Math 184 Homework 2

Spring 2022

This homework is due on gradescope Friday April 15th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Composition Bijections, 30 points)**.** .

(a) *Give a bijection between the set of compositions of* n *into parts of size* 1 *and* 2 *and the set of compositions of* $n+2$ *into parts of size at least* 2*. [15 points]*

(b) *Give a bijection between the set of weak compositions of* n *into* $k+1$ *parts and the set of weak compositions of* k *into* $n+1$ *parts. Hint: use stars and bars. [15 points]*

Question 2 (Sterling Number Inequalities, 25 points)**.** *Prove that for all* $n \geq k > 0$ *that*

$$k^{n-k} \leq S(n,k) \leq k^n/k!$$

Hint: Relate $S(n,k)$ *to the number of functions from* $\{1,2,\ldots,n\}$ *to* $\{1,2,\ldots,k\}$*.*

Question 3 (Partition Identity, 45 points)**.** *Prove that:*

$$p(n) = \sum_{k=1}^{\lfloor\sqrt{n}\rfloor} \sum_{m=0}^{n-k^2} p_{\leq k}(m) p_{\leq k}(n-k^2-m).$$

Where $p_{\leq k}(n)$ *denotes the number of partitions of* n *with at most* k *parts. Hint: Count the number of partitions with a* $k \times k$ *box in the upper left of the Ferrers diagram.*

Question 4 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*