Math 96:
Homework 7

Fall 2022

This homework is due in class on Friday, November 18th. Please complete at least *one* of the following problems (they are sorted roughly in increasing order of difficulty):

1947 A5: a_1, b_1, c_1 are positive numbers whose sum is 1, and for $n = 1, 2, \ldots$ we define

$$a_{n+1} = a_n^2 + 2b_n c_n, b_{n+1} = b_n^2 + 2c_n a_n, c_{n+1}^2 = c_n^2 + 2a_n b_n.$$

Show that a_n, b_n, c_n approach limits as $n \to \infty$ and find these limits.

2002 A3: Let $n \geq 2$ be an integer and T_n be the number of non-empty subsets S of $\{1, 2, 3, \ldots, n\}$ with the property that the average of the elements of S is an integer. Prove that $T_n - n$ is always even.

1957 B5: With each subset X of a set is associated a second subset $f(X)$. The association is such that whenever X contains Y then $f(X)$ contains $f(Y)$. Show that for some set A, $f(A) = A$.

(Note: You can assume that all subsets in this question are subsets of some particular finite set S.)