

# Math 96: Homework 1

Fall 2024

This homework is due in class on Friday, October 4th. Please complete at least one problem below.

**1984 A1:** Let  $A$  be a solid  $a \times b \times c$  rectangular brick in three dimensions, where  $a, b, c > 0$ . Let  $B$  be the set of all points which are distance at most one from some point of  $A$  (in particular,  $B$  contains  $A$ ). Express the volume of  $B$  as a polynomial in  $a, b$ , and  $c$ .

**2004 A1:** Basketball star Shanille O'Keal's team statistician keeps track of the number,  $S(N)$ , of successful free throws she has made in her first  $N$  attempts of the season. Early in the season,  $S(N)$  was less than 80% of  $N$ , but by the end of the season,  $S(N)$  was more than 80% of  $N$ . Was there necessarily a moment in between when  $S(N)$  was exactly 80% of  $N$ ?

**2014 B1:** A base 10 over-expansion of a positive integer  $N$  is an expression of the form

$$N = d_k 10^k + d_{k-1} 10^{k-1} + \dots + d_0 10^0$$

with  $d_k \neq 0$  and  $d_i \in \{0, 1, 2, \dots, 10\}$  for all  $i$ . For instance, the integer  $N = 10$  has two base 10 over-expansions:  $10 = 10 \cdot 10^0$  and the usual base 10 expansion  $10 = 1 \cdot 10^1 + 0 \cdot 10^0$ . Which positive integers have a unique base 10 over-expansion?

**1984 B1:** Let  $n$  be a positive integer, and define

$$f(n) = 1! + 2! + \dots + n!.$$

Find polynomials  $P(x)$  and  $Q(x)$  such that

$$f(n+2) = P(n)f(n+1) + Q(n)f(n)$$

for all  $n \geq 1$ .