Math 96:
Homework 1

Fall 2021

This homework is due in class on Friday, October 1st. Please complete at least one of the following problems:

**1966 B1** Let a convex polygon $P$ be contained in a square of size one. Show that the sum of the squares of the lengths of the sides of $P$ is less than or equal to 4.

**2016 B3** Let $S$ be a finite set of points in the plane such that the area of triangle $ABC$ is at most 1 whenever $A$, $B$, and $C$ are in $S$. Show that there exists a triangle of area 4 that (together with its interior) covers the set $S$.

**1961 B3** Consider four points in the plane, no three of which are collinear, and such that the circle through three of them does not pass through the fourth. Prove that one of the four points can be selected having the property that it lies inside the circle determined by the other three.

**1986 B4** For a positive real number $r$, let $G(r)$ be the minimum value of $|r - \sqrt{m^2 + 2n^2}|$ for all integers $m$ and $n$. Prove or disprove the assertion that $\lim_{r \to \infty} G(r)$ exists and equals zero.