Math 184 Exam 2

Spring 2022

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely (though pay attention to exactly what the question is asking for). You are free to make use of any result in the textbook or proved in class. You may use up to 6 one-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the blank page after this one, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please sit in the seat designated below.

Name:

ID Number:

Seat:

Problem	1	2	3	4	Total
Score					

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Question 1 (Computing Coefficients, 25 points). *What are the first four coefficients (the x^0 coefficient through the x^3 coefficient) of the generating function*

$$(1+x)^{1/3}/(1-x/3)?$$

Question 2 (Recurrence Relation, 25 points). *Suppose that a_n is a sequence of integers so that $a_{n+2} = a_{n+1} - na_n + 1$ for all $n \geq 0$. Letting*

$$A(x) = \sum_{n=0}^{\infty} a_n x^n / n!$$

give a differential equation satisfied by $A(x)$.

Question 3 (Generating Function Product, 25 points). *Let c_n be the number of ways of coloring the numbers $1, 2, \ldots, n$ each red, green or blue so that the number of red numbers equals the number of blue numbers and so that the number of green numbers is odd. Show how to write the exponential generating function for c_n as a product of generating functions whose coefficients can be written down explicitly.*

Question 4 (Catalan Number Parity, 25 points)**.** *Prove that the Catalan number* C_n *is odd if and only if* n *is one less than a power of* 2*.*

Hint: Use the recurrence relation.