Types
Week 4

• General discussion of types
• Type inference
• Type polymorphism
What is a type?

• Examples of types:
  ➤ Integer
  ➤ [Char]
  ➤ Either (Either Char Int) Bool

• Working, informal definition: set of values
  ➤ Where does this definition break down?
A type is: a way to prevent errors

- E.g.,
  ```javascript
  const y = 1;
  y + "w00t";
  ```

- E.g.,
  ```javascript
  function apply(f, x) {
    return f(x);
  }
  ```
A type is: a way to prevent errors

• E.g.,
  --| Function must be applied to 2 Ints
  plus :: Int -> Int -> Int
  plus a b = ...

• E.g.,
  --| Must be applied to a function and
  -- argument that that function can be applied to
  apply :: (a -> b) -> a -> b
  apply f x = f x
A type is: a way to prevent errors

- The world’s most lightweight* and widely-used formal method!
  - Prevent meaningless computations from being expressed or executed
A type is: a method of organization & documentation

• E.g., consider abstract data type for sets

```haskell
data Set k = ...
empty :: Set k
insert :: k -> Set k -> Set k
delete :: k -> Set k -> Set k
member :: k -> Set k -> Bool
```

• E.g., consider type for reading a file

```haskell
readFile :: FilePath -> IO String
```
A type is: a hint to the compiler

- E.g., what should `obj.prop1` be compiled down to?
Who enforces types?

• Consider, for example: arr[200]

  ➤ What happens in JavaScript if arr is null?

  ➤ What happens in C/C++ if arr is of size 10?

  ➤ What happens in Haskell if arr is not an array?
Who enforces types?

• This is language dependent...
  ➤ The compiler at compile time
  ➤ The runtime system at run-time
  ➤ The hardware at run-time
### What are the tradeoffs of each?

<table>
<thead>
<tr>
<th></th>
<th>Compile-time</th>
<th>Run-time checks</th>
<th>Hardware</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pro</strong></td>
<td>No runtime overhead</td>
<td>Permissive</td>
<td>Super fast</td>
</tr>
<tr>
<td><strong>Con</strong></td>
<td>Over approximates</td>
<td>Runtime overhead</td>
<td>Catch bugs late</td>
</tr>
</tbody>
</table>
Compile-time is the best! (Is it?)
The cost of compile-time checking

- Sometimes you give up expressivity

```javascript
function f(x) {
    return x < 10 ? x : x();
}
```

➤ More advanced type systems can “type” this function (dependent types); at what cost?

- Why is this fundamental? A: static analysis approximates — it has to work for every run of the program
Why do we check types? Safety!

• Def: A language is **type safe** if no program is allowed to violate its type distinctions
  
  ➤ Is Haskell type safe? A: yes, B: no
  
  ➤ Is JavaScript type safe? A: yes, B: no
  
  ➤ Is C/C++ type safe? A: yes, B: no
  
• What language features make it hard to guarantee type safety? A: raw pointer/memory access, casts, etc.
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Week 4

• General discussion of types ✓
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Type inference

• What’s the difference between type checking and type inference?
  ➤ E.g.,

        int f(int x) {
            return x + 1;
        }

  ➤ Type checking: checks that `x` is actually used as an int
  ➤ Type inference: based usage infers that `x` is an int
Why study type inference?

• Reduces syntactic overhead of expressive types

• Guaranteed to produce the most general type

• One of the most important language innovations

➤ Even C++ has type inference now!

• Good example of a flow-insensitive static analysis alg
What we’re going to look at

Hindley-Milner type inference for uHaskell!
Hindley-Milner type inference

• [1958] Curry and Feys invented type inference algorithm for the simply typed λ calculus

• [1969] Hindley extended algorithm to richer language and proved it always produced most general type

• [1978] Milner developed Algorithm W

• [1982] Damas prove the algorithm was complete
Hindley-Milner type inference

1. Parse the program
2. Assign type variables to all nodes
3. Generate constraints between type variables
4. Solve constraints (via unification)
5. Read out types of top-level declarations
• Declarations:  \[ d ::= \text{name } p = e \]

• Patterns:  \[ p ::= \text{id} \mid (p, p) \mid p:p \mid [] \]

• Expressions  \[ e ::= n \mid \text{True} \mid \text{False} \mid [] \mid \text{id} \]
\[ \mid (e) \mid e \oplus e \mid e \cdot e \mid (e, e) \]
\[ \mid \text{if } e \text{ then } e \text{ else } e \]

• Types:  \[ \tau ::= \tau \rightarrow \tau \mid [\tau] \mid (\tau, \tau) \]
\[ \mid \text{Bool} \mid \text{Int} \]
Type inference by example

1. Basic idea
2. Polymorphism
3. Data types
4. Type error: cannot unify
5. Type error: occurs check
Ex1. Basic idea

• Example: \( f \ x = 2 + x \)

• Goal: What is the type of \( f \)? Let’s do it informally:

  ➤ \( 2 :: \text{Int} \)

  ➤ \( (+) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int} \)

  ➤ We are applying \( (+) \) to \( x \), we need \( x :: \text{Int} \)

  ➤ Thus: \( f \ x = 2 + x :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int} \)
Ex1. Basic idea

• Step 1: parse program to construct parse tree

➤ $f \ x = 2 + x$

= 

=
Ex1. Basic idea

• Step 2: assign type variables to nodes

\[ f \ x = 2 + x \]
\[ = (+) 2 \ x \]
\[ = ((+) 2) \ x \]
Ex1. Basic idea

- Step 3: add constraints

\[
f(x) = 2 + x = (+) 2 \times = ((+) 2) \times
\]
Generating constraints

• Lambda abstraction ($\lambda x.e$)

$\tau_0 = \tau_1 \tau_2$
Generating constraints

- Lambda abstraction ($\lambda x.e$)

  - $\tau_0 = \tau_1 \rightarrow \tau_2$
Generating constraints

- Function declaration \((f \ x = e)\)

\[\begin{align*}
\tau_0 &= \\
\tau_1 &= \\
\tau_2 &= \\
\end{align*}\]
Generating constraints

• Function declaration \( f \ x = e \)

\[ \tau_0 = \tau_1 \rightarrow \tau_2 \]
Generating constraints

- Function application $(f \ x)$

\[ \tau_0 = \rightarrow \tau_2 \]

\[
\begin{array}{c}
\text{f} \\
\tau_0 \\
\tau_1
\end{array}
\]

\[
\begin{array}{c}
\text{x} \\
\tau_1
\end{array}
\]
Generating constraints

- Function application (f x)

\[ \tau_0 = \tau_1 \rightarrow \tau_2 \]
Generating constraints

- Conditionals \texttt{if x then y else y}
Generating constraints

- Conditionals \texttt{if} \( x \) \texttt{then} \( y \) \texttt{else} \( y \)

\[ \tau_1 = \text{Bool} \]
Generating constraints

• Conditionals \texttt{if } x \texttt{ then } y \texttt{ else } y

\begin{itemize}
  \item \( \tau_1 = \text{Bool} \)
  \item \( \tau_0 = \tau_2 \)
\end{itemize}
Generating constraints

- Conditionals `if x then y else y`
  - $\tau_1 = \text{Bool}$
  - $\tau_0 = \tau_2$
  - $\tau_2 = \tau_3$
Ex1. Basic idea

- Step 4: solve constraints via unification

\[
\begin{align*}
\tau_0 &= \tau_1 \rightarrow \tau_6 \\
\tau_2 &= \tau_3 \rightarrow \tau_4 \\
\tau_4 &= \tau_1 \rightarrow \tau_6 \\
\tau_2 &= \text{Int} \rightarrow \text{Int} \rightarrow \text{Int} \\
\tau_3 &= \text{Int}
\end{align*}
\]
Ex1. Basic idea

• Step 5: read out type

\( \tau_0 = \text{Int} \rightarrow \text{Int} \)
\( \tau_1 = \text{Int} \)
\( \tau_2 = \text{Int} \rightarrow \text{Int} \rightarrow \text{Int} \)
\( \tau_3 = \text{Int} \)
\( \tau_4 = \text{Int} \rightarrow \text{Int} \)
\( \tau_6 = \text{Int} \)

\( f :: \tau_0 \)
\( f :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int} \)
Hindley-Milner type inference

1. Parse the program
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4. Solve constraints (via unification)
5. Read out types of top-level declarations