Math 184 Exam 1

Spring 2022

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely (though pay attention to exactly what the question is asking for). You are free to make use of any result in the textbook or proved in class. You may use up to 6 one-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the blank page after this one, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please sit in the seat designated below.

Name:

ID Number:

Seat:

Problem	1	2	3	4	Total
Score					

This page is left blank for scratch work.

Question 1 (Counting Unions, 25 points)**.** *How many positive integers less than 100 are either divisible by 3 or have 3 as their first digit?*

Question 2 (Permutation Square, 25 points)**.** *Suppose that* π *is the permutation of* [9] *whose canonical cycle notation (without the parentheses) is* 613482957*. What is the canonical cycle notation for* π^2*?*

Question 3 (Binomial Coefficient Bound, 25 points)**.** *Show that for all* $n \geq 0$ *that*

$$\binom{3n}{n} \leq (27/4)^n.$$

Hint: use induction and consider the ratio of $\binom{3(n+1)}{n+1}$ *to* $\binom{3n}{n}$.

Question 4 (Set Partitions with Same Parts, 25 points)**.** *Let S be a collection of set partitions of $[n]$. Show that if the size of S is greater than 2^{n-1}, that it must contain two partitions a and b which share a part in common.*