Math 96:
Homework 3

Fall 2022

This homework is due in class on Friday, October 14th. Please complete at least *one* of the following problems (they are sorted roughly in increasing order of difficulty):

2007 B1: Let f be a (non-constant) polynomial with positive integer coefficients. Prove that if n is a positive integer, then $f(n)$ divides $f(f(n)+1)$ if and only if $n = 1$.

1992 A3: For a given positive integer m, find all triples (n, x, y) of positive integers, with n relatively prime to m, which satisfy

$$(x^2 + y^2)^m = (xy)^n.$$

2017 B3: Suppose that $f(x) = \sum_{i=0}^{\infty} c_i x^i$ is a power series for which each coefficient c_i is 0 or 1. Show that if $f(2/3) = 3/2$, then $f(1/2)$ must be irrational.

2012 B6: Let p be an odd prime number such that $p \equiv 2 \pmod 3$. Define a permutation of the residue classes modulo p by $\pi(x) \equiv x^3 \pmod p$. Show that π is an even permutation if and only if $p \equiv 3 \pmod 4$.