

ANALYTIC SPHERICAL HARMONIC GRADIENTS FOR REAL-TIME RENDERING WITH MANY POLYGONAL AREA LIGHTS

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PROBLEM

Real-time rendering in precomputed radiance transfer (PRT) with many polygonal area lights is challenging because:

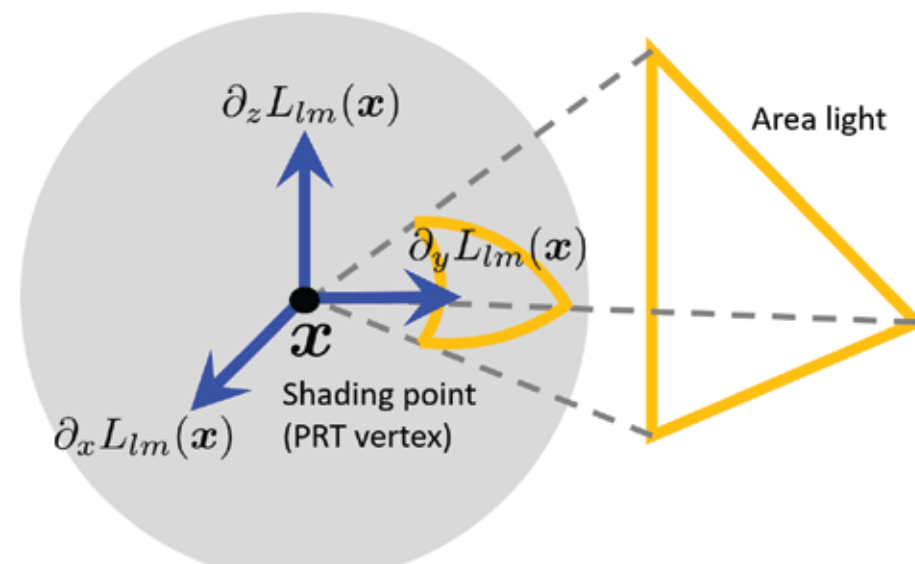
- Near-field area lights result in spatially varying spherical harmonic (SH) lighting coefficients.
- The running time of computing SH lighting coefficients is linear in the number of lights.



To render this scene with 713 area lights, state-of-the-art method [4] takes about 2 minutes, while our method takes only 28 milliseconds.

THEORY

- SH projection from a polygonal area light



SH coefficient (integrating SH basis function over the projected spherical triangle $Q(x)$)

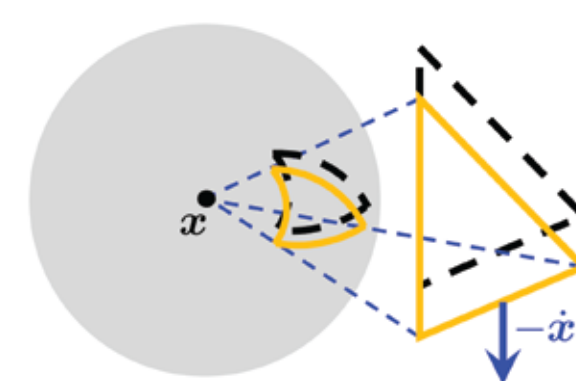
$$L_{lm}(x) = \int_{Q(x)} Y_{lm}(\omega) d\omega$$

SH gradients (partial derivatives w.r.t. the location of the shading point)

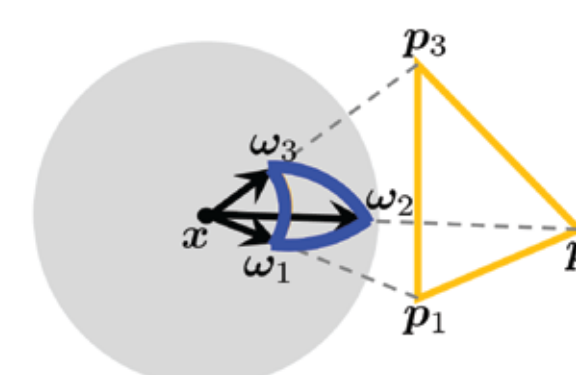
$$\nabla L_{lm}(x) = (\partial_x L_{lm}(x), \partial_y L_{lm}(x), \partial_z L_{lm}(x))$$

- Analytic derivation of SH gradients

$$\partial_z \int_{Q(x)} Y_{lm}(\omega) d\omega = \int_{\partial Q(x)} \langle n_{\perp}, \hat{\omega} \rangle Y_{lm}(\omega) d\ell(\omega) + \int_{Q(x)} \partial_z [Y_{lm}(\omega)] d\omega$$



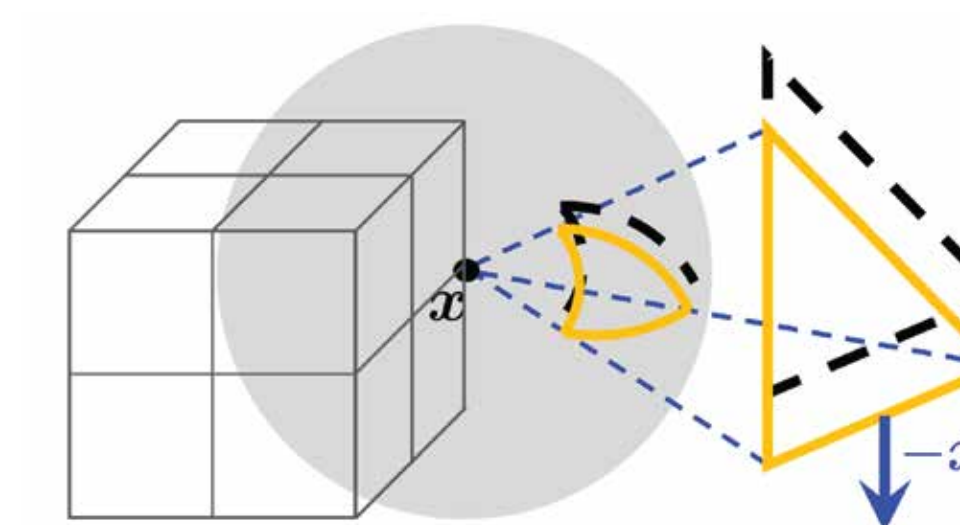
• **Interior term** is zero because the SH basis function is independent of the spatial location.



• **Boundary term** is due to the moving boundary of the integration domain. It is a 1D integral over the edges of the spherical polygon, which can be solved analytically.

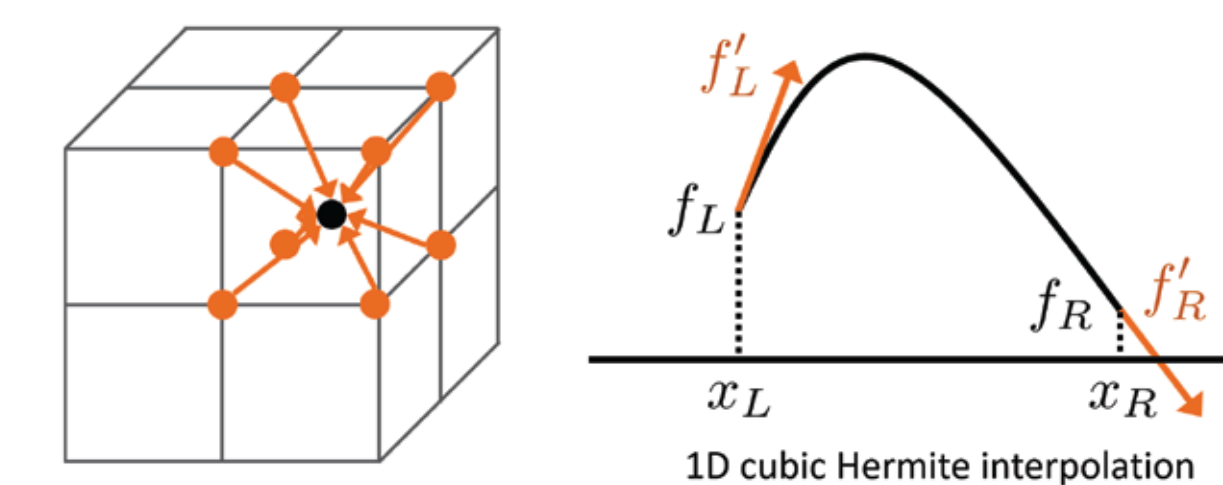
ALGORITHM

- Computing SH coefficients and gradients on a 3D grid



```
// SH Evaluation on a 3D grid
Build a uniform 3D grid of resolution M^3
for each grid point x do
  for each area light in the scene do
    Accumulate L_lm(x) and ∇L_lm(x)
  end for
end for
```

- Interpolating SH coefficients at any intermediate points



```
// Gradient-based Interpolation for PRT vertices
for each vertex v do
  Find its eight adjacent grid points
  for each SH basis (l, m) do
    Fetch L_lm and ∇L_lm at the eight grid points
    Hermite interpolate L_lm(v)
  end for
end for
```

CONTRIBUTIONS

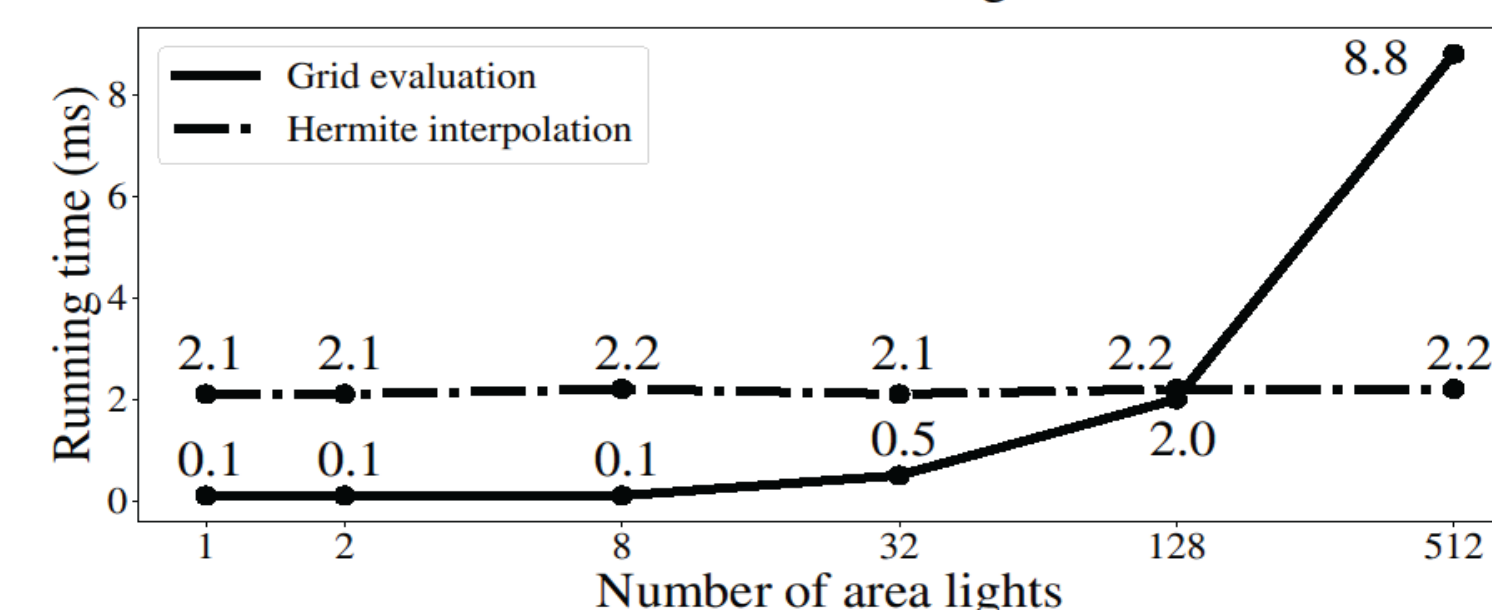
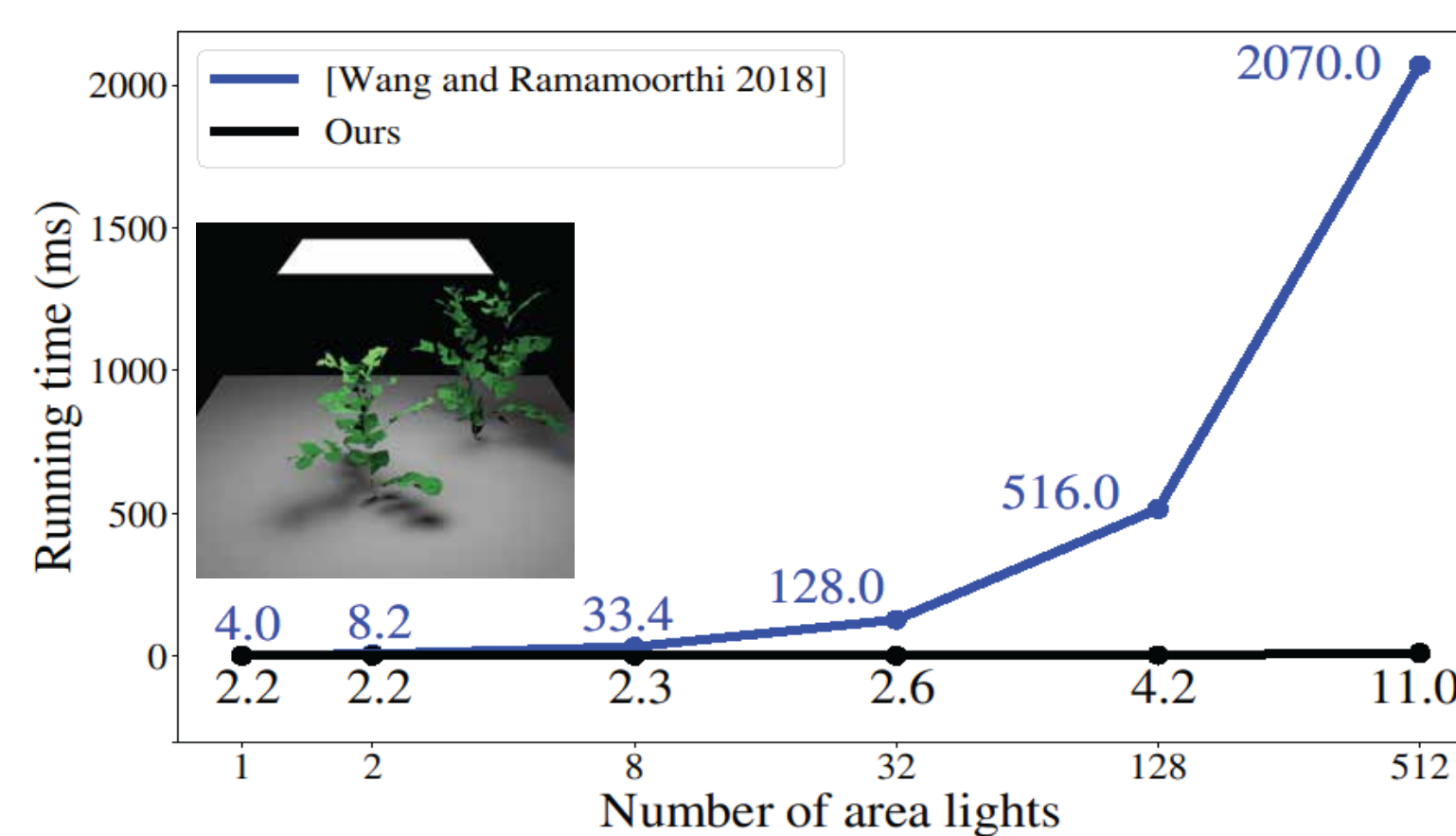
- A novel analytic formula for SH gradients by reducing the problem to a 1D boundary integral.
- An accurate tricubic Hermite interpolation based on the SH gradients from a sparse set of samples.
- Scaling real-time PRT to hundreds of area lights, which was intractable before.

RELATED WORK

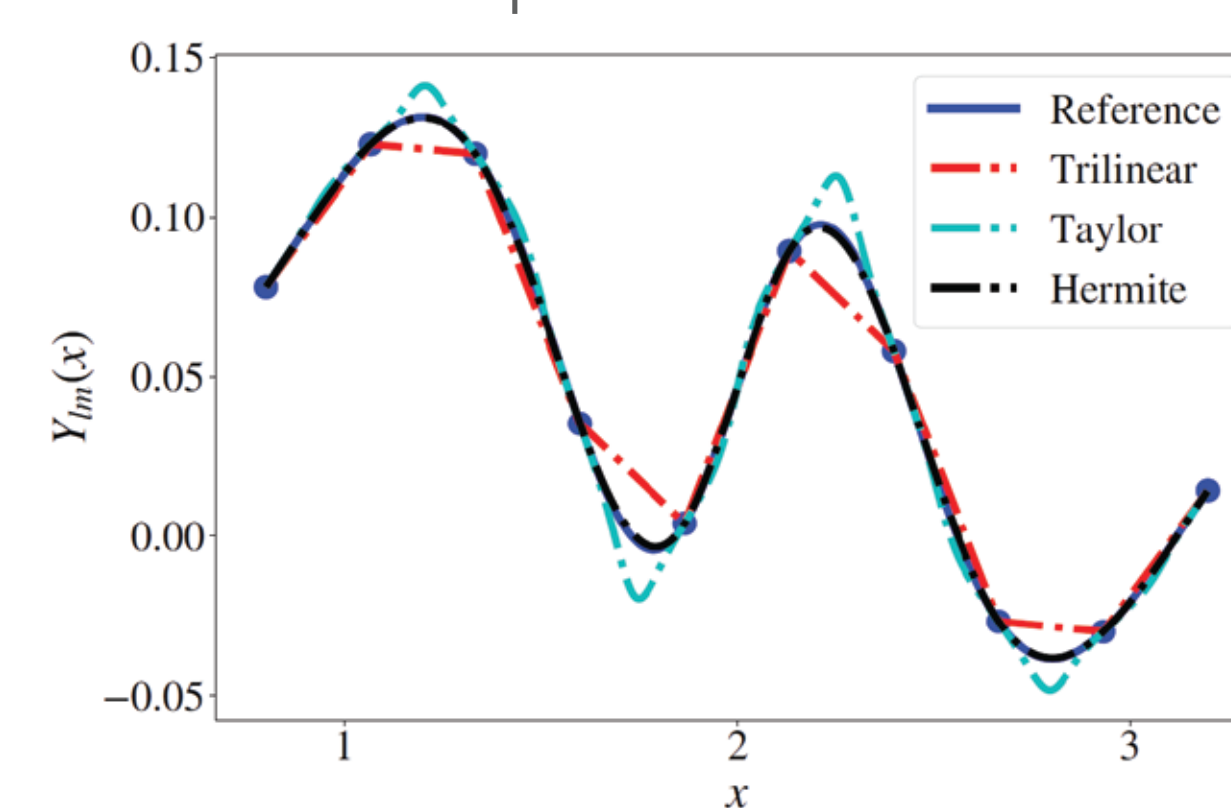
- Precomputed radiance transfer with spherical harmonics [3]: a classic real-time rendering technique.
- Analytic SH coefficients for area lights [1, 4]: making near-field area lighting possible in PRT, but limited to a small number of area lights.
- Reynolds transport theorem [2]: a general theory for differentiating high-dimensional integrals, widely used in differentiable rendering.

RESULTS

- Performance evaluation: our method can scale to hundreds of area lights in real-time



- Accuracy evaluation: our Hermite interpolation is the most accurate method compared to the baselines



- Renderings (generated with an NVIDIA RTX 2080 Ti)



35.8 fps
723 area lights
1.28M polygons



56.5 fps
344 area lights
1.71M polygons

REFERENCES

- [1] L. Belcour, G. Xie, C. Hery, M. Meyer, W. Jarosz, and D. Nowrouzezahrai. 2018. Integrating clipped spherical harmonics expansions. ACM TOG 37, 2 (2018), 19:1–19:12.
- [2] L. Gary Leal. 2007. Advanced Transport Phenomena: fluid mechanics and convective transport processes. Vol. 7. Cambridge University Press.
- [3] P. Sloan, J. Kautz, and J. Snyder. 2002. Precomputed Radiance Transfer for Real-Time Rendering in Dynamic, Low-Frequency Lighting Environments. ACM TOG 21, 3 (2002), 527–536.
- [4] J. Wang and R. Ramamoorthi. 2018. Analytic Spherical Harmonic Coefficients for Polygonal Area Lights. ACM TOG 37, 4 (2018), 54:1–54:11.