Analysis of Planar Light Fields from Homogeneous Convex Curved Surfaces Under Distant Illumination

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http://graphics.stanford.edu/papers/planarlf/
Motivation

Forward Rendering (Computer Graphics)

- Complex Lighting (Environment Maps)
Motivation

Inverse Rendering (Computer Vision and Graphics)

- *Estimate* BRDF, Lighting, *both* BRDF and Lighting
  - Theoretically Possible?
  - Practically Feasible?
Reflection Equation

\[ B(x, \theta'_o) = \int_{-\pi/2}^{\pi/2} L(x, \theta'_i) \rho(\theta'_i, \theta'_o) \cos(\theta'_i) \, d\theta'_i \]
Reflection Equation

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\hat{\rho}(\theta'_i, \theta'_o) = \rho(\theta'_i, \theta'_o) \cos(\theta'_i)
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Approach: Reflection is Convolution

\[
B(\alpha, \theta'_o) = \int_{-\pi/2}^{\pi/2} L(\alpha, \theta'_i) \hat{\rho}(\theta'_i, \theta'_o) \, d\theta'_i
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Approach: Reflection is Convolution

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L(\alpha, \theta'_i) = L(\theta_i) = L(\alpha + \theta'_i)
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Approach: Reflection is Convolution

\[ B(\alpha, \theta'_{o}) = \int_{-\pi/2}^{\pi/2} L(\alpha, \theta'_{i}) \hat{\rho}(\theta'_{i}, \theta'_{o}) \, d\theta'_{i} \]

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\[ B(\alpha, \theta'_{o}) = \int_{-\pi/2}^{\pi/2} L(\alpha + \theta'_{i}) \hat{\rho}(\theta'_{i}, \theta'_{o}) \, d\theta'_{i} \]

CONVOLUTION: \[ B \approx L \otimes \hat{\rho} \]
Related Work

Graphics: Prefiltering Environment Maps
- Qualitative Observation that Reflection is Convolution
- Miller & Hoffman 84, Greene 86
- Cabral Max Springmeyer 87, Cabral Olano Nemec 99

Vision, Perception
- D’Zmura 91: Reflection as Operator in Frequency Space
- Basri & Jacobs: Lambertian Reflection as Convolution
Related Work

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Our Contribution: Formal Analysis in General 2D case
- Key insights extend to 3D (more recent work)
**Fourier Analysis**

\[ L(\theta_i) = \sum_p L_p e^{Ip\theta_i} \]

\[ \hat{\rho}(\theta'_i, \theta'_o) = \sum_p \sum_q \hat{\rho}_{p,q} e^{Ip\theta'_i} e^{Iq\theta'_o} \]

\[ B(\alpha, \theta'_o) = \sum_p \sum_q B_{p,q} e^{Ip\alpha} e^{Iq\theta'_o} \]
Fourier Analysis

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Note: Can fix output direction:

\[ B_p(\theta'_o) = 2\pi L_p \hat{\rho}_{-p}(\theta'_o) \]
Insights

Reflected Light Field is *Convolution* of Lighting, BRDF

Convolution Theorem $\Rightarrow$ Product of Fourier Coefficients

Signal Processing: Filter Lighting using BRDF Filter

Lighting $\leftrightarrow$ Input Signal

BRDF $\leftrightarrow$ Filter

Inverse Rendering is *Deconvolution*
Example: Directional Source at $\theta_i = 0$

$$L(\theta_i) = \delta(\theta_i) \quad L_p = \frac{1}{2\pi}$$

$$B_{p,q} = \hat{\rho}_{-p,q}$$

Reflected Light Field corresponds directly to BRDF

- **Impulse Response** of BRDF filter
Example: Mirror BRDF

\[ \hat{\rho}(\theta_i', \theta_o') = \delta(\theta_i' + \theta_o') \]

\[ \hat{\rho}_{p,q} = \frac{\delta_{p,q}}{2\pi} \]

\[ B_{p,q} = \delta_{p,q} L_{-p} \]

Reflected Light Field corresponds directly to Lighting

Gazing Sphere
Example: Lambertian BRDF

Transfer function is *Clamped Cosine*

No output dependence, drop index $q$

\[ B_p = 2\pi L_p \hat{\rho}_{-p} \]

Lambertian BRDF is *Low-Pass* filter
Properties: Lambertian BRDF Filter

\[ \hat{\rho}_{2p} = \frac{(-1)^{p+1}}{2\pi (4p^2 - 1)} \]

Good approximation using only terms with \( p \leq 2 \)
Phong, Microfacet BRDFs

Rough surfaces blur highlights

Microfacet BRDF is Gaussian
- Hence, Fourier Spectrum also Gaussian
- Similar results for Phong (analytic formulae in paper)
## Inverse Rendering

**Lighting**

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X</strong></td>
<td>Miller &amp; Hoffman 84</td>
</tr>
<tr>
<td></td>
<td>Marschner &amp; Greenberg 97</td>
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</table>

<table>
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<tr>
<th>BRDF</th>
<th>Known</th>
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</tr>
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<tbody>
<tr>
<td>Sato et al. 97, Yu et al. 99, Dana et al. 99 Debevec et al. 00, Marschner et al. 00, ...</td>
<td></td>
<td>Sato et al. 99 (shadows)</td>
</tr>
</tbody>
</table>

Often estimate *Textured* BRDFs (3rd axis of table)
Inverse Rendering

General Complex Illumination?
- Most inverse-BRDF methods use point source
- Outdoor methods: Sato&Ikeuchi94, Yu&Malik98

Well-Posedness, Conditioning?
- Well Posed if unique solution
- Well Conditioned if robust to noisy data

Factorization of BRDF, Lighting (find both)?
- Sato et al. 99 use shadows
Inverse Lighting

\[ L_p = \frac{1}{2\pi} \frac{B_{p,q}}{\hat{\rho}_{-p,q}} \]

Well posed unless \( \hat{\rho}_{-p,q} \) vanishes for all \( q \) for some \( p \).

Well conditioned when Fourier spectrum decays slowly.

- Need high frequencies in BRDF (sharp specularities)
- Ill-conditioned for diffuse BRDFs (low-pass filter)

Mirror    Lambertian
BRDF estimation

\[ \hat{\rho}_{p,q} = \frac{1}{2\pi} \frac{B_{-p,q}}{L_{-p}} \]

Well Posed if all terms in Fourier expansion \( L_{-p} \) nonzero.
Well Conditioned when Fourier expansion decays slowly.
- Need high frequencies in lighting (sharp features)
- Ill-conditioned for soft lighting (low-frequency)

Directional Source  Area Source (same BRDF)
Light Field Factorization

Up to a global scale, Light Field can be factored

- Can simultaneously estimate Lighting, BRDF

Number of Knowns (B) > Number of Unknowns (L, ρ)

- \((B \rightarrow 2D) > (L \rightarrow 1D + \rho \rightarrow 1/2(2D))\)

Explicit Formula in paper
3D

Fourier Series $\rightarrow$ Spherical Harmonics $Y_{lm}(\theta, \phi)$

$\rightarrow$ Representation Matrices of $SO(3)$ $D^l_{mm'}(\alpha, \beta)$
3D

Fourier Series → Spherical Harmonics $Y_{lm}(\theta, \phi)$

→ Representation Matrices of $SO(3)$ $D_{mm'}^l(\alpha, \beta)$

$$L(\theta_i, \phi_i) = \sum_{l,m} L_{lm} Y_{lm}(\theta_i, \phi_i)$$

$$\hat{\rho}(\theta'_i, \phi'_i, \theta'_o, \phi'_o) = \sum_{l,p,q} \hat{\rho}_{lq,pq} Y_{lq}^*(\theta'_i, \phi'_i) Y_{pq}(\theta'_o, \phi'_o)$$

$$B(\alpha, \beta, \theta'_o, \phi'_o) = \sum_{l,m,p,q} B_{lmpq} \left( D_m^l(\alpha, \beta) Y_{pq}(\theta'_o, \phi'_o) \right)$$
3D

Fourier Series → Spherical Harmonics $Y_{lm}(\theta, \phi)$

→ Representation Matrices of $SO(3)$ $D^l_{mm'}(\alpha, \beta)$

$L(\theta_i, \phi_i) = \sum_{l,m} L_{lm} Y_{lm}(\theta_i, \phi_i)$

$\hat{\rho}(\theta_i', \phi_i', \theta_o', \phi_o') = \sum_{l,p,q} \hat{\rho}_{lq,pq} Y^*_{lq}(\theta_i', \phi_i') Y_{pq}(\theta_o', \phi_o')$

$B(\alpha, \beta, \theta_o', \phi_o') = \sum_{l,m,p,q} B_{lmpq} (D^l_{mq}(\alpha, \beta) Y_{pq}(\theta_o', \phi_o'))$

\[\begin{align*}
\text{2D: } B_{pq} &= 2\pi L_p \hat{\rho}_{-p,q} \\
\text{3D: } B_{lmpq} &= L_{lm} \hat{\rho}_{lq,pq}
\end{align*}\]
Implications

Lambertian BRDF

- 2D: Only first 2 Fourier coefficients important
- 3D: First 2 orders of spherical harmonics $\rightarrow$ 99% energy
  
  $\star$ Only the first 9 coefficients are important

- Similar results independently derived by Basri & Jacobs
- Formally, recovery of radiance from irradiance ill-posed

  $\star$ See On the relationship between Radiance and Irradiance: Determining the illumination from images of a convex Lambertian object (submitted)

Phong & Microfacet BRDFs

- Gaussian Filters. Results similar to 2D
Practical Issues (in 3D)

Frequency spectra from Incomplete Irregular Data

Concavities: Self-Shadowing and Interreflection

Textures: Spatially Varying BRDFs
Practical Issues (in 3D)

Frequency spectra from Incomplete Irregular Data

Concavities: Self-Shadowing and Interreflection

Textures: Spatially Varying BRDFs

Issues can be addressed; can derive practical algorithms

- Use Dual Angular and Frequency-space Representations
- Associativity of Convolution
- See *A Signal Processing Framework for Inverse Rendering (submitted)*
Experiment: Cat Sculpture

3 photographs of cat sculpture of known geometry

Microfacet BRDF under complex unknown lighting

Lighting also estimated

Then use recovered BRDF for new view, new lighting
Results: Cat Sculpture

Images below show new view, new lighting

Numerical values verified to within 5%
Implications for Perception

Assume Lambertian BRDF, no shadows

- Perception: Separate Reflectance, Illumination
- Low frequency $\leftrightarrow$ lighting, High frequency $\leftrightarrow$ texture
- Theory formally: lighting $\rightarrow$ only low-frequency effects
- Find high-frequency texture independent of lighting
- But ambiguity regarding low-frequency texture, lighting
Conclusion

Reflection as convolution

Fourier analysis gives many insights

Extends to 3D and results in practical inverse algorithms

Signal-Processing: A useful paradigm for Forward and Inverse Rendering