Math 96:
Homework 1

Fall 2022

This homework is due in class on Friday, September 30th. Please complete at least *one* of the following problems (they are sorted roughly in increasing order of difficulty):

2002 B1: Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability she hits exactly 50 of her first 100 shots?

1987 A2: he sequence of digits

$$123456789101112131415161718192021\ldots$$

is obtained by writing the positive integers in order. If the 10^n-th digit in this sequence occurs in the part of the sequence in which the m-digit numbers are placed, define $f(n)$ to be m. For example, $f(2) = 2$ because the 100th digit enters the sequence in the placement of the two-digit integer 55. Find, with proof, $f(1987)$.

1952 A5: A man has a rectangular block of wood m by n by r inches (m, n, and r are integers). He paints the entire surface of the block, cuts the block into inch cubes, and notices that exactly half the cubes are completely unpainted. Prove that the number of essentially different blocks with this property is finite. (Do *not* attempt to enumerate them.)

1967 A1: Let $f(x) = a_1 \sin(x) + a_2 \sin(2x) + \cdots + a_n \sin(nx)$, where $a_1, a_2, \ldots, a_n$ are real numbers and where n is a positive integer. Given that $|f(x)| \leq |\sin(x)|$ for all real x, prove that

$$|a_1 + 2a_2 + \cdots + na_n| \leq 1.$$

1992 A6: Four points are chosen at random on the surface of a sphere. What is the probability that the center of the sphere lies inside the tetrahedron whose vertices are at the four points? (It is understood that each point is in-dependently chosen relative to a uniform distribution on the sphere.)