Scope
Recall: substitution model

• Way of giving semantics to the $\lambda$-calculus
  ➤ E.g., $(\lambda x.f x x) (\lambda y.z) \rightarrow_\beta f (\lambda y.z) (\lambda y.z)$

• Translate this knowledge to JavaScript functions
  ➤ $(x => f(x)(x)) (y => z) \rightarrow_\beta f(y => z)(y => z)$
Let’s think about this more..

- Why would you not actually want to do function application in this way for a language like JavaScript?
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• Why would you not actually want to do function application in this way for a language like JavaScript?
  ➤ It’s super slow! Why?
Let’s think about this more..

• Why would you not actually want to do function application in this way for a language like JavaScript?
  ➤ It’s super slow! Why?
  ➤ It’s actually nonsensical sometimes! When?
Substitution gone wrong

- Consider variable mutation in JavaScript:

```javascript
let y = 1;
let z = 0;              ...
z++;                   →β. 0++;
console.log(z);        ...
```

- There is nothing wrong with substitution per say
  - It's symbolic evaluation/computation
  - Problem is JavaScript has mutation and not amendable to symbolic evaluation
What can we do?

\[ \lambda \text{-calculus} \]

environment model

machine model
The environment model (by example)

- Anatomy of a scope
- First-order functions
- Free variables
- High-order functions (bonus)
Anatomy of a scope

• What’s the point of a scope (e.g., block scope)?
Anatomy of a scope

• Recall our previous example:

```javascript
let y = 1;
let z = 0;
z++;
console.log(z);
```

• In this model, we associate an environment (activation record) with the code we’re executing
  ➤ Environment contains entries of all variables in scope
  ➤ Environment/stack ptr: points to cur activation record
Anatomy of a scope

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let y = 1;
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- Environment/stack ptr: points to cur activation record

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```javascript

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
</tr>
</tbody>
</table>
```

environment ptr
Anatomy of a scope

• In the environment model, we can distinguish between values and locations
  ➤ r-values: plain old values; we can reason about them using substitution semantics
  ➤ l-values: refer to locations where r-values are stored; they persist beyond single expressions.

• Why is this important?
  ➤ It tells us the kind of values operators like ++ must take. Which does ++ take? A: r-values. B: l-values
Anatomy of a scope

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• Why is this important?
  ➤ It tells us the kind of values operators like ++ must take. Which does ++ take? A: r-values. B: l-values
Anatomy of a scope

• What’s the process for executing `z++`:

```
let y = 1;
let z = 0;
z++;
console.log(z);
```

• Algorithm:
  - Find the current environment
  - Check to see if variable being reference is in env: if so, mutate!
Anatomy of a scope

- What’s the process for executing `console.log(z)`

```javascript
let y = 1;
let z = 0;
z++;
console.log(z);
```

- Algorithm:
  - Find the current environment
  - Check to see if variable being reference is in env: if so, read it!
Anatomy of a scope

• This sounds slow!
  ➤ It is!
  ➤ But remember: this is not the machine model, this is still an abstract model!

• Not too far off from machine model
  ➤ In x86, you dereference %esp to figure out where stack is and use offset to that location
  ➤ In JavaScript, you often do table lookup to find location of variables
The environment model (by example)

- Anatomy of a scope ✅
- First-order functions
- Free variables
- High-order functions (bonus)
When do we create an environment?

- A: every time we enter a new block scope
- B: every time we enter a new function scope
- C: A and B
- D: we don’t create new environments
When do we create an environment?

- A: every time we enter a new block scope
- B: every time we enter a new function scope
- C: A and B
- D: we don’t create new environments
First-order functions

• Consider activation record when calling function:

```javascript
function fact(n) {
    if (n <= 1) {
        return 1;
    } else {
        return n * fact(n-1);
    }
}
fact(3);
```

• What else do we need to keep track of?
First-order functions

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• What else do we need to keep track of?
More bookkeeping

• The parts of an activation record when calling function
  ➤ control link: records where to switch the environment pointer to when we finish evaluating in this scope.
  ➤ Do we need this for block scopes too? A: yes, B:no
  ➤ return value: l-value where the return value of function should be stored
  ➤ parameters: l-value for each formal parameter
  ➤ local variables: l-values for each let+const declaration
More bookkeeping

- The parts of an activation record when calling function
  - **control link**: records where to switch the environment pointer to when we finish evaluating in this scope.
  - Do we need this for block scopes too? A: yes B: no
  - **return value**: l-value where the return value of function should be stored
  - **parameters**: l-value for each formal parameter
  - **local variables**: l-values for each let+const declaration
More bookkeeping

• Do we need anything else besides the control link?
More bookkeeping

- Do we need anything else besides the control link?
  - Yes! Typically activation records will store the return address where to resume code execution — we’ll talk about this in the control flow lecture
Let's look at how evaluation works

- Consider activation records when calling function:

```javascript
function fact(n) {
    if (n <= 1) {
        return 1;
    } else {
        return n * fact(n-1);
    }
}
```

```
fact(3);
```

Do we keep the activation records on the stack after evaluation? A: yes, B: no

<table>
<thead>
<tr>
<th>control</th>
<th>ret</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

Environment pointer

Global env
Let’s look at how evaluation works

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>n</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>n</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
```

```
| environment ptr |
```

```
fact(3);
fact(2);
fact(1);
```
Let’s look at how evaluation works

- Consider activation records when calling function:

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function fact(n) {
    if (n <= 1) {
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  A: yes, B: no
The environment model (by example)

- Anatomy of a scope ✓
- First-order functions ✓
- Free variables
- High-order functions (bonus)
• Should we lookup x via the control link?
  
  ➤ A: yes
  
  ➤ B: no
• Should we lookup x via the control link?

  ➤ A: yes
  ➤ B: no
Free variables

• Consider activation records when calling f:

```javascript
let x = 1;
function f() {
    console.log(x)
}
f();
```
Free variables

- Consider activation records when calling `g`:

```
let x = 1;
function f() {
  console.log(x)
}

function g() {
  let x = 2;
  f();
}

g();
```

- What happens when we follow the control link?
Congrats, you did it!

You invented dynamic scoping!
How do we “fix” this?

• We need more bookkeeping!
  ➤ access link: reference to activation record of closest enclosing lexical scope

• Modify our lookup algorithm:
  ➤ Find the current environment
  ➤ Check to see if variable being reference is in env
  ➤ If not, follow the access link and repeat
Consider activation records when calling `g`:

```javascript
let x = 1;
function f() {
    console.log(x)
}

function g() {
    let x = 2;
    f();
}

g();
```
Consider activation records when calling `g`:

```javascript
let x = 1;
function f() {
  console.log(x)
}

function g() {
  let x = 2;
  f();
}

g();
```
Retry with access links

• Consider activation records when calling g:

```javascript
let x = 1;
function f() {
  console.log(x)
}

function g() {
  let x = 2;
  f();
}

g();
```
Wait, there is some magic here

• How do we know how to wire up the access links?

```javascript
let x = 1;
function f() {
    console.log(x)
}

function g() {
    let x = 2;
    f();
}

g();
```
Functions are data!

The act of defining a function should include the act of recording the access link associated with the function
Treating functions as data

- Let’s look at the example again, with minor rewrite

```javascript
let x = 1;
let f = () => {
    console.log(x)
}

let g = () => {
    let x = 2;
    f();
}

g();
```

- Function as data = closures = (current env ptr, code pointer)
Treating functions as data

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```javascript
let x = 1;
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  console.log(x)
};

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};
g();
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• Function as data = closures = (current env ptr, code pointer)
Treating functions as data

- When we evaluate function, the access link is set to the pointer in the closure

```javascript
let x = 1;
let f = () => {
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let g = () => {
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Treating functions as data

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g();
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Treating functions as data

- When we evaluate function, the access link is set to the pointer in the closure

```javascript
let x = 1;
let f = () => {
    console.log(x) // 1
}

let g = () => {
    let x = 2;
    f();
}

g();
```
The environment model (by example)

- Anatomy of a scope ✓
- First-order functions ✓
- Free variables ✓
- High-order functions (bonus)
Higher-order functions

- Consider the use of high-order `mkCounter` function

```javascript
function mkCounter(c) {
    return () => {
        return c++;
    };
}

let x = mkCounter(0);
let y = mkCounter(2);
console.log(x());
```

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<tr>
<th>control</th>
<th>access</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1</td>
</tr>
</tbody>
</table>
```

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