ON THRESHOLD CIRCUITS FOR PARITY

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ABSTRACT

Motivated by the problem of understanding the limitations of neural networks for representing Boolean functions, we consider size-depth trade-offs for threshold circuits that compute the parity function. It is well-known that the parity function on $n$ variables can be computed by a depth-2 threshold circuit with a quadratic number of edges. It is not hard to construct such a circuit with a linear number of edges in depth $O(\log \log n)$ and in fact there is a construction that provides a transition between these extremes.

Our starting point was an attempt to show that the trade-off exhibited by this construction is the best possible. The result we present in this paper is an almost optimal lower bound $\Omega(n^2/\log^2 n)$ on the number of edges for computing parity function with depth-2 threshold circuits whose weights are bounded by a polynomial in $n$. (We get a much sharper lower bound of $\Omega(n^2)$ in the case when all the weights of the first level gates are nonnegative or of the form $\pm 1$.) While this result falls short of our goal, its proof involves an interesting application of rational approximation theory to complexity theory. The technique holds promise towards a complete answer to this trade-off question and also seems to be a natural and a potentially useful approach to the analysis of threshold circuits in general. Such an approach might also be relevant to the solution of the question whether $TC^0$ (the class of functions computable by bounded depth and polynomial size threshold circuits) is equal to $NC^1$ (the class of functions computable by fan-in 2 log-depth Boolean circuits).

The question of lower bounds on the size of depth-2 circuits for parity is closely related to the following open problem: What is the minimal number of hyperplanes required to intersect all the edges of the unit hypercube? We show that a lower bound of $f(n)$ on the number of hyperplanes would imply an $\Omega(n f(n))$ lower bound on the edge complexity for computing parity with depth-2 circuits.

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