Advanced Computer Graphics

CSE 190 [Winter 2016], Lecture 19

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To Do

- Assignment 3 due Mar 15
  - Should already be well on way
  - Contact us for difficulties etc
- Please fill out CAPE evaluations (Now!)
- Thu lecture will be short/optional or cancelled

Course Outline

- 3D Graphics Pipeline

Modeling (Creating 3D Geometry) → **Rendering** (Creating, shading images from geometry, lighting, materials)

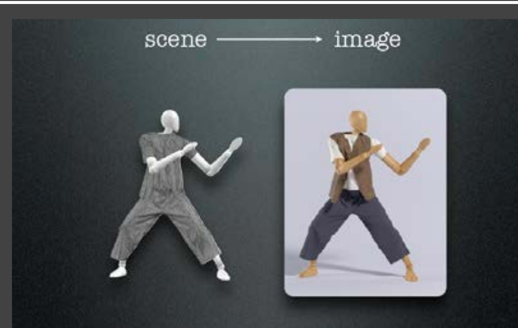
Unit 1: Foundations of Signal and Image Processing
Understanding the way 2D images are formed and displayed, the important concepts and algorithms, and to build an image processing utility like Photoshop
Weeks 1 – 3. **Assignment 1**

Unit 2: Meshes, Modeling
Weeks 3 – 5. **Assignment 2**

Unit 3: Advanced Rendering
Weeks 6 – 8. **(Final Project)**

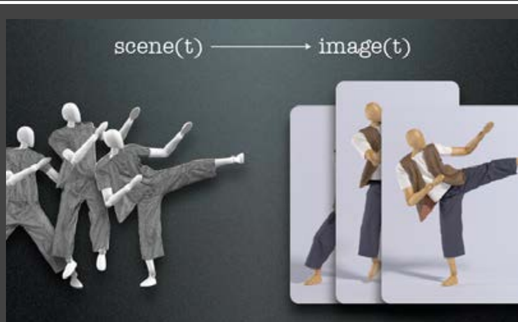
Unit 4: Animation, Imaging
Weeks 9, 10. **(Final Project)**

The Story So Far



Slides courtesy Rahul Narain and James O'Brien

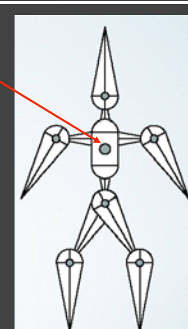
Animation



Forward Kinematics

Root body

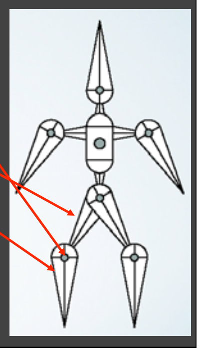
- Position set by global transform
- Root joint: position, rotation
- Other bodies relative to root
- *Inboard* toward the root
- *Outboard* away from the root
- Tree structure: loop joints break "tree-ness"



Inboard and Outboard

Joints
- Inboard body
- Outboard body

Body
- Inboard joint
- Outboard joint (may be several)



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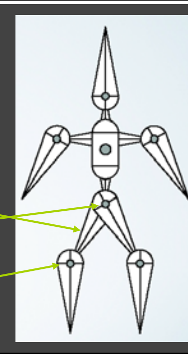
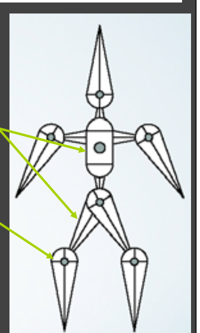


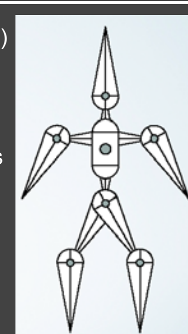
Bodies

Bodies arranged in a tree
- For now, assume no loops
- Body’s parent (except root)
- Body’s child (may have many children)



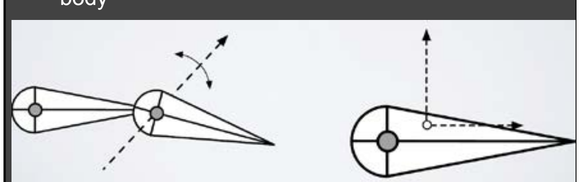
Joints

Interior Joints (typically not 6 DOF)
- Pin – rotate about one axis
- Ball – arbitrary rotation
- Prism – translate along one axis



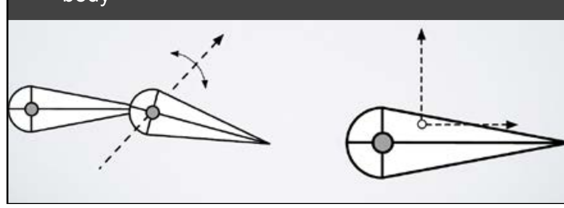
Pin Joints

- Translate inboard joint to local origin
- Apply rotation about axis
- Translate origin to location of joint on outboard body



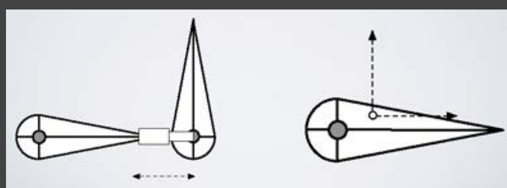
Ball Joints

- Translate inboard point to local origin
- Apply rotation about arbitrary axis
- Translate origin to location of joint on outboard body



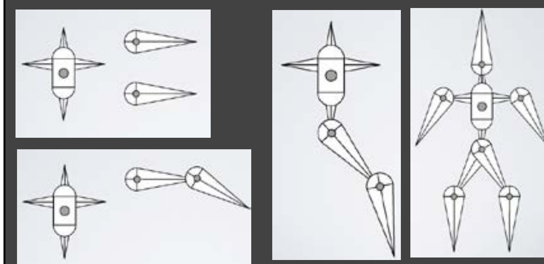
Prism Joint

- Translate inboard joint to local origin
- Translate along axis
- Translate origin to location of joint on outboard

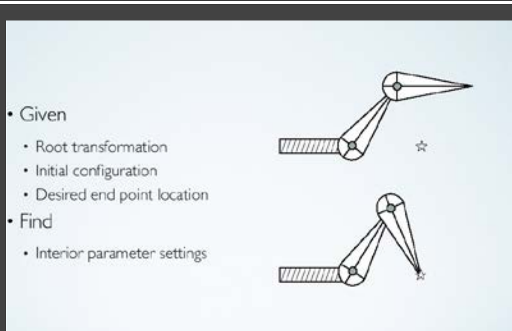


Forward Kinematics

- Composite transformations up the hierarchy



Inverse Kinematics



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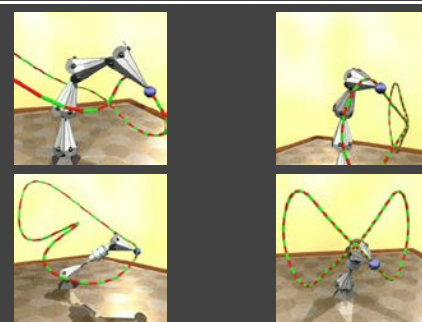
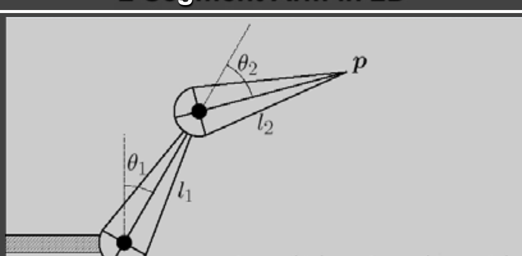


Egon Pasztor

2 Segment Arm in 2D



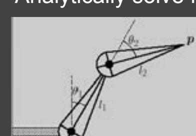
$$p_z = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

$$p_x = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

Warning: Z-up Coordinate System

Direct IK

- Analytically solve for parameters (not general)

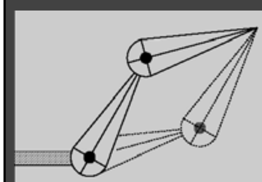


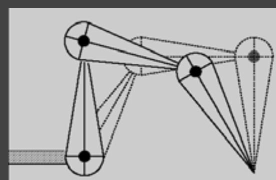
$$\theta_2 = \cos^{-1}\left(\frac{p_z^2 + p_x^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

$$\theta_1 = \frac{-p_z l_2 \sin(\theta_2) + p_x(l_1 + l_2\cos(\theta_2))}{p_x l_2 \sin(\theta_2) + p_z(l_1 + l_2\cos(\theta_2))}$$

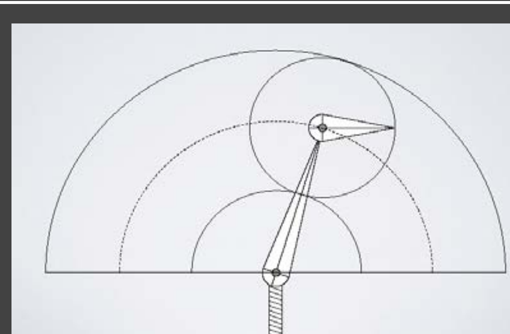
Difficult Issues

- Multiple configurations distinct in config space
- Or connected in config space





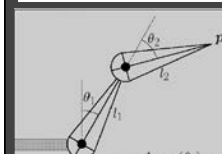
Infeasible Regions



Numerical Solution

- Start in some initial config. (previous frame)
- Define error metric (goal pos – current pos)
- Compute Jacobian with respect to inputs
- Use Newton's or other method to iterate
- General principle of goal optimization

Back to 2 Segment Arm



$$p_z = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$
$$p_x = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

$$\frac{\partial p_z}{\partial \theta_1} = -l_1 \sin(\theta_1) - l_2 \sin(\theta_1 + \theta_2)$$
$$\frac{\partial p_x}{\partial \theta_1} = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$
$$\frac{\partial p_z}{\partial \theta_2} = - l_2 \sin(\theta_1 + \theta_2)$$
$$\frac{\partial p_x}{\partial \theta_2} = + l_2 \cos(\theta_1 + \theta_2)$$

Jacobians and Configuration Space

Dection in Gnfig .Bace

$$\theta_1 = c_1 \theta_*$$
$$\theta_2 = c_2 \theta_*$$
$$\frac{\partial p_z}{\partial \theta_*} = c_1 \frac{\partial p_z}{\partial \theta_1} + c_2 \frac{\partial p_z}{\partial \theta_2}$$

The Jacobian (of p w.r.t. θ)

$$J = \begin{bmatrix} \frac{\partial p_z}{\partial \theta_1} & \frac{\partial p_z}{\partial \theta_2} \\ \frac{\partial p_x}{\partial \theta_1} & \frac{\partial p_x}{\partial \theta_2} \end{bmatrix}$$

$$\frac{\partial \boldsymbol{p}}{\partial \theta_*} = J \cdot \begin{bmatrix} \frac{\partial \theta_1}{\partial \theta_*} \\ \frac{\partial \theta_2}{\partial \theta_*} \end{bmatrix} = J \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Solving for Joint Angles

Solving for c_1 and c_2

$$\boldsymbol{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \qquad \mathrm{d}\boldsymbol{p} = \begin{bmatrix} \mathrm{d}p_z \\ \mathrm{d}p_x \end{bmatrix}$$

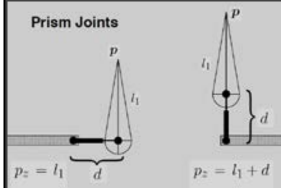
$$\mathrm{d}\boldsymbol{p} = J \cdot \boldsymbol{c}$$
$$\boldsymbol{c} = J^{-1} \cdot \mathrm{d}\boldsymbol{p}$$

Issues

- Jacobian not always invertible
  - Use an SVD and pseudo-inverse
- Iterative approach, not direct
  - The Jacobian is a linearization, changes
- Practical implementation
  - Analytic forms for prism, ball joints
  - Composing transformations
  - Or quick and dirty: finite differencing
  - Cyclic coordinate descent (each DOF one at a time)

Prism and Ball Joints

Prism Joints



$p_z = l_1$

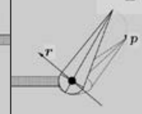
$p_x = d$

$p_z = l_1 + d$

$p_x = 0$

Ball Joints

$$\begin{aligned} \boldsymbol{p} &= \hat{\boldsymbol{r}}(\hat{\boldsymbol{r}} \cdot \boldsymbol{x}) \\ &+ \sin(||\boldsymbol{r}||)(\hat{\boldsymbol{r}} \times \boldsymbol{x}) \\ &- \cos(||\boldsymbol{r}||)(\hat{\boldsymbol{r}} \times (\hat{\boldsymbol{r}} \times \boldsymbol{x})) \end{aligned}$$



More on Ball Joints

Ball Joints (moving axis)

$$\mathrm{d}\boldsymbol{p} = [\mathrm{d}\boldsymbol{r}] \cdot e^{[\boldsymbol{r}]} \cdot \boldsymbol{x} = [\mathrm{d}\boldsymbol{r}] \cdot \boldsymbol{p} = \underbrace{-[\boldsymbol{p}]}\cdot \mathrm{d}\boldsymbol{r}$$

That is the Jacobian for this joint

$$[\boldsymbol{r}] = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}$$

$$[\boldsymbol{r}] \cdot \boldsymbol{x} = \boldsymbol{r} \times \boldsymbol{x}$$

Ball Joints (fixed axis)

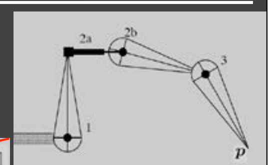
$$\mathrm{d}\boldsymbol{p} = (\mathrm{d}\theta)[\hat{\boldsymbol{r}}] \cdot \boldsymbol{x} = \underbrace{-[\boldsymbol{x}] \cdot \hat{\boldsymbol{r}}}\mathrm{d}\theta$$

That is the Jacobian for this joint

Multiple Links

- IK requires Jacobian
  - Need generic method for building one
- Can't just concatenate matrices

$$\cancel{\tilde{J} = [J_3\ J_{2b}\ J_{2a}\ J_{1b}]}$$



$$\boldsymbol{d} = \begin{bmatrix} d_3 \\ d_{2b} \\ d_{2a} \\ d_{1b} \end{bmatrix}$$

$$\mathrm{d}\boldsymbol{p} \neq \tilde{J} \cdot \mathrm{d}\boldsymbol{d}$$

Composing Transformations

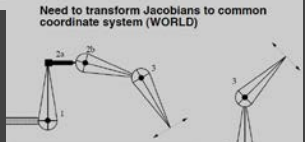
Transformation from body to world

$$X_{0\leftarrow i} = \prod_{j=1}^{i} X_{(j-1)\leftarrow j} = X_{0\leftarrow 1} \cdot X_{1\leftarrow 2} \cdots$$

Rotation from body to world

$$R_{0\leftarrow i} = \prod_{j=1}^{i} R_{(j-1)\leftarrow j} = R_{0\leftarrow 1} \cdot R_{1\leftarrow 2} \cdots$$

Need to transform Jacobians to common coordinate system (WORLD)



$$J_{i,\mathrm{WORLD}} = R_{0\leftarrow(i-1)} \cdot J_i$$

Inverse Kinematics: Final Form

$$J = \begin{bmatrix} R_{0\leftarrow 2b} \cdot J_3(\theta_3, \boldsymbol{p_3}) \\ R_{0\leftarrow 2a} \cdot J_{2b}(\theta_{2b}, X_{2b\leftarrow 3} \cdot \boldsymbol{p_3}) \\ R_{0\leftarrow 1} \cdot J_{2a}(\theta_{2a}, X_{2a\leftarrow 3} \cdot \boldsymbol{p_3}) \\ J_1(\theta_1, X_{1\leftarrow 3} \cdot \boldsymbol{p_3}) \end{bmatrix}^{\mathrm{T}}$$

Note: Each row in the above should be transposed....

$$\boldsymbol{d} = \begin{bmatrix} d_3 \\ d_{2b} \\ d_{2a} \\ d_{1b} \end{bmatrix}$$

$$\boxed{\mathrm{d}\boldsymbol{p} = J \cdot \mathrm{d}\boldsymbol{d}}$$

A Cheap Alternative

- Estimate Jacobian (or parts of it) w. finite diffs.
- Cyclic coordinate descent
  - Solve for each DOF one at a time
  - Iterate till good enough / run out of time

More complex systems

- More complex joints (prism and ball)
- More links
- Other criteria (center of mass or height)
- Hard constraints (e.g., foot plants)
- Unilateral constraints (e.g., joint limits)
- Multiple criteria and multiple chains
- Loops
- Smoothness over time
  - DOF determined by control points of curve (chain rule)

Practical Issues

- How to pick from multiple solutions?
- Robustness when no solutions
- Contradictory solutions
- Smooth interpolation
  - Interpolation aware of constraints

Prior on "good" configurations