Multiple Choice Questions for Review

1. In each case some information is given about a function. In which case is the information *not* sufficient to define a function?

(a) $f \in \underline{4}^{\underline{3}}, \quad 2 \to 3, \quad 1 \to 4, \quad 3 \to 2.$

(b) $f \in \{>, <, +, ?\}^{\underline{3}}, \quad f = (?, <, +).$

(c) $f \in \underline{3}^{\{>,<,+,?\}}, \quad f = (3, 1, 2, 3).$

(d) $f \in \underline{3}^{\{>,<,+,?\}}, \quad f = (3, 1, 2, 3)$. Domain ordered as follows: $>$, $<$, $+$, ?.

(e) $f \in \{>, <, +, ?\}^{\underline{3}}, \quad f = (?, <, +)$. Domain ordered as follows: $3, 2, 1$.

2. The following function is in two line form: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 3 & 7 & 2 & 6 & 4 & 5 & 1 & 8 \end{pmatrix}$. Which of the following is a correct cycle form for this function?

(a) $(1, 8, 9)(2, 3, 7, 5, 6, 4)$

(b) $(1, 9, 8)(2, 3, 5, 7, 6, 4)$

(c) $(1, 9, 8)(2, 3, 7, 5, 4, 6)$

(d) $(1, 9, 8)(2, 3, 7, 5, 6, 4)$.

(e) $(1, 9, 8)(3, 2, 7, 5, 6, 4)$

3. In each case some information about a function is given to you. Based on this information, which function is an injection?

(a) $f \in \underline{6}^{\underline{5}}, \quad \text{Coimage}(f) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$

(b) $f \in \underline{6}^{\underline{6}}, \quad \text{Coimage}(f) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5, 6\}\}$

(c) $f \in \underline{5}^{\underline{5}}, \quad f^{-1}(2) = \{1, 3, 5\}, \quad f^{-1}(4) = \{2, 4\}\}$

(d) $f \in \underline{4}^{\underline{5}}, \quad \big|\text{Image}(f)\big| = 4$

(e) $f \in \underline{5}^{\underline{5}}, \quad \text{Coimage}(f) = \{\{1, 3, 5\}, \{2, 4\}\}$

4. The following function is in two line form: $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 9 & 2 & 4 & 1 & 3 & 6 & 7 \end{pmatrix}$. Which of the following is a correct cycle form for $h = f^3 \circ f^{-1}$?

(a) $(1, 6, 8)(2, 3, 7)(5, 6, 4)$

(b) $(1, 6, 8)(2, 4, 5)(3, 7, 9)$

(c) $(1, 8, 6)(2, 3, 7)(5, 9, 4)$

(d) $(1, 9, 8)(2, 3, 5)(7, 6, 4)$

(e) $(8, 5, 9, 2, 4, 1, 3, 6, 7)$

5. The following permutation is in two line form: $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 4 & 7 & 2 & 9 & 1 & 3 & 5 \end{pmatrix}$. The permutation $g = (1, 2, 3)$ is in cycle form. Let $h = f \circ g$ be the composition of f and g. Which of the following is a correct cycle form for h?

(a) $(1,6,9,5,2,4,7)(3,8)$

(b) $(1,8,3,4,7,2,6)(5,9)$

(c) $(1,8,3,7,4,2,6)(9,5)$

(d) $(1,8,4,3,7,2,6)(9,5)$

(e) $(8,6,4,7,9,1,2)(3,5)$

6. We want to find the smallest integer $n > 0$ such that, for every permutation f on $\underline{4}$, the function f^n is the identity function on $\underline{4}$. What is the value of n?

(a) 4 (b) 6 (c) 12 (d) 24 (e) It is impossible.

7. In the lexicographic list of all strictly decreasing functions in $\underline{9}^{\underline{5}}$, find the successor of 98432.

(a) 98431 (b) 98435 (c) 98521 (d) 98532 (e) 98543

8. The 16 consecutive points $0, 1, \ldots, 14, 15$ have 0 and 15 converted to exterior box boundaries. The interior box boundaries correspond to points $1, 5, 7, 9$. This configuration corresponds to

(a) 9 balls into 5 boxes

(b) 9 balls into 6 boxes

(c) 10 balls into 5 boxes

(d) 10 balls into 6 boxes

(e) 11 balls into 4 boxes

9. The 16 consecutive points $0, 1, \ldots, 14, 15$ have 0 and 15 converted to exterior box boundaries. The interior box boundaries correspond to the strictly increasing functions $1 \le x_1 < x_2 < x_3 < x_4 \le 14$ in lex order. How many configurations of balls into boxes come before the configuration $\bullet|\,|\,|\,|\,\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet$? (Exterior box boundaries are not shown.)

(a) $\binom{13}{3}$ (b) $\binom{13}{4}$ (c) $\binom{14}{3}$ (d) $\binom{14}{4}$ (e) $\binom{15}{3}$

10. Suppose $f \in \underline{7}^{\underline{6}}$. How many such functions have $|\text{Image}(f)| = 4$?

(a) $S(7,4)$ (b) $S(7,4)(6)_4$ (c) $S(6,4)(7)_4$ (d) $S(4,7)(6)_4$ (e) $S(7,4)\,6!$

11. Let X be a random variable with distribution $b(k; n, p)$, $q = 1 - p$. Let $Y = (X+1)^2$. Then $E(Y) =$?

(a) $npq + (np+1)^2$

(b) $2npq + (np+1)^2$

(c) $npq + 2(np+1)^2$

(d) $(npq)^2 + (np+1)^2$

(e) $2npq(np+1)^2$

12. Let X and Y be independent random variables with distribution $b(k; n, a)$ and $b(k; n, b)$ respectively. Let $Z = X + 2Y$. Then, for all $\epsilon > 0$, Tchebycheff's inequality guarantees that $P(|Z - na - 2nb| \ge \epsilon)$ is always less than or equal to what?

(a) $(na(1-a)+nb(1-b))/\epsilon^2$

(b) $(na(1-a)+2nb(1-b))/\epsilon^2$

(c) $(na(1-a)+4nb(1-b))/\epsilon^2$

(d) $(na(1-a)+2nb(1-b))/\epsilon^3$

(e) $(na(1-a)+4nb(1-b))/\epsilon^3$

13. An 800 page book has 400 misprints. If the misprints are distributed uniformly throughout the book, and the Poisson approximation to the binomial distribution is used to calculate the probability of exactly 2 misprints on page 16, which of the following represents the correct use of the Poisson approximation?

(a) $e^{0.5}/8$ (b) $e^{-0.5}/8$ (c) $e^{0.5}/16$ (d) $e^{-0.5}/16$ (e) $e^{-0.5}/32$

14. For 40 weeks, once per hour during the 40 hour work week, an employee of Best Cars draws a ball from an urn that contains 1 black and 9 white balls. If black is drawn, a \$10 bill is tacked to a bulletin board. At the end of the 40 weeks, the money is given to charity. What is the expected amount of money given?

(a) 1000 (b) 1200 (c) 1400 (d) 1600 (e) 1800

15. For 40 weeks, once per hour during the 40 hour work week, an employee of Best Cars draws a ball from an urn that contains 1 black and 9 white balls. If black is drawn, \$10 is tacked to a bulletin board. At the end of the 40 weeks, the money is given to charity. Using the normal approximation, what interval under the standard normal curve should be used to get the area which equals the probability that \$1800 or more is given?

(a) from 1.67 to ∞

(b) from 0 to 1.67

(c) from 0.6 to ∞

(d) from 0 to 0.6

(e) from 0.6 to 1.67

16. A fair coin is tossed three times. Let X be the random variable which is one if the first throw is T (for tails) and the third throw is H (for heads), zero otherwise. Let Y denote the random variable that is one if the second and third throws are both H, zero otherwise. The covariance, $\mathrm{Cov}(X,Y)$ is

(a) 1/8 (b) $-1/8$ (c) 1/16 (d) $-1/16$ (e) 1/32

17. A fair coin is tossed three times. Let X be the random variable which is one if the first throw is T (for tails) and the third throw is H (for heads), zero otherwise. Let Y denote the random variable that is one if the second and third throws are both H, zero otherwise. The correlation, $\rho(X,Y)$ is

(a) 0 (b) 1/3 (c) $-1/3$ (d) 1/8 (e) $-1/8$

18. A fair coin is tossed three times and a T (for tails) or H (for heads) is recorded, giving us a 3-long list. Let X be the random variable which is zero if no T has another T adjacent to it, and is one otherwise. Let Y denote the random variable that counts

the number of T's in the three tosses. Let $h_{X,Y}$ denote the joint distribution of X and Y. $h_{X,Y}(1,2)$ equals

(a) 5/8 (b) 4/8 (c) 3/8 (d) 2/8 (e) 1/8

19. Which of the following is equal to $\text{Cov}(X+Y, X-Y)$, where X and Y are random variables on a sample space S?

(a) $\text{Var}(X) - \text{Var}(Y)$

(b) $\text{Var}(X^2) - \text{Var}(Y^2)$

(c) $\text{Var}(X^2) + 2\text{Cov}(X,Y) + \text{Var}(Y^2)$

(d) $\text{Var}(X^2) - 2\text{Cov}(X,Y) + \text{Var}(Y^2)$

(e) $(\text{Var}(X))^2 - (\text{Var}(Y))^2$

20. Which of the following is equal to $\text{Var}(2X-3Y)$, where X and Y are random variables on S?

(a) $4\text{Var}(X) + 12\text{Cov}(X,Y) + 9\text{Var}(Y)$

(b) $2\text{Var}(X) - 3\text{Var}(Y)$

(c) $2\text{Var}(X) + 6\text{Cov}(X,Y) + 3\text{Var}(Y)$

(d) $4\text{Var}(X) - 12\text{Cov}(X,Y) + 9\text{Var}(Y)$

(e) $2\text{Var}(X) - 6\text{Cov}(X,Y) + 3\text{Var}(Y)$

21. The strictly decreasing functions in $\underline{100}^{\underline{3}}$ are listed in lex order. How many are there before the function (9,5,4)?

(a) 18 (b) 23 (c) 65 (d) 98 (e) 180

22. All but one of the following have the same answer. Which one is different?

(a) The number of multisets of size 20 whose elements lie in $\underline{5}$.

(b) The number of strictly increasing functions from $\underline{20}$ to $\underline{24}$.

(c) The number of subsets of size 20 whose elements lie in $\underline{24}$.

(d) The number of weakly decreasing 4-lists made from $\underline{21}$.

(e) The number of strictly decreasing functions from $\underline{5}$ to $\underline{24}$.

23. Let X be a random variable with Poisson distribution $p(k;\lambda)$ Let $Y = (X+2)(X+1)$. What is the value of $E(Y)$?

(a) $\lambda^2 + 3\lambda + 1$

(b) $\lambda^2 + 3\lambda + 2$

(c) $\lambda^2 + 4\lambda + 2$

(d) $3\lambda^2 + 3\lambda + 2$

(e) $4\lambda^2 + 4\lambda + 2$

Answers: **1** (c), **2** (d), **3** (a), **4** (b), **5** (a), **6** (c), **7** (c), **8** (c), **9** (a), **10** (c), **11** (a), **12** (c), **13** (b), **14** (d), **15** (a), **16** (c), **17** (b), **18** (d), **19** (a), **20** (d), **21** (c), **22** (e), **23** (c).

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