10/9/2015 - Russell's Paradox

\[ f(0) = f(1) + f(2) + \cdots + f(n) \]

\[ f(n) = f(n-1) + f(n) \]

Here, \( f(n) \) is defined as the number of ways to arrange \( n \) objects in a line. The function \( f(n) \) is called the Fibonacci sequence.

\[ f(n) = \frac{1}{\sqrt{5}} \left( \frac{1 + \sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left( \frac{1 - \sqrt{5}}{2} \right)^n \]

This formula, known as Binet's formula, allows us to calculate \( f(n) \) directly without having to compute all the previous terms.

Assume \( f(n) \) can be proven for all \( n \geq 2 \). Then, by induction,

\[ f(n+1) = f(n) + f(n+1) \]

So, \( f(n+1) \) is also true for all \( n \geq 2 \).
Let $\mathcal{E}$ be the set of non-zero elements in $\mathbb{F}_3$. Consider $\mathcal{L} = \{1, \alpha, \alpha^2\}$. Let $\mathbb{F}_3(\alpha) = \{0, 1, \alpha, \alpha^2\}$. Suppose $\mathbb{F}_3(\alpha)$ is a field.

For $\alpha^2 + 1 = 0$, we have $\alpha^2 = -1 = \alpha + 1$. Thus, $\alpha^2 + 1 = 0$.

Now, consider the norm of $\alpha$ in $\mathbb{F}_3(\alpha)$.

Note, integers are not in $\mathbb{F}_3(\alpha)$ as $\alpha$ is not integral.

Let $w$ s.t. $w^3 = 1$. Add $\omega^2 = -1$, except, $\omega^3 = 1$. So $w^3 = 1 = 0$. $(w-1)(w^2 + w + 1) = w^2 = w+1$. $(\omega + 1)(\omega + 1) = \omega^2 = w + 1 + 1 = w + 2$. Thus, $\omega^2 = w + 1$. The characteristic of any extension field $\{1, w, w+1\}$ is $2$.

Consider $\mathbb{F}_3(\alpha) = \mathbb{F}_9$ with $\mathbb{F}_3$. Now, $\mathbb{F} = \{0, w, w^2\} = \{0, \alpha, \alpha^2\}$ as $\alpha \equiv 2, \alpha^2 \equiv -1$. Therefore, $\mathbb{F}_9$ contains all possible fields.