CORRIGENDA AND COMMENTS ON CHAPTER 2

Correction 1 (Page 6). *Just before Equation 11, "A moments reflection ..." should be "A* moment's *reflection ..."*

Correction 2 (Page 12).

$$\mathsf{T} = \prod_{i=0}^{p}{}^{\bullet}\, \mathsf{T}_i$$

should be

$$\mathsf{T} = \sum_{i=0}^{p}{}^{\bullet}\, \mathsf{T}_i$$

Comment 1 (Page 27). *Starting with Equation 13 and throughout the remainder of the chapter, it should be noted that in formulas of this type one needs either the characteristic of the field R to be zero, or that the characteristic of R not be a divisor of $|H|$.*

Comment 2 (Page 30). *In Example 2.5 (c), in the trivial case $m = 1$, $\mathsf{S}_\epsilon = \mathsf{S}_1$ and so $\mathsf{S}_\epsilon \mathsf{S}_1 = \mathsf{S}_1$. Thus one must assume $m \geq 2$.*

Correction 3 (Page 33). *In the second line of the equation in Problem 1,*

$$\ldots = \lambda(\mathsf{S}^{-1}\mathsf{S})_{i1} = \lambda\delta_{i1}$$

should be

$$\ldots = \lambda_1(\mathsf{S}^{-1}\mathsf{S})_{i1} = \lambda_1\delta_{i1}$$

Correction 4 (Page 52). *The lower limit of the sum is missing in the equation:*

$$\varkappa(e^*_\omega) = \frac{1}{|\mathsf{S}_m|} \sum_{\sigma \in \mathsf{S}_m} \epsilon(\sigma) \prod_{t=1}^{m} \left(\sum^{n} b_{it} f_i(e_{\omega\sigma^{-1}(t)}) \right)$$

It should be:

$$\varkappa(e^*_\omega) = \frac{1}{|\mathsf{S}_m|} \sum_{\sigma \in \mathsf{S}_m} \epsilon(\sigma) \prod_{t=1}^{m} \left(\sum_{i=1}^{n} b_{it} f_i(e_{\omega\sigma^{-1}(t)}) \right)$$

Correction 5 (Page 54)**.** *In the last line of Equation 29:*

$$= \delta_{\alpha\beta} \sum_{\sigma \in H_\alpha} \chi(\sigma)$$

the subscript α *of* H_α *is almost cut off, presumably by the page number.*

Correction 6 (Page 59)**.** *In the first set of equations in Exercise 9, the product symbol is missing from last line. It should be:*

$$= \sum_{\omega \in G_{n,n}} \frac{1}{\nu(\omega)} \prod_{t=1}^{n} \lambda_{\omega(t)} |per \mathsf{U}[\omega|1, \ldots, n]|^2$$

Correction 7 (Page 63)**.** *In line 10,*

$$\left(\bigotimes_1^m \mathsf{V}^* \right)^*$$

should be

$$\left(\bigotimes_1^m \mathsf{V} \right)^*$$

Comment 3 (Page 64)**.** *The third equality:*

$$= \frac{1}{|\mathsf{H}|} \sum_{\sigma \in \mathsf{H}} \chi(\sigma) \varphi(f_1, \ldots, f_m)(v_{\sigma^{-1}(1)} \otimes \cdots \otimes v_{\sigma^{-1}(m)})$$

seems unnecessary, and the next equality seems to follow from the definitions.

Correction 8 (Page 69)**.** *In the fourth line of the proof of part (d) of Theorem 4.2, "... are invariant subspaces of* $\bigotimes_1^m \mathsf{T} \ldots$*" should be "... are invariant subspaces of* $\bigotimes_1^m \mathsf{V} \ldots$*"*

Correction 9 (Page 70)**.** *In the second line,* $= (\mathsf{S}u^\otimes, \mathsf{S}(\bigotimes_1^m \mathsf{T})\mathsf{S}v^\otimes)$ *should be* $= (\mathsf{S}u^\otimes, (\bigotimes_1^m \mathsf{T})\mathsf{S}v^\otimes)$.

Correction 10 (Page 78)**.** *In the first line,*

$$\text{tr } \mathsf{C}_m(\mathsf{A}) = \sum_{\alpha \in Q_{m,n}} \prod_{t=1}^{n} \lambda_t^{m_t(\omega)}$$

should be

$$\text{tr } \mathsf{C}_m(\mathsf{A}) = \sum_{\alpha \in Q_{m,n}} \prod_{t=1}^{n} \lambda_t^{m_t(\alpha)}$$

Correction 11 (Page 79)**.** *In Example 4.4 (f),* A *needs to be invertible.*

Correction 12 (Page 85)**.** *In Theorem 4.5 (c), per Exercise 22, the characteristic of* R *must be zero.*

Correction 13 (Page 87)**.** *In the proof of Theorem 4.5 (b) in line 9, the upper limit of the sum is incorrect in:* $y_i = \sum_{j=1}^{n} c_{ij}x_j$*; it should be* $y_i = \sum_{j=1}^{m} c_{ij}x_j$.

Correction 14 (Page 89)**.** *In the first line, in the equality* $z_\theta^* = z^*$ *the stars are clipped at the top.*

Comment 4 (Page 92)**.** *In example 4.6 (d) it is stated "It clearly suffices to prove this for* $\mathsf{T}_2 > 0$*." Try as I might, I can't find an easy (or any) proof of this in general. If it happens to be the case that both* T_1 *and* T_2 *are diagonal with respect to the same orthonormal basis, then the result follows by restricting both operators to the subspace spanned by the eigenvectors of* T_2 *corresponding to non-zero eigenvalues. But there's no reason to suppose that such a basis exists.*

Correction 15 (Page 92)**.** *In the last line "equivalently* $\mathsf{K}(\mathsf{T}_1) \geq \mathsf{K}(\mathsf{T}_2) \geq 0$*" should be "...*$\mathsf{K}(\mathsf{T}_1) \geq \mathsf{K}(\mathsf{T}_2) > 0$*" according to the assumption discussed in the previous comment.*

Correction 16 (Page 94)**.** *In line 10, "...*since $m < r < n$*" should be "...*since $m < r \leq n$*."*

Correction 17 (Page 95)**.** *In lines 2 and 3,* $c_1 = \cdots = c_m = c$ *and* $c_{m+1} = \cdots = c_r = c$ *should be, respectively,* $c_1 = \cdots = c_{m+1} = c$ *and* $c_{m+2} = \cdots = c_r = c$.

Correction 18 (Page 99)**.** *In Exercise 9, one needs to assume that* $char(R) \neq 2$ *in order to use Equation (7) of Section 2.3. In the characteristic 2 case, if* $v_i = v_j$*, just note that if* σ *is a permutation such that* $\sigma(i) = k$ *and* $\sigma(j) = l$*, there is permutation* σ' *such that* $\sigma'(i) = l$, $\sigma'(j) = k$ *and* $\sigma'(p) = \sigma(p)$ *otherwise. Then* $v_{\sigma(1)} \otimes \cdots \otimes v_{\sigma(m)} = v_{\sigma'(1)} \otimes \cdots \otimes v_{\sigma'(m)}$*, and so* $v_{\sigma(1)} \otimes \cdots \otimes v_{\sigma(m)} + v_{\sigma'(1)} \otimes \cdots \otimes v_{\sigma(m)} = 0$*. Pairing all permutations in* S_m *this way yields the result.*

Comment 5 (Page 100)**.** *In Exercise 11, it is stated, "Hence* $\mathrm{d}_\chi^{\mathrm{H}}(\mathsf{X}) = 0$ whenever $\mathrm{get}(\mathsf{X}) = 0$ *implies* $\mathrm{d}_\chi^{\mathrm{H}}(\mathsf{X})$ *is divisible by* $\det(\mathsf{X})$*." another statement I can't easily figure out. Anyway, there's no need to go through these manipulations of multivariable polynomials. The following proof works as well:*

Suppose first that H *is a proper subgroup of* S_n*. Then there exists a transposition* $(pq) \notin S_n$*. Let* X *be a matrix defined as follows:*

$$\begin{aligned} \mathsf{X}_{kk} &= 1, k = 1, \ldots, n; \\ \mathsf{X}_{pq} &= \mathsf{X}_{qp} = 1; \\ \mathsf{X}_{kl} &= 0 \text{ otherwise.} \end{aligned}$$

Since rows p and q of X *are equal,* $\det(\mathsf{X}) = 0$. *Now compute* $\mathrm{d}_\chi^{\mathrm{H}}(\mathsf{X})$. *If* $k \neq p, q$ *and* $\sigma(k) \neq k$, $\mathsf{X}_{k\sigma(k)} = 0$*; hence*

$$\prod_{1}^{n} \chi(\sigma)\mathsf{X}_{j\sigma(j)} = 0$$

So only permutations σ *satisfying* $\sigma(k) = k$ for $k \neq p, q$ *contribute to the value of* $\mathrm{d}_\chi^{\mathrm{H}}(\mathsf{X})$. *But there are only two such permutations: the identity and* (pq)*, and the latter is not in* H *by hypothesis. So* $\mathrm{d}_\chi^{\mathrm{H}}(\mathsf{X}) = \mathsf{X}_{11} \cdots \mathsf{X}_{nn} = 1 \neq 0 = \det(\mathsf{X})$. *So the conclusion is that if* $\mathrm{d}_\chi^{\mathrm{H}}(\mathsf{X}) = 0$ *whenever* X *is singular, then* $H = S_n$. *But then by Exercise 13 of Section 2,* $\chi \equiv 1$ *or* $\chi = \epsilon$ *(the sign function). In the first case,* $\mathrm{d}_\chi^{\mathrm{H}}(\mathsf{X}) = \mathrm{per}(\mathsf{X})$*; if* X *is the matrix with a* 1 *in every entry,* X *is singular but* $\mathrm{per}(\mathsf{X}) = 1$. *So* $\chi = \epsilon$ *and* $\mathrm{d}_\chi^{\mathrm{H}}$ *is the determinant.*

Correction 19 (Page 103)**.** *In Exercise 21, the reader should be asked to, "Show that there exists a* non-zero $f \in \mathcal{V}^* \ldots$*"*

Correction 20 (Page 105)**.** *Two corrections to Exercise 26. First, the hypothesis* $\rho(\mathsf{T}) > m$ *is needed to Apply Theorem 4.6. Second, the hint should read, "The transformation* $\mathrm{i}K(\mathsf{T}) \ldots$*"*

Comment 6 (Pages 107, 108)**.** *In Exercise 31, on Page 107, the decomposition*

$$\mathsf{U}^*\mathsf{AU} = \begin{bmatrix} \lambda_1 & * & \ldots\ldots\ldots\ldots & & & * \\ 0 & \lambda_2 & * & \ldots\ldots\ldots & & * \\ \vdots & \vdots & \vdots & \vdots & \ldots\ldots & \vdots \\ 0 & \ldots\ldots & 0 & \lambda_p & * & * \\ 0 & \ldots\ldots\ldots\ldots\ldots & & & & 0 \\ \vdots & \ldots\ldots\ldots\ldots\ldots & & & & \vdots \\ 0 & \ldots\ldots\ldots\ldots\ldots & & & & 0 \end{bmatrix}$$

is essentially the Schur Decomposition; calling the right side of this equation T*, the Schur Decomposition of* A *is* $\mathsf{A} = \mathsf{UTU}^*$. *On Page 108, I still haven't been able to figure out the statement, "Since* SS^* *is non-singular, it follows that* $\mathsf{LL}^* = 0$*."*