Comprehensive Introduction to Linear Algebra

WEB VERSION

Joel G. Broida

S. Gill Williamson

$$N = \begin{bmatrix} [a_{11} & a_{12} & \cdots & a_{1n}] \\ [a_{21} & a_{22} & \cdots & a_{2n}] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{m1} & a_{m2} & \cdots & a_{mn}] \end{bmatrix} \qquad C = \begin{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} & \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} & \cdots & \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} \end{bmatrix}$$

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Web Preface (2012)

The book, *A Comprehensive Introduction to Linear Algebra* (Addison-Wesley, 1986), by Joel G. Broida and S. Gill Williamson divides naturally into three parts:

Part I - Basic Linear Algebra covers Chapters 0 through 5. Chapters 0 and 1 are for review as needed. Chapters 2 through 5 are suitable for a first course in linear algebra for upper division undergraduates.

Part 2 - Polynomials and Canonical Forms covers Chapters 6 through 8. Chapter 6, Polynomials, will be review to some readers and new to others. Chapters 7 and 8 supplement and extend ideas developed in Part I, Basic Linear Algebra, and introduce the very powerful method of canonical forms.

Part 3 - Operators and Tensors, covers Chapters 9 through 12. Selections from Chapters 9 and 10 are covered in most upper division courses in linear algebra. Chapters 11 and 12 introduce multilinear algebra and Hilbert space.

The original Preface, Contents and Index are included. Three appendices from the original manuscript are included as well as the original Bibliography. For present day readers (2012), Wikipedia articles on selected subjects are generally informative and a good start on the literature.

Preface

As a text, this book is intended for upper division undergraduate and beginning graduate students in mathematics, applied mathematics, and fields of science and engineering that rely heavily on mathematical methods. However, it has been organized with particular concern for workers in these diverse fields who want to review the subject of linear algebra. In other words, we have written a book which we hope will still be referred to long after any final exam is over. As a result, we have included far more material than can possibly be covered in a single semester or quarter. This accomplishes at least two things. First, it provides the basis for a wide range of possible courses that can be tailored to the needs of the student or the desire of the instructor. And second, it becomes much easier for the student to later learn the basics of several more advanced topics such as tensors and infinite-dimensional vector spaces from a point of view coherent with elementary linear algebra. Indeed, we hope that this text will be quite useful for self-study. Because of this, our proofs are extremely detailed and should allow the instructor extra time to work out exercises and provide additional examples if desired.

A major concern in writing this book has been to develop a text that addresses the exceptional diversity of the audience that needs to know something about the subject of linear algebra. Although seldom explicitly acknowledged, one of the central difficulties in teaching a linear algebra course to advanced students is that they have been exposed to the basic background material from many different sources and points of view. An experienced mathematician will see the essential equivalence of these points of view, but these same differences seem large and very formidable to the students. An engineering student for example, can waste an inordinate amount of time because of some trivial mathematical concept missing from their background. A mathematics student might have had a concept from a different point of view and not realize the equivalence of that point of view to the one currently required. Although such problems can arise in any advanced mathematics course, they seem to be particularly acute in linear algebra.

To address this problem of student diversity, we have written a very self-contained text by including a large amount of background material necessary for a more advanced understanding of linear algebra. The most elementary of this material constitutes Chapter 0, and some basic analysis is presented in three appendices. In addition, we present a thorough introduction to those aspects of abstract algebra, including groups, rings, fields and polynomials over fields, that relate directly to linear algebra. This material includes both points that may seem "trivial" as well as more advanced background material. While trivial points can be quickly skipped by the reader who knows them already, they can cause discouraging delays for some students if omitted. It is for this reason that we have tried to err on the side of over-explaining concepts, especially when these concepts appear in slightly altered forms. The more advanced reader can gloss over these details, but they are there for those who need them. We hope that more experienced mathematicians will forgive our repetitive justification of numerous facts throughout the text.

A glance at the Contents shows that we have covered those topics normally included in any linear algebra text although, as explained above, to a greater level of detail than other books. Where we differ significantly in content from most linear algebra texts however, is in our treatment of canonical forms (Chapter 8), tensors (Chapter 11), and infinite-dimensional vector spaces (Chapter 12). In particular, our treatment of the Jordan and rational canonical forms in Chapter 8 is based entirely on invariant factors and the

Smith normal form of a matrix. We feel this approach is well worth the effort required to learn it since the result is, at least conceptually, a constructive algorithm for computing the Jordan and rational forms of a matrix. However, later sections of the chapter tie together this approach with the more standard treatment in terms of cyclic subspaces. Chapter 11 presents the basic formalism of tensors as they are most commonly used by applied mathematicians, physicists and engineers. While most students first learn this material in a course on differential geometry, it is clear that virtually all the theory can be easily presented at this level, and the extension to differentiable manifolds then becomes only a technical exercise. Since this approach is all that most scientists ever need, we leave more general treatments to advanced courses on abstract algebra. Finally, Chapter 12 serves as an introduction to the theory of infinite-dimensional vector spaces. We felt it is desirable to give the student some idea of the problems associated with infinite-dimensional spaces and how they are to be handled. And in addition, physics students and others studying quantum mechanics should have some understanding of how linear operators and their adjoints are properly defined in a Hilbert space.

One major topic we have not treated at all is that of numerical methods. The main reason for this (other than that the book would have become too unwieldy) is that we feel at this level, the student who needs to know such techniques usually takes a separate course devoted entirely to the subject of numerical analysis. However, as a natural supplement to the present text, we suggest the very readable "Numerical Analysis" by I. Jacques and C. Judd (Chapman and Hall, 1987).

The problems in this text have been accumulated over 25 years of teaching the subject of linear algebra. The more of these problems that the students work the better. Be particularly wary of the attitude that assumes that some of these problems are "obvious" and need not be written out or precisely articulated. There are many surprises in the problems that will be missed from this approach! While these exercises are of varying degrees of difficulty, we have not distinguished any as being particularly difficult. However, the level of difficulty ranges from routine calculations that everyone reading this book should be able to complete, to some that will require a fair amount of thought from most students.

Because of the wide range of backgrounds, interests and goals of both students and instructors, there is little point in our recommending a particular course outline based on this book. We prefer instead to leave it up to each teacher individually to decide exactly what material should be covered to meet the needs of the students. While at least portions of the first seven chapters should be read in order, the remaining chapters are essentially independent of each other. Those sections that are essentially applications of previous concepts, or else are not necessary for the rest of the book are denoted by an asterisk (*).

Now for one last comment on our notation. We use the symbol ∎ to denote the end of a proof, and // to denote the end of an example. Sections are labeled in the format "Chapter.Section," and exercises are labeled in the format "Chapter.Section.Exercise." For example, Exercise 2.3.4 refers to Exercise 4 of Section 2.3, i.e., Section 3 of Chapter 2. Books listed in the bibliography are referred to by author and copyright date.

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