

Math 96: Homework 4

Fall 2024

This homework is due in class on Friday, October 25th. Please complete at least one problem below.

1939 A3: Find the cubic equation whose roots are the cubes of the roots of

$$x^3 + ax^2 + bx + c = 0.$$

1999 B2: Let $P(x)$ be a polynomial of degree n such that $P(x) = Q(x)P''(x)$, where $Q(x)$ is a quadratic polynomial and $P''(x)$ is the second derivative of $P(x)$. Show that if $P(x)$ has at least two distinct roots then it must have n distinct roots. [The roots may be either real or complex.]

2004 A4: Show that for any positive integer n there is an integer N such that the product $x_1x_2 \cdots x_n$ can be expressed identically in the form

$$x_1x_2 \cdots x_n = \sum_{i=1}^N c_i (a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n)^n$$

where the c_i are rational numbers and each a_{ij} is one of the numbers 1, 0, 1.