Single scattering

UCSD CSE 272
Advanced Image Synthesis

Tzu-Mao Li

with slides from Jiawen (Kevin) Chen
Today: single scattering
Today: single scattering

- Fast visibility evaluation
- Importance sampling
- Analytical solutions
Today: single scattering

slides mostly borrowed from Jiawen (Kevin) Chen!

- **A Hierarchical Volumetric Shadow Algorithm for Single Scattering**
  - Ilya Baran
  - Jiawen Chen
  - Jonathan Ragan-Kelley
  - Felipe Durand
  - Jakob Lethen
  - Computer Science and Artificial Intelligence Laboratory
  - Massachusetts Institute of Technology

- **Importance Sampling Techniques for Path Tracing in Participating Media**
  - Christopher Kulla
  - Marcos Fujardo
  - Sony Pictures Imageworks, Culver City, USA
  - Solid Angle, Madrid, Spain

- **Real-Time Volumetric Shadows using 1D Min-Max Mipmaps**
  - Jiawen Chen
  - Ilya Baran
  - Felipe Durand
  - Wojciech Jarosz
  - MIT CSAIL
  - Disney Research Zürich

- **A Practical Analytic Single Scattering Model for Real Time Rendering**
  - Bo Sun
  - Columbia University
  - Ravi Ramamoorthi
  - Columbia University
  - Srinivasa G. Narasimhan
  - Carnegie Mellon University
  - Shree K. Nayar
  - Columbia University

- **Practical product sampling for single scattering in media**

- **An Analytical Solution to Single Scattering in Homogeneous Participating Media**
  - Vincent Pogorelov
  - Steven G. Parker
  - University of Utah
  - NVIDIA Corporation

- **fast visibility evaluation**

- **importance sampling**

- **analytical solutions**
Single scattering equation (airlight integral)

\[ L = \int_{0}^{t} T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]
Single scattering equation
(airlight integral)

\[ L = \int_0^t T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]

- Camera transmittance
- Phase function
- Geometry term
- Light transmittance
- Visibility
Shadow mapping

depth map rendered from the light
Simplified scenario

- orthographic camera
- light direction perpendicular to view direction
- visibility only
Brute force complexity: $O(rd)$

https://groups.csail.mit.edu/graphics/mmvs
Visibility = 1D heightfield intersection

depth from shadow map

https://groups.csail.mit.edu/graphics/mmvs
1D Min-Max Mipmap

- binary tree of shadow map depths
- each node stores \textbf{min} and \textbf{max} of children

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1D ray tracing in a mipmap

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$y = 5$

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1D ray tracing in a mipmap

\[ y = 5 \]
\[ 1 \leq 5 < 9 \]

Sum = 0

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1D ray tracing in a mipmap

\[ y = 5 \]

\[ 1 \leq 5 < 9 \]

\[ \text{Sum} = 0 \]

https://groups.csail.mit.edu/graphics/mmvs
1D ray tracing in a mipmap

$y = 5$

$2 \leq 5 < 9$

https://groups.csail.mit.edu/graphics/mmvs
1D ray tracing in a mipmap

$y = 5$

$y: 5 > 2$, shadowed

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1D ray tracing in a mipmap

$y = 5$

https://groups.csail.mit.edu/graphics/mmvs
1D ray tracing in a mipmap

\[ y = 5 \]

https://groups.csail.mit.edu/graphics/mmvs
Generalizing from the perpendicular setting

https://groups.csail.mit.edu/graphics/mmvs
Epipolar rectification

Camera depth map

Light depth map

Rectify

Corresponding epipolar slice

https://groups.csail.mit.edu/graphics/mmvs
Handling transmittance & phase function & textured lights

\[ L = \int_{0}^{t} T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]
Handling transmittance & phase function & textured lights

\[ L = \int_{0}^{t} T_{c} \cdot \sigma_{s} \cdot V \cdot \rho \cdot G \cdot L_{e} \cdot T_{e} \mathrm{d}t' \]

\[ L_{\alpha,\beta} \approx \sum_{\gamma=0}^{N} V_{\alpha,\beta,\gamma} \cdot I_{\beta,\gamma} \]

https://groups.csail.mit.edu/graphics/mmvs
Handling transmittance & phase function & textured lights

\[ L = \int_0^t T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]

\[ L_{\alpha,\beta} \approx \sum_{\gamma=0}^{N} V_{\alpha,\beta,\gamma} \cdot I_{\beta,\gamma} \]

want to compute prefix sum of \( I_{\beta,\gamma} \)

https://groups.csail.mit.edu/graphics/mmvvs
Trick: SVD approximation

\[ I_{\beta,\gamma} = \sum_{i=0}^{M} J_i(\beta)K_i(\gamma) \]

store the prefix sum of \( K_i(\gamma) \) in the tree

https://groups.csail.mit.edu/graphics/mmvs
Runs in real-time!!

Real-Time Volumetric Shadows using 1D Min-Max Mipmaps

Jiwen Chen, Ilya Baran, Frédo Durand, Wojciech Jarosz

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  - Christopher Kulla\textsuperscript{1} and Marcos Fujardo\textsuperscript{2}
  \textsuperscript{1}Sony Pictures Imageworks, Culver City, USA
  \textsuperscript{2}Solid Angle, Madrid, Spain

- A Hierarchical Volumetric Shadow Algorithm for Single Scattering
  - Erya Benaz, Jiawen Chen, Jonathan Roger Kelley, Fieko Durand, Jakob Lehtinen
  - Computer Science and Artificial Intelligence Laboratory, Massachusetts Institute of Technology

- Real-Time Volumetric Shadows using 1D Min-Max Mipmaps
  - Jiawen Chen\textsuperscript{1}, Erya Benaz\textsuperscript{2}, Fieko Durand\textsuperscript{3}, Wojciech Jaroci\textsuperscript{2}
  \textsuperscript{1}MIT CSAIL, \textsuperscript{2}Disney Research Zurich, \textsuperscript{3}Disney Research, USA

- A Practical Analytic Single Scattering Model for Real Time Rendering
  - Bo Sun
  - Columbia University
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  - Shree K. Nayar
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- An Analytical Solution to Single Scattering in Homogeneous Participating Media
  - Vincent Pignedo\textsuperscript{1}
  - Steven G. Parker\textsuperscript{1,2}
  \textsuperscript{1}University of Utah, \textsuperscript{2}NVIDIA Corporation

- Fast visibility evaluation
- Importance sampling
- Analytical solutions
Goal: importance sample single scattering

- given a point on the light, want to sample points on the ray

\[ L = \int_{0}^{t} T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]

- visibility
- geometry term
- camera transmittance
- phase function
- light transmittance
Goal: importance sample single scattering

- sampling transmittance fails to consider the geometry term & phase function

\[ L = \int_{0}^{t} [T_{c}] \cdot \sigma_{s} \cdot V \cdot \rho \cdot G \cdot L_{e} \cdot T_{e} \, dt' \]

- visibility
- geometry term
- phase function
- light transmittance

Equi-angular sampling
importance samples \( \frac{1}{r^2} \)

- change of variable: projects the line onto a circle

\[
\begin{align*}
\text{pdf}(t) &= \frac{D}{(\theta_b - \theta_a)(D^2 + t^2)} \\
t(\xi) &= D \tan \left((1 - \xi)\theta_a + \xi\theta_b\right) \\
\theta_x &= \tan^{-1} \frac{x}{D}
\end{align*}
\]

proposed by Kulla & Fajardo in graphics, known as “track-length estimator” in nuclear engineering

Can we importance sample the whole line integral?

• visibility is hard, but what about the rest?

\[ L = \int_{0}^{t} T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]
Idea: build a 1D table by evaluating the integrand at few points

- important: do this in the angular space, ignore visibility

\[ L = \int_{0}^{t} T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e dt' \]

idea used in Novak et al., Szirmay-Kalos et al., and Villeneuve et al.
Alternative idea: apply a Taylor expansion

- important: do this in the angular space, ignore visibility

\[ L = \int_0^t T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]
Equi-angular usually works pretty well

Villeneuve et al.

http://www.iliyan.com/publications/PracticalProductSampling
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The single-scattering integral can have an analytical solution!

- assuming: constant $\sigma_a$ & $\sigma_s$, isotropic phase function & constant visibility

\[ L = \int_0^t T_c \cdot \sigma_s \cdot V \cdot \rho \cdot G \cdot L_e \cdot T_e \, dt' \]

\[ L_m(x_a,x_b,\bar{\omega}) = I \frac{K_2}{h} e^{K_2(x_a-x_b)} \frac{2}{4\pi} I_0(-H,v_a,v_b) \]
\[ = I \frac{K_2}{h} e^{K_2(x_a-x_b)} \frac{2}{4\pi} \]

\[ \left( \sin(-H)Re\left(Ei(-H(v_b + i)) - Ei(-H(v_a + i))\right) \right) \]
\[ - \cos(-H)Im\left(Ei(-H(v_b + i)) - Ei(-H(v_a + i))\right) \]

\[ H = \sigma_t t \]
\[ Ei(x) = \int_{-\infty}^x \frac{e^t}{t} \, dt \]

http://www.sci.utah.edu/~vpegorar/research/2009_EG.pdf
Analytical solutions can handle simple scenes efficiently

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[audio]
Importance sampling multiple scattering
Next: differentiable rendering

Differentiable Monte Carlo Ray Tracing through Edge Sampling

TZU-MAO LI, MIT CSAIL
MIKA AITTALA, MIT CSAIL
FRÉDO DURAND, MIT CSAIL
JAAKKO LEHTINEN, Aalto University & NVIDIA

Unbiased Warped-Area Sampling for Differentiable Rendering

SAI PRAVEEN BANGARU, Massachusetts Institute of Technology
TZU-MAO LI, Massachusetts Institute of Technology
FRÉDO DURAND, Massachusetts Institute of Technology