Multiple Choice Questions for Review

In each case there is one correct answer (given at the end of the problem set). Try to work the problem first without looking at the answer. Understand both why the correct answer is correct and why the other answers are wrong.

1. Let $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the smallest integer K such that any subset of S of size K contains two disjoint subsets of size two, $\{x_1, x_2\}$ and $\{y_1, y_2\}$, such that $x_1 + x_2 = y_1 + y_2 = 9$?

(a) 8 (b) 9 (c) 7 (d) 6 (e) 5

2. There are K people in a room, each person picks a day of the year to get a free dinner at a fancy restaurant. K is such that there must be at least one group of six people who select the same day. What is the smallest such K if the year is a leap year (366 days)?

(a) 1829 (b) 1831 (c) 1830 (d) 1832 (e) 1833

3. A mineral collection contains twelve samples of Calomel, seven samples of Magnesite, and N samples of Siderite. Suppose that the smallest K such that choosing K samples from the collection guarantees that you have six samples of the same type of mineral is $K = 15$. What is N?

(a) 6 (b) 2 (c) 3 (d) 5 (e) 4

4. What is the smallest $N > 0$ such that any set of N nonnegative integers must have two distinct integers whose sum or difference is divisible by 1000?

(a) 502 (b) 520 (c) 5002 (d) 5020 (e) 52002

5. Let $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$. What is the smallest integer $N > 0$ such that for any set of N integers, chosen from S, there must be two distinct integers that divide each other?

(a) 10 (b) 7 (c) 9 (d) 8 (e) 11

6. The binary relation $R = \{(0, 0), (1, 1)\}$ on $A = \{0, 1, 2, 3, \}$ is

(a) Reflexive, Not Symmetric, Transitive

(b) Not Reflexive, Symmetric, Transitive

(c) Reflexive, Symmetric, Not Transitive

(d) Reflexive, Not Symmetric, Not Transitive

(e) Not Reflexive, Not Symmetric, Not Transitive

7. Define a binary relation $R = \{(0, 1), (1, 2), (2, 3), (3, 2), (2, 0)\}$ on $A = \{0, 1, 2, 3\}$. The directed graph (including loops) of the transitive closure of this relation has

(a) 16 arrows

(b) 12 arrows

(c) 8 arrows

(d) 6 arrows

(e) 4 arrows

8. Let $\mathbb{N}^+$ denote the nonzero natural numbers. Define a binary relation R on $\mathbb{N}^+ \times \mathbb{N}^+$ by $(m,n)R(s,t)$ if $\gcd(m,n) = \gcd(s,t)$. The binary relation R is

(a) Reflexive, Not Symmetric, Transitive

(b) Reflexive, Symmetric, Transitive

(c) Reflexive, Symmetric, Not Transitive

(d) Reflexive, Not Symmetric, Not Transitive

(e) Not Reflexive, Not Symmetric, Not Transitive

9. Let $\mathbb{N}_2^+$ denote the natural numbers greater than or equal to 2. Let mRn if $\gcd(m,n) > 1$. The binary relation R on $\mathbb{N}_2$ is

(a) Reflexive, Symmetric, Not Transitive

(b) Reflexive, Not Symmetric, Transitive

(c) Reflexive, Symmetric, Transitive

(d) Reflexive, Not Symmetric, Not Transitive

(e) Not Reflexive, Symmetric, Not Transitive

10. Define a binary relation R on a set A to be *antireflexive* if xRx doesn't hold for any $x \in A$. The number of symmetric, antireflexive binary relations on a set of ten elements is

(a) 2^{10} (b) 2^{50} (c) 2^{45} (d) 2^{90} (e) 2^{55}

11. Let R and S be binary relations on a set A. Suppose that R is reflexive, symmetric, and transitive and that S is symmetric, and transitive but is **not** reflexive. Which statement is always true for any such R and S?

(a) $R \cup S$ is symmetric but not reflexive and not transitive.

(b) $R \cup S$ is symmetric but not reflexive.

(c) $R \cup S$ is transitive and symmetric but not reflexive

(d) $R \cup S$ is reflexive and symmetric.

(e) $R \cup S$ is symmetric but not transitive.

12. Define an equivalence relation R on the positive integers $A = \{2, 3, 4, \ldots, 20\}$ by $m\ R\ n$ if the largest prime divisor of m is the same as the largest prime divisor of n. The number of equivalence classes of R is

(a) 8 (b) 10 (c) 9 (d) 11 (e) 7

Equivalence and Order

13. Let $R = \{(a,a),(a,b),(b,b),(a,c),(c,c)\}$ be a partial order relation on $\Sigma = \{a,b,c\}$. Let $\preceq$ be the corresponding lexicographic order on Σ^*. Which of the following is true?

(a) $bc \preceq ba$

(b) $abbaaacc \preceq abbaab$

(c) $abbac \preceq abb$

(d) $abbac \preceq abbab$

(e) $abbac \preceq abbaac$

14. Consider the divides relation, $m \mid n$, on the set $A = \{2,3,4,5,6,7,8,9,10\}$. The cardinality of the covering relation for this partial order relation (i.e., the number of edges in the Hasse diagram) is

(a) 4 (b) 6 (c) 5 (d) 8 (e) 7

15. Consider the divides relation, $m \mid n$, on the set $A = \{2,3,4,5,6,7,8,9,10\}$. Which of the following permutations of A is **not** a topological sort of this partial order relation?

(a) 7,2,3,6,9,5,4,10,8

(b) 2,3,7,6,9,5,4,10,8

(c) 2,6,3,9,5,7,4,10,8

(d) 3,7,2,9,5,4,10,8,6

(e) 3,2,6,9,5,7,4,10,8

16. Let $A = \{2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$ and consider the divides relation on A. Let C denote the length of the maximal chain, M the number of maximal elements, and m the number of minimal elements. Which is true?

(a) $C = 3$, $M = 8$, $m = 6$

(b) $C = 4$, $M = 8$, $m = 6$

(c) $C = 3$, $M = 6$, $m = 6$

(d) $C = 4$, $M = 6$, $m = 4$

(e) $C = 3$, $M = 6$, $m = 4$

Answers: **1** (c), **2** (b), **3** (e), **4** (a), **5** (d), **6** (b), **7** (a), **8** (b), **9** (a), **10** (c), **11** (d), **12** (a), **13** (b), **14** (e), **15** (c), **16** (a).

Notation Index

Subject Index

Index