2 t^T H_p(x)t$$
Bias-variance analysis of photon mapping

\[
L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i
\]

\[
bias = E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega)
\]

\[
E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] = \frac{1}{r^2} \int k(t)p(x + rt)dt \quad \quad t = \frac{X - x}{r}
\]

\[
p(x + rt) \approx p(x) + rt \nabla p(x) + r^2 t^T H_p(x) t
\]

\[
\frac{1}{r^2} \int k(t)dt = 1 \quad \int tk(t)dt = 0
\]

normalize kernel s.t. \( x' - x \)
is constrained to a unit circle

\[
\int k(t)p(x + rt)dt \approx p(x) + r^2 \cdot \int t^T H_p(x) t dt
\]
Bias-variance analysis of photon mapping

$$L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i$$

$$E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] \approx p(x) + r^2 \cdot \int t^T H_p(x) t \, dt$$

$$\text{bias} = E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega)$$

normalize kernel s.t. $x' - x$ is constrained to a unit circle

$$\frac{1}{r^2} \int k(t) \, dt = 1 \quad \int tk(t) \, dt = 0$$
Bias-variance analysis of photon mapping

\[
L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i
\]

\[
\text{bias} = E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] E[\gamma] - L(x, \omega)
\]

**normalize kernel s.t.** \(x' - x\)

**is constrained to a unit circle**

\[
E \left[ \frac{1}{r^2} k \left( \frac{X - x}{r} \right) \right] \approx p(x) + r^2 \cdot \int t^T H_p(x) t dt
\]

\[
L(x, \omega) = E[\gamma] p(x)
\]

\[
\frac{1}{r^2} \int k(t) dt = 1 \quad \int t k(t) dt = 0
\]
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

\[ \text{bias} \approx r^2 E[\gamma] \int t^T H_p(x)tdt \]
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

bias \approx r^2 E[\gamma] \int t^T H_p(x) t \, dt

variance \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 \, dt
Bias-variance analysis of photon mapping

\[ L(x, \omega) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r^2} k \left( \frac{x_i - x}{r} \right) \gamma_i \]

\[ \text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t dt \]

\[ \text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 dt \]

Observation:
- variance reduces with \( N \), bias does not
- bias reduces with \( r \), but variance increases with \( r \)
Bias variance trade-off in photon mapping

- high bias, low variance
- low bias, higher variance
Epanechnikov kernel minimizes the variance

\[
k(t) = \begin{cases} 
\frac{3}{4\sqrt{5}} \left( 1 - \frac{1}{5} t^2 \right) & -\sqrt{5} \leq t \leq \sqrt{5} \\
0 & \text{otherwise}
\end{cases}
\]

\[
\text{variance} \approx \left( \text{Var}[\gamma] + E[\gamma]^2 \right) \frac{p(x)}{Nr^2} \int k(t)^2 dt
\]

\[
\frac{1}{r^2} \int k(t) dt = 1 \quad \int tk(t) dt = 0
\]

Silverman 1986
Progressive photon mapping: can we eliminate bias in the limit?

- key idea: slowly reduce $r$ during rendering to remove bias
- can’t reduce too fast, can’t reduce too slow

$$\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t \, dt$$

$$\text{variance} \approx \left( \text{Var}[\gamma] + E[\gamma]^2 \right) \frac{p(x)}{Nr^2} \int k(t)^2 \, dt$$
Progressive photon mapping: can we eliminate bias in the limit?

- allow variance to increase a little bit at each iteration

\[
\frac{\text{Var}_{i+1}}{\text{Var}_i} = \frac{i + 1}{i + \alpha} \quad (\alpha \in (0,1))
\]

\[
\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t dt
\]

\[
\text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 dt
\]
Progressive photon mapping: can we eliminate bias in the limit?

- allow variance to increase a little bit at each iteration

\[
\frac{\text{Var}_{i+1}}{\text{Var}_i} = \frac{i + 1}{i + \alpha} \\
\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}
\]

\[
\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t dt
\]

\[
\text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 dt
\]
Progressive photon mapping: can we eliminate bias in the limit?

- allow variance to increase a little bit at each iteration
  
  \[
  \frac{\text{Var}_{i+1}}{\text{Var}_i} = \frac{i + 1}{i + \alpha} \quad (\alpha \in (0,1))
  \]

- \[
  \frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}
  \]

- variance = \( O \left( \frac{1}{Na} \right) \), bias = \( O \left( \frac{1}{N(1-\alpha)} \right) \) (\( \alpha = \frac{2}{3} \) gives optimal mean square error = bias^2 + variance)

\[
\text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t dt
\]

\[
\text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 dt
\]
Photon mapping is good at SDS light paths
Alternative: directly set $r$ to minimize mean square error

\[ \text{bias} \approx r^2 E[\gamma] \int t^T H_p(x) t \, dt \]

\[ \text{variance} \approx (\text{Var}[\gamma] + E[\gamma]^2) \frac{p(x)}{Nr^2} \int k(t)^2 \, dt \]

mean square error = bias^2 + variance

Adaptive Progressive Photon Mapping

ANTON S. KAPLAN and CARSTEN DACHSBACHER
Karlsruhe Institute of Technology

Anton’s method

(a) progressive photon mapping  (b) adaptive PPM (our method)  (c) PPM  (d) our method
Combining with bidirectional path tracing (VCM/UPS)

- apply multiple importance sampling, but need to match the number of vertices

- idea: perturb the bidirectional path tracing vertex to match, approximate perturbation probability as $\frac{1}{\pi r^2}$
BPT is better at non SDS paths
Can we make photon mapping unbiased?

- surprisingly — yes!

- recall: blurring the integrand doesn’t change the integral if the kernel is properly normalized

- why is photon mapping biased?
  - it usually uses fake BSDF & visibility
  - kernel is not normalized w.r.t. visibility
Unbiased photon mapping:
trace rays to the photon to debias

\[ \int_{\text{surface}} \int_{\text{light paths}} k(x_2, x'_2)f(\bar{x}')d\bar{x}dx'_2 \approx \frac{k(x_2, x'_2)f(\bar{x}')}{p(x_0 \rightarrow x_1 \rightarrow x_2)p(x_4 \rightarrow x_3 \rightarrow x'_2)\int k(x_2, x'_2)d\bar{x}'_2} \]

crucial detail:
which BSDF/geometry term should we use?
- \( G(x_1 \rightarrow x_2)\rho(x_1 \rightarrow x_2 \rightarrow x_3)G(x_2 \rightarrow x_3) \)
- \( G(x_1 \rightarrow x'_2)\rho(x_1 \rightarrow x'_2 \rightarrow x_3)G(x'_2 \rightarrow x_3) \)
- \( G(x_1 \rightarrow x_2)\rho(x_1 \rightarrow x'_2 \rightarrow x_3)G(x'_2 \rightarrow x_3) \)
- \( G(x_1 \rightarrow x_2)\rho(x_1 \rightarrow x_2 \rightarrow x_3)G(x'_2 \rightarrow x_3) \)
Unbiased estimation of a reciprocal integral

\[ \frac{1}{\int f(x)dx} = \frac{1}{1 - F} = 1 + F + F^2 + \cdots \]

can estimate using Russian roulette
Unbiased photon mapping converges faster, but can’t do pure specular paths.
Photon beams for volumetric rendering

• treat a light subpath as infinitely many photons

• treat a camera subpath as infinitely many query points
Combining photon beams, points, and bidirectional path tracing

Unifying Points, Beams, and Paths in Volumetric Light Transport Simulation

Jaroslav Křivánek¹  Iliyan Georgiev²  Toshiya Hachisuka³  Petr Vévoda¹  
Martin Šik¹  Derek Nowrouzezahrai⁴  Wojciech Jarosz⁵  

¹Charles University in Prague  ²Light Transportation Ltd.  ³Aarhus University  ⁴Université de Montréal  ⁵Disney Research Zürich
Photon planes and photon volumes

• infinitely many photons in planes & volumes
Photon cones/cylinders/spheres and photon bunnies

Photon surfaces for robust, unbiased volumetric density estimation

Xi Deng, Shaojie Jiao, Benedikt Bitterli, Wojciech Jarosz

1Dartmouth College

In ACM Transactions on Graphics (Proceedings of SIGGRAPH), 2019
History/bibliography
History / bibliography

Into the Blue: Better Caustics through Photon Relaxation

B. Spencer and M. W. Jones

Progressive Photon Relaxation

2009

Progressive Expectation–Maximization for hierarchical volumetric photon mapping

Wenzel Jakob 1,2,3, Christian Regg 1,2,3, Wojciech Jarosz 1

1Disney Research Zürich 2Cornell University 3ETH Zürich

In Computer Graphics Forum (Proceedings of EGSR), 2011

BEN SPENCER and MARK W. JONES

Visual and Interactive Computing Group, Swansea University

A comprehensive theory of volumetric radiance estimation using photon points and beams

Wojciech Jarosz 1,2,3, Derek Nowrouzezahrai 1,2,3, Iman Sadeghi 1,2,3, Henrik Wann Jensen 1,2,3

1Disney Research Zürich 2UC San Diego 3University of Toronto

In ACM Transactions on Graphics (Presented at SIGGRAPH), 2011

Progressive Photon Mapping: A Probabilistic Approach

Claude Kraus and Matthias Zwicker

University of Bern, Switzerland

2011

Adaptive Progressive Photon Mapping

ANTON S. KAPLANIAN and CARSTEN DACHSBACHER

Karlsruhe Institute of Technology

2012

Improved Stochastic Progressive Photon Mapping with Metropolis Sampling

Jiaying Chen 1,2,3, Bin Xiao 1,2,4, and Jun-Hai Yong 1,2,4

2011

Line Space Gathering for Single Scattering in Large Scenes

Xin Sun *, Kun Zhou 1, Stephen Lin 1, Baining Guo 1

1Microsoft Research Asia 2State Key Lab of CAD&CG, Zhejiang University

2010

Light Transport Simulation with Vertex Connection and Merging

Ilyan Georgiev 1,2,3, Jaroslav Křivánek 2,3, Janos Davidovits 2,3

1Intel VCL, Saarbrücken 2Saitland University 3Saarland University

2012

A Path Space Extension for Robust Light Transport Simulation

Toshiya Hachisuka 1,2,3, Jacopo Pantaleoni 2,4, Henrik Wann Jensen 3,5

1Aarhus University 2NVIDIA Research 3UC San Diego

2012

Path Space Regularization for Holistic and Robust Light Transport

Anton S. Kaplanian and Carsten Dachsbacher

Karlsruhe Institute of Technology, Germany

2013

Progressive photon beams

Wojciech Jarosz 1,2,3, Derek Nowrouzezahrai 1,2,3, Robert Thomas 1,2,3, Peter-Pike Sloan 1,2,3, Matthias Zwicker 1,2,3

1Disney Research Zürich 2Disney Interactive Studios 3University of Bern

In ACM Transactions on Graphics (Proceedings of SIGGRAPH Asia), 2013

Featured in the proceedings inside cover and the papers fast forward videos!
History/biblography

Unbiased Photon Gathering for Light Transport Simulation

Hao Qin*, Xin Sun†, Qiming Hou‡, Baining Guo§, Kun Zhou*

*State Key Lab of CAD&CG, Zhejiang University †Microsoft Research Asia

A Spatial Target Function For Metropolis Photon Tracing

Adrien Gruson, IRISA, University of Rennes 1, France
Michaël Ribardière, XLIM-SIC, University of Poitiers, France
Martin Bix, Charles University, Czech Republic
Jit Vroba, Charles University, Czech Republic
Remi Cozzi, IRISA, University of Rennes 1, France
Kadi Bourouch, IRISA, University of Rennes 1, France
Jaroslav Krivanc, Charles University, Czech Republic
In ACM Trans. Graph, 2016 (Presented at Siggraph 2017).

Gradient-Domain Photon Density Estimation

Binh-Son Hua1, Adrien Gruson2, Derek Nowrouzezahrai3, Toshiya Hachisuka2

1Singapore University of Technology and Design 2The University of Tokyo 3McGill University

Photon surfaces for robust, unbiased volumetric density estimation

Xi Deng3, Shaojie Jiao1, Benedikt Bitterli3, Wojciech Jarosz1

Dartmouth College

In ACM Transactions on Graphics (Proceedings of SIGGRAPH), 2019

Hierarchical Neural Reconstruction for Path Guiding Using Hybrid Path and Photon Samples

ZEHUI LIN, SHENG LI*, XINLU ZENG, and CONGYI ZHANG, Dept. of Computer Science and Technology, Peking University

JINZHU JIA, Dept. of Biostatistics and Center for Statistical Science, Peking University

GUOPING WANG, Dept. of Computer Science and Technology, Peking University

DINESH MANOCHA, University of Maryland at College Park

CPPM: Chi-squared Progressive Photon Mapping

2020
Next time: Metropolis light transport