Math 184 Homework 1

Fall 2026

This homework is due on gradescope Wednesday October 7th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Towers of Hanoi with Extra Pegs, 25 points)**.** *Suppose that you are playing the towers of Hanoi puzzle with $k+2$ pegs and n discs for some positive integers n and k. Show that it is possible to move the entire stack of discs to a new location using at most*

$$2^{k\lceil n^{1/k}\rceil}$$

moves.

Hint: Start by assuming that $n = m^k$ and move blocks of m discs at a time.

Question 2 (Repeating Decimals, 20 points)**.** *Recall that a positive rational number is one that can be written as n/m for some positive integers n and m. Show that every positive rational number can be written as a repeating decimal. That is to say that its decimal representation eventually terminates in a repeating pattern.*

Hint: Try computing the ratio n/m using long division and look at the pattern of remainders left over upon dividing by m.

Question 3 (Rook Configurations, 55 points)**.** *The game of chess is played on an 8×8 board. The rows are labelled $1-8$ and the columns $a-h$ so one can specify a square as $a7, g1$, etc. A rook is a piece that when on a given square can move to a different square on the same row or column as that square (Note: in the real game of chess, the rook cannot jump over other pieces, but we will ignore this). We say that two rooks are attacking each other if they would be able to move onto each others' squares.*

Given the above, solve the following counting problems. For each, please give an numerical answer, the formula used to compute this answer, and a justification of that answer.

(a) How many ways can one place 8 white rooks on the chessboard so that no two attack each other? [5 points]

(b) How many ways can one place 4 white rooks on the chessboard so that no two attack each other? [5 points]

(c) How many ways can one place one white, one black, one red, and one blue rook on the chessboard so that no two attack each other? [5 points]

(d) How many ways can one place four white rooks on distinct squares on the chessboard so that each rook placed attacks each other one? [5 points]

(e) How many ways can one place two white rooks and two black rooks on separate squares of the chessboard so that no white rook attacks a black rook? [10 points]

(f) In how many ways can white rooks be placed on distinct squares of the chessboard so that each row has at least one rook on it? [5 points]

(g) If a rook is located on square $a1$, in how many ways can it travel to $b2$ in exactly 4 moves? [20 points]

Question 4 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*