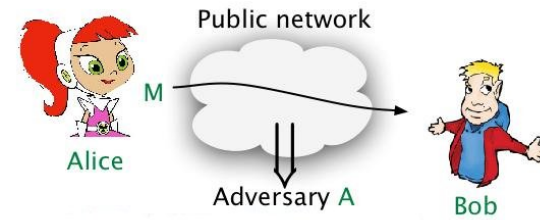


# AUTHENTICATED ENCRYPTION

## So Far ...



We have looked at methods to provide **privacy** and **authenticity** separately:

Goal	Primitive	Security notion
Data privacy	symmetric encryption	IND-CPA
Data authenticity	MAC	UF-CMA

## Authenticated Encryption

In practice we often want **both** privacy and authenticity.

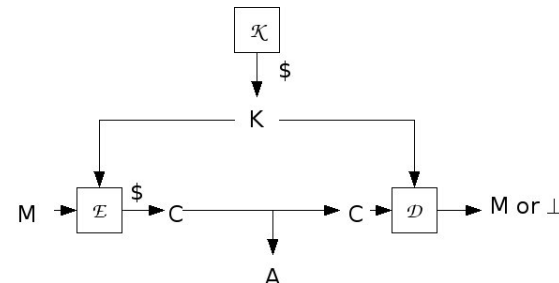
**Example:** A doctor wishes to send medical information  $M$  about Alice to the medical database. Then

- We want **data privacy** to ensure Alice's medical records remain **confidential**.
- We want **authenticity** to ensure the person sending the information is really the doctor and the information was **not modified** in transit.

We refer to this as **authenticated encryption**.

## Authenticated Encryption Schemes

Syntactically, an authenticated encryption scheme is just a symmetric encryption scheme  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  where



## Privacy of Authenticated Encryption Schemes

The notion of **privacy** for symmetric encryption carries over, namely we want IND-CPA.

## Integrity of Authenticated Encryption Schemes

Adversary's goal is to get the receiver to accept a "non-authentic" ciphertext  $C$ .

Integrity of **ciphertexts**:  $C$  is "non-authentic" if it was never transmitted by the sender.

## INT-CTXT

Let  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  be a symmetric encryption scheme and  $A$  an adversary.

Game  $\text{INTCTXT}_{\mathcal{AE}}$

**procedure Initialize**

$K \xleftarrow{\$} \mathcal{K} ; S \leftarrow \emptyset$

**procedure Enc**( $M$ )

$C \xleftarrow{\$} \mathcal{E}_K(M)$

$S \leftarrow S \cup \{C\}$

Return  $C$

**procedure Finalize**( $C$ )

$M \leftarrow \mathcal{D}_K(C)$

if ( $C \notin S \wedge M \neq \perp$ ) then

return true

Else return false

The int-ctxt advantage of  $A$  is

$$\text{Adv}_{\mathcal{AE}}^{\text{int-ctxt}}(A) = \Pr[\text{INTCTXT}_{\mathcal{AE}}^A \Rightarrow \text{true}]$$

## Integrity with privacy

The goal of authenticated encryption is to provide both integrity and privacy. We will be interested in IND-CPA + INT-CTXT.

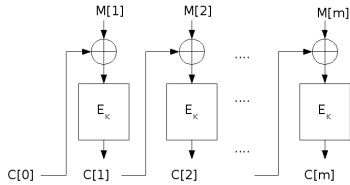
## Plain Encryption Does Not Provide Integrity

### Alg $\mathcal{E}_K(M)$

$C[0] \xleftarrow{\$} \{0, 1\}^n$   
 For  $i = 1, \dots, m$  do  
 $C[i] \leftarrow E_K(C[i-1] \oplus M[i])$   
 Return  $C$

### Alg $\mathcal{D}_K(C)$

For  $i = 1, \dots, m$  do  
 $M[i] \leftarrow E_K^{-1}(C[i]) \oplus C[i-1]$   
 Return  $M$



**Question:** Is CBC\$ encryption INT-CTXT secure?

**Answer:** No, because any string  $C[0]C[1] \dots C[m]$  has a valid decryption.

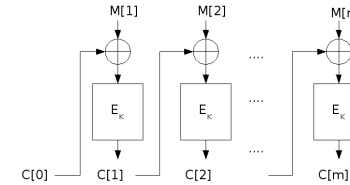
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 Return  $M$

### adversary $A$

$C[0]C[1]C[2] \xleftarrow{\$} \{0, 1\}^{3n}$   
 Return  $C[0]C[1]C[2]$

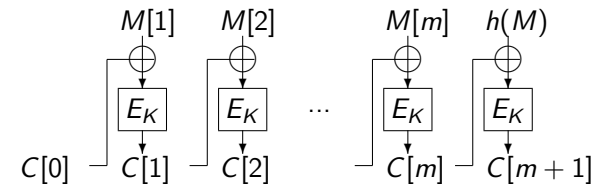
Then

$$\text{Adv}_{\mathcal{SE}}^{\text{int-ctxt}}(A) = 1$$

This violates INT-CTXT.

A scheme whose decryption algorithm **never** outputs  $\perp$  **cannot** provide **integrity!**

## Encryption with Redundancy

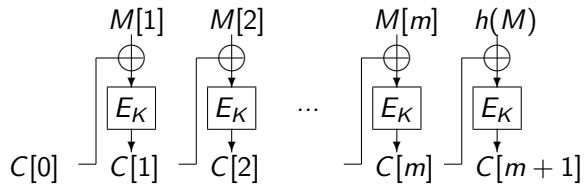


Here  $E: \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n$  is our block cipher and  $h: \{0, 1\}^* \rightarrow \{0, 1\}^n$  is a “redundancy” function, for example

- $h(M[1] \dots M[m]) = 0^n$
- $h(M[1] \dots M[m]) = M[1] \oplus \dots \oplus M[m]$
- A CRC
- $h(M[1] \dots M[m])$  is the first  $n$  bits of  $\text{SHA1}(M[1] \dots M[m])$ .

The redundancy is verified upon decryption.

## Encryption with Redundancy



Let  $E: \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n$  be our block cipher and  $h: \{0, 1\}^* \rightarrow \{0, 1\}^n$  a redundancy function. Let  $\mathcal{SE} = (\mathcal{K}, \mathcal{E}', \mathcal{D}')$  be CBC\$ encryption and define the encryption with redundancy scheme  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  via

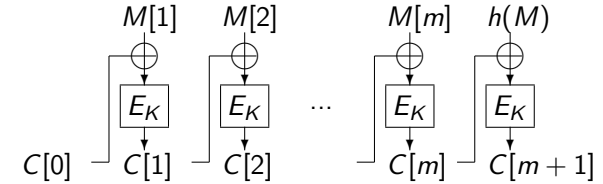
### Alg $\mathcal{E}_K(M)$

$M[1] \dots M[m] \leftarrow M$   
 $M[m+1] \leftarrow h(M)$   
 $C \xleftarrow{\$} \mathcal{E}'_K(M[1] \dots M[m]M[m+1])$   
 return  $C$

### Alg $\mathcal{D}_K(C)$

$M[1] \dots M[m]M[m+1] \leftarrow \mathcal{D}'_K(C)$   
 if  $(M[m+1] = h(M))$  then  
     return  $M[1] \dots M[m]$   
 else return  $\perp$

## Arguments in Favor of Encryption with Redundancy

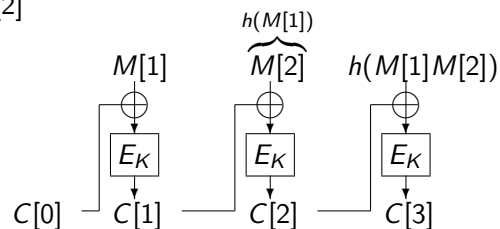


The adversary will have a hard time producing the last enciphered block of a new message.

## Encryption with Redundancy Fails

### adversary $A$

$M[1] \xleftarrow{\$} \{0, 1\}^n; M[2] \leftarrow h(M[1])$   
 $C[0]C[1]C[2]C[3] \xleftarrow{\$} \text{Enc}(M[1]M[2])$   
 Return  $C[0]C[1]C[2]$



This attack succeeds for any (not secret-key dependent) redundancy function  $h$ .

## WEP Attack

A “real-life” rendition of this attack broke the 802.11 WEP protocol, which instantiated  $h$  as CRC and used a stream cipher for encryption [BGW].

What makes the attack easy to see is having a clear, strong and formal security model.

## Generic Composition

Build an authenticated encryption scheme  $\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  by combining

- a given IND-CPA symmetric encryption scheme  $\mathcal{SE} = (\mathcal{K}', \mathcal{E}', \mathcal{D}')$
- a given PRF  $F : \{0, 1\}^k \times \{0, 1\}^* \rightarrow \{0, 1\}^n$

	CBC\$-AES	CTR\$-AES	...
HMAC-SHA1			
CMAC			
ECBC			
⋮			

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- a given PRF  $F : \{0, 1\}^k \times \{0, 1\}^* \rightarrow \{0, 1\}^n$

A key  $K = K_e || K_m$  for  $\mathcal{AE}$  always consists of a key  $K_e$  for  $\mathcal{SE}$  and a key  $K_m$  for  $F$ :

**Alg  $\mathcal{K}$**

$K_e \xleftarrow{\$} \mathcal{K}'$ ;  $K_m \xleftarrow{\$} \{0, 1\}^k$   
 Return  $K_e || K_m$

## Generic Composition Methods

The **order** in which the primitives are applied is important. Can consider

Method	Usage
Encrypt-and-MAC (E&M)	SSH
MAC-then-encrypt (MtE)	SSL/TLS
Encrypt-then-MAC (EtM)	IPSec

We study these following [BN].

## Encrypt-and-MAC

$\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

**Alg  $\mathcal{E}_{K_e || K_m}(M)$**

$C' \xleftarrow{\$} \mathcal{E}'_{K_e}(M)$   
 $T \leftarrow F_{K_m}(M)$   
 Return  $C' || T$

**Alg  $\mathcal{D}_{K_e || K_m}(C' || T)$**

$M \leftarrow \mathcal{D}'_{K_e}(C')$   
 If  $(T = F_{K_m}(M))$  then return  $M$   
 Else return  $\perp$

Security	Achieved?
IND-CPA	
INT-CTXT	

## Encrypt-and-MAC

$\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

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 If  $(T = F_{K_m}(M))$  then return  $M$   
 Else return  $\perp$

Security	Achieved?
IND-CPA	NO
INT-CTXT	

**Why?**  $T = F_{K_m}(M)$  is a deterministic function of  $M$  and allows detection of repeats.

## Encrypt-and-MAC

$\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

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Security	Achieved?
IND-CPA	NO
INT-CTXT	NO

**Why?** May be able to modify  $C'$  in such a way that its decryption is unchanged.

## MAC-then-Encrypt

$\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

**Alg**  $\mathcal{E}_{K_e||K_m}(M)$

$T \leftarrow F_{K_m}(M)$   
 $C \xleftarrow{\$} \mathcal{E}'_{K_e}(M || T)$   
 Return  $C$

**Alg**  $\mathcal{D}_{K_e||K_m}(C)$

$M || T \leftarrow \mathcal{D}'_{K_e}(C)$   
 If  $(T = F_{K_m}(M))$  then return  $M$   
 Else return  $\perp$

Security	Achieved?
IND-CPA	
INT-CTXT	

## MAC-then-Encrypt

$\mathcal{AE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is defined by

<b>Alg</b> $\mathcal{E}_{K_e  K_m}(M)$ $T \leftarrow F_{K_m}(M)$ $C \xleftarrow{\$} \mathcal{E}'_{K_e}(M  T)$ Return $C$	<b>Alg</b> $\mathcal{D}_{K_e  K_m}(C)$ $M  T \leftarrow \mathcal{D}'_{K_e}(C)$ If $(T = F_{K_m}(M))$ then return $M$ Else return $\perp$
---	---

Security	Achieved?
IND-CPA	YES
INT-CTXT	

**Why?**  $\mathcal{SE}' = (\mathcal{K}', \mathcal{E}', \mathcal{D}')$  is IND-CPA secure.

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Security	Achieved?
IND-CPA	
INT-CTXT	

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Security	Achieved?
IND-CPA	YES
INT-CTXT	YES

**Why?** If  $C || T$  is new then  $T$  will be wrong.

## Two keys or one?

We have used separate keys  $K_e, K_m$  for the encryption and message authentication. However, these can be derived from a single key  $K$  via  $K_e = F_K(0)$  and  $K_m = F_K(1)$ , where  $F$  is a PRF such as a block cipher, the CBC-MAC or HMAC.

Trying to directly use the same key for the encryption and message authentication is error-prone, but works if done correctly.



## Exercise

Let  $E = AES$ . Let  $\mathcal{K}$  return a random 128-bit AES key  $K$ . Let  $\mathcal{SE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  where  $\mathcal{E}, \mathcal{D}$  are below. Here,  $X[i]$  denotes the  $i$ -th 128-bit block of a string whose length is a multiple of 128.

### Alg $\mathcal{E}_K(M)$

```

if  $|M| \neq 512$  then return  $\perp$ 
 $M[1] \dots M[4] \leftarrow M$ 
 $C_e[0] \xleftarrow{\$} \{0, 1\}^{128}$   $C_m[0] \leftarrow 0^{128}$ 
for  $i = 1, \dots, 4$  do
   $C_e[i] \leftarrow E_K(C_e[i-1] \oplus M[i])$ 
   $C_m[i] \leftarrow E_K(C_m[i-1] \oplus M[i])$ 
 $C_e \leftarrow C_e[0]C_e[1]C_e[2]C_e[3]C_e[4]$ 
 $T \leftarrow C_m[4]$ ; return  $(C_e, T)$ 
    
```

### Alg $\mathcal{D}_K((C_e, T))$

```

if  $|C_e| \neq 640$  then return  $\perp$ 
 $C_m[0] \leftarrow 0^{128}$ 
for  $i = 1, \dots, 4$  do
   $M[i] \leftarrow E_K^{-1}(C_e[i]) \oplus C_e[i-1]$ 
   $C_m[i] \leftarrow E_K(C_m[i-1] \oplus M[i])$ 
if  $C_m[4] \neq T$  then return  $\perp$ 
return  $M$ 
    
```

## Exercise

1. Is  $\mathcal{SE}$  IND-CPA-secure? Why or why not?
2. Is  $\mathcal{SE}$  INT-CTXT-secure? Why or why not?
3. Is  $\mathcal{SE}$  an Encrypt-and-MAC construction? Justify your answer.

## Generic Composition in Practice

AE in	is based on	which in general is	and in this case is
SSH	E&M	insecure	secure
SSL	MtE	insecure	insecure
SSL + RFC 4344	MtE	insecure	secure
IPSec	EtM	secure	secure
WinZip	EtM	secure	insecure

Why?

- Encodings
- Specific “E” and “M” schemes
- For WinZip, disparity between usage and security model

## Authenticated encryption today

- Dedicated schemes: OCB, OCB $_x$  ( $x=1,2,3$ ), GCM, CCM, EAX
- TLS uses GCM
- CAESAR competition to standardize new schemes:  
<http://competitions.cr.jp.to/caesar.html>