Math 184 Exam 2

Fall 2022

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely (though pay attention to exactly what the question is asking for). You are free to make use of any result in the textbook or proved in class. You may use up to 6 one-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the blank page after this one, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please sit in the seat designated below.

Name:

ID Number:

Seat:

Problem	1	2	3	4	Total
Score					

This page is left blank for scratch work.

Question 1 (Power Series, 25 points)**.** *What are the first five terms (i.e. the x^0 through x^4 terms) of the power series for $(1-2x)^{3/2}$ about $x=0$?*

Question 2 (Subset Counting, 25 points)**.** *How many subsets of* $\{1, 2, \ldots, 7\}$ *either:*

- *Contain* $1, 2,$ *and* $3,$
- *Contain* $3, 4,$ *and* $5,$ *OR*
- *Contain exactly five elements?*

Question 3 (Same Cycle Counting, 25 points)**.** *Let i, j, k be three distinct numbers between* 1 *and* n*. For how many permutations of* $[n]$ *is* k *in the same cycle as* i *but not the same cycle as* j*. Justify your answer.*

Question 4 (Partitions with Distinct Part Sizes, 25 points)**.** *Let $p(n,k)$ denote the number of integer partitions of n with exactly k parts of different sizes. For example, $p(6,2) = 6$ due to the partitions $5+1,\ 4+2,\ 4+1+1,\ 3+1+1+1,\ 2+2+1+1,\ 2+1+1+1+1$. Write the generating function $f(x,y) = \sum_{n,k=0}^{\infty} p(n,k)x^n y^k$ as an infinite product. Hint: For each $m \geq 1$ have a term in your product to account for the number of parts in the partition of size equal to m.*