Limitations on Transformations from Composite-Order to Prime-Order Groups: The Case of Round-Optimal Blind Signatures

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Elliptic curves: what are they and why do we care?

Bilinear groups are cyclic groups G of some finite order that admit a nondegenerate bilinear map $e: G \times G \to G_T$

- Bilinear: $e(x^a,y) = e(x,y)^a = e(x,y^a)$, nondegenerate: $e(x,y) = 1$ for all $y \Leftrightarrow x = 1$
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Historically, we use elliptic curves for two main reasons:

- Functionality: IBE [BF01], functional encryption, etc.
- Efficiency: discrete log problem is harder, can use smaller parameters

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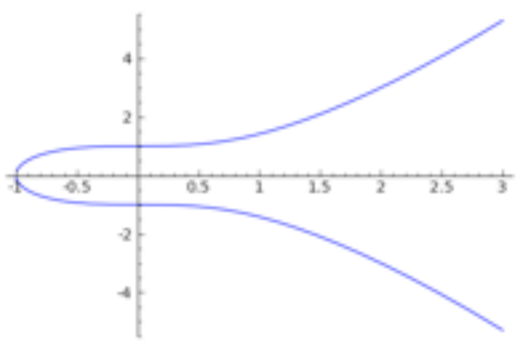

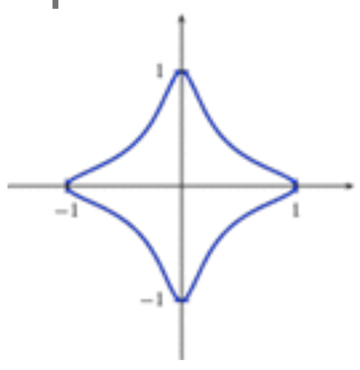

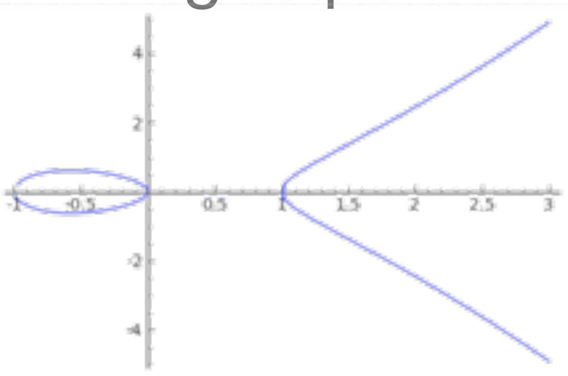

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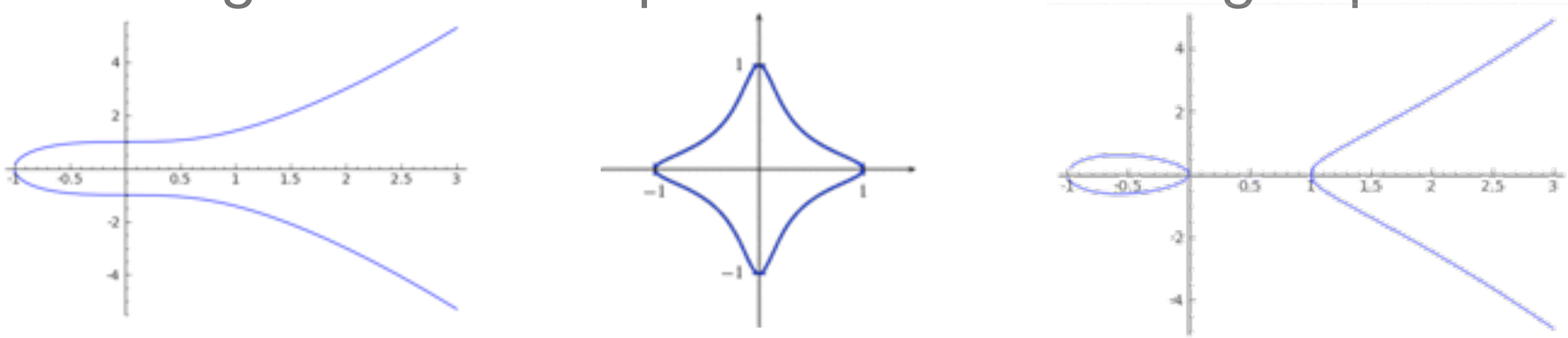

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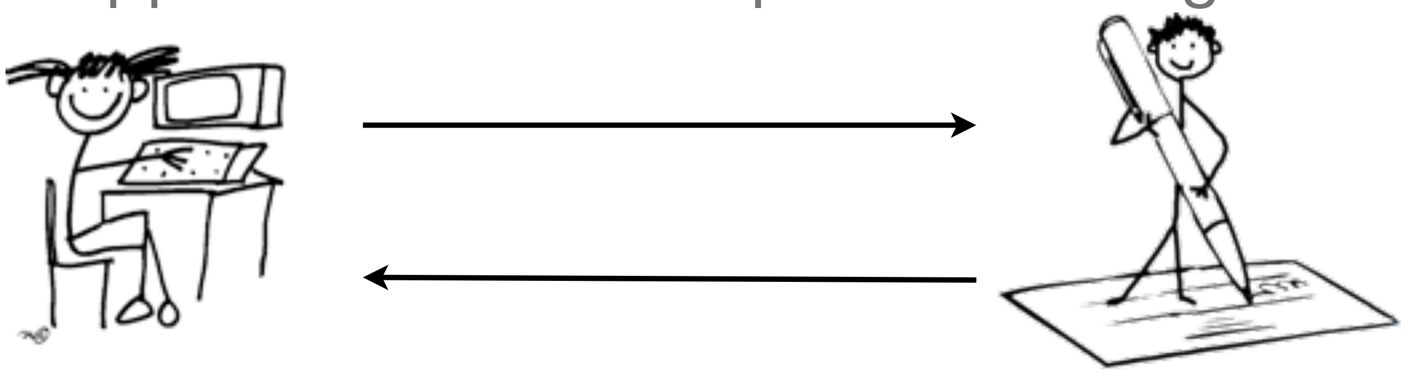

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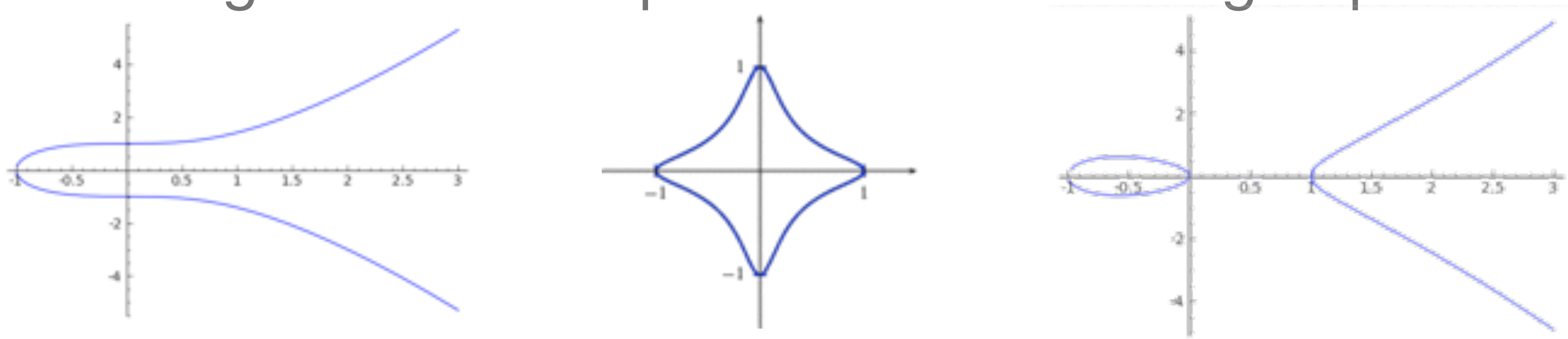

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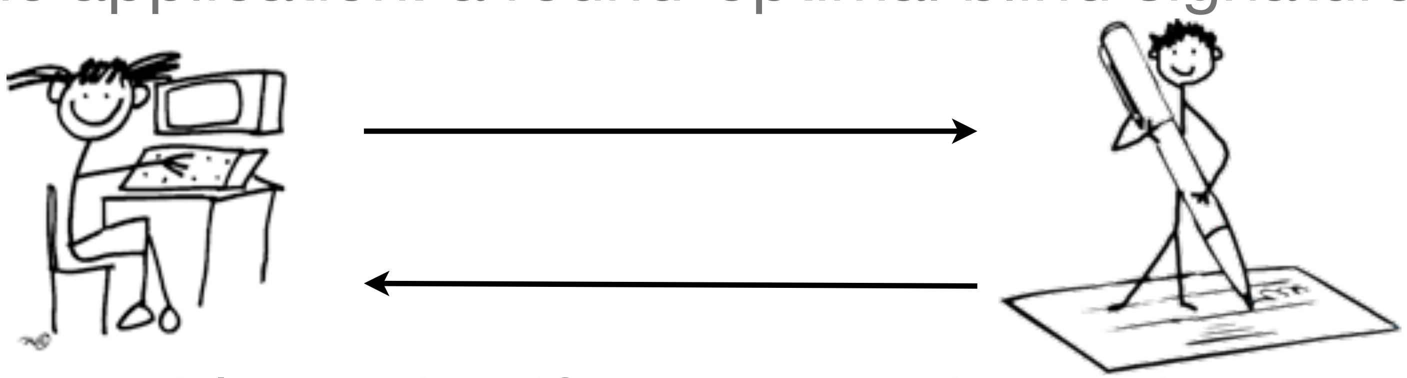

- The problem: what if we want to instantiate our scheme in a prime-order setting instead?

The setting: composite-order groups

- Cyclic groups G and G_T of order $N = pq$, $G = G_p \times G_q$ but p,q are secret
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“somewhat” homomorphic encryption [BGN05]

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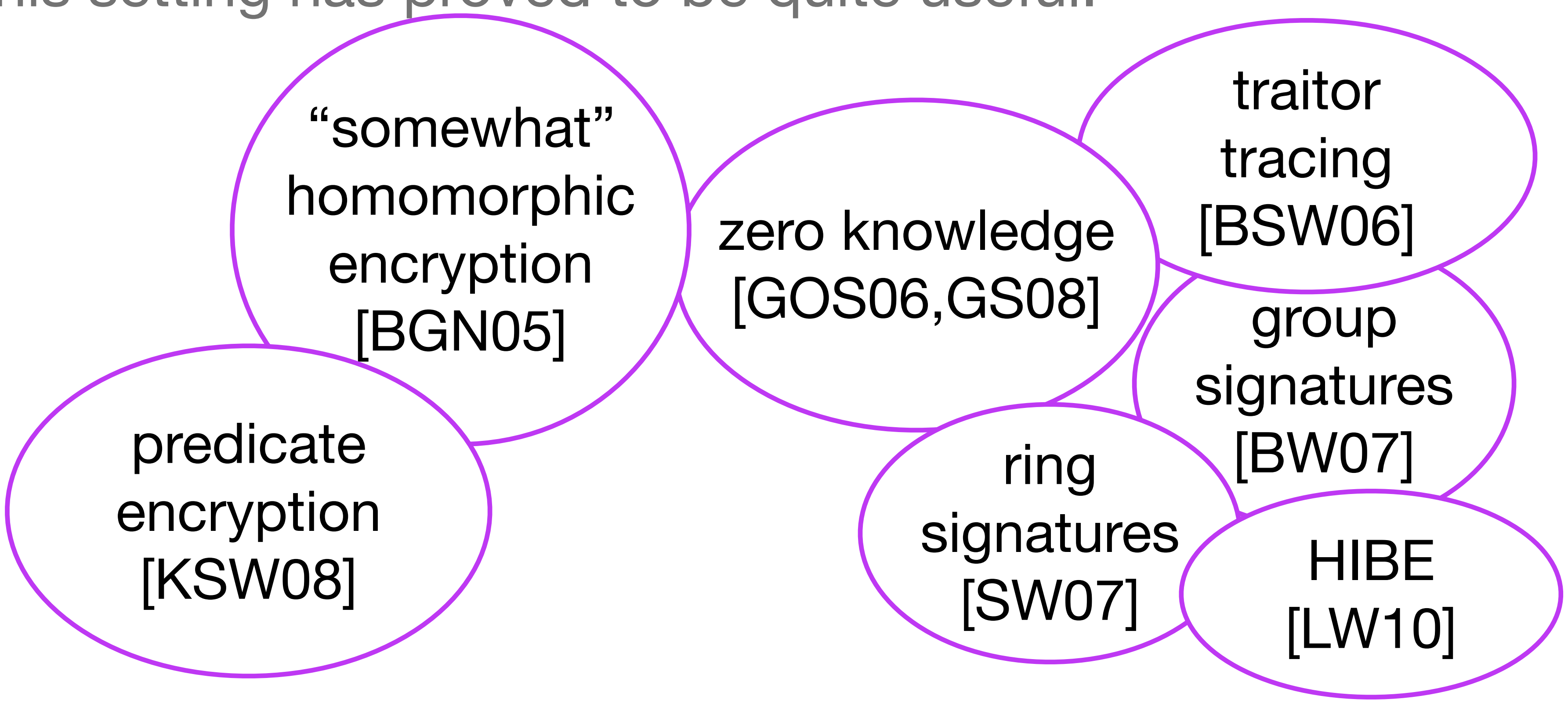

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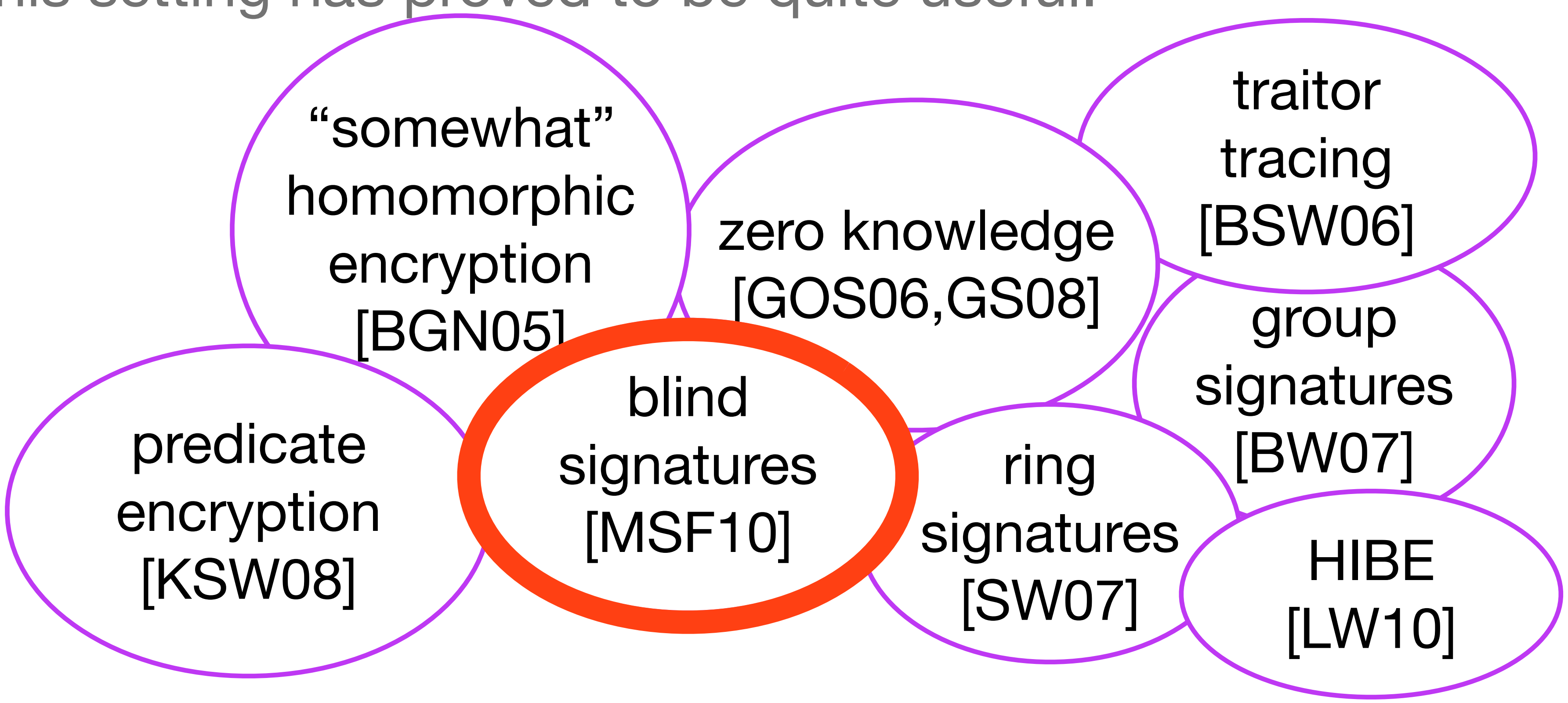

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- Composite-order means bigger: in prime-order groups, can use group of size ~160 bits; in composite-order groups need ~1024 bits (discrete log vs. factoring)

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Previously, people converted schemes in an ad-hoc way [W09,GSW09,LW10]

Freeman [F10] is first to provide a general conversion method

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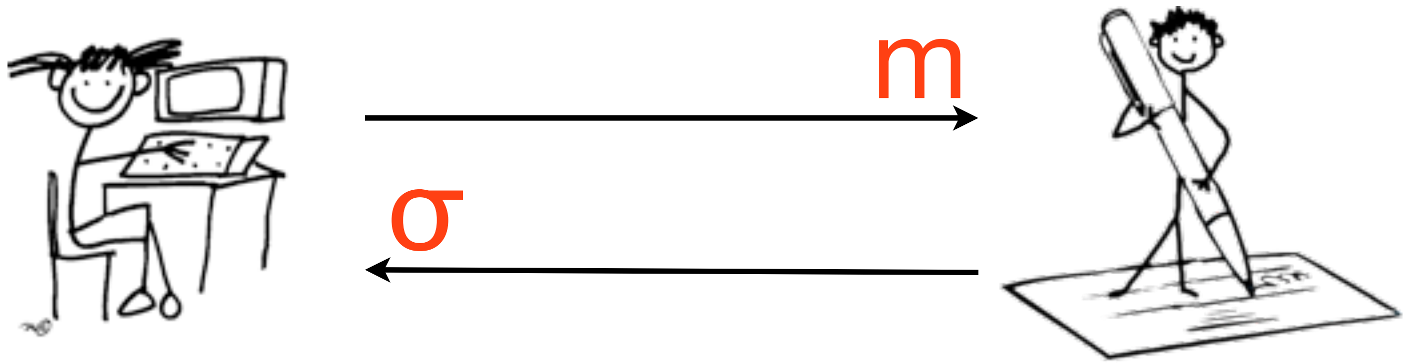

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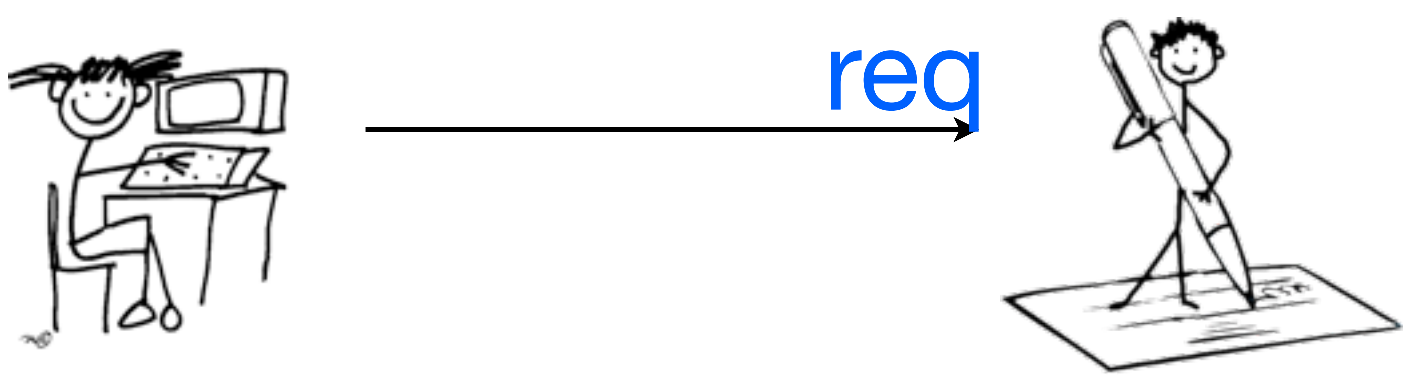

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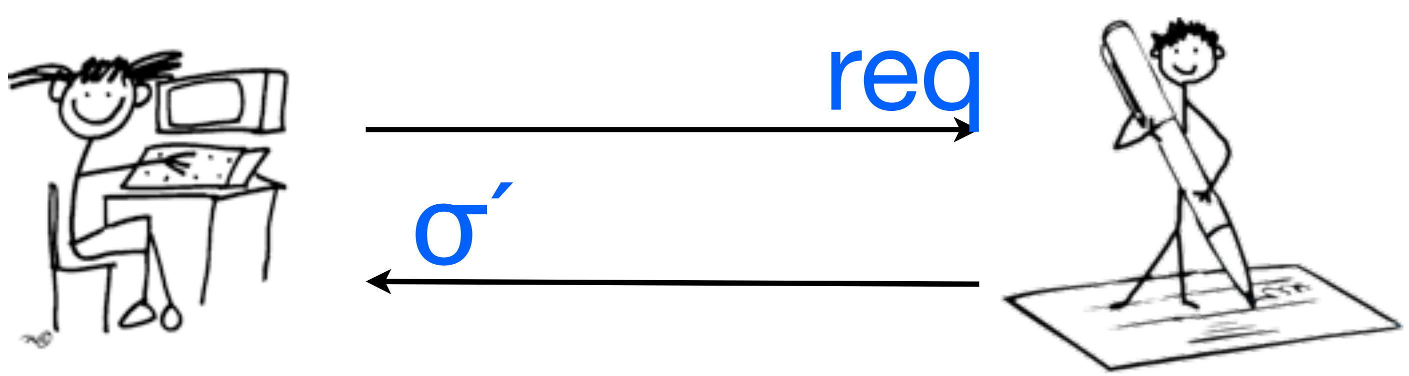

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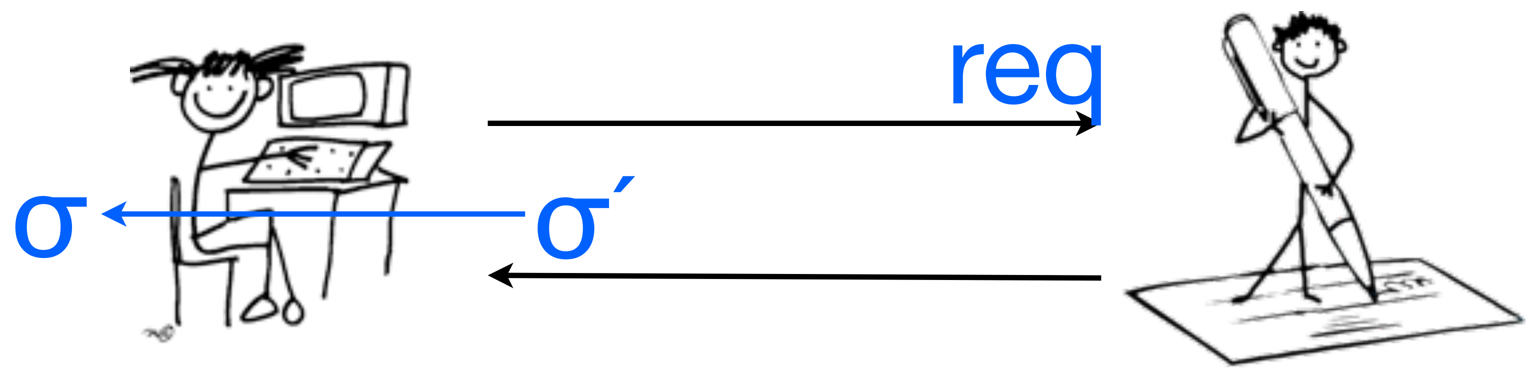

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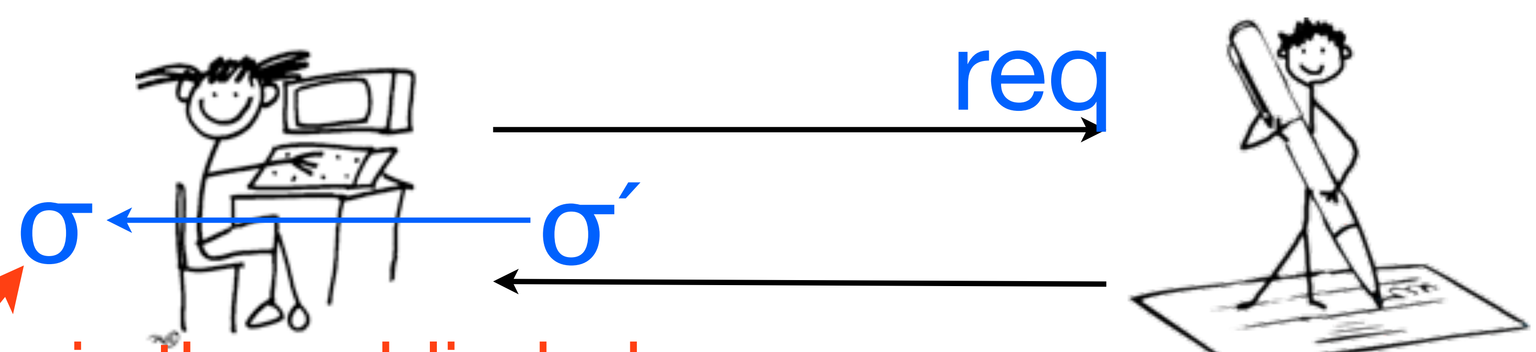

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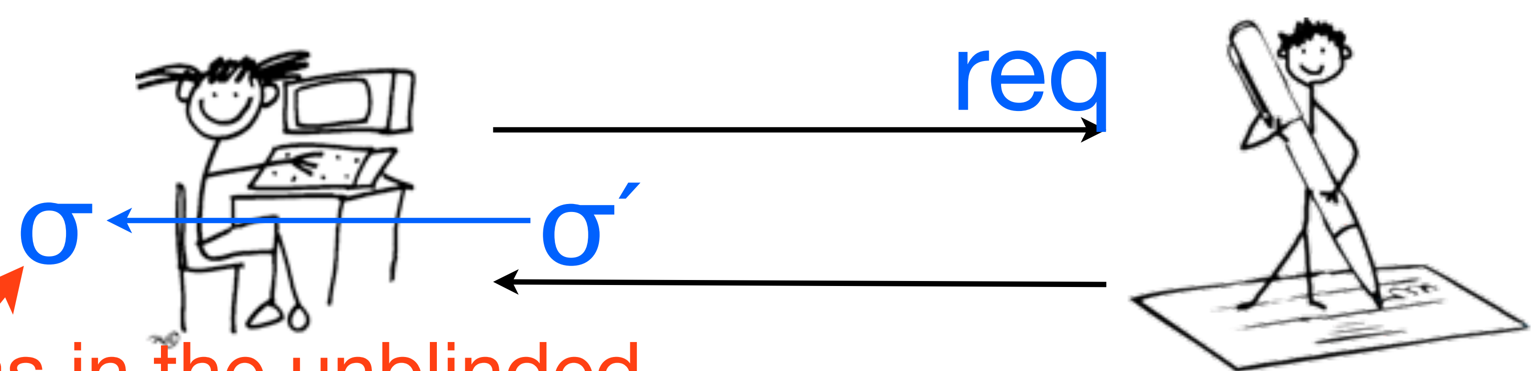

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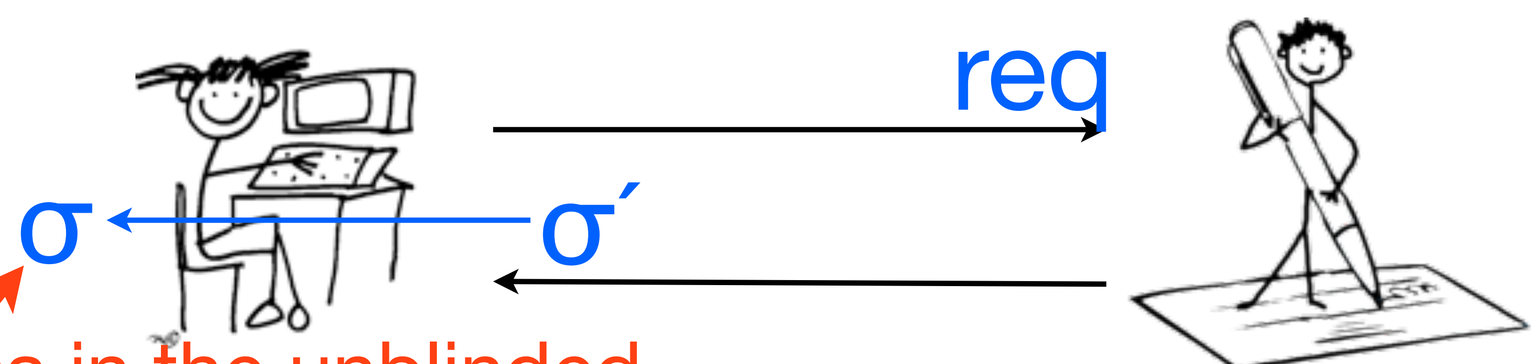

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Still a very active research area [O06,F09,AO10,AHO10,R10,GRSSU11]

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- Benefits: can use composite- and prime-order settings

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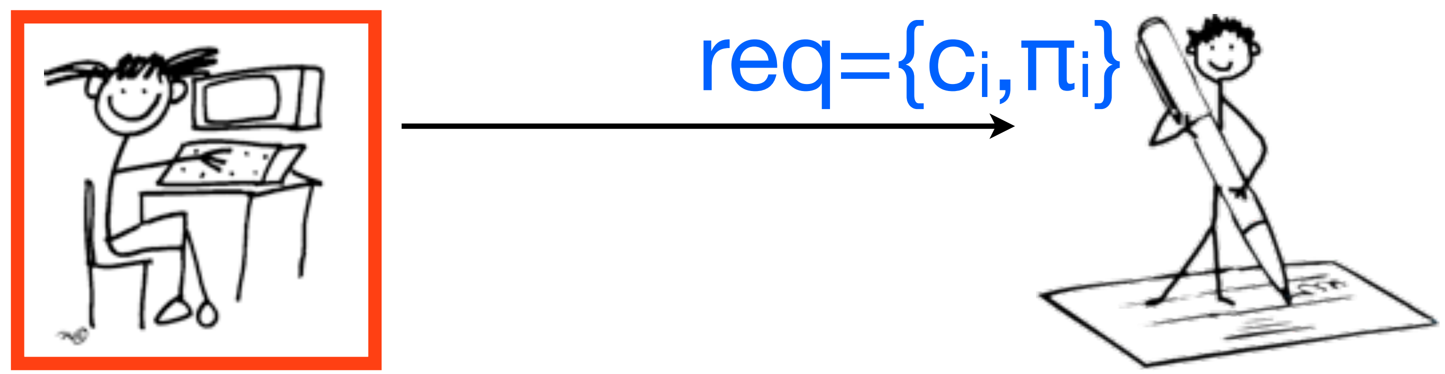

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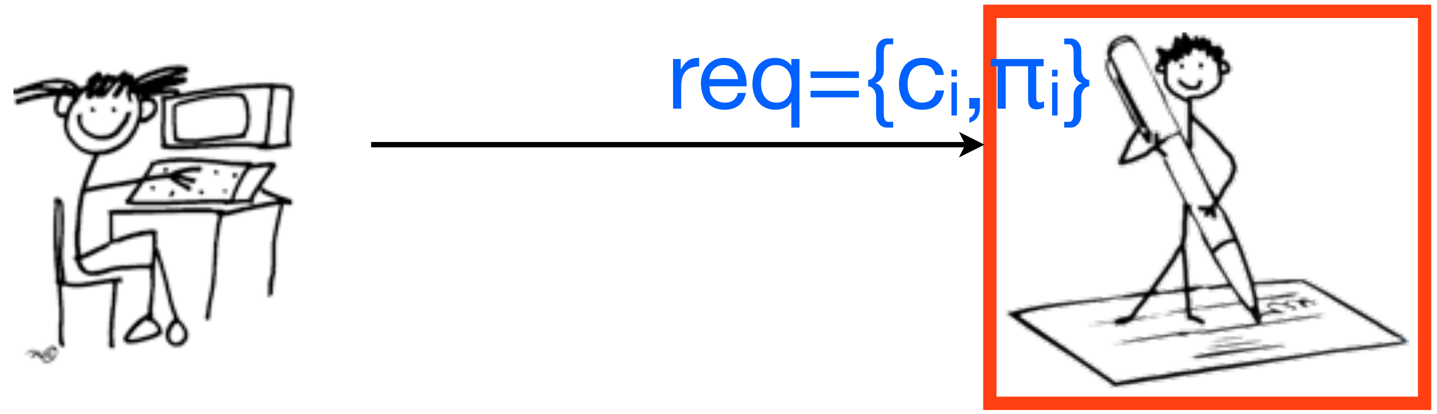

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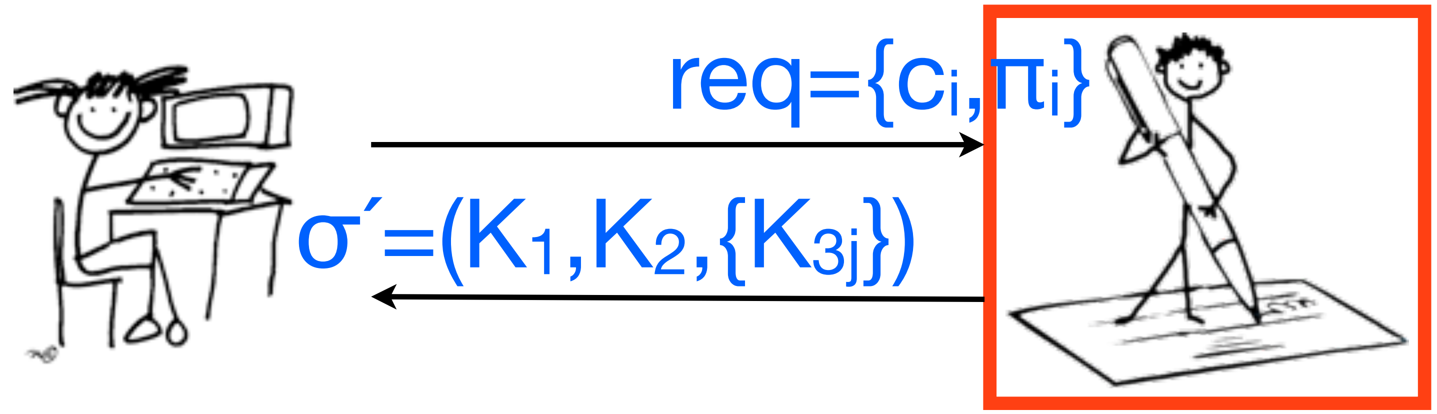

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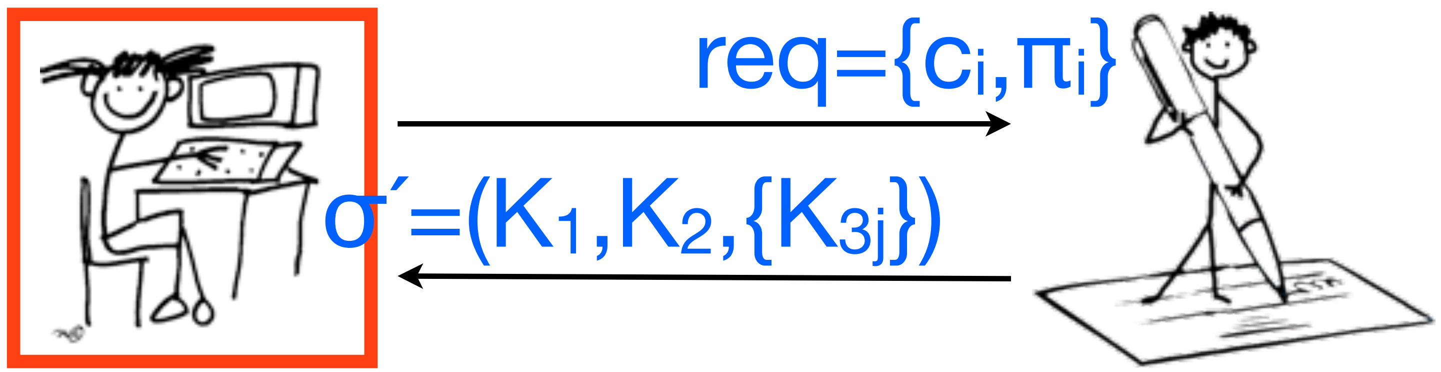

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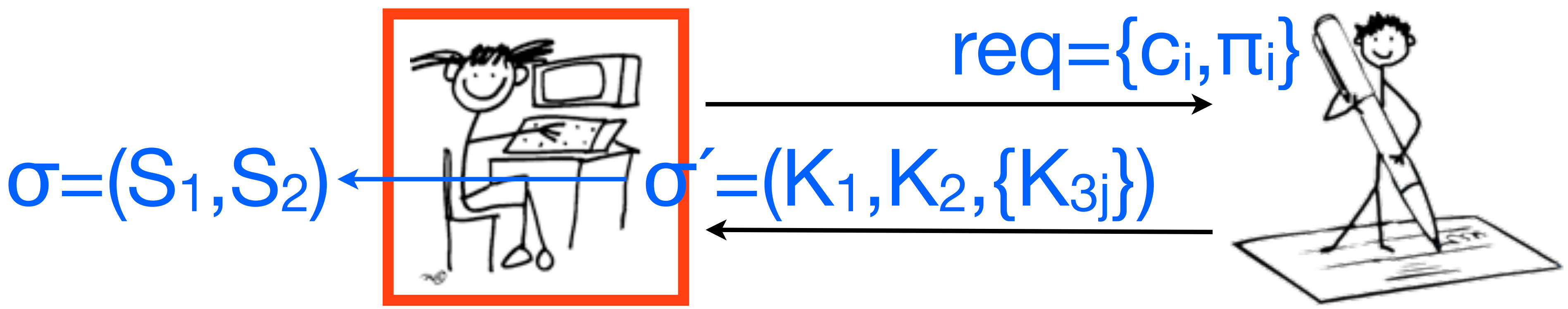

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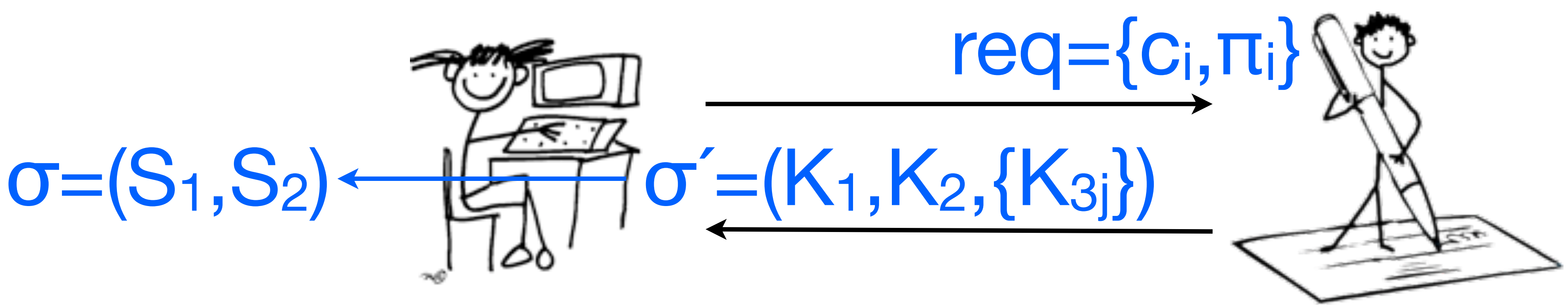

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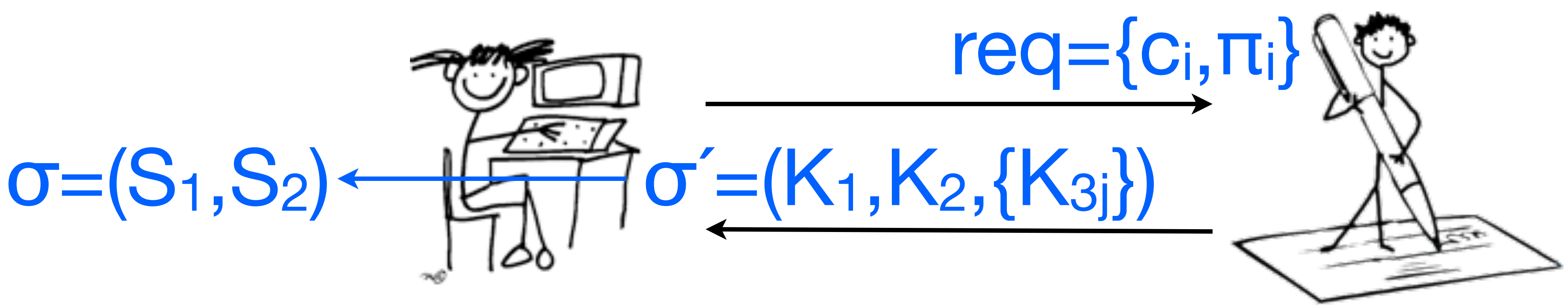

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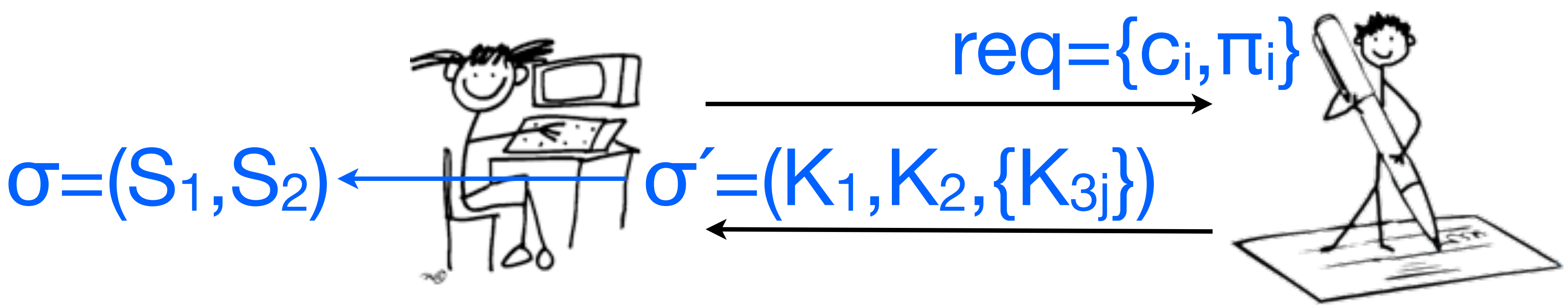

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- Blindness requires only the abstract assumption, ...
- ... but one-more unforgeability requires more.

Projecting and cancelling

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For projecting, we have:

- decomposition $B = B_1 \times B_2$
- map $\pi: B \rightarrow B_2$ such that $\pi(b=b_1{*}b_2) = b_2$
- map π_T such that $\pi_T(E(a,b)) = E(\pi(a),\pi(b))$

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Projecting: $\pi(x) = x^\lambda$ for λ s.t.
$\lambda = 0 \bmod p$
$\lambda = 1 \bmod q$
Then $\pi(g) = \pi(g_p * g_q) = (g^q * g^p)^\lambda = g_q$

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$E(g_p,g_q) = E(g^q,g^p) = E(g,g)^{pq} = E(g,g)^N = 1$

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Freeman [F10] provides generic transformation to prime-order groups for schemes in composite-order groups that require either of these two properties

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Break it up into two lemmas:

- Cancelling shrinks the target space: If we use the DLIN assumption for the indistinguishability of B_1 and B and E is cancelling, then $|E(B,B)| = p$.

- Can’t project with small target: If $|E(B,B)| = p$ then E cannot be projecting.

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We can prove the following theorem:

- If we use the DLIN assumption* for the indistinguishability of B_1 and B and E is cancelling, then E cannot be projecting with overwhelming probability.

Break it up into two lemmas:

- Let E: $B \times B \rightarrow B_T$ be a nondegenerate pairing that is independent of the decomposition $B = B_1 \times B_2$. Then if $B = G^3$, B_1 is a uniformly random rank-2 submodule of B, and E is cancelling, then $|E(B,B)| = p$ with overwhelming probability.

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E is public, if dependent on B_1 could reveal information to help to distinguish it from B

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✓ Can't project with small target: If $|E(B,B)| = p$ then E cannot be projecting.

The problem: what if we want both properties?

We can prove the following theorem:

- If we use the DLIN assumption* for the indistinguishability of B_1 and B and E is cancelling, then E cannot be projecting with overwhelming probability.

Break it up into two lemmas:

E is public, if dependent on B_1 could reveal information to help to distinguish it from B

- Let $E: B \times B \rightarrow B_T$ be a nondegenerate pairing that is independent of the decomposition $B = B_1 \times B_2$. Then if $B = G^3$, B_1 is a uniformly random rank-2 submodule of B, and E is cancelling, then $|E(B,B)| = p$ with overwhelming probability.

If B_1 is *not* random, can't be sure DLIN still holds

✓ Can't project with small target: If $|E(B,B)| = p$ then E cannot be projecting.

Conclusions

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Showed that if we want projecting and cancelling, generic transformations from composite- to prime-order groups fail

- Can't use DLIN (more generally k-Linear [HK07,S07])
- This suggests possible functionality gap

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- This suggests possible functionality gap

Constructed a round-optimal blind signature scheme

- First efficient scheme using ‘mild’ assumptions (non-interactive, static), even including ones in the random oracle model
- Signature scheme demonstrates potential need for both properties

Open problems

Positive:

Negative:

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups

Negative:

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups

Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups

- Prove our scheme secure in prime-order groups

Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
- Prove our scheme secure in prime-order groups

Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups
- Show our scheme is insecure in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
- Prove our scheme secure in prime-order groups
- Show another general conversion from composite- to prime-order groups

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- Prove there can be no projecting and cancelling pairing in prime-order groups
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Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
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Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups
- Show our scheme is insecure in prime-order groups
- Prove that some other properties cannot be achieved in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
- Prove our sch[illegible]
- Show anothe[illegible]e-order groups

Any questions?

Negative:

- Prove there ca[illegible] prime-order groups
- Show our scheme is insecure in prime-order groups
- Prove that some other properties cannot be achieved in prime-order groups