

Math 96: Homework 8

Fall 2024

This homework is due in class on Friday, November 22nd. Please complete at least one problem below.

1954 B3: Let a and b denote real numbers such that $a < b$. The symbol (a, b) will denote the closed interval with end points a, b . Let there be given a collection of closed intervals $(a_1, b_1), \dots, (a_n, b_n)$ such that any two of these closed intervals have at least one point in common. Prove that there exists then a point which is contained in every one of these intervals.

2009 A1: Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?

1964 B2: Let S be a set of $n > 0$ elements, and let A_1, A_2, \dots, A_k be a family of distinct subsets, with the property that any two of these subsets meet. Assume that no other subset of S meets all of the A_i .

Prove that $k = 2^{n-1}$.