Math 96:
Homework 3

Fall 2022

This homework is due in class on Friday, October 28th. Please complete at least *one* of the following problems (they are sorted roughly in increasing order of difficulty):

1942 A2: If a polynomial $f(x)$ is divided by $(x-a)^2(x-b)$, where $a \neq b$, derive a formula for the remainder.

1952 A3: Develop necessary and sufficient conditions which ensure that r_1, r_2, r_3 and r_1^2, r_2^2, r_3^2 are simultaneously roots of the equation $x^3 + ax^2 + bx + c = 0$.

1972 B4: Let n be an integer greater than 1. Show that there exists a polynomial $P(x, y, z)$ with integral coefficients such that $x \equiv P(x^n, x^{n+1}, x + x^{n+2})$.

2007 B5: Let k be a positive integer. Prove that there exist polynomials $P_0(n), P_1(n), \dots, P_{k-1}(n)$ (which may depend on k) such that for any integer n,

$$\left\lfloor \frac{n}{k} \right\rfloor^k = P_0(n) + P_1(n) \left\lfloor \frac{n}{k} \right\rfloor + \cdots + P_{k-1}(n) \left\lfloor \frac{n}{k} \right\rfloor^{k-1}.$$

($\lfloor a \rfloor$ means the largest integer $\leq a$.)