Student ID _____ **Name** _____

Signature _____

By filling in the above and signing my name, I confirm I will complete this exam with the utmost integrity and in accordance with the Policy on Integrity of Scholarship.

Final
CSE 21
Fall 2012

Page 1	_____	**(11 points)**
Page 2	_____	**(13 points)**
Page 3	_____	**(20 points)**
Page 4	_____	**(10 points)**
Page 5	_____	**(15 points)**
Page 6	_____	**(8 points)**
Page 7	_____	**(32 points)**
Page 8	_____	**(20 points)**
Page 9	_____	**(7 points)**
Page 10	_____	**(11 points)**
SubTotal	_____	**(147 points = 100%)**
Page 11	_____	**(7 points) Extra Credit (5%)**
Total	_____	

This exam is to be taken by yourself with closed books, closed notes, no electronic devices.
You are allowed both sides of an 8.5"x11" sheet of paper handwritten by you.

Let $S(n) = 1^2 + 2^2 + 3^2 + \ldots + n^2$ be the sum of the first n perfect squares. Find a recurrence relation for $S(n)$.

$$S(n) = \begin{cases} \underline{\qquad\qquad\qquad\qquad} & \text{if } n = 1 \\ \underline{\qquad\qquad\qquad\qquad} & \text{if } n > 1 \end{cases}$$

Calculate the first 6 terms for this recurrent relation. Then to the right calculate the sequence of differences between these terms. You may not need all the slots on the right.

n		Sequences of differences			
1	_____ = _____				
		_____			
2	_____ = _____		_____		
		_____		_____	
3	_____ = _____		_____		_____
		_____		_____	
4	_____ = _____		_____		_____
		_____		_____	
5	_____ = _____		_____		
		_____			
6	_____ = _____				

Based on the sequence of differences (above) what is a good guess for the closed-form solution to the recurrence relation above?

$f(n) =$ _____

A) $n^4 - n^2 + 2$	B) $n(2n + 1)$	C) $2^n + n$
D) $4n - 2n$	E) $n(n^3 - n)$	F) $\frac{n(2n+1)(n+1)}{6}$

Verify this with a proof by induction. Prove $S(n) = f(n)$ for all n _____ .

<u>Proof</u> (Induction on n):

_____: If $n =$ ____, the recurrence relation says $S($__$) =$ ____,

and the closed-form solution says $f($__$) =$ _____ = ____, so $S($__$) = f($__$)$.

_____: Suppose as inductive hypothesis that $S($_____$) =$ _____

for some k _____.

_____: Using the recurrence relation, $S(k) =$ _____, by 2nd part of RR

= _____, by IHOP

= _____

= _____

So, by induction, $S(n) = f(n)$ for all _____ (as _____).

Write the function definition for the recurrence relation below.

$$T(n) = \begin{cases} 2 & \text{if } n = 1 \\ T(n-1) + 2n & \text{if } n > 1 \end{cases}$$

```
/* Precondition: n >= 1 */
int T( int n )
{
  ____________________________________________

  ____________________________________________

  ____________________________________________

  ____________________________________________
}
```

Which general recursive decompositions discussed in class are most appropriate for the following algorithms:

Sum as defined _____
Sum(L) = x + Sum(L') [where L = L',x]
|L'| = |L| - 1

Sum as defined _____
Sum(L) = Sum(X) + Sum(Y) [where L = (X,Y)]
|X| = |Y| (+/- 1)

The "quicker" power2 function (hw16) _____

Snow White has 55 one-dollar bills, which she wishes to divide up among the seven different dwarves. Each dwarf may receive any (integral) number of bills, from 0 to 55. How many different ways can she distribute this money?

Hugo and Viviana work in an office with eight other coworkers. Out of these 10 workers, their boss needs to choose a group of four to work together on a project. Suppose Hugo and Viviana absolutely refuse, under any circumstances, to work together. Under this restriction, how many different working groups of four can be formed?

How many different strings can be formed by rearranging the letters in BOOKKEEPER, using all the letters?

How many different strings of length 12 can be formed from a set of 26 refrigerator magnets A-Z (no duplicates)?

A standard combination lock that you might use on a locker has numbers on the dial numbered 0-39. The combination to open the lock uses three of these numbers (turn right to first number, turn left to second number, turn right to third number). Your friend has forgotten the combination. How many 3-number combinations are possible if your friend knows for sure that none of the three numbers of the combination is the digit 0?

While you are performing brute force attempts on different combination in the above question, your friend also remembers that in addition to none of the digits being 0 there are no repeated digits (like 1-1-1 or 1-5-1) - in other words the 3 digits of the combination are distinct. How many 3-digit combinations are possible now?

What is the value of C(7,2)? Your answer should be an actual number. _____

What is the value of P(6,4)? Your answer should be an actual number. _____

In Section 3.5 we discussed the Search and the BSearch algorithms. If a list contains n elements, …

Search will require approximately _____ comparison(s) while

BSearch will require approximately _____ comparison(s).

How many (nonempty) strings of at most length 4 can be formed from a 26-symbol alphabet?

How many strings of length 5 can be formed from a 15-symbol alphabet where no 2 adjacent symbols are the same?

How many 5-digit zip codes are possible? Zip codes can contain all zeros.

How many 5-digit zip codes contain only even digits? 0 is an even number.

How many 5-digit zip codes have at least one odd digit?

How many 5-digit zip codes have all different digits (no duplicates)?

How many 5-digit zip codes start with an even digit and end with an even digit?

How many 5-digit zip codes can be formed with at least one duplicate digit (for example, 00489, 58868, and 33997)?

C(38,5) is the same value as C(38, ____)

C(42,1) is the same value as C(42, ____) = _____ (actual #)

C(17,17) is the same value as C(17, ____) = _____ (actual #)

P(n,n) = _____ (in terms of n)

P(n,1) = _____ (in terms of n)

P(n,0) = _____

An urn contains 10 balls numbered 1-10. Four balls are drawn from the urn as described below, and the numbers on the balls are recorded. How many ways are there to do this …

… if the four balls are drawn one at a time and not replaced before the next one is drawn?

… if the four balls are drawn all at once?

… if the four balls are drawn one at a time and replaced before the next one is drawn?

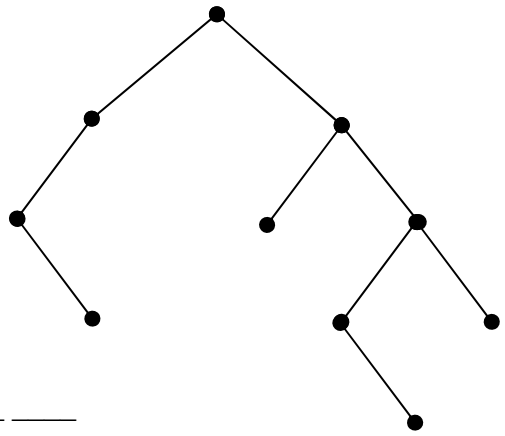
If you pick 30 cards out of a standard 52 card deck, at least how many of them must be the same face value (ace, two, three, … , queen, king)?

How many different possible ten character passwords are there made up of upper (A-Z) and lower (a-z) alpha characters and digits (0-9) and ten punctuation characters with the restriction that at least one character must be an upper-case letter, at least one character must be a lower-case letter, at least one character must be a digit, at least one character must be a punctuation character, and the remaining characters can come from any of those groups with no restriction on repeated characters (repeated characters are allowed)?

There are 15 customers and 4 cashiers. How many ways can the customers line up to the cashiers, if the order of each line does not matter.

Given the binary tree to the right

Specify the output for the following traversals



Preorder traversal: ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

Inorder traversal: ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

Postorder traversal: ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

Breadth-first traversal: ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

In Craps, a Field Bet is a bet that the next roll will be a 2, 3, 4, 9, 10, 11, or 12. It pays even money (1-to-1) for 3, 4, 9, 10, and 11, and pays double (2-to-1) for 2 and 12.

What is the probability of rolling a 2 or 12 (assume two fair 6-sided dice)?

P(X= 2 or 12) = _____ **Non**-reduced fractions are preferred.

What is the probability of rolling a 3, 4, 9, 10, or 11 (assume two fair 6-sided dice)?

P(X= 3, 4, 9, 10, or 11) = _____ **Non**-reduced fractions are preferred.

For a $1 bet on the Field, the payout for rolling a 2 or 12 is 2-to-1 (for example, $1 bet pays $2 + the original $1 bet for a total of $3) and the payout for rolling a 3, 4, 9, 10, or 11 is 1-to-1 (for example, $1 bet pays $1 + the original $1 bet for a total of $2), the Expected Value of the amount of money you will pull off the table in terms of P(X=x) is

E(X) = 3 * P(X = 2 or 12) + 2 * P(X = 3, 4, 9, 10, or 11) + 0 * P(X != 2, 3, 4, 9, 10, 11, or 12)

Now replace the P(X=x) values with their numeric probabilities keeping your answer in terms of fractions vs. decimals. **Non**-reduced fractions are preferred.

E(X) = 3 * _____ + 2 * _____ + 0 * _____ = _____

If your bet is $1 (costs you $1 to play), what is your expected return each time you make this kind of bet? Express your answer as a positive or negative **non**-reduced fraction.

E(X) - 1 = _____

A simple Pass Line bet Expected Value is approximately -1/70 or about -1.41%. Which is a better overall bet for you who is placing the bet - the Pass Line or the Field?

_____

Big-Oh provides a(n) _____ bound on the growth rate of a function while big-Omega provides a(n) _____ bound on the growth rate of a function.

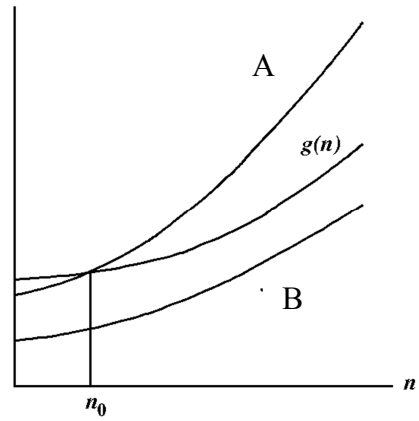
Given $K_1 f(n) \leq g(n) \leq K_2 f(n)$, we say that "$g$ is big-_____ of f" or "g is order f."

$K_1 f(n)$ represents the big-_____ of f while $K_2 f(n)$ represents the big-_____ of f.

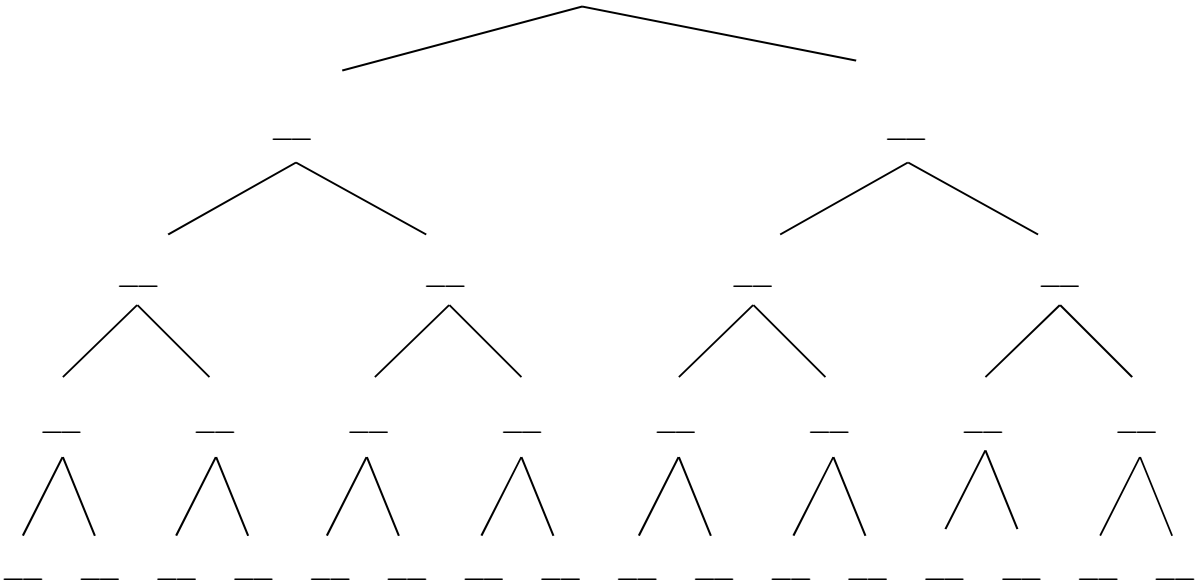
With respect to the graph to the right,

the function labeled _____ represents big-Omega of f and

the function labeled _____ represents big-Oh of f.



A fair coin is flipped 4 times. Use the decision tree below to fill in the different possibilities with each flip putting an H for Heads on the left and T for Tails on the right. Use this decision tree to calculate the probability that at least 2 consecutive Tails occur during the process of flipping the coin 4 times. <u>Just one H or T per line.</u>



How many sequences of flips contain at least 2 consecutive Tails? _____

What is the probability that at least 2 consecutive Tails occur when flipping the coin 4 times? _____

What is the probability that at least 3 consecutive Heads occur when flipping the coin 4 times? _____

What is the probability that the Heads and Tails alternate when flipping the coin 4 times? _____

A random number generator produces a sequence of 20 digits (0,…,9).

a) What is the probability that the sequence contains all 0s?

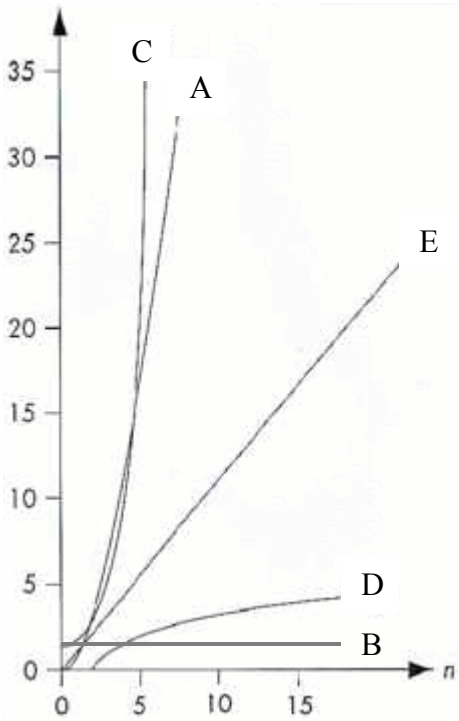
b) What is the probability that the sequence contains at least one 7?

Consider the following algorithm:

$x \leftarrow 1$
$\boldsymbol{for}\ i \in \{1,2,3,4\}\ do$
  $\boldsymbol{for}\ j \in \{1,2,3,4\}\ do$
    $x \leftarrow x + x$
  $\boldsymbol{for}\ k \in \{1,2,3\}\ do$
    $x \leftarrow x + 1$
    $x \leftarrow x + 5$

Count the number of + operations done by this algorithm. _____

Match the big-Oh complexity class value to the graph. Write the letter associated with the function to the right that describes the big-Oh runtime complexity for that function.



____ $O(2^n)$

____ $O(\log_2 n)$

____ $O(n)$

____ $O(n^2)$

____ $O(1)$

Between which two plot lines would we find

$O(n \log_2 n)$ - between ____ and ____

$O(n^3)$ - between ____ and ____

What is the <u>name</u> of the complexity class for each labeled function? (Not big-Oh – the English word name for the complexity class.)

A) _____

B) _____

C) _____

D) _____

E) _____

Match the algorithms with their recurrence relations and their run time complexities. Use the letters and numbers from the boxes to the right.

<u>Algorithm</u>	<u>Recurrence Relation</u>	<u>Time Complexity</u>
Binary Search	_____	_____
Sequential Search	_____	_____
Merge Sort	_____	_____
Towers of Hanoi	_____	_____
Bubble Sort	_____	_____
Selection Sort	_____	_____
Binary Tree Traversal	_____	_____
Fibonacci sequence (recursive)	_____	_____
Array access		_____
Fibonacci sequence (iterative as discussed in class and homework)		_____

A) $T(n) = 2\,T(n-1) + O(1)$
B) $T(n) = 2\,T(n/2) + O(1)$
C) $T(n) = 2\,T(n-1) + O(n)$
D) $T(n) = T(n-1) + O(n)$
E) $T(n) = 2\,T(n/2) + O(n)$
F) $T(n) = T(n/2) + O(1)$
G) $T(n) = T(n-1) + T(n-2) + O(1)$
H) $T(n) = T(n-1) + O(1)$

1) $O(\log_2 n)$
2) $O(n!)$
3) $O(n^2)$
4) $O(n)$
5) $O(2^n)$
6) $O(1)$
7) $O(n \log_2 n)$

If you have an algorithm that when you double the number of elements it …

doubles the number of comparisons, this algorithm is most likely in what big-Oh complexity class? _____

squares the number of comparisons, this algorithm is most likely in what big-Oh complexity class? _____

does not change the number of comparisons, this alg. is most likely in what big-Oh complexity class? _____

increases the number of comparisons by 1, this alg. is most likely in what big-Oh complexity class? _____

quadruples the number of comparisons, this alg. is most likely in what big-Oh complexity class? _____

it increases the number of comparisons by slightly more than double (2.x times) for sufficiently large *n* (say *n* >= 16), this algorithm is most likely in what big-Oh complexity class? _____

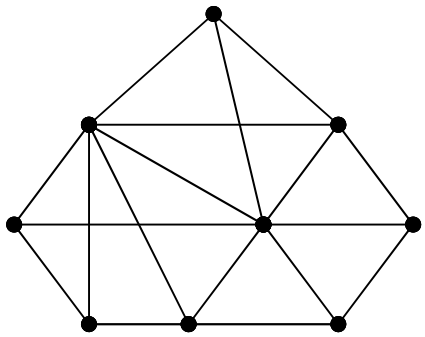
If you write a program which contains an array initialized with *n* random unsorted elements that you sort using Merge sort and then perform 2048 binary searches on this sorted array, what is the overall run time complexity of the program in terms of *n*? O(_____)

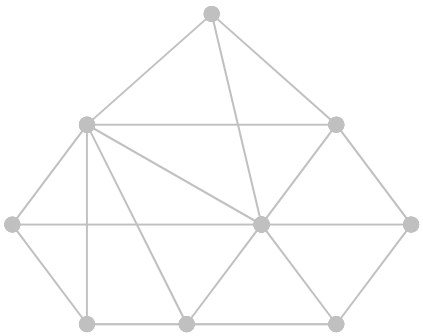
What if you quadruple the number of binary searches? O(_____)

According to the Lower Bound Theorem, the best (most efficient) worst case complexity for comparison-based <u>sort</u> is Θ(_____)

According to the Lower Bound Theorem, the best (most efficient) worst case complexity for comparison-based <u>search</u> is Θ(_____)

Construct a minimum spanning tree from the following network. Use the grayed network on the right to construct your msp. Hint: 9 vertices so msp should have 8 edges.





What is the total weight of the minimum spanning tree? _____

Is there more than one minimum spanning tree in this graph (yes or no)? _____

Given the sorted array of ints in an array as: 0, 1, 3, 4, 8, 10, 12, 14 what are the values of lo, mid, and hi indexes for each recursion in the binary search algorithm when searching the entire array for the target value 9?

<u>lo</u>	<u>mid</u>	<u>hi</u>	What is the return value of binarySearch() in this case? _____
0		7	- starting min and max indexes are given for you

_____ is the collection of all problems whose solutions can be checked (but not necessarily solved) in polynomial time.

_____ is the collection of all problems that can be solved with an algorithm whose complexity is, at most, polynomial.

Given the initial order of ints in an array as: 2, 6, 9, 8, 1, 0, 5, 3, 4 what is the order of the elements after <u>5 iterations</u> of the selection sort algorithm covered in class and one of the HW exercises?

____ ____ ____ ____ ____ ____ ____ ____ ____

Consider the following algorithm:

```
for ( i = 1; i <= 4; ++i )
{
  beep;

  for ( j = 1; j <= 5; ++j )
  {
    beep;
  }

  for ( k = 1; k <= 2; ++k )
  {
    for ( l = 1; l <= 3; ++l )
    {
      beep;
    }

    for ( m = 1; m <= 5; ++m )
    {
      beep;
    }
  }
}
```

How many times does the `beep` statement get executed? _____

Who is Rick's favorite rapper? _____.

Consider the following algorithm:

$i \leftarrow 0$
$j \leftarrow 0$
$\boldsymbol{while}\ i < n\ \boldsymbol{do}$
{
  $i \leftarrow i + 1$
  $j \leftarrow j + 2$
}

Show that the statement $j = 2i$ is an invariant for this while loop. Use $\underline{i}$ and $\underline{j}$ to denote the values of i and j <u>before</u> the body of the while executes and use $\bar{i}$ and $\bar{j}$ to denote the values of i and j <u>after</u> execution. To prove that $j = 2i$ is an invariant, we need to show that

$$\underline{j} = 2\underline{i} \Rightarrow \bar{j} = 2\bar{i}$$

A regular expression cannot be written to correctly recognize only valid strings that are palindromes or only valid strings in the form $a^n b^n$ because regular expressions cannot _____.

In grep (and vim), what metacharacter is used to match 1 or more of the preceding element? _____

In grep (and vim), what metacharacter is used to specify/match the end of the line? _____

In grep (and vim), what metacharacter is used to match 0 or more of the preceding element? _____

In grep (and vim), what metacharacter is used to match any character? _____

S is the start symbol. a and b are a terminal symbols.

1) $S \rightarrow aSa$
$S \rightarrow aba$

2) $S \rightarrow SbS$
$S \rightarrow a$

3) $S \rightarrow Sb$
$S \rightarrow a$

4) $S \rightarrow abS$
$S \rightarrow ab$

5) $S \rightarrow aSb$
$S \rightarrow ab$

6) $S \rightarrow abSab$
$S \rightarrow ab$

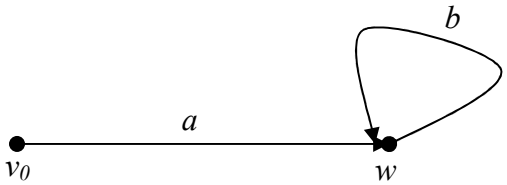
7) None of the above

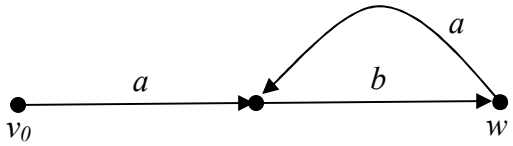
Which context-free grammar correctly recognizes only words of the language $(ab)^n$ for $n \geq 1$? _____

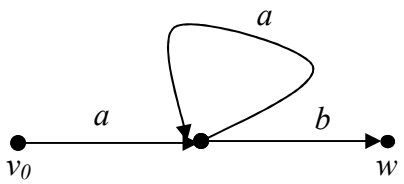
Which context-free grammar correctly recognizes only words of the language $a^n b a^n$ for $n \geq 1$? _____

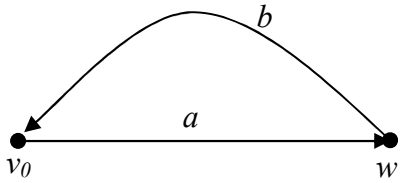
Which context-free grammar correctly recognizes only words of the language $a^n b^n$ for $n \geq 1$? _____

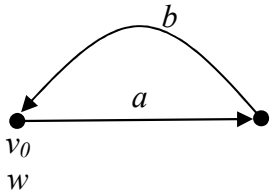
v_0 is the start node. w is a terminal node. A node labeled with both v_0 and w is both a start and terminal node.

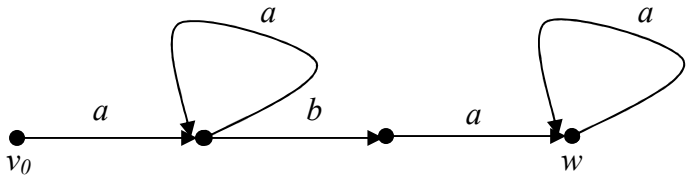
A)


B)


C)


E)


F)


G)


D) None of the above

Which finite state automaton correctly recognizes only words of the language $(ab)^n$ for $n \geq 1$? _____

Which finite state automaton correctly recognizes only words of the language $a(ba)^n$ for $n \geq 1$? _____

Which finite state automaton correctly recognizes only words of the language $a^n b a^n$ for $n \geq 1$? _____

Extra Credit

Match the person to what the person is famous for. (1/2 point each)

A) Edsger Dijkstra
B) Al Aho
C) John McCarthy
D) Niklaus Wirth
E) Donald Knuth
F) Alan Perlis
G) Grace Hopper
H) Richard Stallman
I) Brian Kernighan
J) Bjarne Stroustrup
K) Alan Turing
L) C.A.R. Hoare
M) James Gosling
N) Ken Thompson
O) Linus Torvalds
P) John von Neumann
Q) Ron Graham
R) Gordon Moore
S) John Backus
T) Gene Amdahl
U) Dennis Ritchie

_____ Known as the father of the C++ programming language.

_____ Co-founded Intel and described trend in number of transistors.

_____ Known as the father of the Linux operating system.

_____ Known as the father of the Java programming language.

_____ Known as the father of the GNU Project and GCC.

_____ Known as the father of the C programming language.

_____ Known as the father of Lisp and garbage collection.

_____ Known as the father of the analysis of algorithms.

_____ Credited as being the father of Computer Science.

_____ Helped popularize the term debugging.

_____ Invented the single-source shortest path algorithm and the semaphore used in operating systems.

_____ The speedup of a program using multiple processors in parallel computing is limited by the time needed for the sequential fraction of the program.

_____ Developed Unix along with Kernighan & Ritchie, created grep, co-designed the Go programming language at Google.

_____ Has a number named after him that at one time was the largest number ever used in a serious mathematical proof.

Scratch Paper

Scratch Paper