Math 184 Homework 2

Fall 2022

This homework is due on gradescope Friday October 7th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Other Pigeonhole Principle Generalizations, 30 points)**.** *Suppose that each of n pigeons are assigned to one of m holes. For which values of m and n can each of the following be guaranteed?*

(a) There are at least n (possibly overlapping) pairs of pigeons that share the same hole. [10 points]

(b) There are at least $n/2$ pigeons that share a hole with some other pigeon. [10 points]

(c) There are at least 2 holes each with at least 2 pigeons. [10 points]

Question 2 (Bishop Packing, 20 points)**.** *In chess a bishop can move diagonally any number of squares. Prove that if 15 bishops are placed on an 8×8 chessboard that at least two will be attacking each other.*

Hint: Try to divide the board into 14 regions so that any two bishops in the same region would attack each other.

Question 3 (Counting Subsets, 50 points)**.** *In the following $[n]$ will be used to denote the set $\{1, 2, 3, \ldots, n\}$. For each of the following, give a formula for the number of sets of the type specified along with the actual number.*

(a) How many subsets of $[10]$ are there? [5 points]

(b) How many subsets of $[10]$ are there of size exactly 5? [5 points]

(c) How many subsets of $[12]$ consist of exactly 4 even numbers and 3 odd numbers? [5 points]

(d) How many subsets of $[12]$ have the same number of even and odd numbers in them? [5 points]

(e) How many subsets of $[20]$ contain exactly one element with each last digit? [5 points]

(f) How many subsets of $[20]$ contain at least one element with each last digit? [5 points]

(g) How many subsets of $[10]$ contain exactly one multiple of 3 and no other elements within 1 of that multiple of 3? [5 points]

(h) How many subsets of $[20]$ have largest and smallest elements that differ by at most 5? [5 points]

(i) How many subsets of $[10]$ contain no pair of consecutive numbers? Hint: Figure out the number of such subsets of $[n]$ for smaller values of n as well. [5 points]

(j) How many subsets of $[12]$ contain no pair of elements where one is exactly twice the other? [5 points]

Question 4 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*