# Math 184 Homework 1

Fall 2022

This homework is due on gradescope Friday September 30th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

**Question 1** (Ackermann Function, 50 points)**.** *The Ackermann function is a function $A(m, n)$ that takes as input two integers $m, n \geq 0$ and returns an integer. For all $n, m \geq 0$, it satisfies the recurrence relation:*

$$\begin{aligned} A(0, n) &= n + 1 \\ A(m + 1, 0) &= A(m, 1) \\ A(m + 1, n + 1) &= A(m, A(m + 1, n)). \end{aligned}$$

*Prove that the above recurrence relations uniquely define the value of $A(m, n)$ for all integers $m, n \geq 0$. Hint: You will want to use induction, but you will need to be careful about what you are inducting on and how you set it up.*

**Question 2** (Generalized Binary Representation, 50 points)**.** *Let $a_1 < a_2 < a_3, \ldots$ be a sequence of positive integers with $a_1 = 1$ and $a_n < a_{n+1} \leq 2a_n$. Show that any positive integer $m$ can be written as a sum of distinct elements of this sequence.*

**Question 3** (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*