Math 184 Homework 4

Fall 2022

This homework is due on gradescope Friday October 28th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Distinct Parts Partitions, 20 points)**.** *Show that the number of partitions of n into exactly k distinct parts is*

$$p_k(n - k(k-1)/2).$$

Question 2 (Permutation Orders, 20 points)**.** *Let π be a permutation of $[n]$. Show that $\pi^{n!}$ is the identity permutation (i.e. the permutation σ of $[n]$ where $\sigma(i) = i$ for all $1 \leq i \leq n$).*

Question 3 (Ambiguous Permutations, 30 points)**.** *We have discussed two ways to encode a permutation π of $[n]$ as an ordered list of the numbers from 1 to n: either by listing the values $\pi(1), \pi(2), \ldots, \pi(n)$ in order or by writing π in canonical cycle notation and removing the parentheses. Call a permutation π* ambiguous *if each of these representations gives rise to the same sequence of numbers. Show that the number of ambiguous permutations of $[n]$ is given by the Fibonacci number F_n where $F_1 = 1, F_2 = 2$, and $F_n = F_{n-1} + F_{n-2}$ for all $n > 2$.*

Question 4 (Random Permutations and Coin Flips, 30 points)**.** *Consider the following two random processes:*

1. *Pick a uniform random permutation π of $[n]$ and count the number of cycles in π.*
2. *Flip n coins $C_1, C_2, \ldots, C_n$ where C_i has a probability $1/i$ of coming up heads, independently of the other coins and count the total number of heads.*

Show that these processes give rise to the same distribution over outcomes.

Question 5 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*