Math 96:
Homework 2

Fall 2026

This homework is due in class on Friday, October 9th. Please complete at least *one* problem below. Problems are roughly sorted in increasing order of difficulty.

2006 A4: Let $S = \{1, 2, \ldots, n\}$ for some integer $n > 1$. Say a permutation π of S has a *local maximum* at $k \in S$ if

(i) $\pi(k) > \pi(k+1)$ for $k = 1$;

(ii) $\pi(k-1) < \pi(k)$ and $\pi(k) > \pi(k+1)$ for $1 < k < n$;

(iii) $\pi(k-1) < \pi(k)$ for $k = n$.

(For example, if $n = 5$ and π takes values at $1, 2, 3, 4, 5$ of $2, 1, 4, 5, 3$, then π has a local maximum of 2 at $k = 1$, and a local maximum of 5 at $k = 4$.) What is the average number of local maxima of a permutation of S, averaging over all permutations of S?

1961 A5: Let Ω be a set of n points, where $n > 2$. Let Σ be a nonempty subcollection of the 2^n subsets of Ω that is closed with respect to unions, intersections, and complements (that is, if A and B are members of Σ, then so are $A \cup B$, $A \cap B$, $\Omega - A$ and $\Omega - B$, where $\Omega - B$ denotes all points in Ω but not in B). If k is the number of members of Σ, what are the possible values of k?

1996 A3: Suppose that each of 20 students has made a choice of anywhere from 0 to 6 courses from a total of $6t$ courses offered. Prove or disprove: there are 5 students and 2 courses such that all 5 have chosen both courses or all 5 have chosen neither course.

2006 B2: Prove that for every set $X = \{x_1, x_2, \ldots, x_n\}$ of real numbers, there exists a nonempty subset S of X and integer m such that

$$\left| m + \sum_{s \in S} s \right| \leq \frac{1}{n+1}.$$