Math 184 Final Exam

Fall 2022

Instructions: Do not open until the exam starts. The exam will run for 180 minutes. The problems are roughly sorted in increasing order of difficulty. You may use up to 10 one-sided pages of notes, and may not use the textbook nor any electronic aids.

All questions require a numerical, integer answer to be entered in the appropriate answer box for the question. Please take extra care to ensure that your written answers are legible. Some questions have hints designed to help you check your work.

Please sit in the seat designated below.

Name:

ID Number:

Seat:

This page is left blank for scratch work. Do not remove from your exam blooklet.

Question 1 (Canonical Cycle Notation, 25 points)**.** *A permutation of* $[9]$ *when written in function notation is* 879231465*. What is this permutation when written in canonical cycle notation? Express your answer as a single* 9*-digit number without parentheses.*

Hint: The fifth digit of your answer should be 6*.*

Answer =

Question 2 (Coefficient Calculation, 25 points)**.** *What is the coefficient of* xy^3z^5 *in* $(x + y + z)^9$*?*
Hint: The answer should be between 500 *and* 600*.*

$$\boldsymbol{Answer} =$$

Question 3 (Partition Correspondence, 25 points)**.** *The partition* $17 = 5 + 5 + 3 + 2 + 2$ *is self-conjugate. What is the corresponding partition of* 17 *into distinct, odd parts? Express your answer as a number obtained by listing the parts in descending order (note that each part should be a 1-digit number). For example, the partition* $8 + 7 + 6$ *would correspond to the answer* 876*.*

$$\boldsymbol{Answer} = \boxed{}$$

Question 4 (Promotion Fatigue, 25 points)**.** *A particular store runs a number n of different promotions in February, each lasting a single day. How big does n need to be to guarantee that every year at least three of these promotions will take place during the same calendar week (a calendar week runs from Sunday until the next Saturday)?*

$$\boldsymbol{Answer} =$$

Question 5 (Set Union Size Range, 25 points)**.** *Suppose that* A, B, *and* C *are three sets with* $|A| = 100$, $|B| = 75$, $|C| = 80$ *and* $|A \cap B| = 55$, $|B \cap C| = 30$. *What is the smallest possible size of* $A \cup B \cup C$*?*

Answer = ☐

Question 6 (Cycle Notation Count, 25 points)**.** *On average, in how many ways can a permutation* π *of* [9] *be written in (not necessarily canonical) cycle notation?*

Hint: The last digit of the answer is 6*.*

***Answer* =**

Question 7 (Set Partition Counting, 25 points)**.** *Let* f_n *be the number of ways to partition a set* $[n]$ *into subsets* A, B, C *so that:*

- *The elements of* A *are colored red, blue and green*
- B *is non-empty*
- $|C|$ *is even.*

Let

$$F(x) = \sum_{n=0}^{\infty} f_n x^n / n!$$

be the corresponding exponential generating function. What is $F(\log(3))$*, where* $\log(3)$ *denotes the natural logarithm of* 3*?*

Hint: The answer is between 75 *and* 100*. Furthermore, the middle digit of* f_{10} *is a* 4*.*

Answer $=$ $\boxed{}$

Question 8 (One Even Part, 25 points)**.** *Let* g_n *be the number of weak compositions of* n *where exactly one part of the composition is even. For example,* $g_2 = 4$ *due to the compositions* $2, 1+1+0, 1+0+1, 0+1+1$*. Let* $G(x)$ *be the generating function*

$$G(x) = \sum_{n=0}^{\infty} g_n x^n.$$

What is $G(1/2)$*?*

Hint: First find the generating function for the number of such weak compositions where the even part is 0*. This should be a composition of simpler generating functions. Then write the final answer as a product of this and something else.*

Answer =