Math 96:
Homework 2

Fall 2021

This homework is due in class on Friday, October 8th. Please complete at least one of the following problems:

1941 B2: Evaluate the following limits:
\[
\lim_{n \to \infty} \left( \frac{1}{\sqrt{n^2 + 1^2}} + \frac{1}{\sqrt{n^2 + 2^2}} + \ldots + \frac{1}{\sqrt{n^2 + n^2}} \right);
\]
\[
\lim_{n \to \infty} \left( \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \ldots + \frac{1}{\sqrt{n^2 + n}} \right);
\]
\[
\lim_{n \to \infty} \left( \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \ldots + \frac{1}{\sqrt{n^2 + n}} \right).
\]

1981 B1: Find:
\[
\lim_{n \to \infty} \left[ \frac{1}{n^5} \sum_{h=1}^{n} \sum_{k=1}^{n} (5h^4 - 18h^2k^2 + 5k^4) \right].
\]

Note: Unfortunately, approximating the sum by an integral is not enough here.

1986 A2: What is the unit (i.e. rightmost) digit of \([\frac{10^{20000}}{100000 + 3}]\)? Here \([x]\) is the greatest integer \(\leq x\).