Math 96:
Homework 1

Fall 2026

This homework is due in class on Friday, October 2nd. Please complete at least *one* problem below. Problems are roughly sorted in increasing order of difficulty.

1961 B2: Let α and β be given positive real numbers, with $\alpha < \beta$. If two points are selected at random from a straight line segment of length β, what is the probability that the distance between them is at least α?

1991 B1: For each integer $n \geq 0$, let $S(n) = n - m^2$ where m is the greatest integer with $m^2 \leq n$. Define a sequence $(a_k)_{k=0}^{\infty}$ by $a_0 = A$ and $a_{k+1} = a_k + S(a_k)$ for $k \geq 0$. For what positive integers A is this sequence eventually constant?

2001 B1: Let n be an even positive integer. Write the numbers $1, 2, \ldots, n^2$ in the squares of an $n \times n$ grid so that the k^{th} row, from left to right, is

$$(k-1)n+1, (k-1)n+2, \ldots, (k-1)n+n.$$

Color the squares of the grid so that half the squares in each row and in each column are red and the other half are black (a checkerboard coloring is one possibility). Prove that for each coloring, the sum of the numbers on the red squares is equal to the sum of the numbers on the black squares.

1966 B1: A convex polygon P is contained in a square of size one. Show that the sum of the squares of the sides of P is less than or equal to 4.

1981 A2: Two distinct squares of the 8 by 8 chessboard C are said to be adjacent if they have a vertex or side in common. Also, g is called a C-gap if for every numbering of the squares of C with all the integers $1, 2, \ldots, 64$ there exist two adjacent squares whose numbers differ by at least g. Determine the largest C-gap g.

1961 B4: For a fixed positive integer n let $x_1, x_2, \ldots, x_n$ be real numbers satisfying $0 \leq x_i \leq 1$ for $k = 1, 2, \ldots, n$. Determine the maximum value, as a function of n, of the sum of the $n(n-1)/2$ terms:

$$\sum_{\substack{i,j=1 \\ i<j}} |x_i - x_j|.$$

1986 B4: For a positive real number r, let $G(r)$ be the minimum value of $|r - \sqrt{m^2 + 2n^2}|$ for all integers m and n. Prove or disprove the assertion that $\lim_{r\to\infty} G(r)$ exists and equals 0.

1991 A4: Does there exist an infinite sequence of closed discs $D_1, D_2, D_3, \ldots$ in the plane, with centers $c_1, c_2, c_3, \ldots$, respectively such that

(i) the c_i have no limit point in the finite plane,

(ii) the sum of the areas of the D_i is finite, and

(iii) every line in the plane intersects at least one of the D_i?