

T. C. HU

Career Summary

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Unlike many computer scientists who concentrate their efforts on discovering new NP-complete problems, trying to prove  $P \neq NP$  or establishing new theoretical models, Professor Hu's efforts have been concentrated on:

- (1) inventing new algorithms that can solve well-known problems more efficiently than existing algorithms;
- (2) unifying the central concepts of existing algorithms and extending cost functions; and
- (3) discovering efficient algorithms with error bounds for NP-complete problems.

Examples of (1) would include his paper, A6, which introduced a data structure known as the Gomory-Hu tree that reduce the problem complexity from  $\binom{n}{2}$  to  $n-1$ , and other results summarized below.

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Typical of (2) include A41 which extends the applicability of Huffman's tree and the Hu-Tucker tree to a wide class of cost functions, and A87 which combines the shortest paths and minimum spanning tree algorithms.

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One striking feature of Professor Hu's research is that many of his papers either: (a) is the first paper on a new concept or problem; or (b) present an algorithm which remains the best for its problem since its publication 10, 20 or 30 years ago.

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