CSE 150 – Homework 1

Out: Tue Aug 5
Due: Fri Aug 8 @ 5pm

1.1 Kullback-Leibler distance

Often it is useful to measure the difference between two probability distributions over the same random variable. For example, as shorthand let

$$\begin{aligned} p_i &= P(X = x_i|E), \\ q_i &= P(X = x_i|E') \end{aligned}$$

denote the conditional distributions over the random variable X for different pieces of evidence $E \neq E'$. The Kullback-Leibler (KL) distance between these distributions is defined as:

$$\mathrm{KL}(p, q) = \sum_i p_i \log(p_i/q_i).$$

(a) By sketching graphs of $\log x$ and $x - 1$, verify the inequality

$$\log x \leq x - 1,$$

with equality if and only if $x = 1$. Confirm this result by differentiation of $\log x - (x - 1)$. (Note: all logarithms in this problem are *natural* logarithms.)

(b) Use the previous result to prove that $\mathrm{KL}(p, q) \geq 0$, with equality if and only if the two distributions p_i and q_i are equal.

(c) Provide a counterexample to show that the KL distance is not a symmetric function of its arguments:

$$\mathrm{KL}(p, q) \neq \mathrm{KL}(q, p).$$

Despite this asymmetry, it is still common to refer to $\mathrm{KL}(p, q)$ as a measure of distance. Many algorithms in machine learning are based on minimizing KL distances between probability distributions.

1.2 Conditional independence

Show that the following three statements about random variables X, Y, and Z are equivalent:

$$\begin{aligned} P(X, Y|Z) &= P(X|Z)P(Y|Z) \\ P(X|Y, Z) &= P(X|Z) \\ P(Y|X, Z) &= P(Y|Z) \end{aligned}$$

Do become fluent with all these ways of expressing that X is conditionally independent of Y given Z.

1.3 Creative writing

Attach events to the binary random variables X, Y, and Z that are consistent with the following patterns of common sense reasoning. You may use different events for the different parts of the problem.

(a) Explaining away:

$$\begin{aligned} P(X=1|Y=1) &> P(X=1), \\ P(X=1|Y=1,Z=1) &< P(X=1|Y=1) \end{aligned}$$

(b) Accumulating evidence:

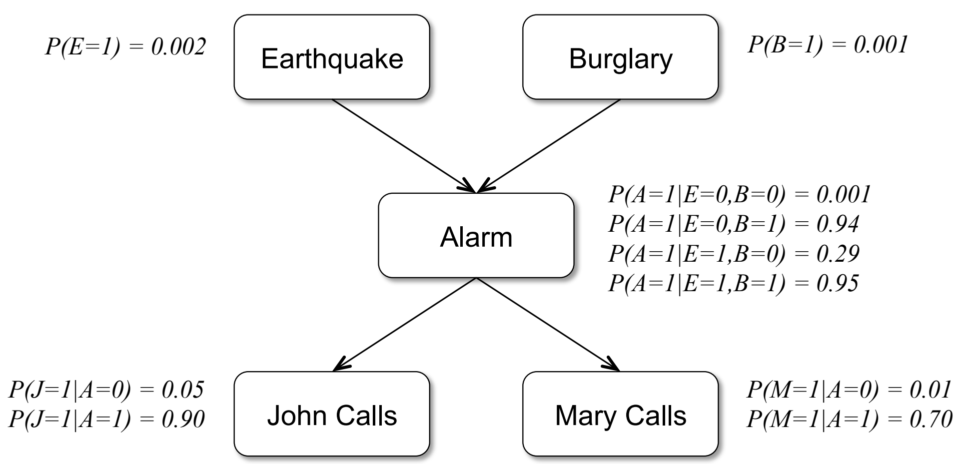
$$P(X=1) < P(X=1|Y=1) < P(X=1|Y=1,Z=1)$$

(c) Conditional independence:

$$\begin{aligned} P(X=1,Y=1) &\neq P(X=1)P(Y=1) \\ P(X=1,Y=1|Z=1) &= P(X=1|Z=1)P(Y=1|Z=1) \end{aligned}$$

1.4 Probabilistic inference

Recall the probabilistic model that we described in class for the binary random variables $\{E = \text{Earthquake}, B = \text{Burglary}, A = \text{Alarm}, J = \text{John Calls}, M = \text{Mary Calls}\}$. We also expressed this model as a belief network, with the directed acyclic graph (DAG) and conditional probability tables (CPTs) shown below:



Compute numeric values for the following probabilities, exploiting relations of conditional independence as much as possible to simply your calculations. You may re-use numerical results from lecture, but otherwise show your work. Be careful not to drop significant digits in your answer; give final answers to three decimal places.

(a) $P(E\!=\!1\|A\!=\!1)$	(c) $P(A\!=\!1\|M\!=\!1)$	(e) $P(A\!=\!1\|J\!=\!0)$
(b) $P(E\!=\!1\|A\!=\!1,B\!=\!0)$	(d) $P(A\!=\!1\|M\!=\!1,J\!=\!0)$	(f) $P(A\!=\!1\|J\!=\!0,B\!=\!1)$

Consider your results in (b) versus (a), (d) versus (c), and (f) versus (e). Do they seem consistent with common sense patterns of reasoning?