CORRIGENDA AND COMMENTS ON CHAPTER 3

Comment 1 (Page 1). *The properties*

$$r(u+v) = ru + rv$$
$$(r+s)u = ru + su$$

of the algebra $\mathfrak{A}$ *follow from the fact that* $\mathfrak{A}$ *is an* R*-module.*

Comment 2 (Page 18). *At the bottom of the page, in the statement, "It follows that since* $e^{\wedge}_{\omega}$*,* $\omega \in Q_{m,n}$ *is a basis of* $\overset{m}{\wedge} V$*,* $c_\omega = 0$*,* $\omega \in Q_{m,n}$*," the fact that* $e^{\wedge}_{\omega}$*,* $\omega \in Q_{m,n}$ *is a basis follows from Theorem 3.2 of Chapter 2.*

Comment 3 (Page 19). *Equation 32 is, strictly speaking, an isomorphism rather than an equality. This applies as well to the equation*

$$q(V_0^m) = V^{(m)}$$

near the bottom of Page 22 and again in the sentence following Equation 37.

Comment 4 (Page 26, Theorem 1.5). *In the statement of the theorem,* $h(z \wedge w) = h(z)h(w)$ *is superfluous since it is already stated that* h *is a homomorphism. The same comment applies to Theorem 1.6 on page 27.*

Comment 5 (Page 36, Exercise 7). *"... if* X *is a spanning set..." is unnecessary. The polynomial ring* $\mathbb{K}[x]$ *over the field* $\mathbb{K}$ *is generated by the linearly dependent set* $\{1, x, 1+x\}$ *which does not span* $\mathbb{K}[x]$*, but it's not possible to define a homomorphism that takes on arbitrary values on the elements of this generating set.*

Comment 6 (Page 39, Exercise 12). *The equality* $0 = a^{-1}(ab_g) = b_g$ *requires that* $\mathfrak{A}$ *be associative.*

Correction 1 (Page 40, Exercise 16). *"... each of length* $n+1$*." should be "... each of length* $\geq n+1$*."*

Correction 2 (Page 45, Exercise 24). $\bigotimes_1^m V$ *should be* $\times_1^m V$*.*

Correction 3 (Page 49, Exercise 28).

$$J_n \sum_{k=0}^{m} c_k J_n A^k$$

should be

$$\sum_{k=0}^{m} c_k J_n A^k;$$

there's an extra occurrence of J_n*. Also, the next to last sentence should start out, "But* $J_n A^{\frac{1}{2}} = cJ_n \ldots$*"; the subscript* n *is missing from the second* J*.*

Correction 4 (Page 49, Exercise 29)**.** *Example 4.6 (d) of Chapter 2 is unrelated to the van der Waerden Conjecture, which is mentioned in Exercise 9 of Section 2.3. At the end of the next to last line,* $A^{(j)}$ *should be* $A_{(j)}^{\frac{1}{2}}$*.*

Correction 5 (Page 59)**.** *The next to last expression should be*

$$q\left(\sum_{k=1}^{m} x_1 \otimes \cdots \otimes Ax_k \otimes \cdots \otimes x_m\right).$$

The subscripts 1 *and* k *are missing.*

Correction 6 (Page 62)**.** *The end of the third line of text following Equation (22) should be* $\sigma^{-1}(k)$*.*

Comment 7 (Page 63)**.** *In the equation*

$$\pi_\omega(T_1, \ldots, T_p) = \otimes_{j=1}^{m} X_j$$

the subscript ω *isn't defined. It seems to mean the* p*-tuple* $(\omega^1, \ldots, \omega^p)$*.*

Comment 8 (Page 64)**.** *In Equation (26),* $\prod$ *indicates composition of the* $\pi_{\omega^i(j)}$*.*

Comment 9 (Page 65)**.** *The reader should be aware that it is not necessarily true that* $\sigma\omega^i(1) < \ldots < \sigma\omega^i(r_i)$*. What* is *true is that* $\{\sigma\omega^i(1), \ldots, \sigma\omega^i(r_i)\}$ *is the image of some element of* $Q_{r_i,m}$*.*

Correction 7 (Page 70)**.** *The last equation should be*

$$\varphi\left(\sum_{k=1}^{t-1} r_k + j\right) = \omega^t(j)$$

(φ *and not* φ^{-1} *of the sum).*

Correction 8 (Page 70)**.** *The last sentence on the page, carrying over to Page 71, is not correct. In fact, there is precisely* one *permutation* φ *that satisfies the preceeding equalities. However, there are* $r_1! \cdots r_p!$ *permutation* φ *that satisfy*

$$\varphi\left(\sum_{k=1}^{t-1} r_k + j\right) \in \{\omega^t(1), \ldots, \omega^t(r_t)\}, \quad j = 1, \ldots, r_t, \quad t = 1, \ldots, p$$

Comment 10 (Page 72, Equation 33)**.** *Here also,* ω *is implicitly defined as* $(\omega^1, \ldots, \omega^p)$ *where* $\omega^i \in Q_{r_i,m}$*.*

Correction 9 (Page 75)**.** *In the last paragraph, the equation* $T_i e^{i\alpha(i)} = \lambda_{i\alpha(i)} e_{i\alpha(i)}$ *should be* $T_i e^{i\alpha(j)} = \lambda_{i\alpha(j)} e_{i\alpha(j)}$ *with* i *and* j *varying appropriately.*

Correction 10 (Page 77)**.** *The next to last equation is missing the summation on the left side; it should be*

$$= \frac{|H|}{\nu(\alpha)} \sum_{\omega \in Q_{r,m}} \cdots$$

Also see comments 7 and 10 about ω.

Comment 11 (Page 81)**.** *In the equation following Equation (44), the equality on the right is true but a little confusing. The partial derivation*

$$D(\overbrace{T_1, \dots, T_1}^{r_1}, \dots, \overbrace{T_p, \dots, T_p}^{r_p})$$

corresponds to the partition $1 + \cdots + 1 = m$*, while the partial derivation*

$$D(T_1, \dots, T_p)$$

corresponds to the partition $r_1 + \cdots + r_p = m$*.*

Comment 12 (Page 91)**.** *Equation 58 follows because the contrapositive of what was just shown is: if* f *is a non-trivial linear functional on* $\langle \prod_1^m A, A \in M_n(R) \rangle$*, it is non-trivial on* $\langle \prod_1^m A, A \in M_n(R), \det A \neq 0 \rangle$*. But if* $\mathcal{X} \subseteq \mathcal{Y}$ *are subspaces of a vector space* $\mathcal{V}$*, the containment is proper (i.e.* $\mathcal{X} \neq \mathcal{Y}$*) iff there is an* $f \in \mathcal{V}^*$ *that is non-trivial on* $\mathcal{Y}$ *such that* $\mathcal{X} \subseteq \ker f$

Correction 11 (Page 92, Exercise 2)**.** *"If* $\deg v = 0$*, say* $v = r$*, then the left side of (3) is* $rh(v)$ *and the right side is* $h(v)r$*." This should be: "If* $\deg v = 0$*, say* $v = r$*, then the left side of (3) is* $rh(u)$ *and the right side is* $h(u)r$*."*

Correction 12 (Page 97, Exercise 14)**.** *By hypothesis,* $f(a_1, \dots, a_p) = 0$ *whenever* $g_t(a_1, \dots, a_p) \neq 0, \quad t = 1, \dots, r$*, so if any of the* g_t *are the zero polynomial, the problem is vacuous. The injunction to discard such* g_t *is superfluous.*

Correction 13 (Page 99, Exercise 16)**.** $D_1 D_2$ *is a derivation of* $\mathfrak{A}$ *of degree* $p_1 + p_2$*. The problem omits the degree of* $D_1 D_2$*.*

Correction 14 (Page 104, Exercise 25)**.** *Should start, "Let* $\varphi \in M_2(\mathcal{V}, R, S_2, 1) \dots$*" The group character is identically* 1*, not the sign of the permutation.*

Correction 15 (Page 110, Exercise 32)**.** *In the first line of the page,* z_{jk} *should be* z_{kj}*.*

Comment 13 (Page 110, Exercise 32)**.** *In the last part of the hint, which starts out, "Write $(-1)^j = -(-1)^{j-1} \ldots$" it's not clear (to me) what M. had in mind. The following works:*

$$\sum_{k=1}^{m}(-1)^{k-1}\left[\sum_{j=1}^{k-1}(-1)^{j-1}\varphi(x_j,x_k)z_{jk}+\sum_{j=k+1}^{m}(-1)^{j}\varphi(x_k,x_j)z_{kj}\right] =$$

$$\sum_{k=1}^{m}\left[\sum_{j=1}^{k-1}(-1)^{j+k}\varphi(x_j,x_k)z_{jk}-\sum_{j=k+1}^{m}(-1)^{j+k}\varphi(x_k,x_j)z_{kj}\right] =$$

$$\sum_{k=1}^{m}\sum_{j=1}^{k-1}(-1)^{j+k}\varphi(x_j,x_k)z_{jk}-\sum_{k=1}^{m}\sum_{j=k+1}^{m}(-1)^{j+k}\varphi(x_k,x_j)z_{kj}$$

Interchange the order of summation in the second term:

$$\begin{aligned}\sum_{k=1}^{m}\sum_{j=k+1}^{m}(-1)^{j+k}\varphi(x_k,x_j)z_{kj} &= \sum_{j=1}^{m}\sum_{k=1}^{j-1}(-1)^{j+k}\varphi(x_k,x_j)z_{kj}\\ &= \sum_{k=1}^{m}\sum_{j=1}^{k-1}(-1)^{j+k}\varphi(x_j,x_k)z_{jk}\end{aligned}$$

where the last equality results from interchanging j and k. Note also that this calculation does't use symmetry of φ since the interchange of x_k and x_j occurs because of interchanging subscripts, not arguments.

Correction 16 (Page 112, Exercise 36)**.** *The next to last line of the page should read:*

$$\sigma_1\{\alpha^t(1),\ldots,\alpha^t(r_t)\}=\{\gamma^t(1),\ldots,\gamma^t(r_t)\},\qquad t=1,\ldots,p$$

In any case, σ_1 never appears in the proof.

Comment 14 (Page 114, Exercise 37)**.** *There is a parenthetical comment: "since σ^{-1} and σ both run over H" but σ^{-1} never appears in the proof.*

Correction 17 (Page 118, Exercise 44)**.** *The recommended computation is quite confusing. In the very first equality of the chain, $\sum_{k=1}^{p}(\mu_k T_k e_i, e_j) = \sum_{k=1}^{p}\mu_k(A_k)_{ji}$. So there needs to be some operation to gather all the entries of the A_k into matrices in this expression. I couldn't figure out how to fix this. Further down, there are two consecutive sums in which$(D(T_1,\ldots,T_p)e^\star, e^\star)$ is missing modulus bars.*

Correction 18 (Page 120, Exercise 45)**.** *In the multiple equalities following the phrase, "and subtracting we have" in the right side of the first equality the range of summation of* *the second sum should be* $k > m$. *The sum over the range* $k < m$ *appears twice on the right of the next equality.*

Correction 19 (Page 121, Exercise 47)**.** *In the first equation, the range of summation should be* $\varphi(2k-1) < \varphi(2k)$ *(see Page 123).*

Correction 20 (Page 124)**.** *Example 2.1 should be (a) to (c); Example 2.6 should be pp. 65 - 71.*